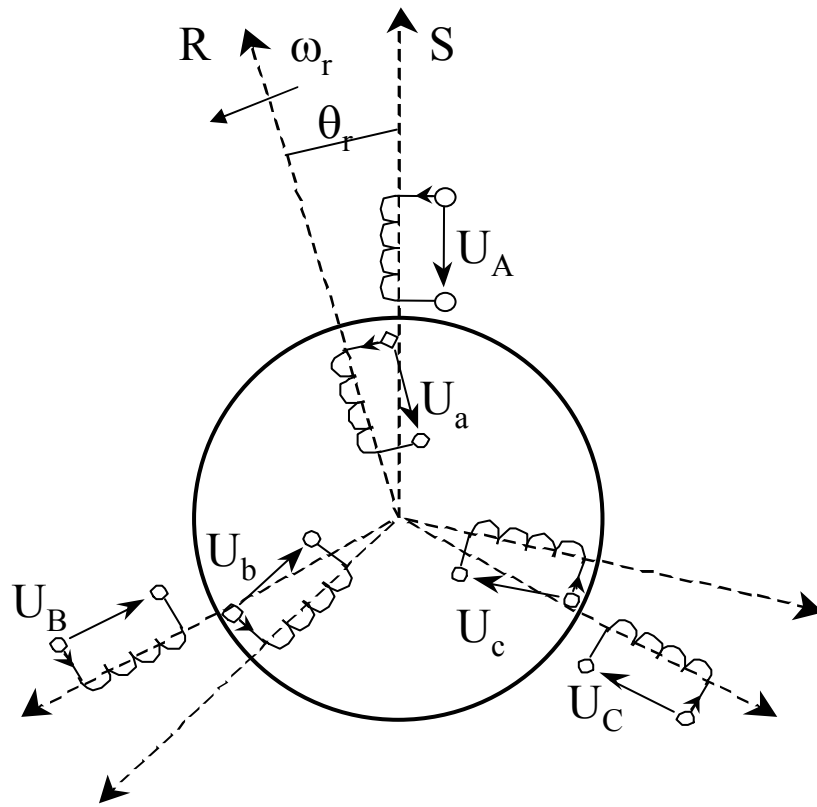


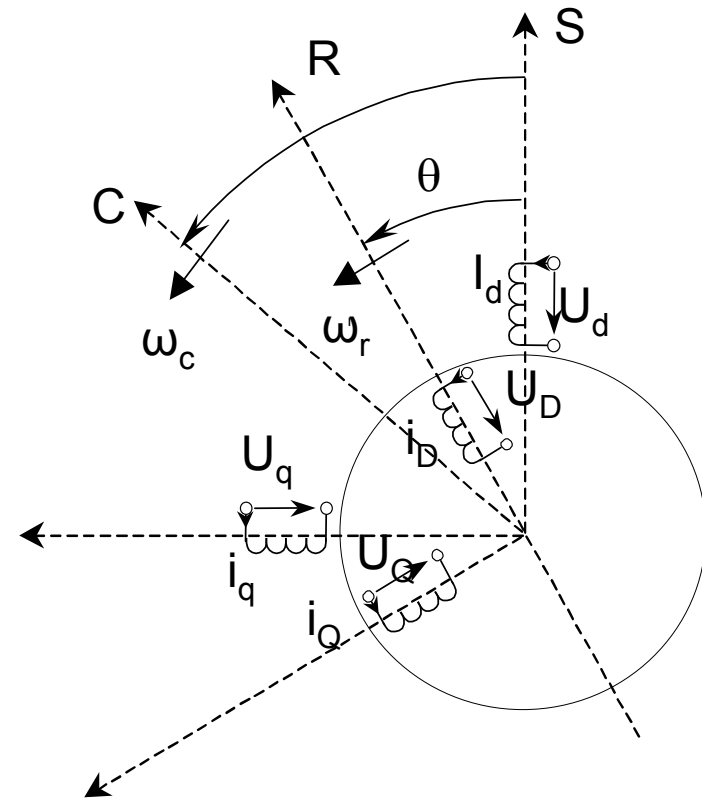
# Pornirea motoarelor de inductie

Metode de pornire

# Modelul masinii de inductie



Modelul de circuit trifazat



Modelul bifazat ortogonal

## Ecuatiile fazoriale ale masinii de inductie

$$\underline{u}_S = R_S \cdot \underline{i}_S + \frac{d}{dt} \underline{\Psi}_S + j \cdot \omega_c \cdot \underline{\Psi}_S$$

$$0 = R_R \cdot \underline{i}_R + \frac{d}{dt} \underline{\Psi}_R + j \cdot (\omega_c - \omega) \cdot \underline{\Psi}_R$$

$$\underline{\Psi}_S = \underline{\Psi}_{S\sigma} + \underline{\Psi}_{Sm} = L_{S\sigma} \cdot \underline{i}_S + M \cdot (\underline{i}_S + \underline{i}_R)$$

$$\underline{\Psi}_R = \underline{\Psi}_{R\sigma} + \underline{\Psi}_{Rm} = L_{R\sigma} \cdot \underline{i}_R + M \cdot (\underline{i}_S + \underline{i}_R)$$

$$\underline{i}_m = \underline{i}_S + \underline{i}_R$$

$$C = p \cdot \Re\{j \cdot \underline{\Psi}_S \cdot \underline{i}_S^*\} = p \cdot \Re\{j \cdot \underline{\Psi}_m \cdot \underline{i}_S^*\} = p \cdot \Re\{j \cdot M \cdot \underline{i}_R \cdot \underline{i}_S^*\}$$

$$C = \frac{J}{p} \cdot \frac{d\omega}{dt} - F_v \cdot \omega - C_{sr}$$

$$\frac{d\Psi_m}{dt} = \frac{d\Psi_m}{di_m} \cdot \frac{di_m}{dt} = M_t \cdot \frac{di_m}{dt}$$

## Ecuatiile modelului bifazat ortogonal

$$\underline{u}_S = u_d + j \cdot u_q$$

$$\underline{i}_S = i_d + j \cdot i_q$$

$$\underline{\Psi}_S = \Psi_d + j \cdot \Psi_q$$

$$\Psi_d = L_{S\sigma} \cdot i_d + \Psi_{dm} = L_{S\sigma} \cdot i_d + M \cdot i_{md}$$

$$\Psi_q = L_{S\sigma} \cdot i_q + \Psi_{qm} = L_{S\sigma} \cdot i_q + M \cdot i_{mq}$$

$$\Psi_D = L_{R\sigma} \cdot i_D + \Psi_{dm} = L_{R\sigma} \cdot i_D + M \cdot i_{md}$$

$$\Psi_Q = L_{R\sigma} \cdot i_Q + \Psi_{qm} = L_{R\sigma} \cdot i_Q + M \cdot i_{mq}$$

$$u_d = R_S \cdot i_d + \frac{d\Psi_d}{dt} - \omega_c \cdot \Psi_q$$

$$u_q = R_S \cdot i_q + \frac{d\Psi_q}{dt} + \omega_c \cdot \Psi_d$$

$$0 = R_R \cdot i_D + \frac{d\Psi_D}{dt} - (\omega_c - \omega) \cdot \Psi_Q$$

$$0 = R_R \cdot i_Q + \frac{d\Psi_Q}{dt} + (\omega_c - \omega) \cdot \Psi_D$$

$$C = p \cdot (\Psi_d \cdot i_q - \Psi_q \cdot i_d)$$

# Expresiile curentilor

$$\underline{i}_S = \frac{L_R \cdot \underline{\Psi}_S - M \cdot \underline{\Psi}_R}{L_S \cdot L_R - M^2}$$

$$\underline{i}_S = \frac{\omega_S \cdot \underline{\Psi}_S - k_R \cdot \omega_S \cdot \underline{\Psi}_R}{\omega \cdot L_S \cdot \sigma_{SR}}$$

