RANSAC

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LECTURE PLAN

Courtesy: O. Chum, J. Matas

1.

2.

3.

The RANSAC Algorithm [Fischler, Bolles '81]

In:	$U = \{x_i\}$	set of data points, $\|U\| = N$
	$f(S) : S \to p$	function f computes model parameters p given a sample S from U
	$\rho(p, x)$	the cost function for a single data point x
Out:	p^*	p^*, parameters of the model maximizing the cost function

RANSAC Algorithm

$k := 0$

Repeat until P$\{$better solution exists$\} < \eta$
(a function of C^* and no. of steps k)

$k := k + 1$

I. Hypothesis

(1) select randomly set $S_k \subset U$, $|S_k| = m$

(2) compute parameters $p_k = f(S_k)$

II. Verification

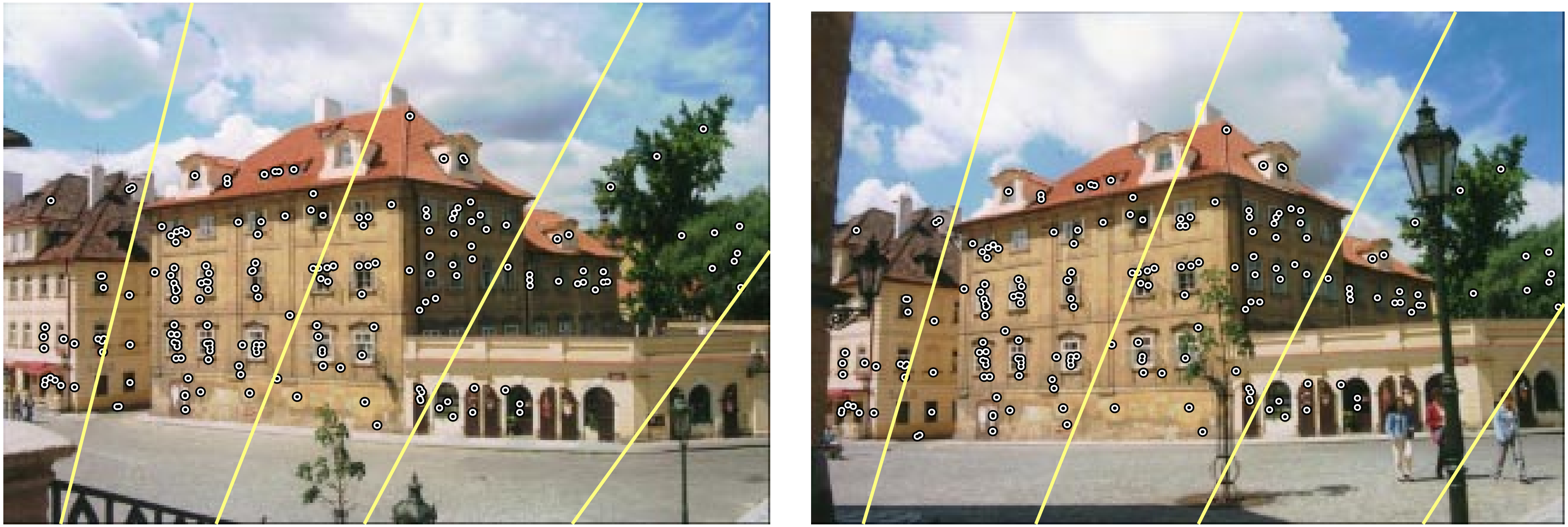
(3) compute cost $C_k = \sum_{x \in U} \rho(p_k, x)$

(4) if $C^* < C_k$ then $C^* := C_k$, $p^* := p_k$

end

Example I: Epipolar geometry estimation by RANSAC

- U : a set of correspondences, i.e. pairs of 2D points — data points
- $m = 7$ — sample size
- f : seven-point algorithm - gives 1 to 3 independent solutions — model parameters
- ρ : thresholded Sampson's error — cost function



