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MODELLING AND SIMULATION OF PHOTOVOLTAIC CELLS

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Abstract: A photovoltaic cell converts the solar energy into the electrical energy by the photovoltaic effect. Solar cells are widely used in terrestrial and space applications. The photovoltaic cells must be operated at their maximum power point. The maximum power point varies with illumination, temperature, radiation dose and other ageing effects. In this paper, we present four models for a photovoltaic cell. Each method was evaluated and their strengths/weaknesses were identified. Two empirical models was modeled and simulated in Mathcad. The empirical models developed are validating through the comparison of the obtained characteristics with the ones given by the manufacturers of the PV panels.

Key words: *photovoltaic (PV) cell, maximum power point (MPP), I-V curves, model.*

I. INTRODUCTION

In order to create low power remote and independent electronic devices it is necessary to collect and convert energy directly from the environment. This is very important in order to maintain a continuous operation. A good solution is the use of a photovoltaic device [1]

A photovoltaic (PV) cell converts the solar energy into the electrical energy by the photovoltaic effect. The heat does not participate constructively in this process. Heat actually limits the performance of these fine layers, and the presence of excess heat is a sign of deterioration in a PV cell. Most solar cells are built from silicon, and the presence of impurities influences their performance. Solar cell efficiencies vary from 6% for amorphous silicon-based solar cells to 42.8% with multiple-junction research lab cells. Solar cell energy conversion efficiencies for commercially available multicrystalline Si solar cells are around 14-19%.

The major advantages of using PV cells are: short lead time for designing and installing a new system, output power matching with peak load demands, static structure, no moving parts, longer life, no noise, high power capability per unit of weight, inexhaustible and pollution free, highly mobile and portable because of its light weight [2].

Solar arrays are used in many terrestrial and space applications. For best utilisation, the photovoltaic cells must be operated at their maximum power point (MPP). However, the MPP varies with illumination, temperature, radiation dose and other ageing effects.

The block diagram of typical used for battery charger is presented in figure 1.

The weather and load changes cause the operation of a PV system to vary almost all the times. A dynamic tracking method is necessary to ensure maximum power is extracted from the PV cells.

Due to the mismatch between load line and operating characteristic of the solar cells, the power available from the solar cells is not always fully extracted. This can be demonstrated by figure 2. Maximum power point tracking (MPPT) is a control technique to adjust the terminal voltage of PV panels so that maximum power can be extracted.

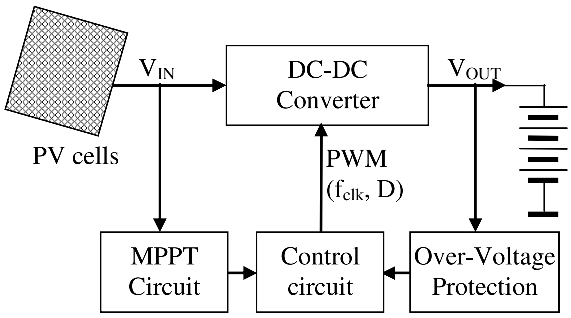

Figure 1. Schematic diagram of a battery charger with MPPT.

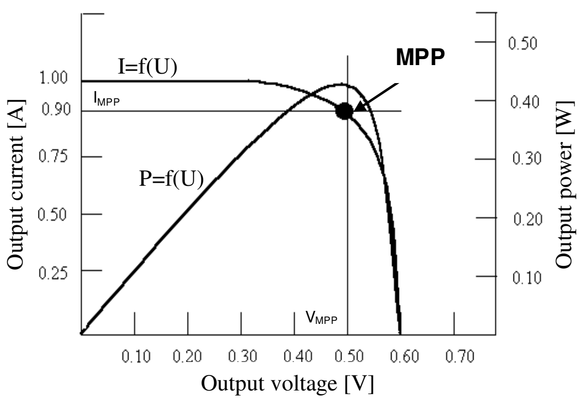

Figure 2. Typical I-V and P-V characteristics of photovoltaic cell.