$$\underline{i}_R = \frac{L_S \cdot \underline{\Psi}_R - M \cdot \underline{\Psi}_S}{L_S \cdot L_R - M^2}$$

$$\underline{i}_R = \frac{\omega_S \cdot \underline{\Psi}_R - k_S \cdot \omega_S \cdot \underline{\Psi}_S}{\omega \cdot L_S \cdot \sigma_{SR}}$$

$$L_S \cdot L_R - M^2 = L_{S\sigma} \cdot L_{R\sigma} + M(L_{S\sigma} + L_{R\sigma}) = L_S \cdot L_R \cdot \sigma_{SR}$$

$$k_S = \frac{M}{L_S}$$

$$k_R = \frac{M}{L_R}$$

$$\sigma_{SR} = 1 - \frac{M^2}{L_S \cdot L_R} = 1 - k_S \cdot k_R$$

$$s_k = \frac{R_R}{\omega_S \cdot L_S \cdot \sigma_{SR}}$$

$$\varepsilon_k = \frac{R_S}{\omega_S \cdot L_S \cdot \sigma_{SR}}$$

## Ecuatiile operationale

$$\underline{U}_S = (\varepsilon \cdot \omega_S + p + j \cdot \omega_S) \underline{\Psi}_S - \varepsilon \cdot \omega_S \cdot k_R \cdot \underline{\Psi}_R$$

$$0 = -s_k \cdot \omega_S \cdot k_S \cdot \underline{\Psi}_S + (s_k \cdot \omega_S + p + j \cdot s \cdot \omega_S) \underline{\Psi}_R$$

fluxurile

$$\underline{\Psi}_S = \frac{(s_k \cdot \omega_S + p + j \cdot s \cdot \omega_S) \underline{U}_S}{(\varepsilon \cdot \omega_S + p + j \cdot \omega_S) \cdot (s_k \cdot \omega_S + p + j \cdot s \cdot \omega_S) - \varepsilon \cdot s_k \cdot k_S \cdot k_R \cdot \omega_S^2}$$

$$\underline{\Psi}_R = \frac{s_k \cdot k_S \cdot \omega_S \cdot \underline{U}_S}{(\varepsilon \cdot \omega_S + p + j \cdot \omega_S) \cdot (s_k \cdot \omega_S + p + j \cdot s \cdot \omega_S) - \varepsilon \cdot s_k \cdot k_S \cdot k_R \cdot \omega_S^2}$$

Ecuatia caracteristica

$$(\varepsilon \cdot \omega_S + p + j \cdot \omega_S) \cdot (s_k \cdot \omega_S + p + j \cdot s \cdot \omega_S) - \varepsilon \cdot s_k \cdot k_S \cdot k_R \cdot \omega_S^2 = 0$$

# Constantele de timp si frecventele de oscilatii

Radacinile ecuatiei caracteristice:

$$p_{1,2} = \frac{\omega_s}{2} \left\{ -[(s_k + \varepsilon) + j(1 + s)] \pm \sqrt{[(s_k + \varepsilon) + j(1 + s)]^2 - 4 \cdot [\varepsilon \cdot s_k \cdot \sigma + j(s_k + \varepsilon \cdot s)]} \right\}$$

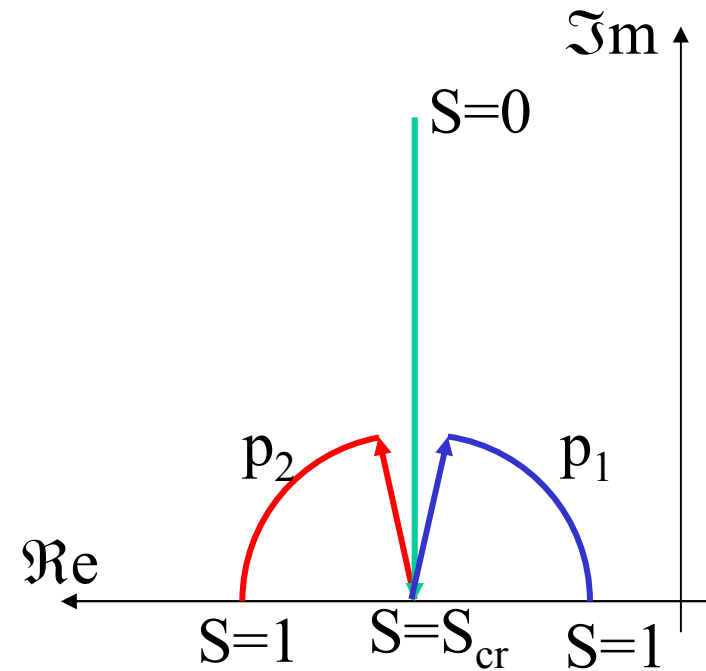
Forma generala a radacinilor

$$p_{1,2} = -\frac{1}{T(s)} \pm j \cdot \rho(s)$$

$S = S_c$  Doua radacini duble reale

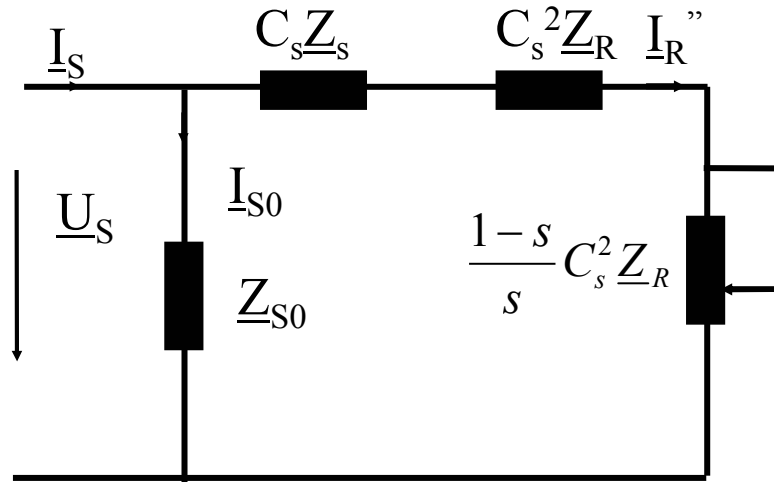
$s < s_c$  Partea reala este constanta  
Pot apare oscilatii

$S > S_c$  Variaza si partea reala si imaginara



Pot apare oscilatii

# Pornirea prin cuplarea directă la rețea



Regim tranzitoriu

$$i_S = i_{S0} + i_R''$$

Curentul de mers în gol:

$$i_{S0} = C_{S0} \cdot e^{-\frac{t}{T_{S0}}} + I_{S0m} \cdot \sin(\omega \cdot t + \gamma_{S0})$$

