Volume 51, Number 1, 2010

ACTA TECHNICA NAPOCENSIS
Electronics and Telecommunications

ASYMMETRIC DIFFUSION AND FUSION TECHNIQUES FOR IMAGE RESTORATION

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Abstract: We propose a novel method for restoring and enhancing images. The method is based on the Partial Differential Equations (PDE) framework and image fusion techniques. The degraded image is processed independently with two PDE based diffusion filters with different theoretical properties, an asymmetric forward diffusion equation and an evolved shock filter, and then fusion is performed at an intermediate results level. The procedure is iterated for a predefined number of steps and the restored or enhanced image is obtained as the output of the last fusion step. Through application samples we show that the method can be successfully applied for restoring images composed of oriented patterns and degraded by additive Gaussian noise and Gaussian blur.

Keywords: *Diffusion, partial differential equations, shock filters, image fusion*

I. INTRODUCTION

Introduced in the early 90s, the Partial Differential Equations (PDE) framework continues to play an important role in low-level or high-level image processing. Its main advantages – extreme precision, nonlinearity, steerability, the coexistence of smoothing or enhancement processes acting adaptively on the same pixel – have aroused the interest of many researchers and lead to the development of powerful PDE based image processing methods with a broad range of applications.

The PDE framework allows the implementation of virtually any operator that can be then applied for various image processing tasks. The domain was practically born together with the introduction of the Perona-Malik anisotropic diffusion equation [1] that models the image restoration process through an iterative filter that relates time with spatial derivatives:

$$\frac{\partial U}{\partial t} = div[c(U, x, y, t)\nabla U] . \qquad (1)$$

Using eq.(1), the luminance function of a gray scale image –*U(x,y,t)*- is smoothed or enhanced selectively in each pixel. Intra-region smoothing is favored and inter-region smoothing is penalized using a diffusivity function:

$$c(U, x, y, t) = g(|\nabla U(x, y, t)|) = \frac{1}{1 + \left(|\nabla U(x, y, t)| / K\right)^2} \qquad (2)$$

The function *g( )* takes as arguments the gradient vector norms $|\nabla U(x, y, t)|$ and a threshold K and plays the role of an edge detector. The behavior of the filter can be easily understood if the equation is put in terms of second order directional derivatives along two directions pointing in the direction of edges ($\vec{\xi}$) and along a vector ($\vec{\eta}$) collinear to the gradient vector. By already classical results it can be shown that eq. (1) is equivalent in each pixel to:

$$\frac{\partial U}{\partial t} = c_\xi U_{\xi\xi} + c_\eta U_{\eta\eta}, \ \vec{\eta} = \frac{\nabla U}{|\nabla U|}, \ \vec{\xi} \perp \vec{\eta}. \qquad (3)$$

For a constant diffusivity (*c(.)*=ct.), the filter corresponds to the isotropic diffusion equation that models the image smoothing process through an analogy with heat propagation in a homogenous medium.

The behavior of the diffusion equation for Perona-Malik like diffusion functions is fundamentally different. Figure 1 shows a plot of the diffusion coefficients along the two diffusion axes; it can be easily noticed that, meanwhile on the $\vec{\xi}$ direction the diffusion coefficient (c_ξ) is always positive, in the directions orthogonal to edges the diffusion coefficient (c_η) becomes negative for norms superior to *K*.

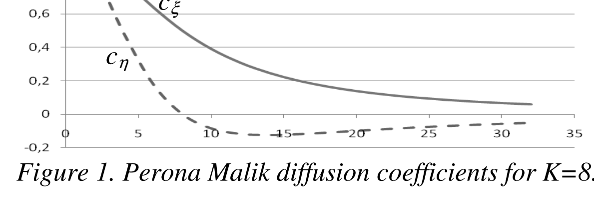

Figure 1. Perona Malik diffusion coefficients for K=8.

