COMPLEX MODEL OF A SERIES LOAD INDUCTION HEATING INVERTER

T. SZELITZKY, I. INOAN, M. TULBURE, A.O. NEAGA,
Department of Automatic Control, Faculty of Automation and Computer Science, Technical University of Cluj Napoca
Cluj Napoca, Romania, (e-mail:sztibi82@yahoo.com, iuliainoan@yahoo.com,
miron.tulbure@aut.utcluj.ro,adrian.neaga@aut.utcluj.ro)

Abstract: The design of robust controllers require a special type of system model, called augmented plant (system) model, which includes as uncertainties the plant parameter variations. In this article the accuracy of the classic describing function based model for a series load induction heating inverter will be analyzed and updated to include the parameter uncertainties. Additive and multiplicative type parameter uncertainties will be introduced and their effect over the system model will be analyzed.

Keywords: *Robust control, Induction heating, series load inverter*

I. INTRODUCTION

For a system engineer, the most important information about a process is the model of the system. The model can be obtained by solving the equations which describes the behavior of the process or by identification. For the obtained model, easy controllers can be designed to ensure stability, disturbance attenuation and quick system response.

During modeling high frequency harmonics and parameter uncertainties are usually deliberately neglected to obtain a reduced order system model.

The present article will analyze the accuracy of the reduced order model and will propose a new model which incorporates parameter uncertainties for a series resonant load induction heating inverter with three input variables and five available output variables.

The article is divided in six parts. The introduction and a short analysis on the effects of the harmonics over the main variables is followed by the introduction of the uncertainties in the system. In part four simulations will be shown about the newly obtained model. Then conclusions will be drawn.

II. SQUARE WAVE APROXIMATION USING FOURIER ANALYSIS

For modeling purposes it is considered that the load is supplied by a square wave voltage [1, 2] with variable duty cycle. In practice this is a trapezoidal wave with different rise and fall times.

These rise and fall times play an important role in ensuring zero voltage and/or zero current commutations. They are easily altered by appropriate power transistor drivers and snubber circuits.

If the rise and fall times can be neglected in comparison with switching period, the square wave approximation approach can be used. In figure 1 the Fourier analysis of a variable duty cycle square wave signal can be observed. The signal has high order harmonics as presented in figure 1. The harmonics effect over the load can be estimated using the Bode plot presented in figure 2.

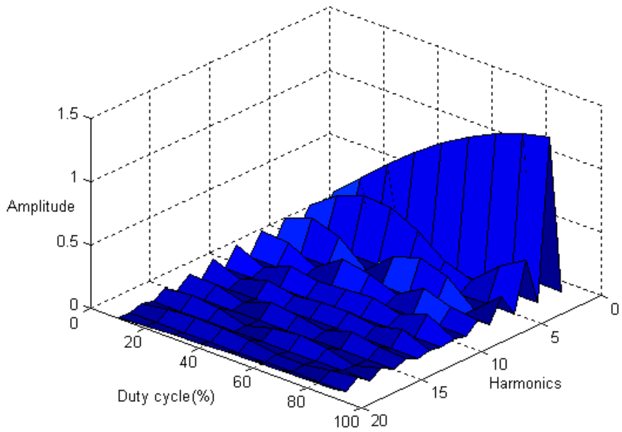

Figure 1. Fourier analysis of a square wave signal.

The peak appears at the natural oscillating frequency. On both sides the plot decreases rapidly. The slope depends by the circuit's quality factor.

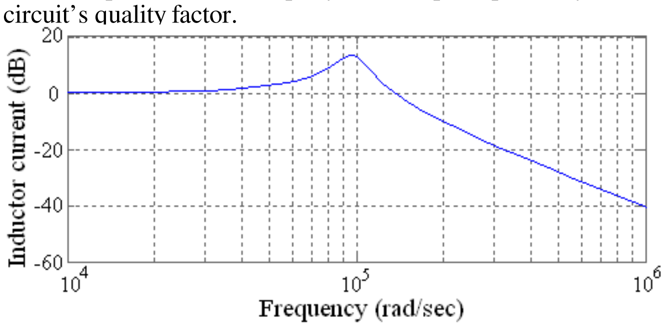

Figure 2. RLC circuit Bode plot.

The inductor voltage in function of supply voltage and quality factor can be expressed as [3]:

$$u_L = Q_C \frac{\frac{\omega}{\omega_0}}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}} U \quad (1)$$

The capacitor voltage in function of supply voltage and quality factor can be expressed as:

$$u_C = Q_C \frac{\frac{\omega_0}{\omega}}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}} U \quad (2)$$

Inductor current in function of maximal current and quality factor can be expressed as:

$$I = Q_C \frac{1}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}} I_0 \quad (3)$$

If quality factor Q is high enough the second and higher harmonics of the supply voltage can be neglected as presented in figure 3.