$$I_{S0m} = \frac{\sqrt{2} \cdot U_S}{Z_{S0}}$$

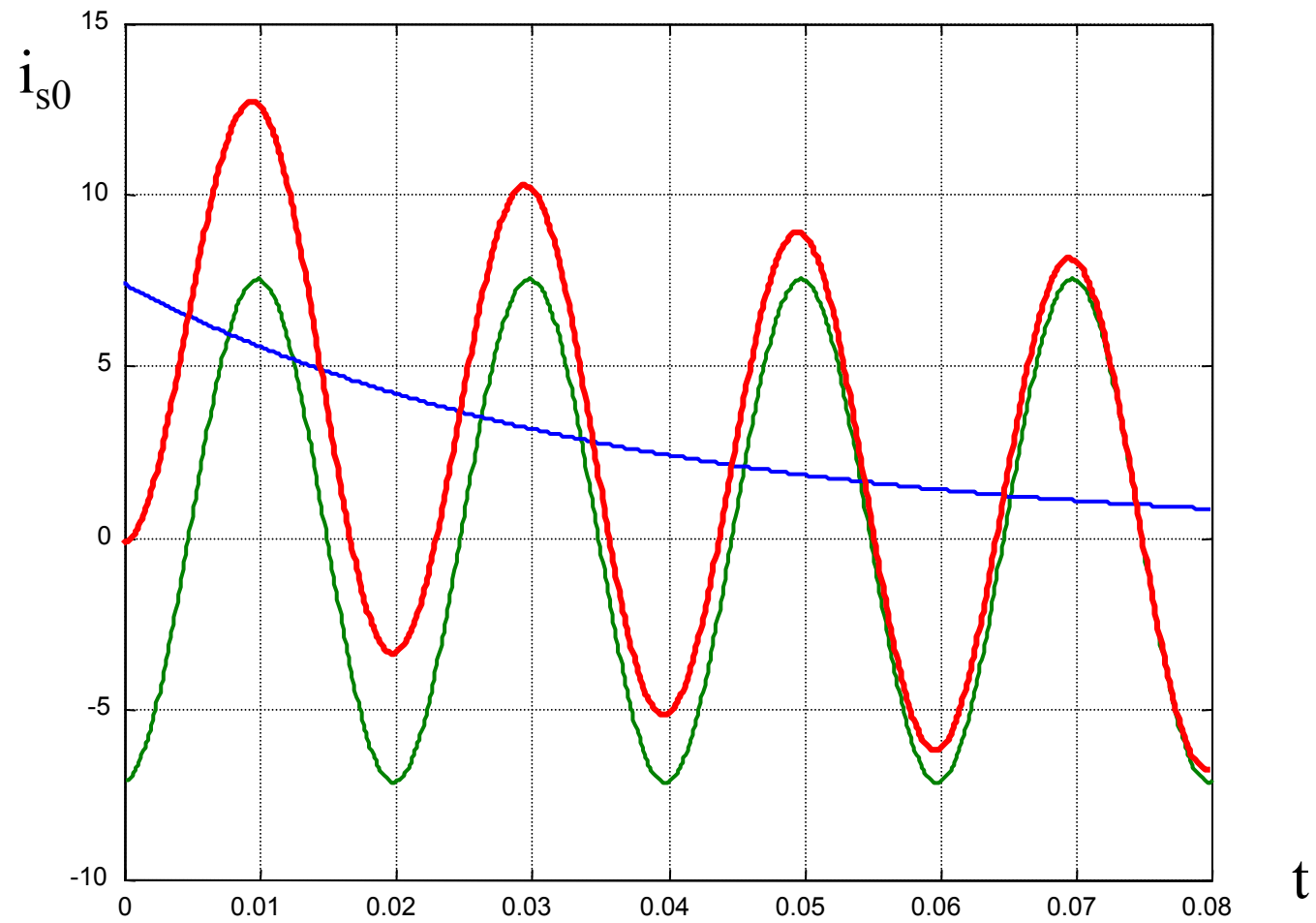
$$\operatorname{tg}(\gamma_{S0} - \alpha) = -\omega \cdot T_{S0}$$

$$T_{S0} = \frac{L_{S0}}{R_{S0}}$$

$$C_{S0} = -I_{S0m} \cdot \sin \gamma_{S0}$$



# Curentul de mers în gol tranzitoriu



# Pornirea prin cuplarea directă la rețea

Curentul rotoric

$$i_R'' = C_R \cdot e^{-\frac{t}{T_{Ssc}}} + I_{Rm} \cdot \sin(\omega \cdot t + \gamma_{Ssc}) \quad I_{Rm} = \frac{\sqrt{2} \cdot U_S}{C_S \cdot Z_{Ssc}}$$

$$\operatorname{tg}(\gamma_{Ssc} - \alpha) = -\omega \cdot T_{Ssc}$$

$$T_{Ssc} = \frac{L_{Ssc}}{R_{Ssc}}$$

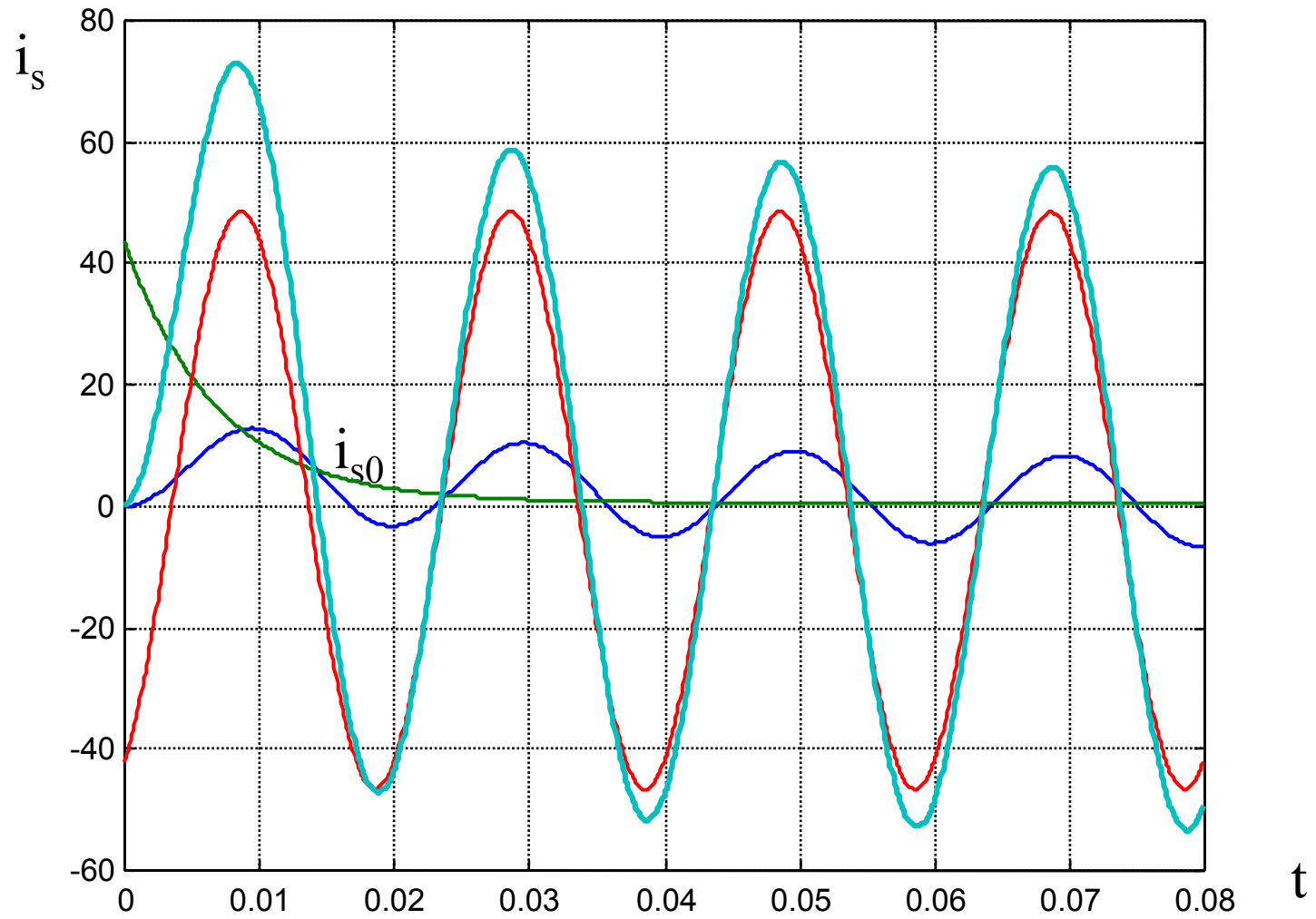
$$C_R = -I_{Rm} \cdot \sin \gamma_{Ssc}$$

Curentul statoric de pornire

$$i_S = -I_{S0m} \cdot \sin \gamma_{S0} \cdot e^{-\frac{t}{T_{S0}}} + I_{S0m} \cdot \sin(\omega \cdot t + \gamma_{S0}) \\ - I_{Rm} \cdot \sin \gamma_{Ssc} \cdot e^{-\frac{t}{T_{Ssc}}} + I_{Rm} \cdot \sin(\omega \cdot t + \gamma_{Ssc})$$