Manuscript received February 22, 2010; revised March 16, 2010

The diffusion function was chosen by the authors as in eq. (2) for allowing edge enhancement to take place. Such a function induces negative diffusion coefficients in the direction orthogonal to edges whenever the gradient norms are above the threshold *K*. This deliberate inversion of a forward diffusion process that is smoothing out noise is characteristic for another family of PDE based filters – shock filters – that was specially designed to deal with blur like image degradations. In its simple form such a filter can be put in terms of the following equation:

$$\frac{\partial U}{\partial t} = -sign\left(U_{\eta\eta}\right)\cdot\left|\nabla U\right| . \tag{4}$$

Eq.(4) is due to the seminal work of Osher and Rudin [2] and it can be made stable only in the numerical domain by appropriate numerical schemes relying on slope limiter functions.

Both filters given by eq.(3) and (4) were generalized since their introduction by several authors. The filter (3) was made more robust with respect to noise in [3]; the modification is related to a simple Gaussian pre-smoothing of the input image, prior to the estimation of the diffusivity function:

$$g(\left|\nabla U\right|) \to g\left(\left|\nabla U_\sigma\right|\right) = g\left[\nabla(\left|G_\sigma * U\right|)\right]. \tag{5}$$

Eq.(5) avoid noise amplification but still allows edge enhancement to take place for relatively small standard deviations (σ) of the Gaussian filter.

As far as shock filters are concerned major improvements were introduced in [4], [5] and [6]. The approach in [4] also uses a Gaussian pre-smoothing for making the filter more robust with respect to noise; the modified shock filter equation is:

$$\frac{\partial U}{\partial t} = -sign(G_\sigma * U)_{\eta\eta}\left|\nabla U\right| . \tag{6}$$

The approach proposed in [5] employs robust diffusion directions estimated via a structure tensor based approach for steering the action of the shock filter. The equation:

$$\frac{\partial U}{\partial t} = -sign(G_{\sigma_s} * U)_{vv}\left|\nabla U\right| , \tag{7}$$

reverses the diffusion process along the direction of the eigenvector $\vec{v}$ that corresponds to the largest eigenvalue of the structure tensor:

$$J_\rho(\nabla U_\sigma) = \begin{pmatrix} G_\rho * (\frac{\partial U_\sigma}{\partial x})^2 & G_\rho * \frac{\partial U_\sigma}{\partial x}\frac{\partial U_\sigma}{\partial y} \\ G_\rho * \frac{\partial U_\sigma}{\partial x}\frac{\partial U_\sigma}{\partial y} & G_\rho * (\frac{\partial U_\sigma}{\partial y})^2 \end{pmatrix} . \tag{8}$$

The method has essentially two parameters: the structure scale σ_s determining the size of flow like patterns in the underlying image and the integration scale ρ that establishes the size of the orientation averaging window. The use of a structure tensor for steering the diffusion axis is closely related to the seminal work of Weickert in tensor driven diffusion processes [7]; the same reference includes a comprehensive review of the most important PDE models for image processing. The same approach was later used by Tshumperlé et al. in proposing image smoothing techniques using tensor-based diffusion axis and admitting a directional interpretation for the tensor driven case similar to (3) [8-9].

Other authors judged that the combination of the effects of more than one PDE could prove useful. We only mention here the filter introduced in [10] directly in terms of a directional interpretation:

$$\begin{aligned} \frac{\partial U}{\partial t} &= \alpha_f (U - U_0) + \alpha_r [h_\tau(\left|G_\sigma * \nabla U\right|)U_{\xi\xi} + U_{\eta\eta}] \\ &\quad - \alpha_e [1 - h_\tau(\left|G_\sigma * \nabla U\right|)] sign(G_\sigma * I)_{\eta\eta}\left|\nabla U\right| \end{aligned} . \tag{9}$$

Eq. (9) combines directional with isotropic smoothing and edge enhancement through the use of a fuzzy edge detector function *h()* and, as the authors show in their original publication, the method can successfully deal with AWGN and Gaussian blur degradation scenarios. We refer to the original publication [10] for a full description of the method's parameters.