Example II: Line detection by RANSAC

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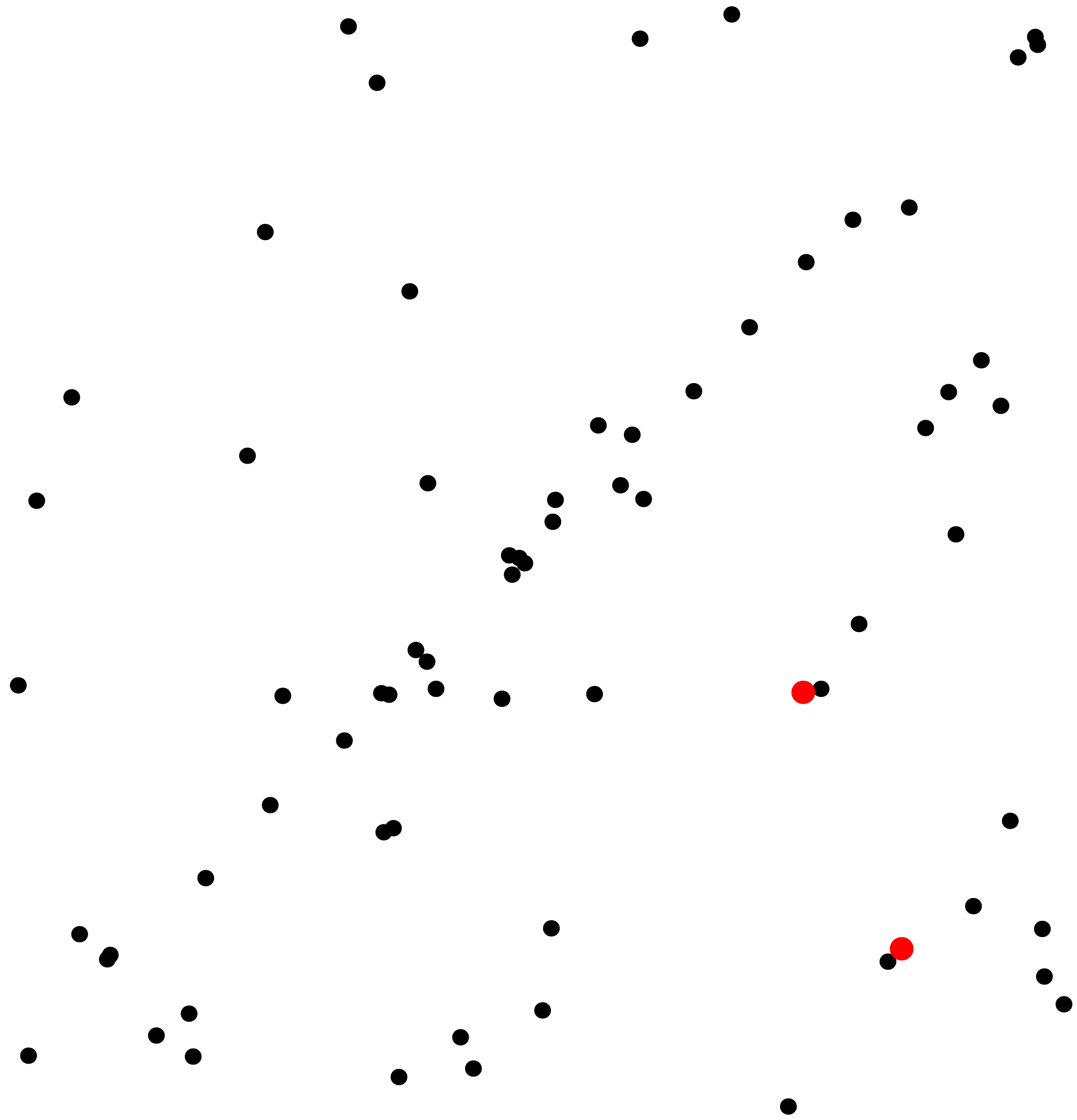
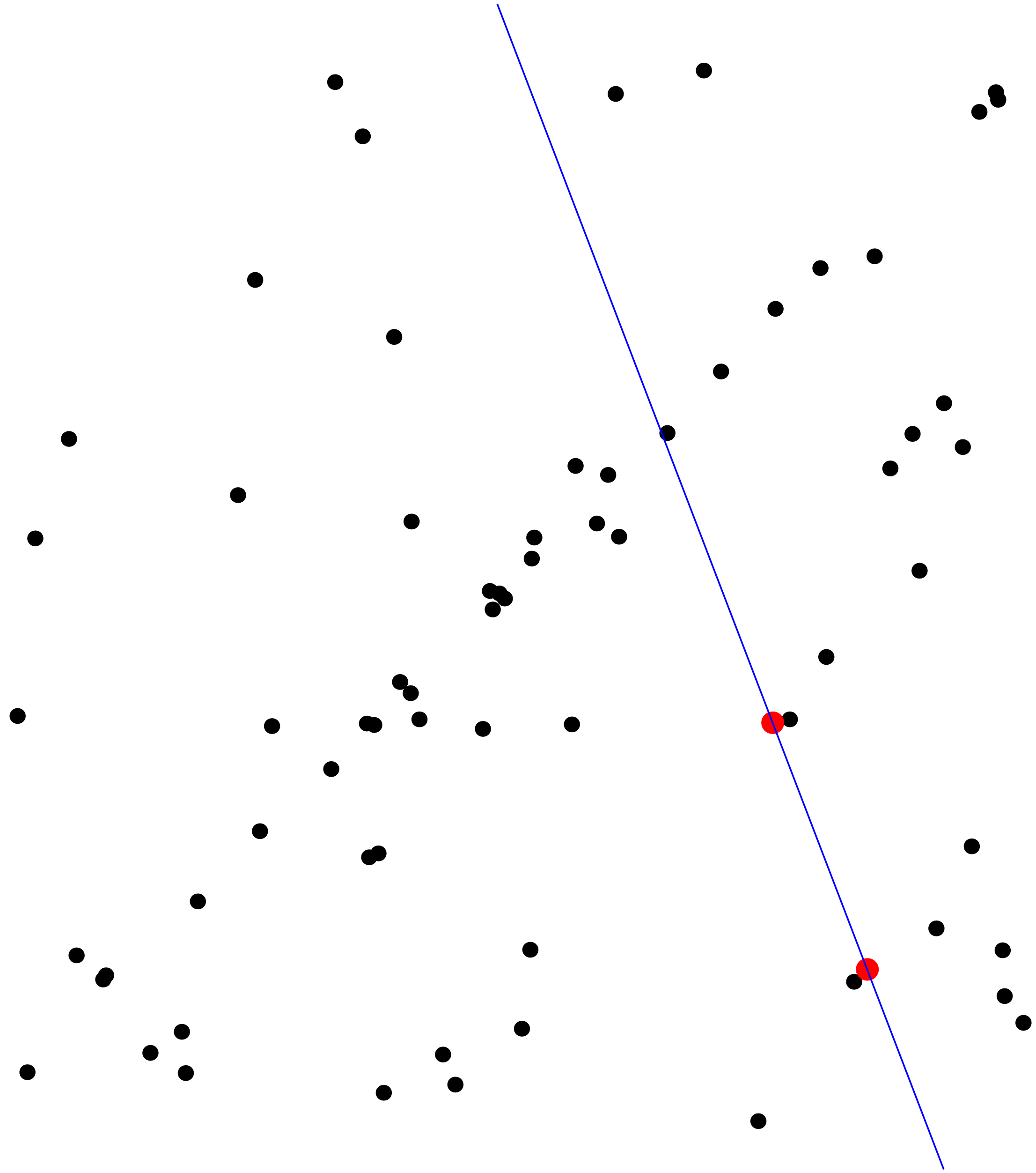