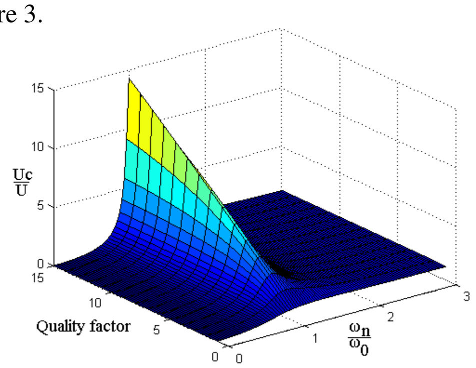

Figure 3. Capacitor voltage gain vs quality factor and ω_n/ω_0

The shape of U_C/U, U_L/L and I_0/I is similar. The graph is shifted to left or right depending by the studied output variable.

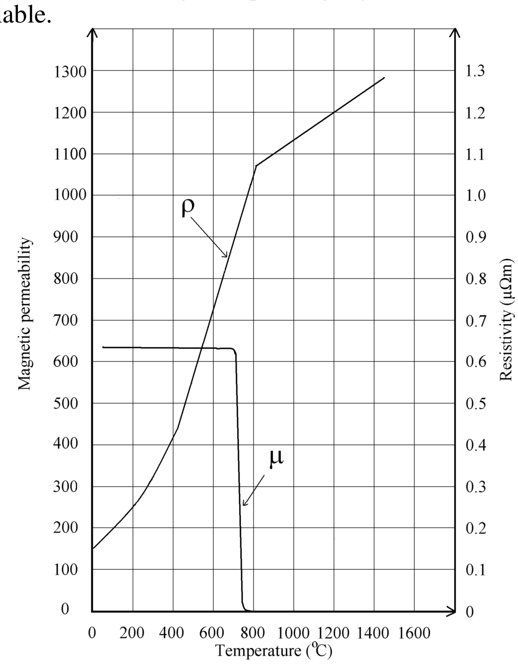

Figure 4. Resistivity and magnetic permeability dependency with temperature.

III. INVERTER MODEL WITH UNCERTAINTIES

During the heating cycle parameter variations can be observed [4]. The variations are because resistivity and magnetic permeability are temperature dependent. Figure 4 presents the resistivity and magnetic permeability temperature dependence for iron.

Variations can be included in the model, which makes available the selection of parameter of interest using perturbation blocks [6].

For robust control typically three types of parameter uncertainties are used: additive uncertainty, multiplicative uncertainty and coprime factorization [7, 8](figure 5). In this article only the first two modeling techniques will be presented for magnetic (as Fe,Co) materials. For nonmagnetic materials(as Cu, Al) the perturbation block responsible for the inductivity variations has to be initialized with 0. During heating of magnetic materials, the resonant circuit's equivalent resistor and inductor values will be considered with uncertainties, while for nonmagnetic materials only the equivalent resistor will exhibit modifications.

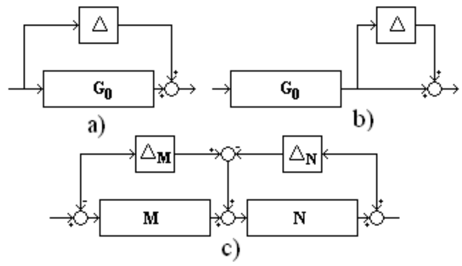

Figure 5. Additive, multiplicative uncertainties and coprime factorization.

To develop the new models, we started from [1, 2]. The resonant circuit (figure 6) is supplied by a variable duty cycle square wave voltage which was approximated by the first harmonic of the Fourier expansion as presented in section II.

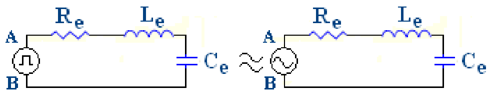

Figure 6. RLC series circuit supplied by the first harmonic of the Fourier expansion of the square wave.