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The open circuit voltage of the PV module (V_{oc}) is the point of intersection of the curve with the horizontal axis and it varies little with solar radiation changes. A rise in temperature produces a decrease in voltage. Short circuit current (I_{sc}), the point of intersection of the curve with the vertical axis, is directly proportional to solar radiation and is relatively steady with temperature variations. Actually, the photovoltaic module acts like a constant current source for most parts of its I-V curve [3].

For a given solar radiation and operating temperature, the output power depends on the value of the load. As the load increases, the operating point moves along the curve towards the right. Only one load value produces a PV maximum power. The maximum power points line has a relatively constant output voltage at varying solar radiation conditions.

II. MODELING OF THE PV CELLS

Modeling is basic tool of the real system simulation. For modeling, it is necessary to analyze the influence of different factors on the photovoltaic cells and to take in consideration the characteristics given by the producers. The mathematical models for photovoltaic cells are based on the theoretical equations that describe the operation of the photovoltaic cells and can be developed using the equivalent circuit of the photovoltaic cells. The empirical models rely on different values extracted from the I-V characteristic of the photovoltaic cells and they approximate the characteristic equation of the solar panels using an analytical function.

For the photovoltaic systems modeling, we analyze the influence of different factors on the solar panels and to consider the characteristics given by the producers. The mathematical models for PV arrays are based on the theoretical equations that describe the functioning of the PV cells and can be developed using the equivalent circuit of the PV cells. The empirical models rely on different values extracted from the I-V curve of the PV arrays and they approximate the characteristic equation of the solar panels using an analytical function.

There are many models for a photovoltaic cell. This paragraph presents an overview of the different equivalent circuits of the solar cell.

a) The One-Diode-Model for PV cells

The One-Diode -Model is the most simple and the most used model for PV cells (figure 3).

The simplified equivalent circuit of a solar cell consists of a diode and a current source which are connected in parallel. The current source generates the photo current I_{Ph}, which is directly proportional to the solar irradiance F_s [W/m^2], ambient temperature T_a [ºC], and two output parameters: current I_s [A] and voltage V_s [V]. The p-n transition area of the solar cell is equivalent to a diode.

The characteristic equation of the one diode model could be derived from Kirchhoff's current law:

$$I_s = I_{ph} - I_d - I_{sh} \tag{1}$$

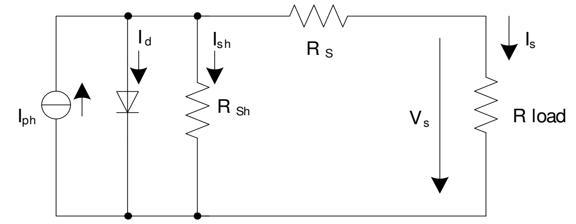

Figure 3. The equivalent circuit for the one diode model.

With:
I_{ph} – photo current;
I_d – diode current;
I_{sh} – diode reverse saturation current;
I_S – load current.
The photocurrent I_{ph} is given by the equation:

$$I_{ph} = P_1 \cdot F_s \cdot \left[1 + P_2 \cdot (F_s - F_0) + P_3 \cdot (T_j - T_0)\right] \tag{2}$$

Where F_0 = 1000W/m^2, T_0 = 298.15K, P_1 [Am^2/W], P_2 [m^2/W] and P_3 [1/K] are constants, usually given by the producers, and T_j is the junction temperature.