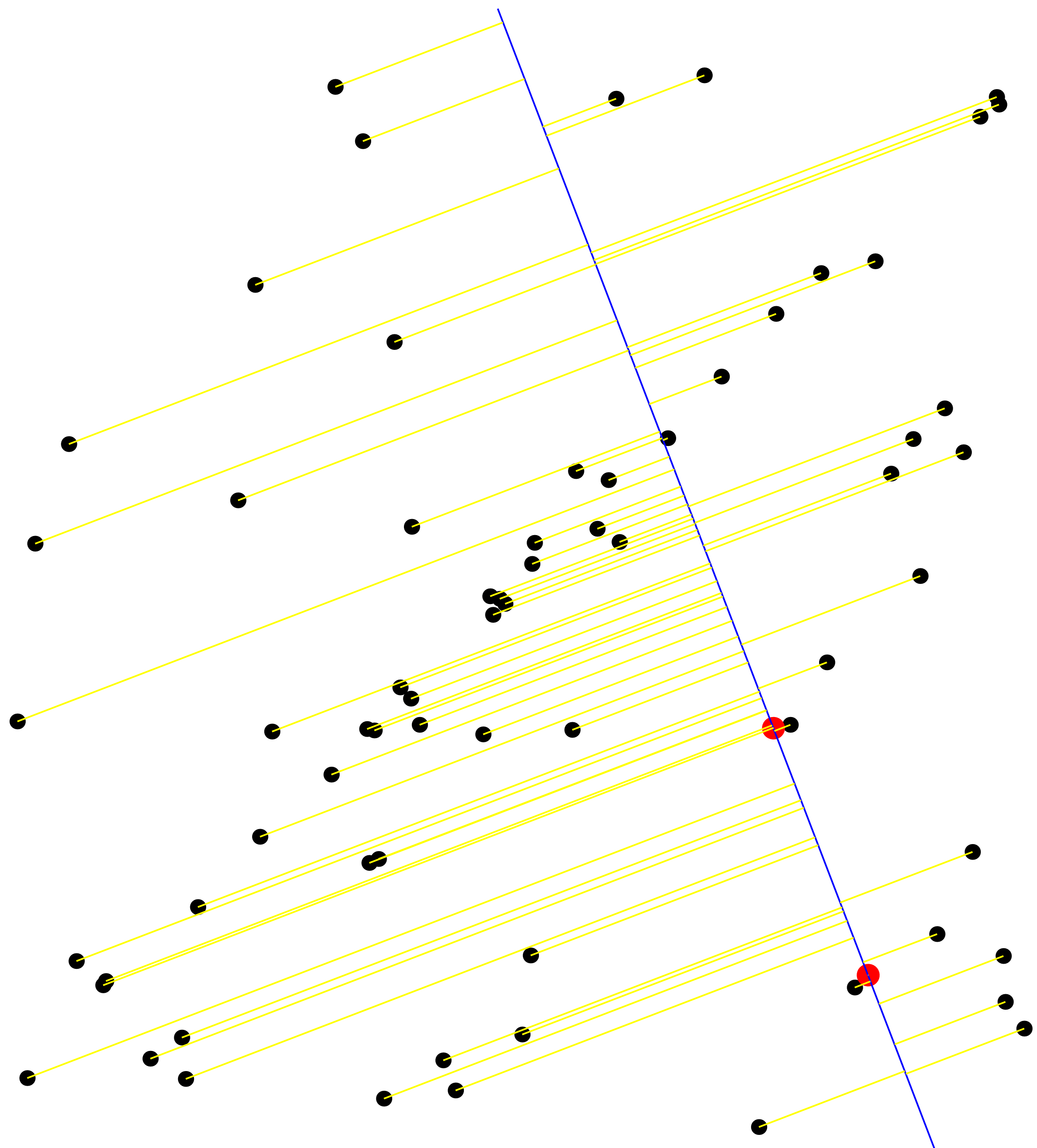
- Randomly select two points

Example II: Line detection by RANSAC



- Randomly select two points
- The hypothesised model is the