For the RLC series circuit the following equations can be derived using Kirchhoff's law:

$$u_{AB} = L\frac{di}{dt} + iR + u_C, \quad (4)$$

$$C\frac{du_C}{dt} = i \quad (5)$$

Using the same notations as in [1] i(t) and $u_C(t)$ can be approximated as:

$$i(t) = i_{\sin}(t)\sin(\omega t) + i_{\cos}(t)\cos(\omega t) \quad (6)$$

$$u_C(t) = u_{C\sin}(t)\sin(\omega t) + u_{C\cos}(t)\cos(\omega t) \quad (7)$$

Introducing equations 4 and 5 in 6 and 7 the following system model was obtained in matrix form:

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1/C & \omega & 0 \\ -1/L & -R/L & 0 & \omega \\ -\omega & 0 & 0 & 1/C \\ 0 & -\omega & -1/L & -R/L \end{bmatrix}}_{A} x + \begin{bmatrix} 0 \\ -\frac{2}{\pi L}(\cos(h)-1) \\ 0 \\ \frac{2}{\pi L}\sin(h) \end{bmatrix} U_{DC} \quad (8)$$

The following state variables were used:

$$x = [u_{C\sin} \quad i_{\sin} \quad u_{C\cos} \quad i_{\cos}]^T \tag{9}$$

Were h represents the duty cycle and varies between 0 and 2π. This model represents the large signal model of a series load induction heating inverter. The following output variables can be selected: capacitor voltage (maxim or rms), inductor current (maxim or rms), output power and phase lead/lags between supply voltage and load current or capacitor voltage.

$$y_{Ucap\max} = \sqrt{u_{C\sin}^2 + u_{C\cos}^2}, \; y_{UcapRMS} = \sqrt{u_{C\sin}^2/2 + u_{C\cos}^2/2}$$
$$y_{I_{AB\max}} = \sqrt{i_{\sin}^2 + i_{\cos}^2}, \; y_{IABRMS} = \sqrt{i_{\sin}^2/2 + i_{\cos}^2/2} \tag{10}$$
$$y_{POW} = R\left(\frac{i_{\sin}^2}{2} + \frac{i_{\cos}^2}{2}\right) \; y_{U_{AB}\angle I_{AB}} = \arctan\frac{i_{\cos}}{i_{\sin}}, \; y_{U_{AB}\angle U_C} = \arctan\frac{u_{C\cos}}{u_{C\sin}}$$

To obtain the small signal model of the inverter, all variables were perturbed and the following results:

$$\dot{x} = Ax + \begin{bmatrix} U_{C\cos} \\ I_{\cos} \\ -U_{C\sin} \\ -I_{\sin} \end{bmatrix}\hat{\omega} + \begin{bmatrix} 0 \\ -\frac{2}{\pi L}(\cos(h)-1) \\ 0 \\ \frac{2}{\pi L}\sin(h) \end{bmatrix}\hat{U}_{DC} + \begin{bmatrix} 0 \\ \frac{2}{\pi L}U_{DC}\sin(h) \\ 0 \\ \frac{2}{\pi L}U_{DC}\cos(h) \end{bmatrix}\hat{h} \tag{11}$$

For the small signal model the output matrix takes the following form:

$$y_{Ucap\max\&RMS} = \begin{bmatrix} U_{C\sin}/\sqrt{U^2_{C\sin} + U^2_{C\cos}} & 0 & U_{C\cos}/\sqrt{U^2_{C\sin} + U^2_{C\cos}} & 0 \end{bmatrix}x$$

$$y_{IAB\max\&RMS} = \begin{bmatrix} 0 & I_{\sin}/\sqrt{I^2_{\sin} + I^2_{\cos}} & 0 & I_{\cos}/\sqrt{I^2_{\sin} + I^2_{\cos}} \end{bmatrix}x$$
$$y_{POW} = [0 \quad RI_{\sin} \quad 0 \quad RI_{\cos}]x \tag{12}$$
$$y_{U_{AB}\angle I_{AB}} = \begin{bmatrix} 0 & \frac{1}{1+\left(I_{\cos}/I_{\sin}\right)}\frac{-I_{\cos}}{I^2_{\sin}} & 0 & \frac{1}{1+\left(I_{\cos}/I_{\sin}\right)}\frac{1}{I_{\sin}} \end{bmatrix}x$$
$$y_{U_{AB}\angle U_C} = \begin{bmatrix} \frac{1}{1+\left(U_{C\cos}/U_{C\sin}\right)}\frac{-U_{C\cos}}{U^2_{C\sin}} & 0 & \frac{1}{1+\left(U_{C\cos}/U_{C\sin}\right)}\frac{1}{U_{C\sin}} & 0 \end{bmatrix}x$$

The above presented system's block diagram is shown in figure 7.