The method we are proposing is similar in spirit with eq. (9); it combines smoothing processes with shock filter-based actions for dealing with AWGN and Gaussian blur degradations. Instead of being proposed as eq. (9), i.e. as a unique PDE that modifies its action depending on semi-local spatial information, the method is developed using a fusion –diffusion framework and it is presented in the next section.

II. PROPOSED METHOD

II.1 The fusion-diffusion framework for image restoration and enhancement

The approach we are proposing is build around the fusion-diffusion model developed in [11-12].

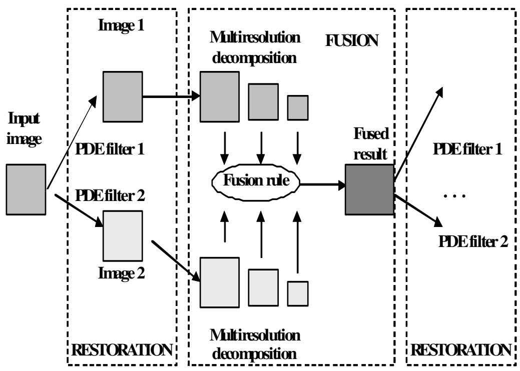

Figure 2. Fusion–diffusion framework for image restoration and enhancement.

The fusion-diffusion framework uses two non-linear based PDE filters for processing an input image in a restoration step and then combines the intermediate results in a fusion step, with the restoration-fusion procedure being iterated for a predefined number of steps. The fundamental ideas, as they were introduced in [11], are to take explicitly the scale parameter *t* into-account and to use non-linear PDE models as constituent filters. The PDE models that are to be considered as constituent filters are supposed to be able to generate nonlinear multiscale representations of the processed image. By combining results at coarser scales (higher *t* values), artifacts that may occur due to the influence of the noise at finer scales can be eliminated and better results can be obtained [12].

Figure 3 shows such a comparison between a linear and a nonlinear multiscale representation of an image degraded by an AWGN and Gaussian blur scenario. The first column presents, respectively, the input image, the result of a linear Gaussian convolution with a standard deviation of 1.5 and the result obtained with a non linear operator at the same observation scale $t=\sigma^2/2$. The right column shows the response of a Sobel edge detector; at a fine scale (original image) edges cannot be detected as the operator produces spurious results due to noise. At coarser scales (figure 3.d and 3.f) relevant structures in the image can be detected and local decision policies to smooth more or to enhance can be imposed. The advantage of using nonlinear operators is that they allow better structure preservation without corner or edge displacement (figure 3.f) than linear operators (figure 3.d).

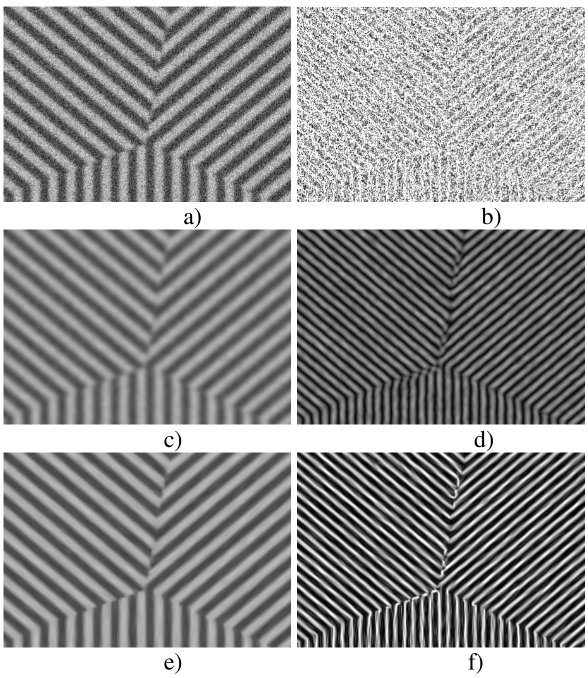

Figure 3. Linear vs. non-linear scale space at t=1.5. a) Original image; b)Sobel operator response for a); c) Gaussian convolution; d) Sobel response for c); e) Non linear operator result; f) Sobel response for e).