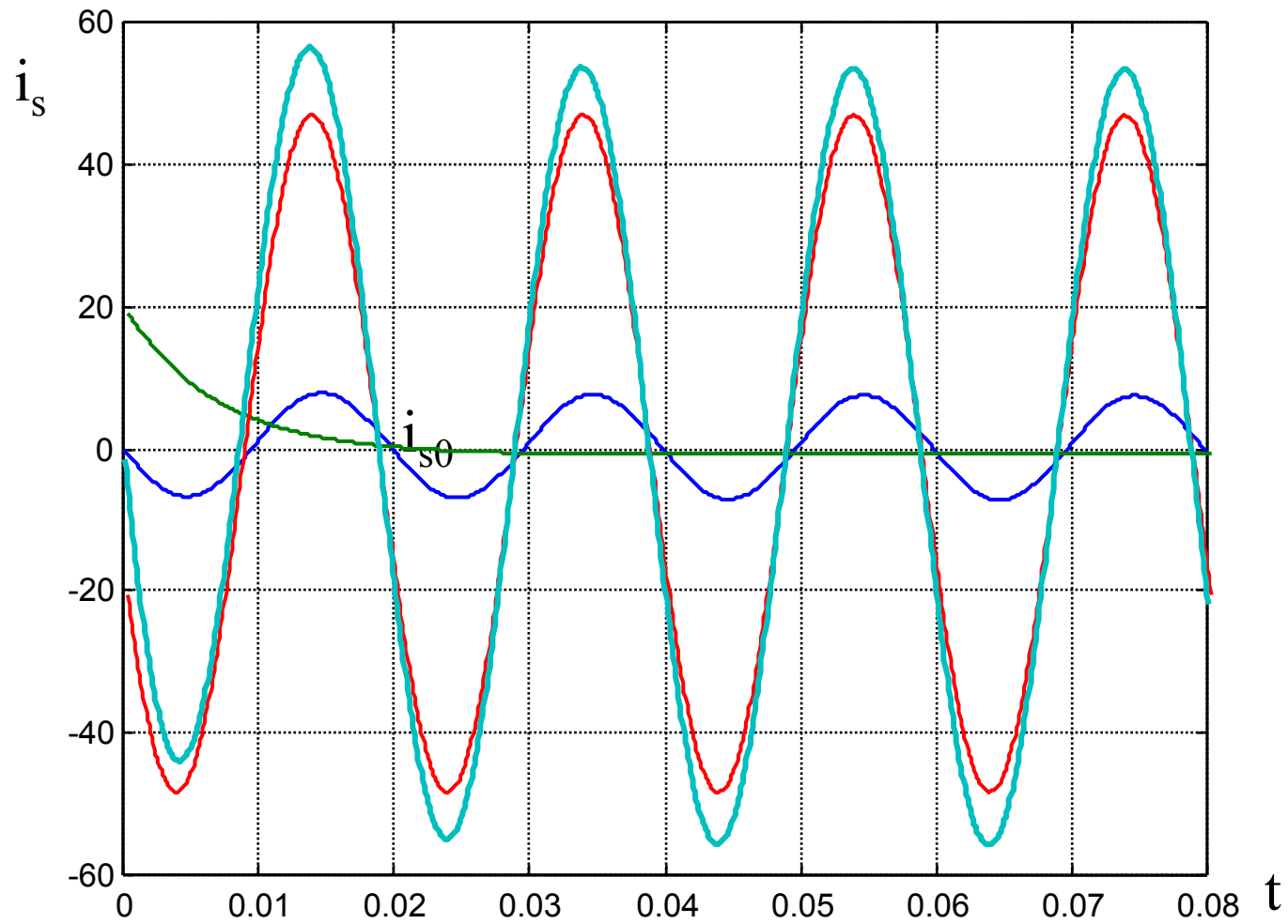
# Curentul statoric tranzitoriu

$$\alpha = 0$$

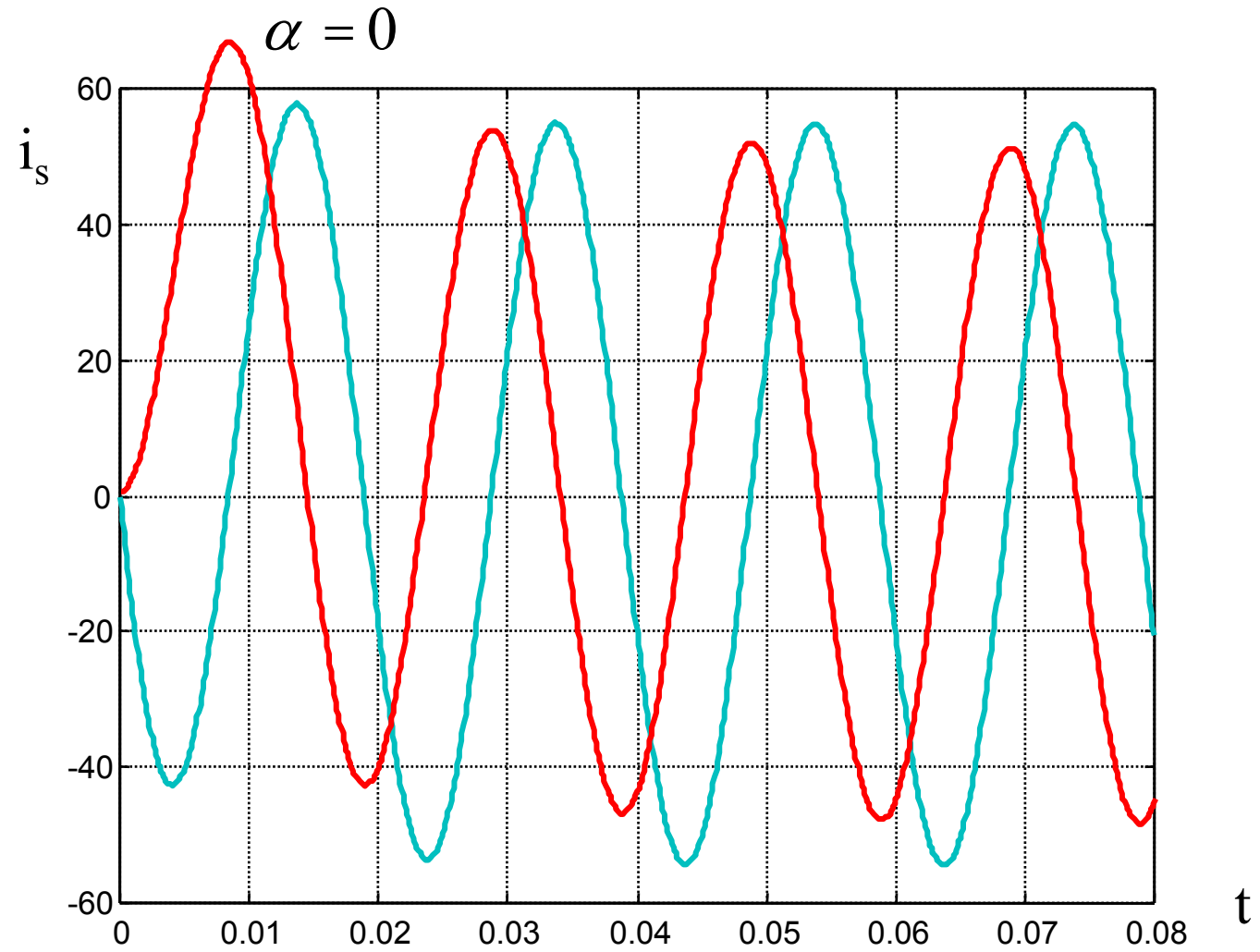


# Curentul statoric tranzitoriu

$$\alpha = \frac{\pi}{2}$$



# Curentul statoric tranzitoriu

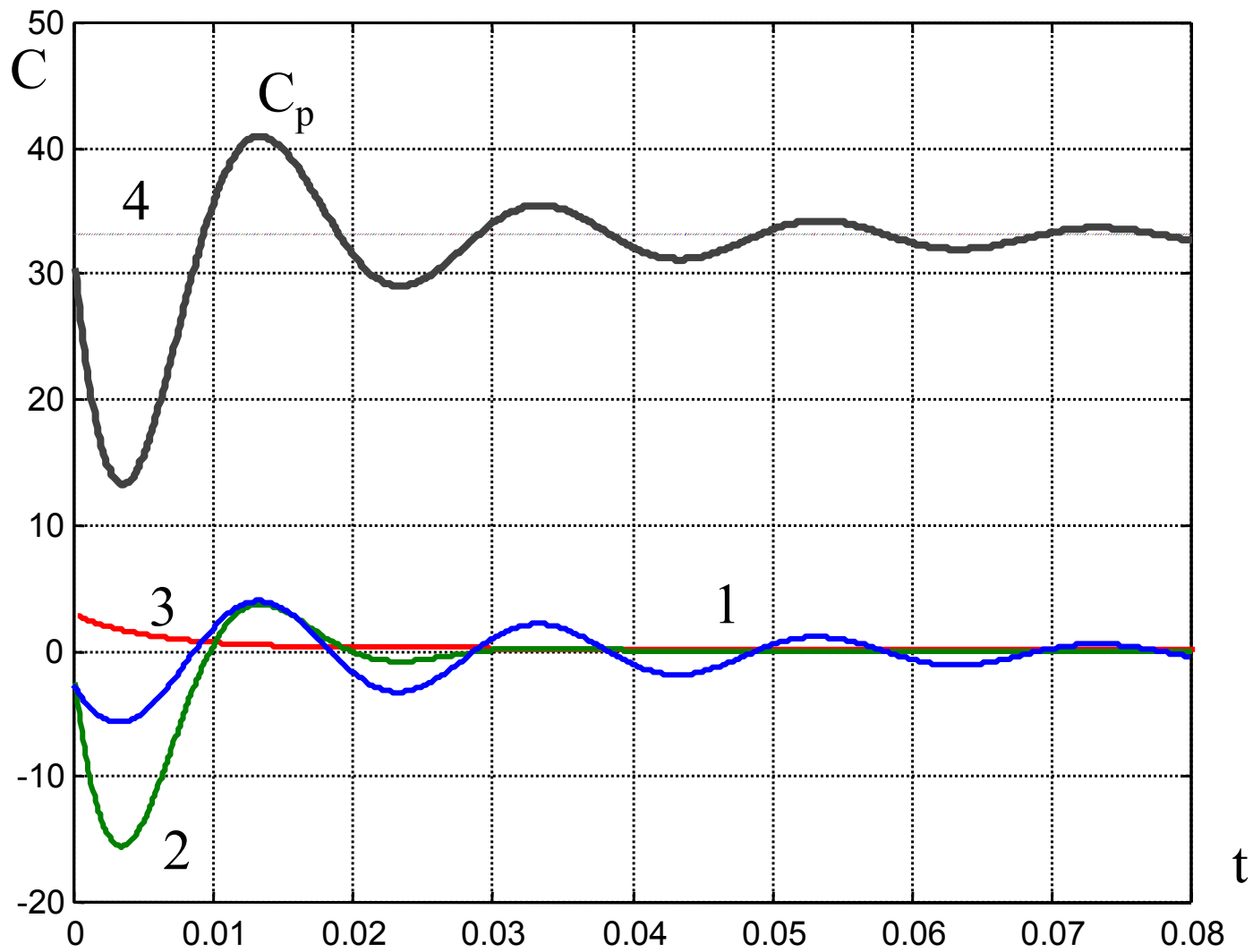


# Cuplul de pornire

Cuplul este proporțional cu produsul curenților

$$i_{S0} = C_{S0} \cdot e^{-\frac{t}{T_{S0}}} + I_{S0m} \cdot \sin(\omega \cdot t + \gamma_{S0})$$
$$T_1 \cdot e^{-\frac{t}{T_{S0}}} \cdot \sin(\omega \cdot t + \gamma_1) \quad T_2 \cdot e^{-\frac{t}{T_{Rsc}}} \cdot \sin(\omega \cdot t + \gamma_2) \quad T_3 \cdot e^{-\left(\frac{t}{T_{S0}} + \frac{t}{T_{Rsc}}\right)} \quad T_4$$
$$i_R'' = C_R \cdot e^{-\frac{t}{T_{Ssc}}} + I_{Rm} \cdot \sin(\omega \cdot t + \gamma_{Ssc})$$

# Cuplul de pornire



# Pornirea prin cuplarea directă la rețea

## Curentul de pornire.

Alunecarea  $s = 1$  regim staționar

$$\underline{I}_{Sp} = \underline{U}_S \cdot \left( \frac{1}{\underline{Z}_{Sm}} + \frac{1}{\underline{C}_S \cdot \underline{Z}_{Ssc}} \right) = I_{Rp} \cdot e^{-j \cdot \varphi_{sc}} \cdot \left( \frac{\underline{Z}_{Ssc}}{\underline{Z}_{Sm}} e^{-j \cdot \varphi_{0sc}} + 1 \right)$$