line passing through the two points

Example II: Line detection by RANSAC



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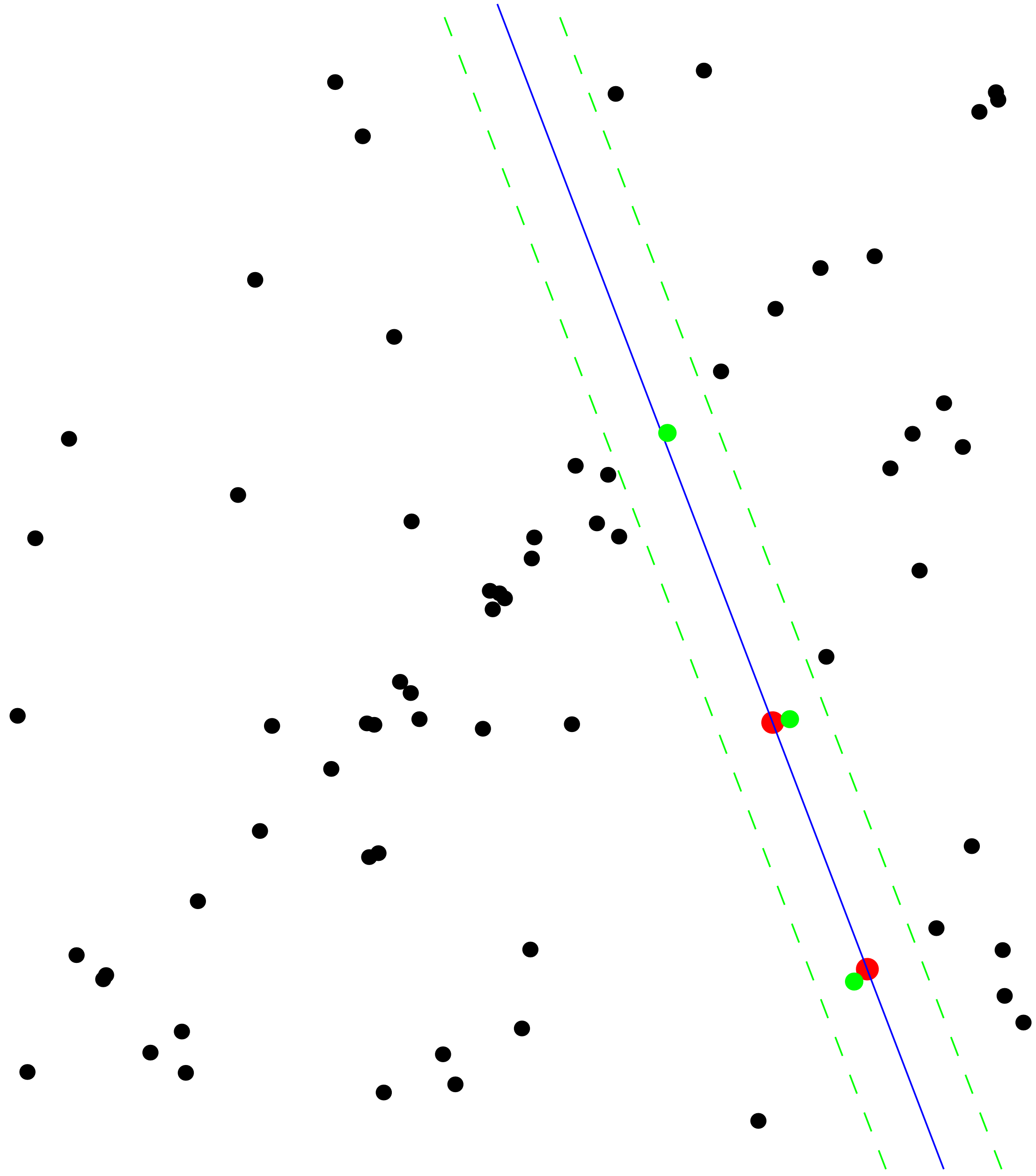

