SPECTRAL CHARACTERIZATION OF TWO FAMILIES OF FIBER OPTIC TRANSMISSION CODES

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Abstract: Because of the advantages it offers, such as high bandwidth, electromagnetic interference immunity, light weight, etc, the optical fiber is replacing the classic coaxial or twisted pair links. Two families of fiber optic transmission codes are presented and investigated in terms of their spectral properties. The D.C. component carried by these codes affects the power budget of the signal and results in baseline wander that determines an increase of the bit error rate. However, they have the advantage of simple coding and decoding procedures and circuits, which are easy to implement.

Key words: *Fiber optic codes, power spectral density, coding factor, autocorrelation function.*

I. INTRODUCTION

There is a variety of codes that were developed especially for optical fiber digital transmission channels. Because of the advantages it offers, such as high bandwidth, electromagnetic interference immunity, light weight, etc. the optical fiber is replacing the classic coaxial or twisted pair links [11]. The most important feature of such a code is that of meeting transparency requirements for various input binary sequences. The transparency is the property of code that ensures the transmission of any binary string and is also known as BSI (*bit-sequence independence*).

Another important feature is the simplicity of the coder and decoder circuits. Since these circuits operate at high frequencies, chip power consumption and crosstalk increase tremendously with the frequency. Also, the bipolar transistor speed margin decreases with frequency and wide-band technologies such as cascode connections or double emitter follower are needed.

The mBnB codes [6], though balanced and some designed to have good synchronization properties, require complex electronics for encoding and decoding

Other codes considered so far for fiber optic transmission, such as mB1C [4], DmB1M [3] and mB1I [2] are unbalanced. As a consequence, their power spectral densities (p.s.d.) are not zero at D.C.

The D.C. component carried by these codes affects the power budget of the signal [5] and results in baseline wander that determines an increase of the bit error rate. However, they have the advantage of simple coding and decoding procedures and circuits, which are easy to implement.

The complexity of implementation increases non-linearly with *m*. This is quite a problem as, in order to increase the code efficiency, *m* should be large.

II. DmB1M CODE FAMILY

These codes were introduced by Yamada e.a. [1] in 1983. Like *mB1C* codes [2, 3] they are one bit insertion codes intended for high-speed optical transmission.

A block of *m* bits is converted into a group of *m+1* bits, by adding a supplementary bit, which is a mark *M*. An

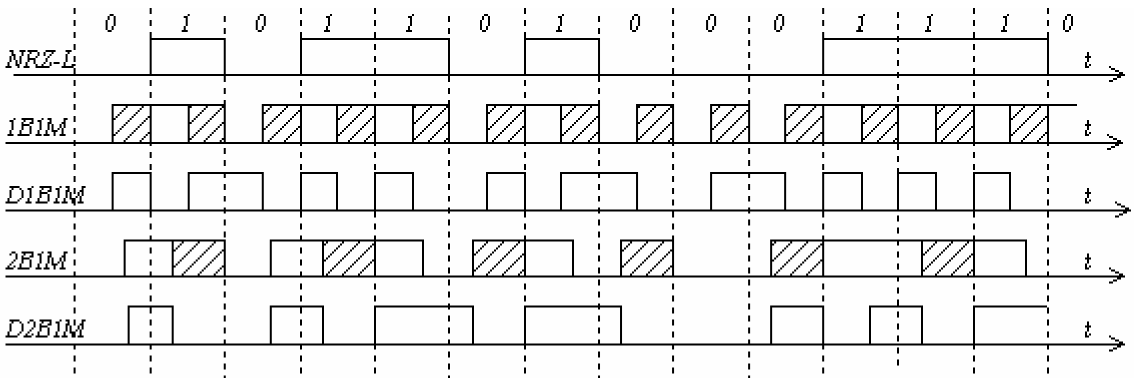

Figure 1 Coding example for DmB1M codes (m=1 and m=2)

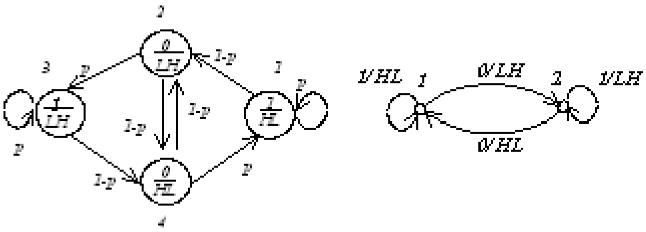

Figure 2 FSTD of D1B1M code (Moore and Mealy)

mB1M code was obtained in this way.

