COMPLEX MODEL OF A SERIES LOAD INDUCTION HEATING INVERTER

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Abstract: The design of robust controllers requires a special type of system model, called augmented plant (system) model, which includes as uncertainties the plant parameter variations. In this article the accuracy of the classic describing function based model for a series load induction heating inverter will be analyzed and updated to include the parameter uncertainties. Additive and multiplicative type parameter uncertainties will be introduced and their effect over the system model will be analyzed.

Keywords: Robust control, Induction heating, series load inverter

I. INTRODUCTION
For a system engineer, the most important information about a process is the model of the system. The model can be obtained by solving the equations which describes the behavior of the process or by identification. For the obtained model, easy controllers can be designed to ensure stability, disturbance attenuation and quick system response.

During modeling high frequency harmonics and parameter uncertainties are usually deliberately neglected to obtain a reduced order system model.

The present article will analyze the accuracy of the reduced order model and will propose a new model which incorporates parameter uncertainties for a series resonant load induction heating inverter with three input variables and five available output variables.

The article is divided in six parts. The introduction and a short analysis on the effects of the harmonics over the main variables is followed by the introduction of the uncertainties in the system. In part four simulations will be shown about the newly obtained model. Then conclusions will be drawn.

II. SQUARE WAVE APPROXIMATION USING FOURIER ANALYSIS
For modeling purposes it is considered that the load is supplied by a square wave voltage [1, 2] with variable duty cycle. In practice this is a trapezoidal wave with different rise and fall times.

These rise and fall times play an important role in ensuring zero voltage and/or zero current commutations. They are easily altered by appropriate power transistor drivers and snubber circuits.

If the rise and fall times can be neglected in comparison with switching period, the square wave approximation approach can be used. In figure 1 the Fourier analysis of a variable duty cycle square wave signal can be observed. The signal has high order harmonics as presented in figure 1. The harmonics effect over the load can be estimated using the Bode plot presented in figure 2.

The peak appears at the natural oscillating frequency. On both sides the plot decreases rapidly. The slope depends by the circuit’s quality factor.

Figure 1. Fourier analysis of a square wave signal.

Figure 2. RLC circuit Bode plot.
The capacitor voltage in function of supply voltage and quality factor can be expressed as:

$$u_C = Q_C \frac{\omega_0}{\omega_0} u$$

The inductor current in function of maximal current and quality factor can be expressed as:

$$I = Q_I \frac{1}{1 + Q} I_0$$

If quality factor $Q$ is high enough the second and higher harmonics of the supply voltage can be neglected as presented in figure 3.

III. INVERTER MODEL WITH UNCERTAINTIES

During the heating cycle parameter variations can be observed [4]. The variations are because resistivity and magnetic permeability are temperature dependent. Figure 4 presents the resistivity and magnetic permeability temperature dependence for iron.

Variations can be included in the model, which makes available the selection of parameter of interest using perturbation blocks [6].

For robust control typically three types of parameter uncertainties are used: additive uncertainty, multiplicative uncertainty and coprime factorization [7, 8](figure 5). In this article only the first two modeling techniques will be presented for magnetic (as Fe,Co) materials. For nonmagnetic materials (as Cu, Al) the perturbation block responsible for the inductivity variations has to be initialized with 0. During heating of magnetic materials, the resonant circuit’s equivalent resistor and inductor values will be considered with uncertainties, while for nonmagnetic materials only the equivalent resistor will exhibit modifications.

To develop the new models, we started from [1, 2]. The resonant circuit (figure 6) is supplied by a variable duty cycle square wave voltage which was approximated by the first harmonic of the Fourier expansion as presented in section II.

For the RLC series circuit the following equations can be derived using Kirchhoff’s law:

$$u_{ab} = L \frac{di}{dt} + iR + u_c$$

$$C \frac{du_c}{dt} = i$$

Using the same notations as in [1] $i(t)$ and $u_c(t)$ can be approximated as:

$$i(t) = i_{\text{sin}}(t) \sin(\omega t) + i_{\text{cos}}(t) \cos(\omega t)$$

$$u_c(t) = u_{\text{sin}}(t) \sin(\omega t) + u_{\text{cos}}(t) \cos(\omega t)$$

Introducing equations 4 and 5 in 6 and 7 the following system model was obtained in matrix form:

$$\begin{bmatrix} 0 & 1/C & \omega & 0 \\ -1/L & -R/L & 0 & \omega \\ -\omega & 0 & 0 & 1/C \\ 0 & -\omega & -1/L & -R/L \end{bmatrix} x + \begin{bmatrix} -2 \pi \omega \cos(h) - 1 \\ 0 \\ \frac{2 \pi \omega}{2 \pi \sin(h)} \end{bmatrix} U_{ac}$$
The following state variables were used:

\[ x = [u_{C_{\text{sin}}}, i_{\text{sin}}, u_{C_{\text{cos}}}, i_{\cos}] ^T \]  \hspace{1cm} (9)

Were \( h \) represents the duty cycle and varies between 0 and 2\( \pi \).