$$I_{Rp} = \frac{U_S}{C_S \cdot Z_{Ssc}} \quad \underline{k}_p = \left( \frac{Z_{Ssc}}{Z_{Sm}} e^{-j \cdot \varphi_{0sc}} + 1 \right) \cdot e^{-j \cdot \varphi_{sc}}$$

$$\underline{I}_{Sp} = I_{Rp} \cdot \underline{k}_p$$



# Curentul de pornire la cuplarea directă

$$\underline{Z}_{Ssc} = 2.5 + j5.4$$

$$\underline{Z}_m = 2.8 + j39.8$$

$$\underline{k}_p = 0.4310 + j1.0613$$

$$\underline{k}_p = 1.1455 \cdot e^{-j1.185}$$

$$I_{Rp} = \frac{U_S}{C_S \cdot Z_{Ssc}} = 33.6421 \text{ A}$$

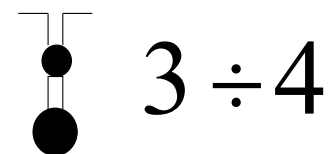
$$I_{Sp} = I_{Rp} \cdot |\underline{k}_p| = 38.537 \text{ A}$$

$$1.185 \text{ rad} = 67,9^\circ$$



Curentul de pornire raportat

$$i_p = \frac{I_{Sp}}{I_{SN}}$$



# Pornirea prin cuplarea directă la rețea

## Cuplul de pornire

$$C_p = \frac{2 \cdot C_k \cdot (1 + \varepsilon)}{\frac{1}{s_k} + \frac{s_k}{1} + \varepsilon} = 2 \cdot C_k \frac{(1 + \varepsilon) \cdot s_k}{1 + \varepsilon \cdot s_k + s_k^2}$$