Advanced PDE based filters like eq.(9) are combining the effect of different types of PDEs (forward diffusion, shock PDE etc) through weighed averaging with weights computed also on scale-space like simplifications of the processed image. However, unlike our model, simplifications of the processed image are obtained through use of linear Gaussian preconvolutions that can lead to elimination of small scale details or edge displacement.

The model in figure 2 is extremely general since it allows a complete freedom for the choice of the constituent filters and for the fusion technique to be employed for combining the results. Preliminary results included in [11] and [12] were based on symmetric diffusion filters and scalar shock filters.

We design our new method to work in the AWGN and Gaussian blur scenario by choosing the constituent filters to be able to process efficiently oriented patterns and to have complementary actions of smoothing and, respectively, enhancement. The choice of these filters is motivated in the following subsection.

II.2 Constituent PDE filters

For inducing selective noise smoothing with edge and junction preservation we employ a state-of-the art directional PDE model developed in [13]. The PDE that describes the filter is given by the following equation:

$$\begin{aligned}\frac{\partial U}{\partial t} = 0.5 \cdot [1 - f(\frac{\partial_+ U}{\partial u}, \frac{\partial_- U}{\partial u})] \frac{\partial_+}{\partial u}[g(U_{\sigma_u})U_u] + \\ + \max\left[0, f(\frac{\partial_+ U}{\partial u}, \frac{\partial_- U}{\partial u})\right] \frac{\partial}{\partial u}[g(U_{\sigma_u})U_u] + \\ + \frac{\partial}{\partial v}[g(U_{\sigma_v})U_v]\end{aligned} \quad (10)$$

The smoothing PDE is based on an asymmetric orientation estimation operator (IRON-Isotropic Recursive Oriented Network) shown in [14] to minimize orientation estimation errors on corners and junctions. The purpose of the function *f* appearing in eq. (10) is to detect possible asymmetric configurations along the structure's directions (i.e. junctions and corners) and it employs a formulation inspired from the *minmod* function used in hyperbolic equations [15]:

$$f(a,b) = \mathrm{sgn}(a) \cdot \mathrm{sgn}(b) . \quad (11)$$

Depending on the semi-local asymmetric orientation obtained via a unique IRON orientation estimation step, eq.(10) induces one-sided or two-sided smoothing or even enhancement processes in the maximum homogeneity direction ($\vec{u}$) computed by the IRON operator. In the orthogonal direction ($\vec{v}$), for Perona–Malik like diffusivities (2), eq.(10) smoothes or enhances edges depending on the relationship between the directional derivative along this direction and the threshold K.

The method was proven to be superior experimentally and statistically than most classical and recent PDE based filters in [13] but it cannot handle blur like distortions. The result in figure 3 e) was produced with this filter.

The deblurring filter is derived from equation (7) and it has the following expression:

$$\frac{\partial U}{\partial t} = -sign(G_{\sigma_r} * U)_{vv}|U_v| \quad (12)$$

In contrast to (7) that corresponds to continuous scale morphological erosions and dilations with a disk shaped element, eq.(12) acts only on the edge orthogonal directions and effectively inverts the diffusion process only on the directions orthogonal to edges and produces deblurring results without introducing artifacts and preserving corners.

The difference in terms of behavior on junctions and corners between the original formulation of the filter and the proposed shock PDE is shown in figure 4; despite the robust, modulo 2π, orientation information, on such type of structures the original PDE (7) reduces to $\frac{\partial U}{\partial t} = \pm|\nabla U|$ and, by acting in all directions of the space, introduces visible artifacts.