This is differentially encoded in order to have the marks coded as transitions and ensure that there is at least a transition during *m+1* bits. An example illustrating the coding procedure for D1B1M and D2B1M is given in Fig. 1. The shaded pulses represent inserted marks.

The finite state transition diagram (FSTD) of the *D1B1M* code is represented in Fig 2.

The p.s.d. of the *DmB1M* codes for the equiprobable case is identical with that of the corresponding *mB1C* code. If the probability of a mark varies, then the p.s.d. will differ.

Moreover, the *mB1C* codes [3] present a discrete harmonic spectrum with the fundamental frequency $f_{bit}/(m+1)$, in opposition with *DmB1C* codes, where such spectrum is not present. The coding factors of several members of *DmB1M* family were obtained, starting from a Mealy-type FSTD, such as the one represented in Fig.2 for D1B1M using the approach in [8], [10] and [12]and are shown below.

They are represented in Fig.3 for the equiprobable case $p = 0.5$. They present a strong D.C. component, which is a result of the unbalanced character of these codes.

$$C_1(f,p)=\frac{8p(1-p)\sin^2 x/2}{1-2p+2p^2+(1-2p)\cos 2x}$$

$$C_2(f,p)=\frac{2p(1-p)(3-6p+6p^2+4p^2\cos 4x/3)-4p(1-3p+2p^2)\cos 8x/3}{1-4p+12p^2-16p^3+8p^4+(1-2p)^2\cos 4x}$$

where $x=\pi\cdot f_n$

$$C_3(f,p)=\frac{4p(1-p)(a+b\cos 3x/2+c\cos 3x+d\cos 9x/2}{3(1-6p+30p^2-80p^3+120p^4-96p^5+32p^6+(1-2p)^3\cos 6x)}$$

$a=6-24p+56p^2-64p^3-32p^4$ $\quad b=3-12p+8p^2+8p^3-16p^4$

$c=2p(1-p)(1-2p+4p^2)$ $\quad d=-12p(1-p)(1-2p)^2$

$$C_4(f,p)=\frac{2p(1-p)(a+b\cos 8x/5+c\cos 16x/5+d\cos 24x/5+e\cos 32x/5}{1-8p+56p^2-224p^3+560p^4-896p^5+896p^6-512p^7+128p^8+(1-2p)^4\cos 8x)}$$

$a=5-30p+110p^2-240p^3+320p^4-240p^5+80p^6$ $\quad c=-2p^2(1-2p)(5-10p+8p^2)$

$b=4(1-6p+15p^2-20p^3+8p^4+8p^5-8p^6$ $\quad d=2(1-2p)^2(1-2p+3p^2)$

$e=4(1-2p)^3p(1-p)$

$$C_5(f,p)=\frac{4p(1-p)(a+b\cos 5x/3+c\cos 10x/3+d\cos 5x+e\cos 20x/3+f\cos 25x/3}{5(1-2p+2p^2)(g+(1-2p)^5\cos 10x)}$$