$$s_k = \frac{R_R}{\sqrt{R_S^2 + X_{Ssc}^2}} \cong \frac{R_R}{Z_{Ssc}}$$

$$\varepsilon = \frac{R_S \cdot s_k}{R_R} \cong \frac{R_S}{Z_{Ssc}}$$

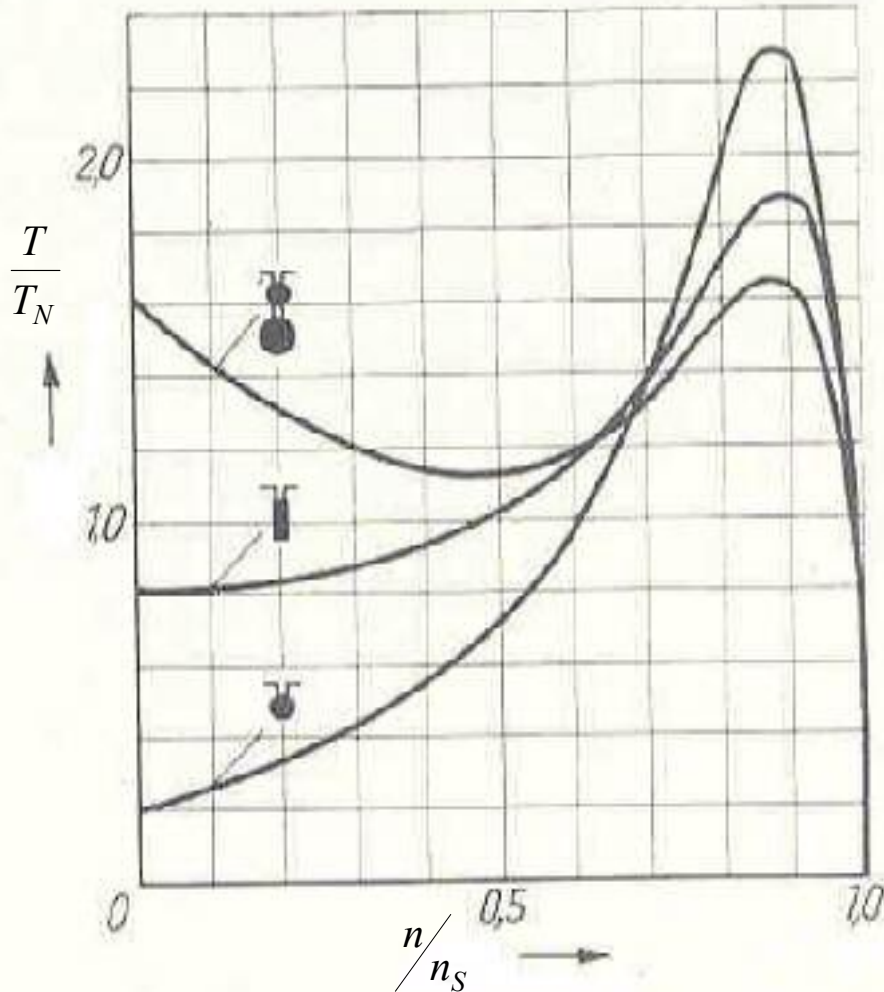
$$C_k = \frac{p \cdot m_S}{\omega_S} \frac{U_S^2}{2 \cdot c_S} \frac{1}{R_S \pm \sqrt{R_S^2 + (X_{S\sigma} + c_S \cdot X_{R\sigma})^2}}$$

$$C_k \cong \frac{p \cdot m_S}{\omega_S} \frac{U_S^2}{2 \cdot c_S} \frac{1}{R_S \pm Z_{Ssc}} \approx k_c \cdot \frac{U_S^2}{Z_{Ssc}} \frac{1}{\varepsilon \pm 1}$$

$$k_c = \frac{m_s}{2} \frac{p}{\omega_s}$$

# Cuplul de pornire la cuplarea directă

Cuplul de pornire



● 0.2 ÷ 0.8

▮ 0.8 ÷ 1.4

● 1.4 ÷ 2

Modulul curenului rotoric

$$I_{Rp} = \frac{(\epsilon + 1) \cdot C_k}{k_c \cdot c_s \cdot U_s}$$

## Metode de pornire

$$\underline{I}_{Sp} = \frac{U_S}{C_S \cdot Z_{Ssc}} \underline{k}_p$$

### **Reducerea tensiunii de alimentare**

- comutator stea triunghi
- autotransformator

### **Marirea impedantei de scurtcircuit**

- reactante in circuitul statoric
- rezistente in circuitul rotoric

# Pornire cu comutator stea- triunghi

Metoda se poate aplica numai în cazul motoarelor care în mod normal funcționează în conexiunea triunghi.

Curentul de pornire în cazul cuplării directe în conexiunea triunghi:

$$\underline{I}_{p\Delta} = \sqrt{3} \cdot \frac{U_S}{Z_{Ssc}} \underline{k}_p \cdot e^{-j\frac{\pi}{3}}$$

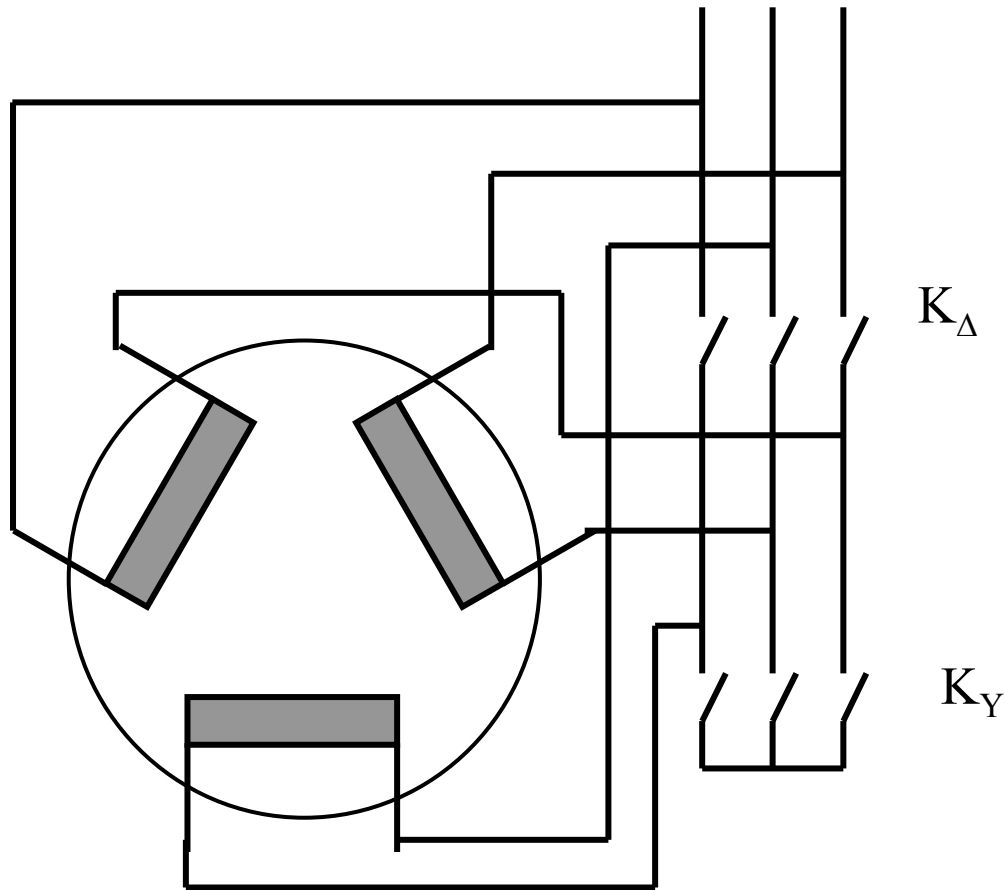
La conectarea în stea a înfășurărilor se reduce tensiunea