The currents I_d can be calculated using:

$$\begin{cases} I_d = I_{sat}\left[exp\left(\dfrac{e_0}{a_f \cdot N_s \cdot k} \cdot \dfrac{V_s + R_s \cdot I_s}{T_j}\right) - 1\right] \\ I_{sat} = P_4 \cdot T_j^3 \cdot exp\left(-E_g / k \cdot T_j\right) \end{cases} \tag{3}$$

where: $e_0 = 1{,}60217733 \cdot 10^{-19}$ [C] is the charge of the electron; $a_f = 1 \div 5$ is the diode "ideally factor", usualy is 1; N_s represents the number of cells in series; $k = 1{,}380658 \cdot 10^{-23}$ [J/K] is Boltzmann's constant; R_s [Ω] models the series resistance, E_g [eV] is the band gap, and P_4 [A/K^3] is a correction factor. The parameters (P_4 [A/K^3], R_s [Ω], R_{sh} [Ω]) can be obtained from the solar panel's datasheet.

The diode reverse saturation current I_{sh} is given by:

$$I_{sh} = \frac{V_s + R_s \cdot I_s}{R_{sh}} \tag{4}$$

b) The Two-Diode-Model for PV cells

An even more exact modeling could be achieved by the Two-Diode-Model (figure 4). Here two different diodes with different diode ideally factors a_f are connected in parallel. This model has the modelling advantage of a better accuracy but it has the disadvantage of relying more parameters to implement.

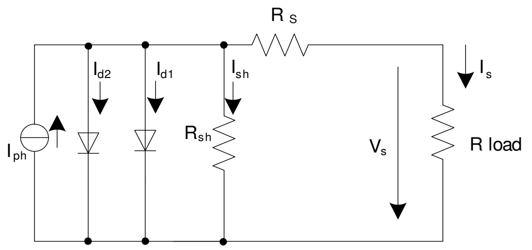

Figure 4. The equivalent circuit for the two diode model.

The equations that describe the equivalent circuit can also be derived from the node-law of Kirchhoff:

$$\begin{cases} I_s = I_{ph} - I_{d1} - I_{d2} - I_{sh} \\ I_{ph} = \left(P_1 + P_2 \cdot T_j\right) \cdot F_s \\ I_{sat1,2} = P_{01,2} \cdot T_j^3 \cdot exp\left(-E_g / k \cdot T_j\right) \end{cases} \tag{5}$$

where $I_{sat1,2}$ is the saturation current.

The set of parameters P_1 [Am^2/W], P_2 [Am^2/WK], P_{01} [A/K^3], and P_{02} [$A/K^{5/2}$] can be obtained in this case also from the solar panel's datasheet.

c) The first empirical model for PV cells

The One-Diode-Model and the Two-Diode-Model use a large number of parameters. It is difficult to determine many of these parameters. So, the experimental models was developed.

The first empirical model proposed relies on a small number of parameters: V_{oc}, I_{sc} and maximum power P_{MPP} of the solar panel. With another set of three parameters, $\left(\partial V_{oc}/\partial T_j,\ \partial V_{oc}/\partial E_s,\ \partial I_{sc}/\partial T_j\right)$, the effects of the junction temperature and light intensity can be taken into consideration.

The equation that describes the I-V characteristic is given by:

$$I_s = I_{ph} - I_d \tag{6}$$

Using the approximation $I_{ph} \approx I_{sc}$ and substituting the constant $e_0/a_f \cdot k \cdot T_j$ with A, the expression of the I-V curve becomes:

$$I_s = I_{sc} \cdot \left[1 - \left(\frac{I_{sat}}{I_{sc}}\right) exp\left[A\left(V_s + I_s \cdot R_s\right)\right]\right] \tag{7}$$

If I_s=0 then V_s =V_{oc} and the factor A can be calculated by:

$$V_s\Big|_{I_s=0} = V_{oc} = \frac{1}{A} ln\left(\frac{I_{sc}}{I_{sat}}\right) \Rightarrow A = \frac{1}{V_{oc}} ln\left(\frac{I_{sc}}{I_{sat}}\right) \tag{8}$$