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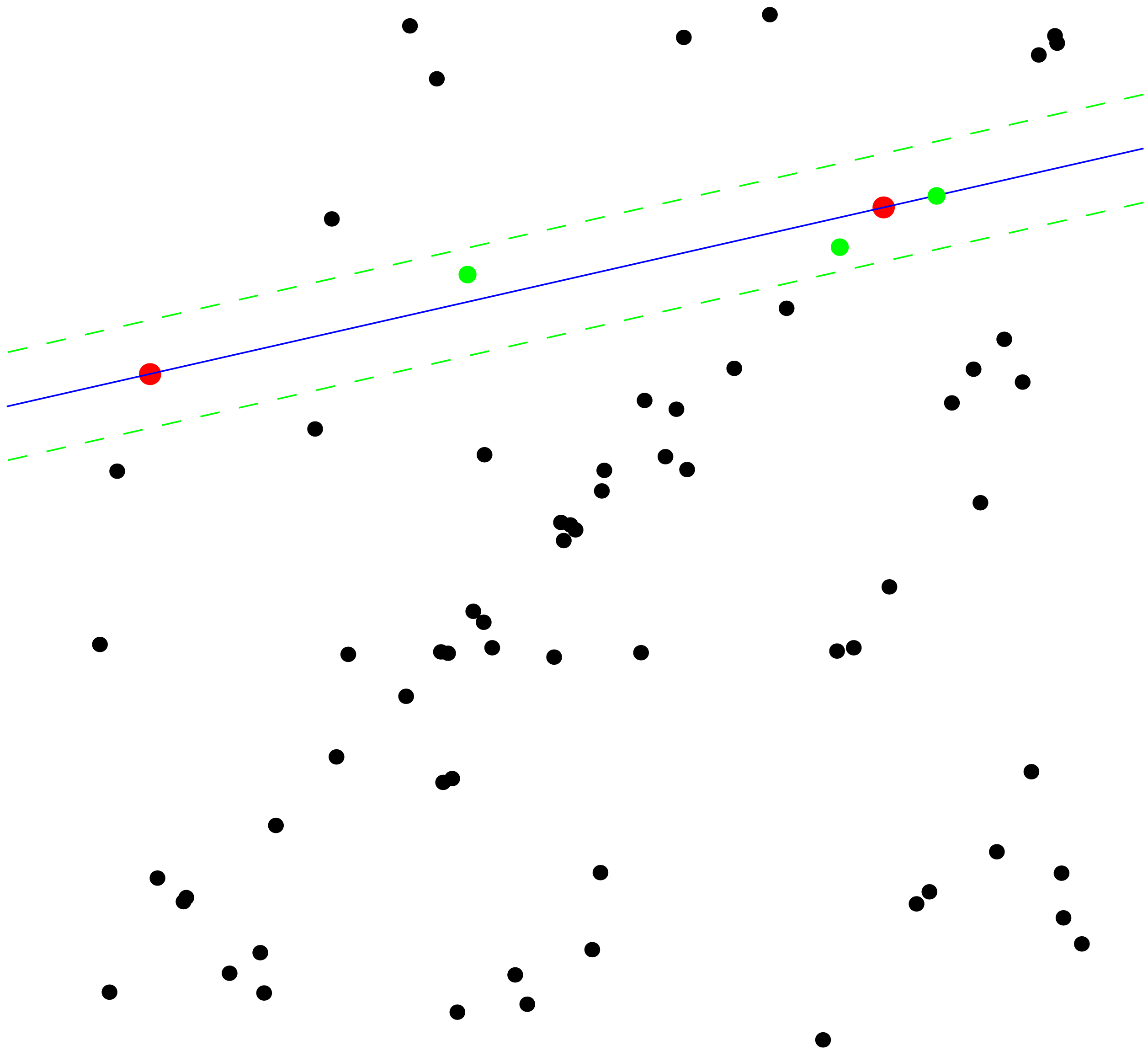
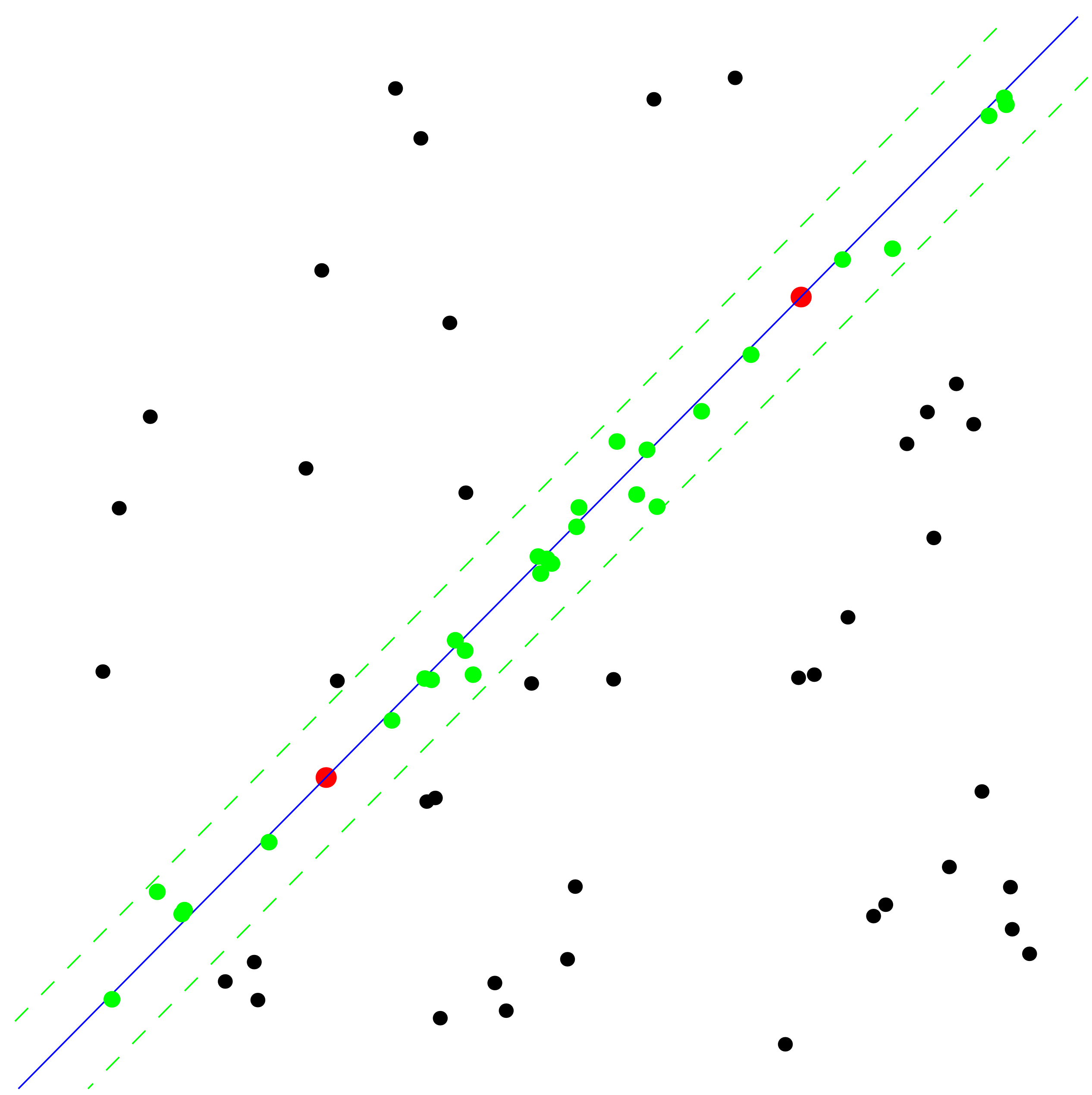