$$\underline{I}_{pY} = \frac{U_S}{\sqrt{3} \cdot Z_{Ssc}} \underline{k}_p$$

Raportul curenților de pornire, considerând neglijabilă saturația

$$r_{i\Delta Y} = 3 \cdot e^{-j\frac{\pi}{3}}$$

# Pornire cu comutator stea triunghi

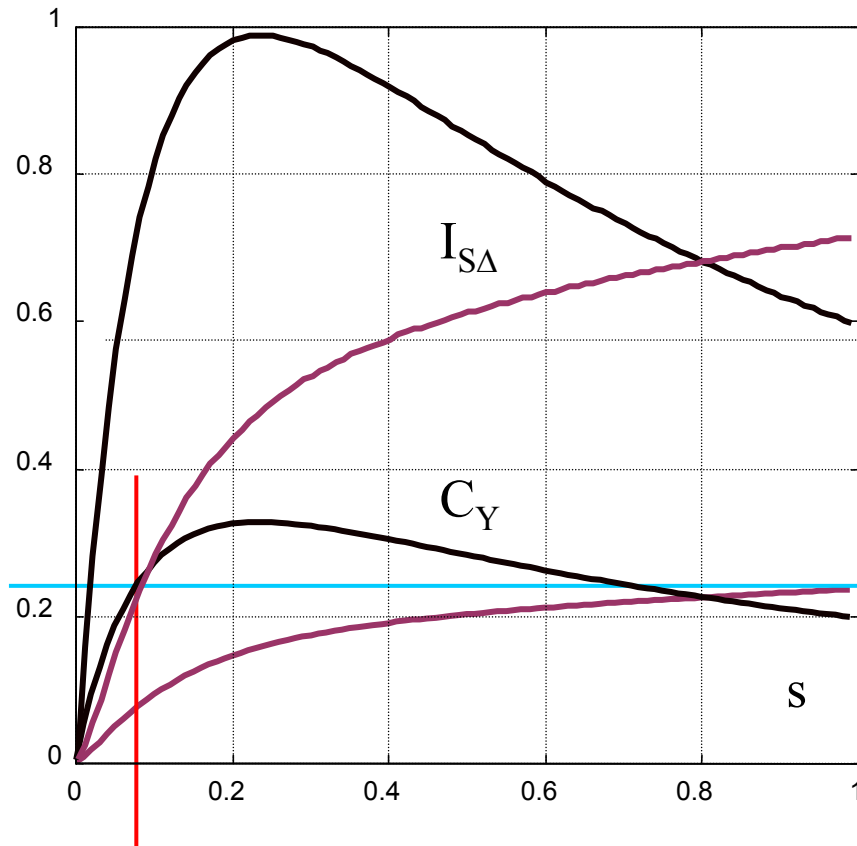


Cuplul de pornire se reduce proporțional cu pătratul tensiunii, deci:

$$r_{C\Delta Y} = \left( \frac{U_S}{\frac{U_S}{\sqrt{3}}} \right)^2 = 3$$

$$C_p = 2 \cdot k_c \cdot U_S^2 \frac{R_R}{Z_{Ssc}^2 + R_S \cdot R_R + R_R^2}$$

# Pornire cu comutator stea triunghi

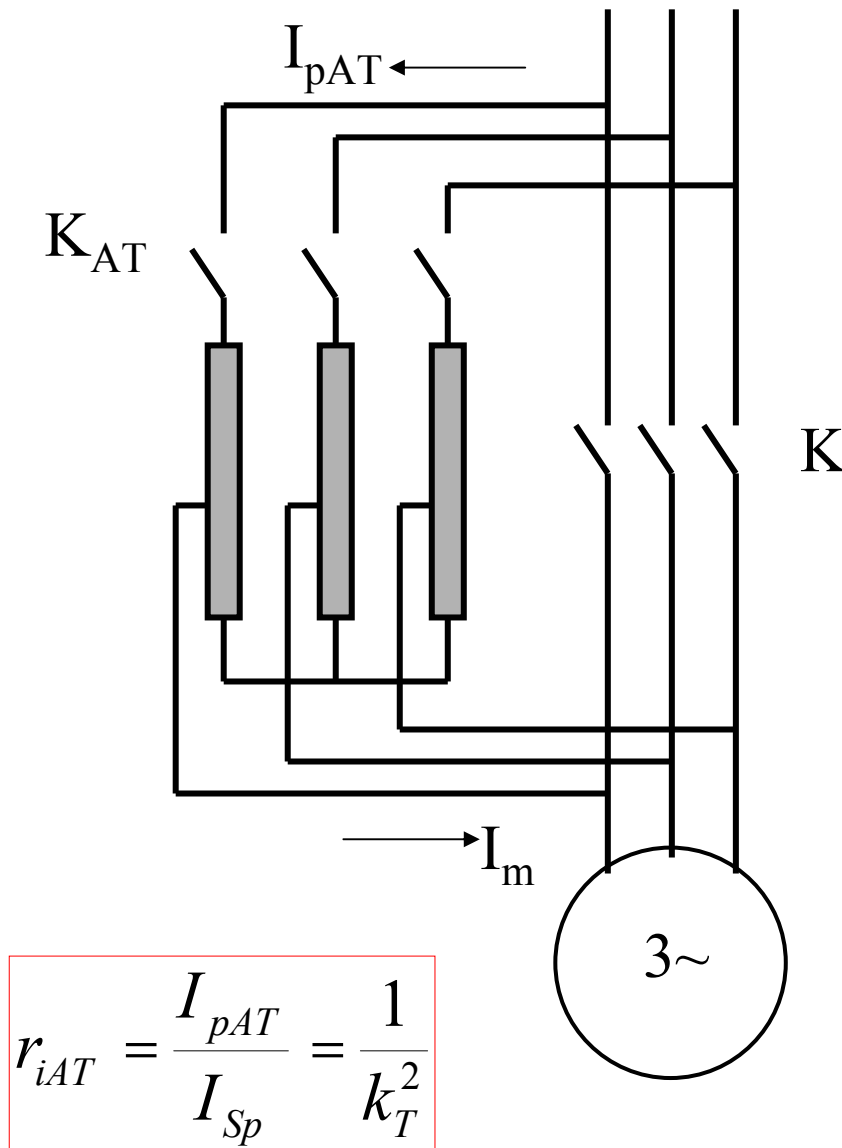


Schimbarea conexiunii.  
alunecarea trebuie să fie  
mai mică decât valoarea  
din figură

Curentul la schimbarea  
conexiunii nu trebuie să  
depășească curentul de  
pornire.

Valabil la oricare metodă  
de pornire.

# Pornire cu autotransformator



$$r_{iAT} = \frac{I_{pAT}}{I_{Sp}} = \frac{1}{k_T^2}$$

Tensiunea aplicată  
motorului

$$U_m = \frac{U_S}{k_T}$$

Curentul prin motor

$$\underline{I}_m = \frac{I_{Rp}}{k_T} k_p$$

Curentul de pornire  
curentul în primarul  
autotransformatorului

$$\underline{I}_{pAT} = \frac{\underline{I}_m}{k_T} = \frac{1}{k_T^2} \underline{I}_{Sp}$$



# Pornire cu autotransformator

Cuplul de pornire

$$C_{pAT} = 2 \cdot k_c \cdot \left( \frac{U_S}{k_T} \right)^2 \frac{R_R}{\left| \underline{Z}_{AT} + \underline{Z}_{Ssc} \right|^2 + (R_S + R_{AT}) \cdot R_R + R_R^2}$$