$a=15-120p+600p^2-1920p^3+4128p^4-5952p^5+5568p^6-3072p^7+768p^8$

$b=15-120p+480p^2-1200p^3+1824p^4-1536p^5+384p^6+384p^7-256p^8$

$c=4(1-2p)(1-6p+8p^2+8p^3-48p^4+64p^5-32p^6$

$d=-3(1-2p)^2(1-4p+16p^2-24p^3+16p^4)$

$e=-2(1-2p)^3(3-6p+8p^2)$

$f=-20p(1-p)(1-2p)^4$

$g=1-8p+72p^2-320p^3+896p^4-1600p^5+1728p^6-1024p^7+256p^8$

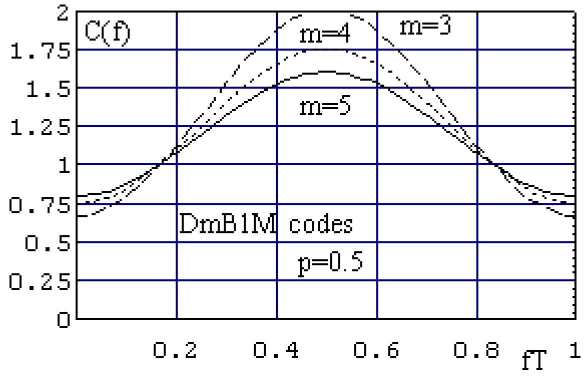

Figure 3 Coding factor of DmB1M codes

III. mB1I CODE FAMILY

These codes were introduced by Stojanović and Smiljanić [4] in 1997. They show better D.C. - and low-frequency suppression, as compared with *mB1C* and *DmB1M* codes.

Table I Coding Rules of mB1I Code

Disparity		Transmitted word
d_o	d_e	
+	-	$a_1 a_2 a_3 a_4 \cdots a_{m-1} a_m 0$
+	+	$a_1 \bar{a}_2 a_3 \bar{a}_4 \cdots a_{m-1} \bar{a}_m 1$
-	+	$a_1 a_2 a_3 a_4 \cdots a_{m-1} a_m 0$
-	+	$a_1 \bar{a}_2 a_3 \bar{a}_4 \cdots a_{m-1} \bar{a}_m 1$

The implementation of coding and decoding circuits is a little more complicated.

The better D.C. attenuation is accompanied by a greater average number of transitions per code-word [9] and consequently, better synchronization capabilities. The length of consecutive identical symbols is limited to $m+1$.

Also, a better balance of marks and spaces in the codeword is achieved, as compared with *mB1C* and *DmB1M* codes.

In *mB1I* coding, a group of *m* bits is converted into a codeword of length *m+1*, by adding an extra bit and inverting or not the even bits. A group of *m* bits at the time moment *j* can be written as

$$w_j = (a_1 a_2 a_3 \cdots a_m)_j \tag{2}$$

where $m = 4k+2$, $k = 1,2,\cdots$.

The disparities of the odd and even bits [4] are given by

$$d_o = \sum_{k=1}^{m/2} a_{2k-1}$$
$$d_e = \sum_{k=1}^{m/2} a_{2k} \tag{3}$$

If $d_o \cdot d_e < 0$, the transmitted word will be unchanged, except for the addition of an *m+1* bit equal to 0.

If $d_o \cdot d_e > 0$, i.e. the odd- and even order disparities are of the same sign, an inversion of the even bits is performed and the extra added bit will be a *'1'*. The coding rules are illustrated in Table I for a generic code word $[a_1 a_2 \cdots a_m]$.

In Table II an example illustrating the coding rules for *2B1C*, *D2B1M* and *2B1I* codes is given.

The decoding procedure is quite simple. After detecting and extracting the $(m+1)$ bit, the coded word is obtained directly if the *m+1* bit is zero or, by inverting the even bits only, if the *m+1* bit is a '*1*'.

The *mB1I* codes show also error-monitoring features by checking the disparities between the even and odd bits. The coding factor of *2B1I* code was obtained as:

$$C(f,p) = 8p(1-p)(1-p+p^2-p\cdot\cos 4x/3)$$
$$-4p(1-3p+2p^2)\cos 8x/3 \tag{4}$$