In standard test conditions (T_0=25°C and F_0=1000W/m^2), the ratio I_{sc}/I_{sat} can be approximated with 10^9. Replacing A and making the notation $B = ln\left(I_{sc}/I_{sat}\right)$ the value of, the expression of V_s becomes:

$$V_s = V_{oc} \cdot \left[1 + \frac{1}{B} \cdot ln\frac{I_{sc} - I_s}{I_{sc}}\right] - R_s \cdot I_s \tag{9}$$

At maximum power point (MPP) the value of the current I_S is I_{MPP} and $V_s = P_{MPP}/I_{MPP}$. The values of R_s and I_{MPP} can be obtained by resolving the system:

$$\begin{cases} \frac{P_{MPP}}{I_{MPP}} = V_{oc} \cdot \left[1 + \frac{1}{B} \cdot ln\frac{I_{sc} - I_{MPP}}{I_{sc}}\right] - R_s \cdot I_{MPP} \\ \frac{P_{MPP}}{I_{MPP}^2} = \frac{V_{oc}}{B} \cdot \left(\frac{1}{I_{sc} - I_{MPP}}\right) + R_s \end{cases} \tag{10}$$

d) The second empirical model for PV cells

The second empirical model proposed begins with the equation:

$$i(v) = I_{sc}\left(1 - \left(v/V_{oc}\right)^k\right) \tag{11}$$

where:

$$k = ln\left(1 - \frac{I_1}{I_{sc}}\right) \Big/ ln\left(\frac{V_1}{V_{oc}}\right) \tag{12}$$

and the values (I_1, V_1) are measured close to the region of MPP.

The empirical value of parameter k, was obtained by optimizing the model in such a way that the experimental and theoretical I-V curves would coincide at the MPP. A useful outcome of this method is the ability to predict the MPP using the equations:

$$\begin{cases} V_{MPP} = V_{oc} \cdot \frac{1}{\left(1 + k\right)^{1/k}} \\ I_{MPP} = I_{sc} \cdot \frac{k}{1 + k} \end{cases} \tag{13}$$

As the light intensity increases, the current increases linear, and the output voltage increases logarithmic. A rise of 25ºC in temperature produces a rise of 1% in current, and a decrease of 2mV/°C per cell in voltage. These relations can be modeled by relations:

$$\begin{cases} V(F,T) = V_0 + \dfrac{nkT}{q} ln\left(\dfrac{F}{F_0}\right) - n\alpha(T - T_0) \\ I(F,T) = I_0 \cdot \left(\dfrac{F}{F_0} - \beta(T - T_0)\right) \end{cases} \quad (14)$$

where: T_0=25°C is the standard temperature of the PV cells; F_0=1000W/m^2 is the standard light intensity; T [°C] is the operating temperature; F is the operating light intesity; n is the number of solar cells connected in series; V_0 is the voltage of the cells at T_0 and F_0; I_0 is the current delivered by the cells at T_0 and F_0; β is the temperature coefficient of the current, usually considered 0.0004; α is the temperature coefficient of the voltage, usually considered 0.002.

III. SIMULATION OF THE PV CELLS

Both empirical models proposed were implemented in order to simulate them. For the PV cells simulating, one chose the solar panel ASE 30-DG-UT (32W) manufactured by ASE Americas, Inc. This panel has the following specifications:

- Maximum Output power: P_{MPP} = 32W;
- Short circuit current: I_{sc} = 0.6A;
- Open circuit voltage: V_{oc}=95V;
- Voltage at maximum power: V_{MPP}=68V;
- Current at maximum power: I_{MPP}=0.47A.

The I-V characteristic given by the producer for this solar panel is shown in figure 5.