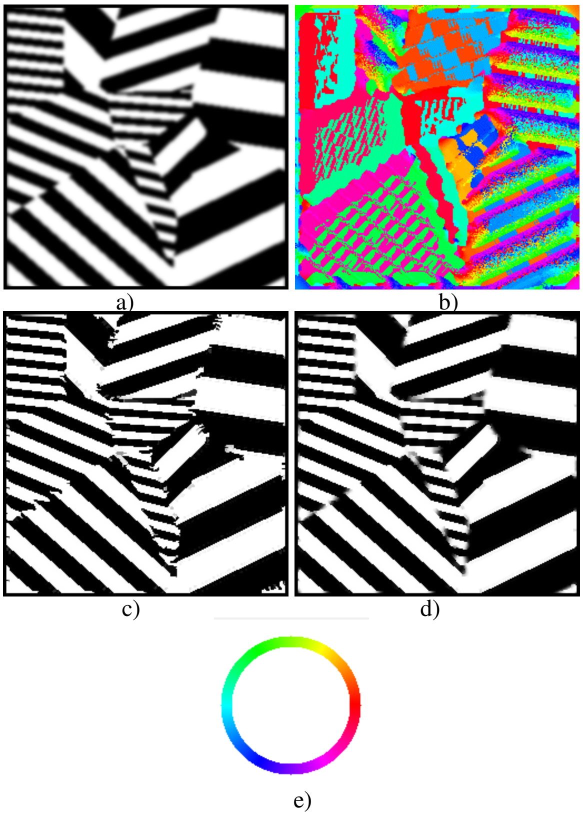

Figure 4. Gaussian blur elimination using semi-local scale-based orientation information. a) Blurred image; b)IRON based orientation;c) Result obtained using eq. (7); d) Result obtained using eq. (12); e) Circular palette of colors used to represent orientation.

By restricting its action on a single direction, the proposed shock filter formulation act as 1D erosion/dilation equation and does not produce such undesirable effects.

II.3 Fusion step

Besides the choice of the PDE based filters the design of the fusion rule is susceptible of influencing the results. We employ a modified selection/weighted averaging fusion scheme performed on the Discrete Wavelet Transform (DWT) domain that allows:

- selection of the solution of a PDE for a given scale whenever one of the results is significantly more pertinent than the other;
- weighted averaging of the results obtained using the constituent PDEs for spatial locations where the results are similar.

The processed results are first decomposed in subband images using the maximum possible number of decomposition levels (depending on the size of the input image) and then neighborhood (W) saliency measures are computed [16]:

$$E_k = \sum_{(i,j)\in W} \left|D_{i,j}{}^k\right|, \quad (13)$$

with $D_{i,j}{}^k$ denoting the wavelet coefficients at spatial locations (i,j) on the k input image.

The match measure reflects the resemblance of the two results and is computed using the classical formula:

$$M = \frac{2E_1E_2}{E_1{}^2 + E_2{}^2}. \quad (14)$$

The fused coefficients in the subbands obtained by at least one high pass filtering operation are obtained through a selection rule whenever the match measure falls below a given threshold (α):

$$D_{i,j}^{fus} = D_{i,j}{}^{\min}, \quad (15)$$

or through a weighted averaging scheme for high match measures:

$$\begin{cases} D_{i,j}^{fus} = w_{\max} D_{i,j}{}^{\min} + w_{\min} D_{i,j}{}^{\max} \\ w_{min}=0.5-0.5\ (1-M)/(1-\alpha) \\ w_{max}=1-w_{min} \end{cases} . \quad (16)$$

In (15) and (16) $D_{i,j}{}^{\min/\max}$ are denoting the signed wavelet coefficients at spatial locations (i,j) corresponding to the pair wise minimum/maximum values.

It should be noted that the theoretical properties of the constituent filters allowed us to modify the match/saliency rule. Both of them are edge preserving filters and the choice of a *min* like mathematical operation does not lead to destruction of edges. In selection mode the fusion rule selects the DWT coefficient that corresponds to the least oscillatory solution among the two available and in weighted averaging mode it favors also elimination of local, spurious, oscillations.

II.4 Numerical scheme

For the numerical scheme Gaussian filters were truncated at 3 times their standard deviation and the IRON operator was implemented using shear transformation based image rotations [14]. The spatial derivatives corresponding to the asymmetric directional diffusion filter (10) have been approximated by using central or forward finite differences and interpolation schemes as given in [13] and in the references therein. The second-order directional derivative U_{vv} was approximated by the following formula:

$$U_{vv} = U_{xx}\sin^2(\theta) - 2U_{xy}\sin(\theta)\cos(\theta) + U_{yy}\cos^2(\theta)\ , \quad (17)$$

with θ denoting the direction of the vector pointing on the maximum homogeneity $(\vec{u})$ obtained by the IRON operator and standard approximations for the second-order derivatives along the *x* and *y* axis [15].