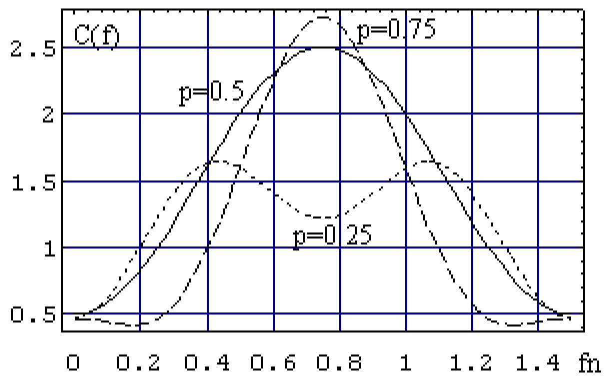

Figure 4 Coding factor of 2B1I code

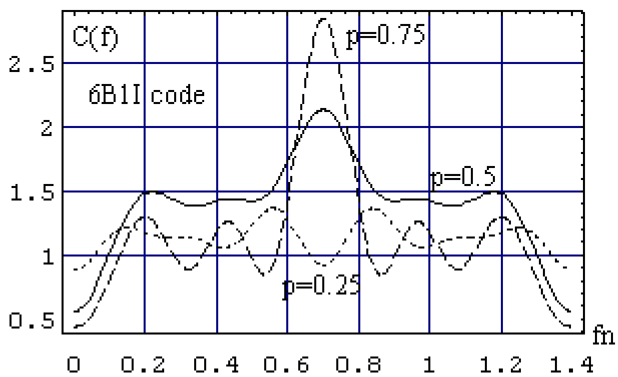

Figure 5 Coding factor of 6B1I code

where $x = \pi fT$ using the approach in [8], [10] and [12]. It is represented in Fig.4 for three values of the probability *p*.

Figure 4 illustrates the coding factor of *6B1I* code for three values of the probability *p*. A 3D view of the coding factor of *6B1I* code is represented in figure 5.

Table II A Comparison of Several Codes

Input data	2B1C	D2B1M	2B1I
00	$--+$	$--+$ or $++-$	$-++$
01	$-+-$	$-+-$ or $+-+$	$-+-$
10	$+-+$	$++-$ or $--+$	$+--$
11	$++-$	$+-+$ or $-+-$	$+-+$

A comparison of coding factors of D5B1M (5B1C) and 6B1I is illustrated in figure 6 for the equiprobable case ($p = 0.5$). Here the frequency in the coding factor was not normalised ($f_n = f\,T$ in order to scale both representations to cover the interval $(0,T)$, *T* being the duration of a bit in the input data.

As the m*B1I* codes achieve a larger number of transitions per codeword and a good balance of marks and spaces, as compared with *DmB1M* and *mB1C* codes, they are more random like. As a consequence, the p.s.d. is more flat in the bandwidth $0 - f_n$ (see figure 6).

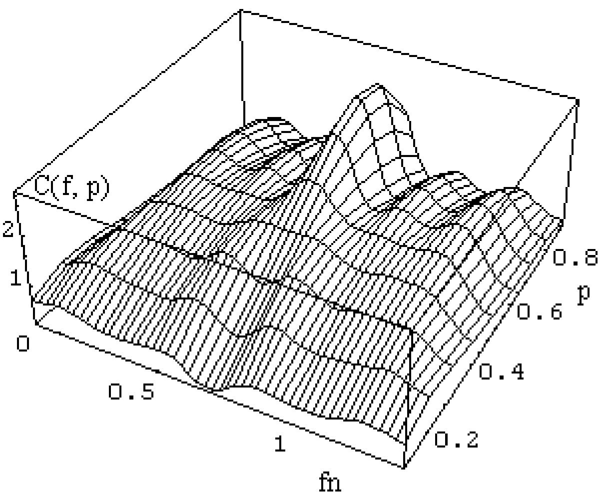

Figure 6 3D view of coding factor of 6B1I code

This random-like feature manifested in the flatness of the coding factor is kept for other values of the probability p as well ($p \neq 0.5$), as evidenced in figure 5.

On the other hand, the higher timing content (a larger number of transitions per codeword) determines a peak at the frequency $0.5 f_n$ or $1/2T$ in the non-normalised case.

In the limit, repetitive patterns, such as $+-/+-/\cdots$ would determine a discrete component or line on the repetition frequency $1/2T$.

IV. CONCLUSIONS

We presented and investigated the spectral properties of two families of fiber optic transmission codes. Due to their unbalanced character they show a strong D.C. component.

Their coding factor and power spectral densities were derived as a function both of the normalized frequency f_n and the probability of a mark p at the coder input.

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