RANSAC Time Complexity

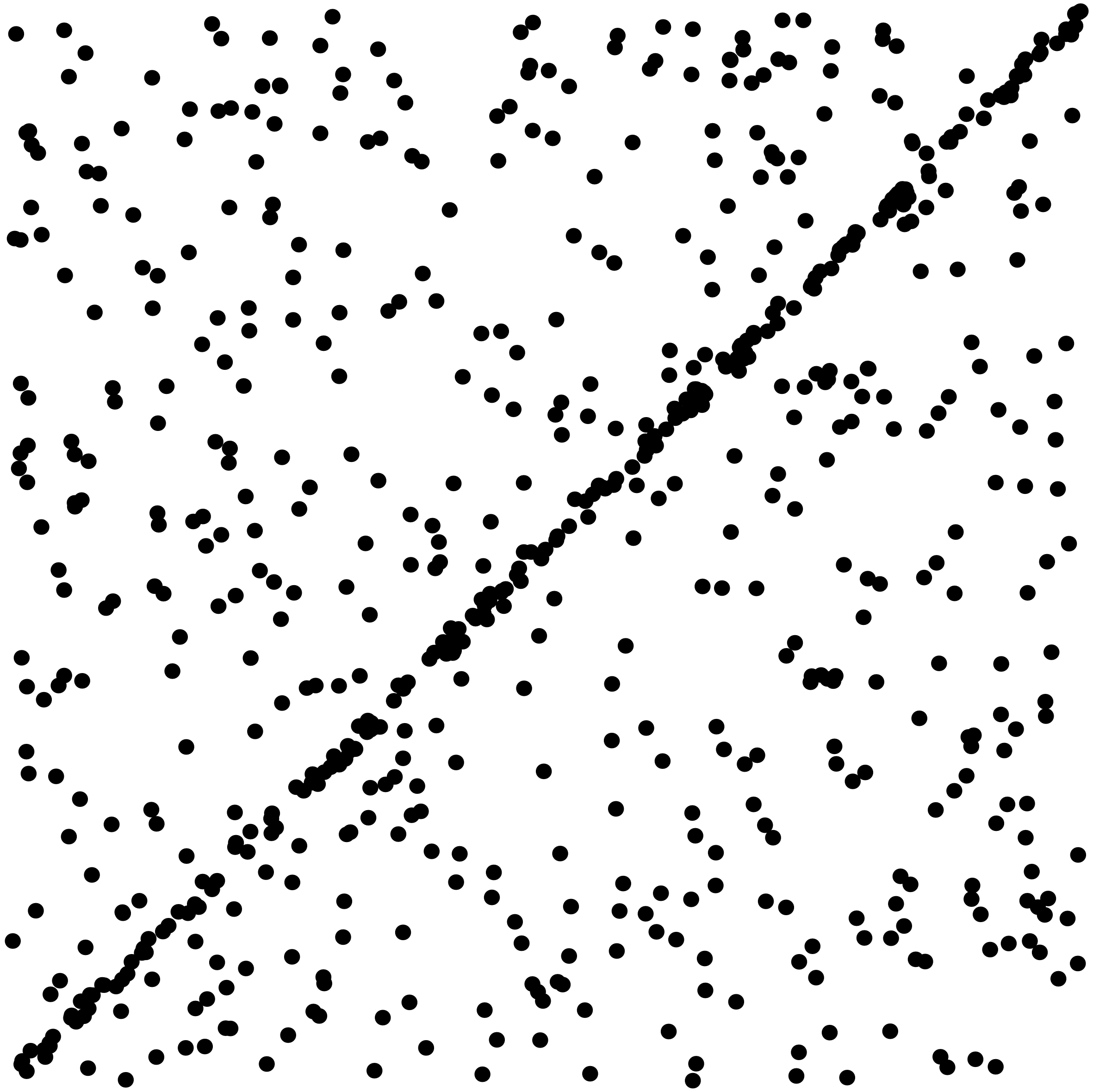


Uncontaminated sample

RANSAC time: $J = k(t_M + N)$

Depends on:

The Number of Data Points



- For each hypothesised model, all the data points are verified
- The more data points the longer RANSAC takes