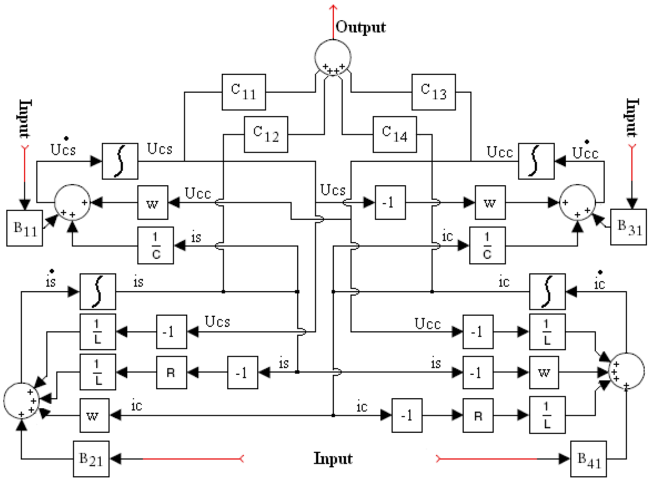

Figure 7. Inverter model block diagram.

The system presented in figure 7 will be altered to include the parameter uncertainties. The simplest case is for nonmagnetic materials, in which only the equivalent resistors value is changing with temperature. For magnetic materials also the inductivity will exhibit variations. For additive and multiplicative uncertainties the blocks containing non fixed values will be changed with [9]:

- Additive uncertainties

$$G_p = G_0 + \Delta \tag{13}$$

- Multiplicative uncertainties

$$G_p = [I + \Delta]G_0 \tag{14}$$

Were:

- G_0 represents the nominal model
- Δ perturbation block

For the resistor the following upper linear fractional transformations can be defined:

$$F_{uADDITIVE}(R,\Delta) = R_0 + 1\cdot\Delta\cdot(1-0\cdot\Delta)^{-1}\cdot 1 \tag{15}$$
$$F_{uMULTIPLICATIVE}(R,\Delta) = R_0 + P_R\cdot\Delta\cdot(1-0\cdot\Delta)^{-1}R_0 \tag{16}$$

In interconnected matrix form:

$$G_{ADDITIVE} = \begin{bmatrix} 0 & 1 \\ 1 & R_0 \end{bmatrix}, \; G_{MULTIPLICATIVE} = \begin{bmatrix} 0 & R_0 \\ P_R & R_0 \end{bmatrix} \tag{17}$$

Since in the model the inductivity appears always in the denominator, the entire fraction was submitted for upper linear fractional transformation. For 1/L the following upper linear fractional transformations were defined:

$$F_{uADDITIVE}(1/L,\Delta) = 1/L_0 + 1\cdot\Delta\cdot(1-0\cdot\Delta)^{-1}\cdot 1 \tag{18}$$
$$F_{uMULTIPLICATIVE}(1/L,\Delta) = 1/L_0 + -P_L\cdot\Delta\cdot(1+P_L\cdot\Delta)^{-1}1/L_0 \tag{19}$$

In interconnected matrix form:

$$G_{ADDITIVE} = \begin{bmatrix} 0 & 1 \\ 1 & 1/L_0 \end{bmatrix}, \; G_{MULTIPLICATIVE} = \begin{bmatrix} -P_L & 1/L_0 \\ -P_L & 1/L_0 \end{bmatrix} \tag{20}$$

The generalized system with uncertainties is given by[4]:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1w(t) + B_2u(t) \\ z(t) &= C_1x(t) + D_{11}w(t) + D_{12}u(t) \\ y(t) &= C_2x(t) + D_{21}w(t) + D_{22}u(t) \end{aligned} \tag{21}$$

Were:

- The state matrix A remains the same
- Input matrix B_2 is the same with the B matrix presented in the small signal model. The B_2 has to be chosen accordingly to the selected input variable.

- Output matrix C_2 is the same with C presented in the small signal model. C_2 has to be chosen accordingly with the desired output variable.
- Matrix D_{22}=D from small signal model, for this case initialized with 0
- Matrixes B_1, C_1, D_{11}, D_{12} and D_{21} will have the following forms accordingly to the selected uncertainty type.

Additive uncertainty:

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -R_0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -R_0 & -1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1/L_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1/L_0 \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, D_{21} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \qquad (22)$$

Multiplicative uncertainty:

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ P_L & R_0 P_L & -P_R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_L & R_0 P_L & -P_R \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1/L_0 & 0 & 0 & 0 \\ 0 & 1/L_0 & 0 & 0 \\ 0 & R_0/L_0 & 0 & 0 \\ 0 & 0 & 1/L_0 & 0 \\ 0 & 0 & 0 & 1/L_0 \\ 0 & 0 & 0 & R_0/L_0 \end{bmatrix} \qquad (23)$$

$$D_{11} = \begin{bmatrix} -P_L & 0 & 0 & 0 & 0 & 0 \\ 0 & -P_L & 0 & 0 & 0 & 0 \\ 0 & 0 & -R_0 P_L & 0 & 0 & 0 \\ 0 & 0 & 0 & -P_L & 0 & 0 \\ 0 & 0 & 0 & 0 & -P_L & 0 \\ 0 & 0 & 0 & 0 & -R_0 P_L & 0 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, D_{21} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$