This model represents the large signal model of a series load induction heating inverter. The following output variables can be selected: capacitor voltage (maxim or rms), inductor current (maxim or rms), output power and phase lead/lags between supply voltage and load current or capacitor voltage.

\[
\begin{align*}
\gamma_{\text{cap max}} &= \sqrt{V_{C_{\text{cap}}}^2 + u_{C_{\text{cos}}}^2} \cdot \gamma_{\text{cap RMS}} = \sqrt{\frac{V_{C_{\text{cap}}}^2 + u_{C_{\text{cos}}}^2}{2}} \\
\gamma_{\text{cap RMS}} &= \sqrt{V_{C_{\text{cap}}}^2 + u_{C_{\text{cos}}}^2} \cdot \gamma_{\text{cap MAX}} = \sqrt{V_{C_{\text{cap}}}^2 + u_{C_{\text{cos}}}^2} \\
\gamma_{\text{output}} &= R \left( \frac{i_{\text{sin}}^2}{2} + \frac{i_{\cos}^2}{2} \right) \gamma_{\text{output max}} = \arctan \frac{i_{\text{sin}}}{i_{\cos}} , \gamma_{\text{output RMS}} = \arctan \frac{u_{C_{\text{cos}}}}{u_{C_{\text{sin}}}}
\end{align*}
\]  \hspace{1cm} (10)

To obtain the small signal model of the inverter, all variables were perturbed and the following results:

\[
\begin{align*}
\dot{x} &= A \cdot x + 0 \cdot (\cos(h) - 1) \cdot U_{\text{dc}} + 0 \cdot \frac{2}{\pi} U_{\text{dc}} \cdot \sin(h) \\
0 &= \frac{2}{\pi} U_{\text{dc}} \cdot \cos(h)
\end{align*}
\]  \hspace{1cm} (11)

For the small signal model the output matrix takes the following form:

\[
\begin{align*}
\gamma_{\text{cap max}} &= \left[ U_{C_{\text{cap}}} + \sqrt{V_{C_{\text{cap}}}^2 + u_{C_{\text{cos}}}^2} \right] \cdot \gamma_{\text{cap RMS}} = \sqrt{\frac{V_{C_{\text{cap}}}^2 + u_{C_{\text{cos}}}^2}{2}} \\
\gamma_{\text{output max}} &= \left[ 0 \cdot \sqrt{I_{C_{\text{cap}}}^2 + U_{C_{\text{cap}}}^2} + 0 \cdot \sqrt{I_{C_{\text{cap}}}^2 + U_{C_{\text{cap}}}^2} \right] x \\
\gamma_{\text{output RMS}} &= \left[ 0 \cdot R \cdot I_{C_{\text{cap}}} + 0 \cdot R \cdot I_{C_{\text{cap}}} \right] x \\
\gamma_{\text{output max}} &= \left[ 0 \cdot \sqrt{I_{C_{\text{cap}}}^2 + U_{C_{\text{cap}}}^2} + 0 \cdot \sqrt{I_{C_{\text{cap}}}^2 + U_{C_{\text{cap}}}^2} \right] x \\
\gamma_{\text{output RMS}} &= \left[ 0 \cdot \sqrt{I_{C_{\text{cap}}}^2 + U_{C_{\text{cap}}}^2} + 0 \cdot \sqrt{I_{C_{\text{cap}}}^2 + U_{C_{\text{cap}}}^2} \right] x
\end{align*}
\]  \hspace{1cm} (12)

The above presented system’s block diagram is shown in figure 7.

The system presented in figure 7 will be altered to include the parameter uncertainties. The simplest case is for nonmagnetic materials, in which only the equivalent resistors value is changing with temperature. For magnetic materials also the inductivity will exhibit variations. For additive and multiplicative uncertainties the blocks containing non fixed values will be changed with [9]:

- Additive uncertainties
  \[ G_{p} = G_{0} + \Delta \]  \hspace{1cm} (13)

- Multiplicative uncertainties
  \[ G_{p} = G_{0} \cdot \left( 1 + \Delta \right) \]  \hspace{1cm} (14)

Were:
- \( G_{0} \) represents the nominal model
- \( \Delta \) perturbation block