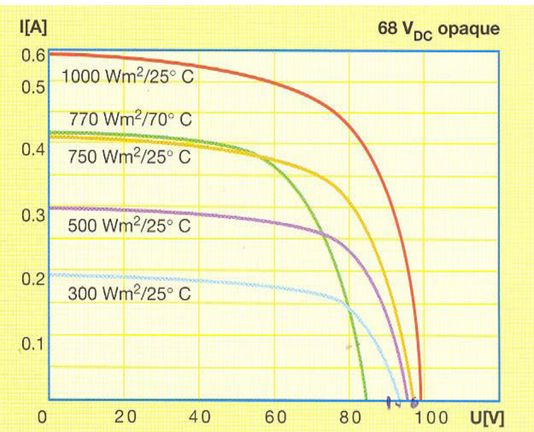

Figure 5. The I-V curves for ASE 30-DG-UT (32W).

For *the first empirical model*, knowing the values of V_{oc}, I_{MPP} and V_{MPP}, from equations (10) results:

$$\begin{cases} B = \left[\dfrac{I_{MPP}}{I_{sc} - I_{MPP}} + ln\left(\dfrac{I_{sc} - I_{MPP}}{I_{sc}}\right)\right] \Big/ \left(2\dfrac{V_{MPP}}{V_{oc}} - 1\right) \\ R_s = \dfrac{P_{MPP}}{I_{MPP}^2} - \dfrac{V_{oc}}{B}\left(\dfrac{1}{I_{sc} - I_{MPP}}\right) \end{cases} \quad (15)$$

The load voltage expression as dependent on load current I_s, load resistance R_s and parameter B will be:

$$V_s(I_s, B, R_s) = V_{oc}\left[1 + \frac{1}{B} ln\left(\frac{I_{sc} - I_s}{I_{sc}}\right)\right] - R_s \cdot I_s \quad (16)$$

For comparison, a set of 20 values (v_i, i_i) for current and voltage was extracted from I-V characteristics given by the producer, and then both, model and experimental I-V curves were represented in figure 6.

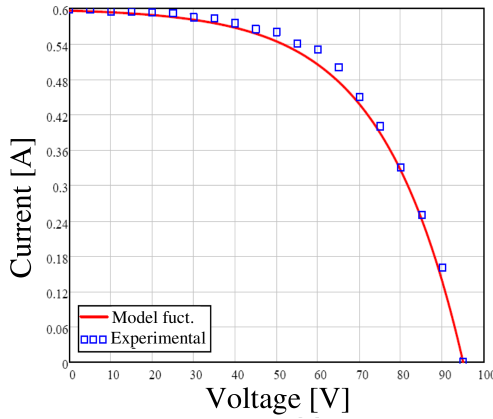

Figure 6. The I-V curve before optimization, for the first empirical model.

Using a classical method for minimizing errors, a function ε_{r1}, having variables B and R_s was defined:

$$\varepsilon_{r1}(B, R_s) = \sum_i \left|\frac{v_i - V_s(i_i, B, R_s)}{v_i + V_s(i_i, B, R_s)}\right| \quad (17)$$

The I-V characteristic using the optimum values determined for B and R_s is presented in figure 7.

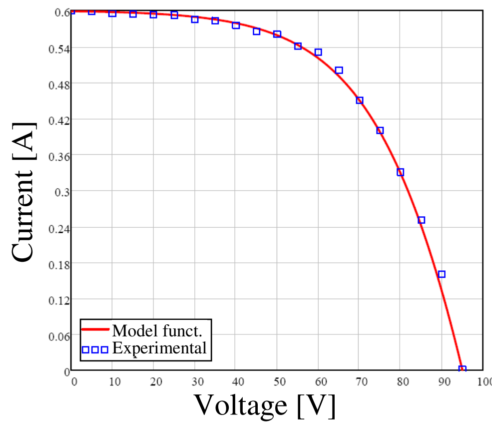

Figure 7. The I-V curve after optimization, for the first empirical model.

For *the second empirical model*, considering the values I_1= 0.3A and V_1 = 60V the empirical value of the parameter k = 1.508 will be obtained with (12). If the values (I_1, V_1) are closer to MPP, then the I-V curve is closer to the one given by the producer.