III. RESULTS

III.1 Synthetic images

We first illustrate the efficiency of the proposed method in restoring a synthetic image composed of oriented patterns and deliberately degraded by Gaussian blur (σ=1) and by additive Gaussian noise (σ=25) (PSNR=17.08dB). In order to find a best filtered result all the considered methods were tuned with a full search in the space of parameters.

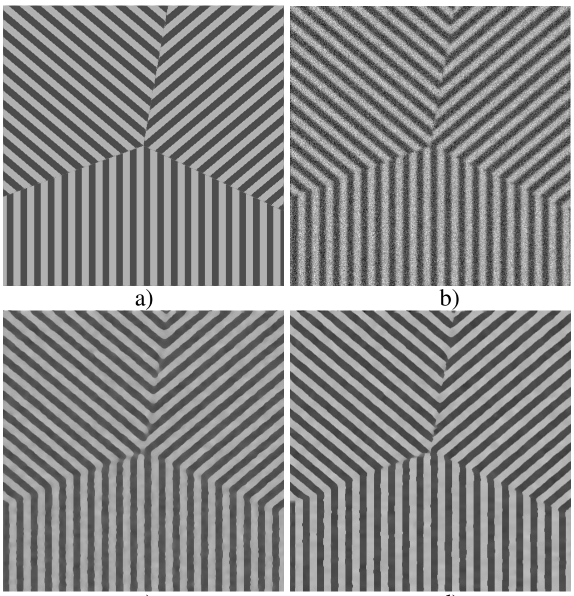

Figure 5. Restoration of a synthetic image
a) Original image; b) Blurred noisy image;
c) Result obtained using eq. (9)- PSNR=24.21dB;
d) Result obtained using the proposed method – PSNR=25.71dB.

The best results has been obtained with the proposed method using 5 fusion steps and 20 iterations per each restoration step with an α=0.75 match threshold. The increased precision in junction restoration, coupled with an efficient deblurring action, are easily noticeable on the results shown in figure 5 d) and they are in correspondence with the superior PSNR of the processed image.

The classical way of combining PDE based filters given by eq, (9) fails in obtaining comparable results due to the formulation of the PDE itself (the existence of a curvature driven term that rounds corners) and to the linear behavior of the operator used to decide if directional or isotropic diffusion is to be performed.

III.2 Real images

We also show how the proposed method and the approaches discussed in section 1 are performing in restoring a real image. We use the standard fingerprint image in the same AWGN and Gaussian blur degradation scenario.