For the resistor the following upper linear fractional transformations can be defined:

\[
F_{\text{additive}} (R, \Delta) = R_{p} + 1 \cdot (1 - 0 \cdot \Delta)^{-1} \cdot 1
\]  \hspace{1cm} (15)

\[
F_{\text{additive}} (R, \Delta) = R_{p} \cdot \Delta \cdot (1 - 0 \cdot \Delta)^{-1} \]  \hspace{1cm} (16)

In interconnected matrix form:

\[
\begin{align*}
G_{\text{additive}} &= \left[ \begin{array}{c} 0 & 1 \\ 1 & R_{p} \end{array} \right] \cdot G_{\text{nominal}} \cdot \left[ \begin{array}{c} 0 & R_{p} \\ 1 & R_{p} \end{array} \right] R_{p} \cdot \Delta \cdot (1 - 0 \cdot \Delta)^{-1} \]  \hspace{1cm} (17)
\]

Since in the model the inductivity appears always in the denominator, the entire fraction was submitted for upper linear fractional transformation. For 1/L the following upper linear fractional transformations were defined:

\[
F_{\text{additive}} (1/L, \Delta) = 1/L_{p} + 1 \cdot (1 - 0 \cdot \Delta)^{-1} \cdot 1
\]  \hspace{1cm} (18)

\[
F_{\text{additive}} (1/L, \Delta) = 1/L_{p} \cdot \Delta \cdot (1 + 0 \cdot \Delta)^{-1} \cdot 1/L_{p} \]  \hspace{1cm} (19)

In interconnected matrix form:

\[
\begin{align*}
G_{\text{additive}} &= \left[ \begin{array}{c} 0 & 1 \\ 1 & 1/L_{p} \end{array} \right] \cdot G_{\text{nominal}} \cdot \left[ \begin{array}{c} 0 & 1/L_{p} \\ 1 & 1/L_{p} \end{array} \right] R_{p} \cdot \Delta \cdot (1 - 0 \cdot \Delta)^{-1} \]  \hspace{1cm} (20)
\]

The generalized system with uncertainties is given by[4]:

\[
\begin{align*}
\dot{x}(t) &= A x(t) + B \cdot \Phi(t) + B \cdot \Psi(t) \\
\dot{y}(t) &= C x(t) + D \cdot \Phi(t) + D \cdot \Psi(t) \\
y(t) &= C x(t) + D \cdot \Phi(t) + D \cdot \Psi(t)
\end{align*}
\]  \hspace{1cm} (21)

Were:
- The state matrix \( A \) remains the same
- Input matrix \( B_{2} \) is the same with the \( B \) matrix presented in the small signal model. The \( B_{2} \) has to be chosen accordingly to the selected input variable.
Additive uncertainty:

\[
B_i = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
-1 - R_0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad C_i = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
D_{i1} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad D_{i1} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

Multiplicative uncertainty:

\[
B_i = \begin{bmatrix}
P_L & R_0 P_L & -P_L \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}, \quad C_i = \begin{bmatrix}
- \frac{R_L}{L_0} & 0 & 0 & 0 & 0 \\
0 & - \frac{R_L}{L_0} & 0 & 0 & 0 \\
0 & 0 & - \frac{R_L}{L_0} & 0 & 0 \\
0 & 0 & 0 & - \frac{R_L}{L_0} & 0 \\
0 & 0 & 0 & 0 & - \frac{R_L}{L_0} \\
\end{bmatrix}
\]

\[
D_{i1} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad D_{i1} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

IV. SIMULATION RESULTS

Simulations have been carried out to test the accuracy of the model. To obtain the equivalent inductivity and resistivity of the inductor, finite element methods were applied. For one kilogram iron heating equipment the following load variations were obtained:

\[
R \text{ varies between } 0.46-0.76 \Omega \\
L \text{ varies between } 24.5-33.5 \mu H
\]

For additive uncertainties based on the R and L variations the following values were defined: \( R_0 = 0.46 \Omega \), \( L_0 = 24.5 \mu H \), \( \delta_R = [0.36] \), \( \delta_L = [0.32] \)

For multiplicative uncertainties: \( R_0 = 0.61 \Omega \), \( L_0 = 29.3 \mu H \), \( p_R = 0.3 \), \( p_L = 0.14 \), \( \delta_R = [0.35] \)

The model G and \( \delta \) was initialized accordingly to the desired input and output variables and the type of uncertainty modeled.

To simulate the behavior of the system using additive uncertainties, the input variable was chosen to be the duty cycle of the power transistors and for output variable the load current.

V. CONCLUSIONS

The newly developed model which includes the parameter variations as uncertainties, offers the advantage of easy parameter selection for simulation. Advanced controllers can be designed using \( H_\infty \) and \( H_2 \) design algorithms with the newly presented model.

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