The comparative representation of the I-V curves obtained from the experimental values and the proposed function for model are presented in figure 8.

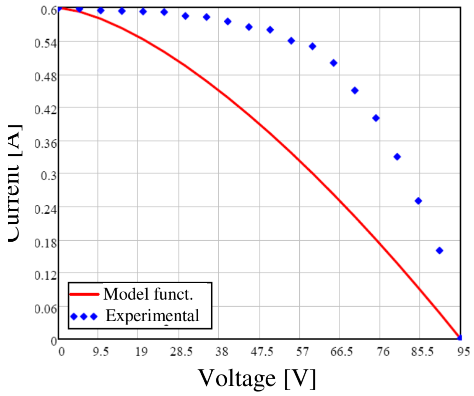

Figure 8. The I-V curve before optimization, for the second empirical model.

Defining an error function ε_{r2}:

$$\varepsilon_{r2}(k) = \sum_i \left| \frac{i_i - il(v_i, k)}{i_i + il(v_i, k)} \right| \quad (18)$$

and minimizing the errors, the optimum value of the parameter k is determined, $k_{ac} = 4.647$.

The I-V curve after the optimization is shown in figure 9.

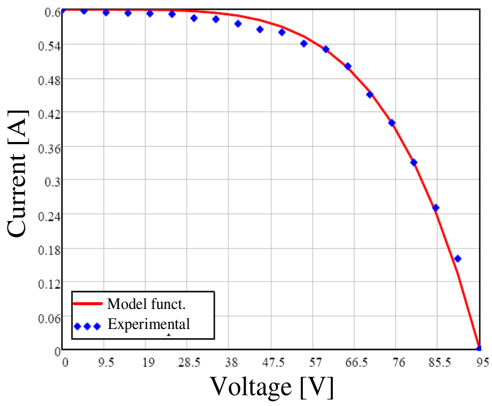

Figure 9. The I-V curve after optimization, for the second empirical model.

If the optimum value of the parameter k is k_{ac}, from the equation 11, the expression of the power delivered by the solar panel becomes:

$$p(v) = i(v) \cdot v = I_{sc}\left(1 - (v/V_{oc})^{k_{ac}}\right) \cdot v \quad (19)$$

Using the Mathcad function 'maximize' the value of the V_{MPP} was determined and a linear function i(v) was defined to establish the MPP:

$$i(v) = \frac{i(V_{MPP})}{V_{MPP}} \cdot v \quad (20)$$

Intersecting the function i(v) with the I-V curve obtained for the solar panel, the MPP can be indicated. The output power versus output voltage is presented in figure 10.

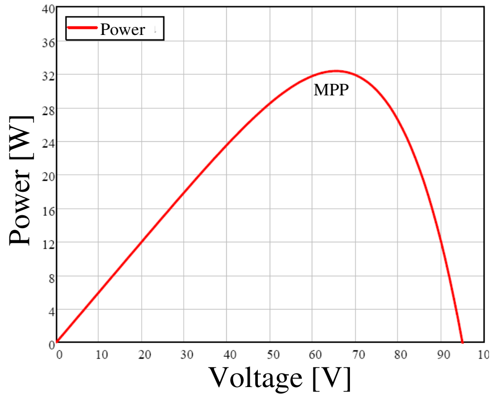

Figure 10. Output power versus output voltage.