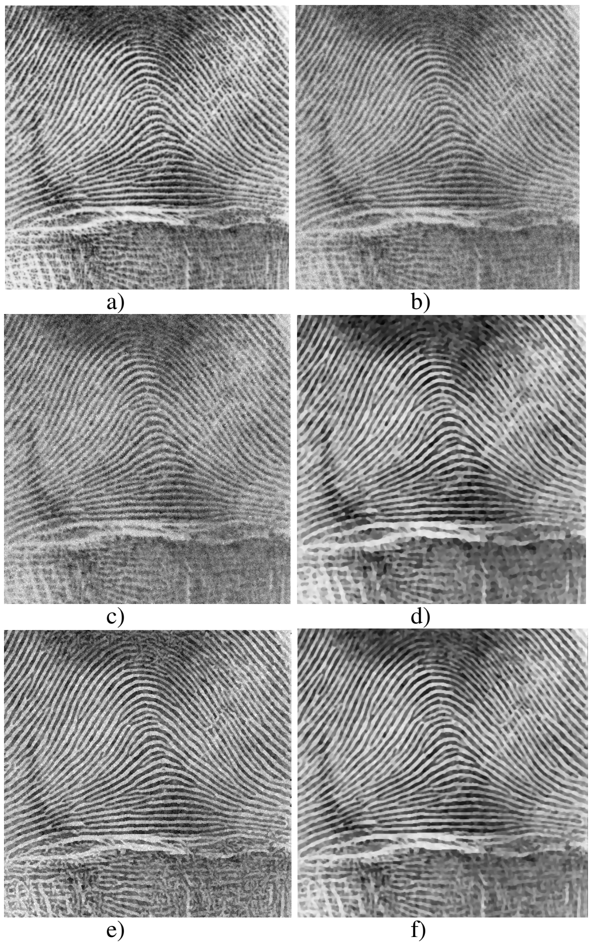

Figure 6. Restoration of a real image
a) Input image; b) Blurred noisy image PSNR=21.19dB;
c) Result obtained using eq. (4)- PSNR=17.19dB;
d) Result obtained using eq. (9)- PSNR=23.15dB;
e) Result obtained using eq. (7)- PSNR=15.81dB;
f) Result using the proposed method – PSNR=24.68dB.

The classical shock filter formulation cannot cope with noise and instead of eliminating it amplifies it; this explains the lower PSNR value than of the degraded image (figure 6.c). By employing forward diffusion eq. (9) smoothes out noise but produces topological modifications due to the presence of a curvature driven term; the resulting PSNR is however higher with 2dBs (figure 6.d). The method given by equation (7) was conceived only to deal with flow-like patterns and it is not endowed with a smoothing term. The result shown for this filter in figure 6.e) was obtained for the same observation scale that we used for the proposed method and the lowest PSNR value accounts for structural modifications and noise amplification. Once again (figure 6.f) our method proves to be superior, both in terms of PSNR value and in terms of visual quality.

We show in figure 7 a supplementary result obtained in the restoration of a heavily degraded image using equation (9) and, respectively, our method. The same effects are observable; the results were obtained after a full search in the space of parameters.

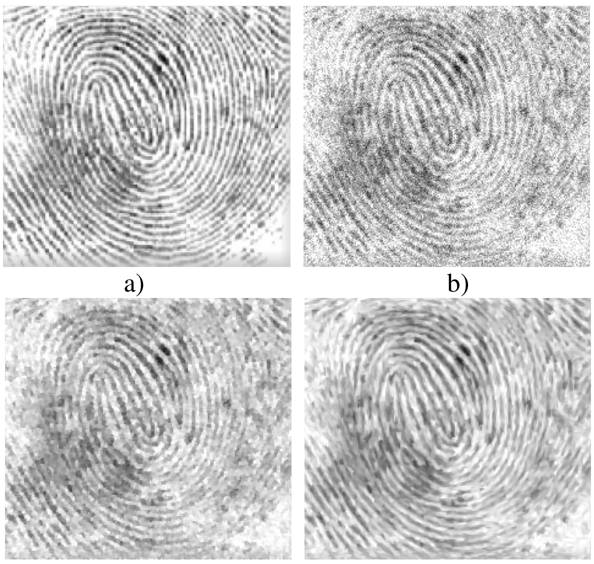

Figure 7. Restoration of a heavily degraded image a)Original image; b) Blurred noisy image- PSNR=17.33; c) Result obtained using eq. (9)- PSNR=20.32dB; f) Result using the proposed method– PSNR=21.27dB.

IV. CONCLUSIONS

We propose an efficient image restoration and enhancement approach based on the PDE framework and image fusion techniques. The filter considers asymmetric information in characterizing and handling corners and combines through fusion the actions of smoothing and shock filter PDEs at an intermediate results level. By explicitly taking into account the observation scale at which the intermediate results are obtained the method is capable of producing high quality results. We have shown that the new filter outperforms other classical and recent PDE methods dedicated for noise filtering and blur elimination.

Further work will adapt the model for the 3D and color images and will concentrate in providing a method for an automatic choice of parameters.

AKNOWLEDGEMENT

This work was supported by CNCSIS-UEFISCSU, project number IDEI 908/2007.

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