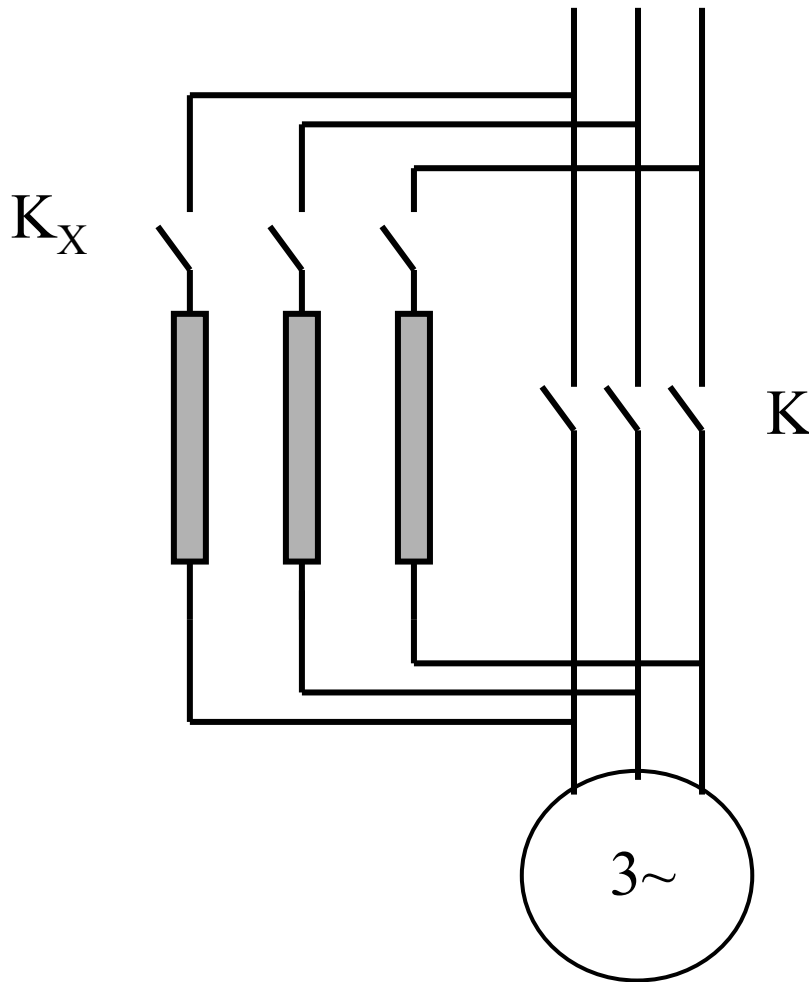
$$r_{CAT} = \frac{C_{pAT}}{C_p} = \frac{1}{k_T^2} \frac{Z_{Ssc}^2 + R_S \cdot R_R + R_R^2}{\left| \underline{Z}_{AT} + \underline{Z}_{Ssc} \right|^2 + (R_S + R_{AT}) \cdot R_R + R_R^2}$$

Se reduce ceva mai mult decât proporțional cu  $k_T^2$

$$\frac{Z_{AT}}{Z_{Ssc}} = \frac{u_{scAT}}{\cos \varphi_S} \frac{S_{Nmotor}}{S_{NAT}}$$

$$\frac{Z_{AT}}{Z_{Ssc}} = \frac{0.06}{0.9} < 0.1$$

## Pornire cu reactanță înseriată



Se modifică  $Z_{Ssc}$

$$\underline{Z}_{Ssc} \Rightarrow \underline{Z}_{Ssc} + j \cdot X$$

Se modifică  $\varphi_{sc}$  și  $\underline{k}_p$

$$\varphi_{sc}^* = \text{arctg} \frac{R_{sc}}{X_{sc} + X}$$

$$\underline{k}_p = \left( \frac{(\underline{Z}_{Ssc} + jX)}{\underline{Z}_{Sm}} + 1 \right) \cdot e^{-j \cdot \varphi_{sc}^*}$$

Curentul se reduce dependent de  $X$

$$I_{Rp} = \frac{U_S}{C_S \cdot |\underline{Z}_{Ssc} + jX|}$$

## Pornire cu reactanță înseriată

$$r_{iB} = \frac{Z_{Ssc}}{|Z_{Ssc} + jX|} \left| 1 + \frac{Z_{Ssc} + jX}{Z_{Sm}} \right|$$

Cuplul se reduce dependent de **X**

$$C_{pB} = 2 \cdot k_c \cdot U_S^2 \frac{R_R}{|jX + Z_{Ssc}|^2 + R_S \cdot R_R + R_R^2}$$

$$r_{CB} = \frac{C_{pB}}{C_p} = \frac{Z_{Ssc}^2 + R_S \cdot R_R + R_R^2}{|jX + Z_{Ssc}|^2 + (R_S + R_B) \cdot R_R + R_R^2}$$

# Pornire cu rezistență în rotor

Numai la motoare cu rotor bobinat

$$\underline{Z}_{Ssc}^* \Rightarrow (R_{Ssc} + R_p) + j \cdot X_{Ssc}$$

Se modifică  $\underline{k}_p$  și  $\varphi_{sc}$

$$\varphi_{sc}^* = \operatorname{arctg} \frac{R_{sc} + R}{X_{sc}}$$

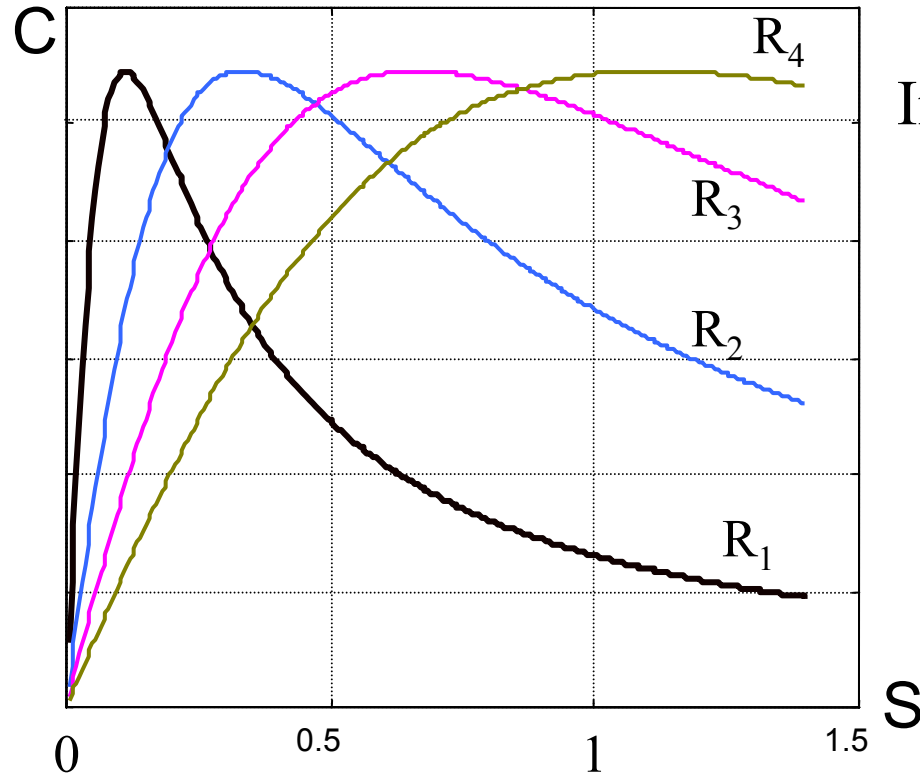
$$\underline{k}_p = \left( \frac{(\underline{Z}_{Ssc} + R)}{\underline{Z}_{Sm}} + 1 \right) \cdot e^{-j \cdot \varphi_{sc}^*}$$

Se reduce curentul de pornire

$$I_{Rp} = \frac{U_S}{C_S \cdot |\underline{Z}_{Ssc} + R|}$$

$$r_{iR} = \frac{Z_{Ssc}}{|\underline{Z}_{Ssc} + R|} \left| 1 + \frac{\underline{Z}_{Ssc} + R}{\underline{Z}_{Sm}} \right|$$

# Pornire cu rezistență în rotor



In expresia cuplului  $R_R^* \Rightarrow R_R + R_p$

La început cuplul crește, atinge maxim la  $R_R + R_p = Z_{Ssc}$

$$C_{pR} = 2 \cdot k_c \cdot U_S^2 \frac{R_R^*}{\left(Z_{Ssc}^*\right)^2 + R_S \cdot R_R^* + R_R^{*2}}$$

$$r_{CR} = \frac{C_{pR}}{C_p} = \frac{Z_{Ssc}^2 + R_S \cdot R_R + R_R^2}{\left|R + Z_{Ssc}\right|^2 + (R_R + R_B) \cdot R_S + (R + R_R)^2} \frac{R_R + R}{R_R}$$

# Reducerea cuplului și curentului la pornire cu reactanță și rezistență