For determining *the influence of the light intensity* on the PV cells, three light intensities were considered: F_0=1000W/m^2, F_1=300W/m^2 and F_2=700W/m^2. The corresponding voltage values for intensities F_1 şi F_2 are given by the producer (V_{F1}=98V; V_{F2}=92V). The expression for paramater k corresponding to the minimal error (I_1 and V_1 are substituted with I_{MPP} and V_{MPP}) becomes:

$$k = ln\left(1 - \frac{I_{MPP}}{I_{sc}}\right) \Big/ ln\left(\frac{V_{MPP}}{V_{oc}}\right) \quad (21)$$

The expression describing the I-V characteristics for different light intensities becomes:

$$i(v, F) = I_{sc0}(F) \cdot \left[1 - \left(\frac{v}{V_{oc0}(F)}\right)^k\right] \quad (22)$$

where: $V_{oc0}(F) = V_{oc} + k_1 \cdot ln\left(\frac{F}{F_0}\right)$, $I_{sc0}(F) = I_{sc} \cdot \frac{F}{F_0}$,

and $k_1 = \left(V_{F_1} - V_{F_2}\right) / ln(F_1/F_2)$.

The expression of the output power will be:

$$p(v, F) = i(v, F) \cdot v \quad (23)$$

The corresponding I-V curves and the output power variation are presented in figures 11 and 12.

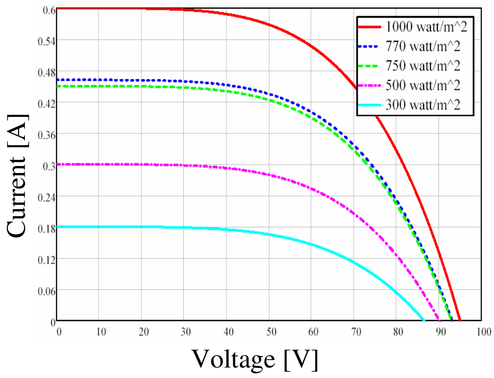

Figure 11. The light influence on the I-V curves.

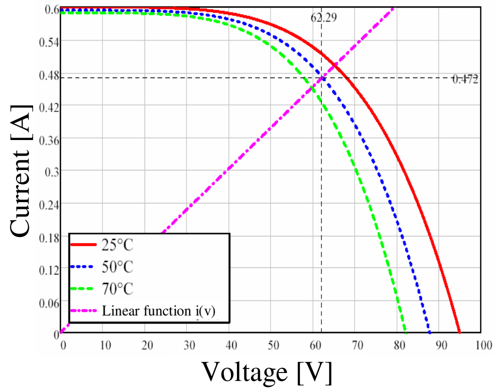

Figure 12. The temperature influence on the I-V curves.

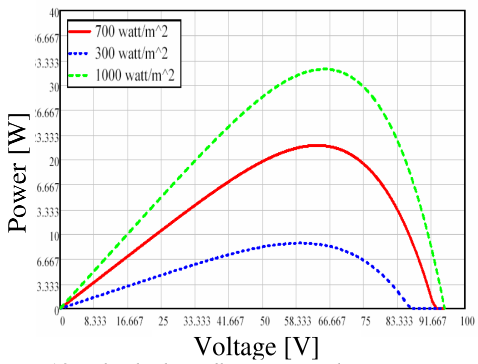

Figure 12. The light influence on the output power.

For determining the influence of the temperature, n PV cells are considered being connected in series. The expression describing the I-V characteristic for different operating temperatures is:

$$il(v,T) = I_{sc}(T)\cdot\left[1-\left(\frac{v}{V_{oc}(T)}\right)^k\right] \tag{24}$$

where: $\begin{cases} I_{sc}(T) := I_{sc}\cdot[1-\beta(T-T_0)] \\ V_{oc}(T) := V_{oc} - n\cdot\alpha(T-T_0) \end{cases}$.

The points of maximum power are found at the intersection between function i(v) and the I-V characteristics as in figure 13.

IV. CONCLUSIONS

In this paper, we present four photovoltaic cells models. Two empirical models proposed were implemented and simulate in Mathcad software packages. For the PV cells simulating, one chose the solar panel ASE 30-DG-UT (32W) manufactured by ASE Americas, Inc.

The I-V curves, before and after optimizations, are presented for both empirical models. After optimization, model and experimental I-V curves are very closed.

Also, it is presented the environmental parameters (light and temperature) influence on the I-V curves and on the output power.

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