Randomized RANSAC Algorithm

$k := 0$

Repeat until P{better solution exists} $< \eta$

$k := k + 1$

I. Hypothesis

(1) select randomly set $S_k \subset U$, $|S_k| = m$

(2) compute parameters $p_k = f(S_k)$

II. Preverification

(3) perform test based on $d \ll N$ data points

(4) continue verification only if the test is passed

III. Verification

(5) compute cost $C_k = \sum_{x \in U} \rho(p_k, x)$

(6) if $C^* < C_k$ then $C^* := C_k$, $p^* := p_k$

end

GOOD SAMPLE

$P_I = \varepsilon^m$ probability that all of m sampled data points are inliers, i.e. the probability of taking a good sample

α probability that the good sample passes the preverification test

$\bar{t}_\alpha$ the average number of data points tested in false negative test

$\bar{k} = 1/(\varepsilon^m \alpha)$ the average number of samples before first good sample

CONTAMINATED SAMPLE

$P_O = 1 - \varepsilon^m$ probability that at least one sampled data point is outlier
$P_O \gg P_I$

β the probability that contaminated sample passes the test
$\beta \ll \alpha$

$\bar{t}_\beta$ the average number of data points tested in correct negative test
$\bar{t}_\beta \ll N$

average R-RANSAC time:

$$J = \frac{1}{\varepsilon^m \alpha}\Big(t_M + P_I\ (\alpha N + (1-\alpha)\,\bar{t}_\alpha) + P_O\ (\beta N + (1-\beta)\,\bar{t}_\beta)\Big)$$

Standard RANSAC:

$$\alpha = 1, \qquad \beta = 1 \quad \Longrightarrow \quad J = \frac{1}{\varepsilon^m}(t_M + N)$$

Choosing the Preverification Test : The $T_{d,d}$ Test

$\mathbf{T}_{d,d}$ definition: test passed if all d randomly selected data points from $U \setminus S$ are consistent with the model parameters p

The time spent on the R-RANSAC with $T_{d,d}$

$$J(T_{d,d}) = \frac{1}{\varepsilon^m \, \varepsilon^d} \left(t_M + \varepsilon^m \left(\varepsilon^d N + \frac{1-\varepsilon^d}{1-\varepsilon} \right) + (1-\varepsilon^m) \left(\delta^d N + \frac{1-\delta^d}{1-\delta} \right) \right)$$

where δ is the probability that data point is consistent with a "random" model

$$\alpha = \varepsilon^d$$
$$\beta = \delta^d$$

Optimal d^* satisfying $\frac{\partial J(T_{d,d})}{\partial d} = 0$

$$d^* = \ln \left(\frac{\ln \varepsilon (t_M + 1)}{N \, (\ln \delta - \ln \varepsilon)} \right) / \ln \delta$$

Randomized RANSAC is faster than the standard one, if $J(T_{0,0}) > J(T_{1,1})$

$$N > (t_M + 1) \frac{1-\varepsilon}{\varepsilon - \delta}$$

- 1500 correspondences, 900 outliers, 600 inliers

d	samples	models	tests	inliers	time
0	1866	4569	6821218	600	25.0
1	4717	11536	16311	600	6.0
2	11849	28962	33841	600	15.1

Epipolar Geometry Estimation: Short Baseline Experiment

- 676 correspondences, approx. 60% of inliers
- tentative corr. by Harris operator and cross-correlation
- Leuven castle dataset

d	samples	models	tests	inliers	time
0	480	1146	766875	343	2.6
1	960	2301	83953	342	1.4

Epipolar Geometry Estimation: Wide Baseline Experiment

- 413 correspondences, less than 40% of inliers
- tentative corr. by WBS algorithm [Matas, Chum, Urban, Pajdla '01]
- BOOKSHELF dataset

d	samples	models	tests	inliers	time
0	3094	7582	3078184	161	12.9
1	6366	15583	178217	164	8.7

- Benefits of RANSAC randomization of hypothesis verification studied.
- A statistical preverification test was proposed.
- The increased performance experimentally verified.

Remaining problems

- The parameter d is fixed at 1, not optimal
- Estimating ε, δ and consequently d during the sampling process

Instead of the number of steps k we can use the average number of samples selected before sampling uncontaminated sample

$$\bar{k} = \frac{1}{\varepsilon^m \alpha}$$

since they differ only by a constant multiplication
α is the probability that the good S (i.e. $\forall x \in S$ are inliers) passes the test

$$\begin{aligned} \eta &= (1 - \varepsilon^m \, \alpha)^k \; < \; e^{-\varepsilon^m \alpha \, k} \\ -\log \eta \cdot \bar{k} = \frac{-\ln \eta}{\varepsilon^m \, \alpha} &> \quad k \end{aligned}$$

where $-\ln \eta$ is predefined constant.