IV. SIMULATION RESULTS

Simulations have been carried out to test the accuracy of the model. To obtain the equivalent inductivity and resistivity of the inductor, finite element methods were applied. For one kilogram iron heating equipment the following load variations were obtained:

R varies between 0.46-0.76 Ω

L varies between 24.5-33.5 μH

For additive uncertainties based on the R and L variations the following values were defined: R_0=0.46Ω, L_0=24.5μH, δ_R=[0…0.3Ω], δ_L=[0…9μH].

For multiplicative uncertainties: R_0=0.61Ω, L_0=29.3μH, p_R=0.3, p_L=0.14, δ=[-1…1]

The model G and Δ was initialized accordingly to the desired input and output variables and the type of uncertainty modeled.

To simulate the behavior of the system using additive uncertainties, the input variable was chosen the duty cycle of the power transistors and for output variable the load current.

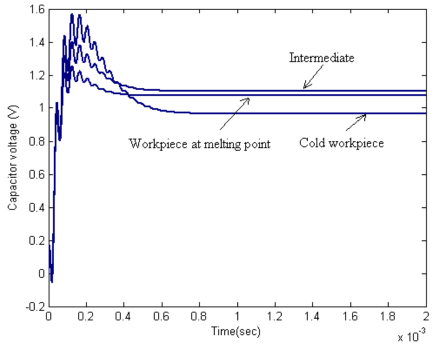

Figure 8. System step response for additive uncertainties.

In figure 8 the step response of the new model with additive uncertainties is presented. The input variable is increased by 5%, for which the system begins a 0.6milliseconds transient. Three cases were selected using the perturbation blocks which are: workpiece at room temperature, intermediate and melting point.

To simulate the behavior of the inverter using the model with multiplicative uncertainties input variable is considered: pulsation of the power switches and output variable: capacitor voltage.

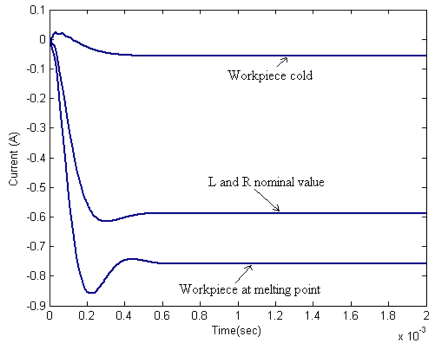

Figure 9. System step response for multiplicative uncertainties.

In figure 9 the 50Hz step response of the new model is presented for three cases: workpiece at room temperature, at L_0 and R_0 and at melting point. The transient takes about the same time for both systems.

V. CONCLUSIONS

The newly developed model which includes the parameter variations as uncertainties, offers the advantage of easy parameter selection for simulation. Advanced controllers can be designed using H_∞ and H_2 design algorithms with the newly presented model.

VI. ACKNOWLEDGMENT

This paper was supported by the project "Doctoral studies in engineering sciences for developing the knowledge based society-SIDOC", contract no. POSDRU/88/1.5/S/60078, project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

REFERENCES

[1] L. Grajales, F.C. Lee *Control System Design and Small-Signal Analysis of a Phase-Shift-Controlled Series-Resonant Inverter for Induction Heating,* Power Electronics Specialists Conference, PESC 1995, Atlanta, pages 450-456

[2] M. Tomse, N.D. Trip, A., Schiop, C. Gordan, *Modeling and Simulation of a series Resonant Inverter,* International Conference

on Computer as a Tool, EUROCON 2007, Warsaw, pages 1512-1517
[3] M. Iordache, L. Dumitru, *Modern theory of electric circuits vol.2*(in Romanian), Editura All, Bucuresti
[4] A.E. Sluhotkii, S.E. Riskin, *Inductors for induction heating*(in Romanian), Editura tehnica, Bucuresti, 1982, Romania
[5] J.M. Espi, A.E. Navarro, J. Maicas, J. Ejea, S. Casans, *Control Circuit Design of the L-LC Resonant Inverter for Induction heating,* Power Electronics Specialists Conference, PESC2000, pp.1430-1435
[6] K. Zhou, *Esentials of robust control*, Prentice-Hall, New Jersey 2008
[7] E.H. Dulf, *Robust control* (in Romanian), Editura Mediamira, 2007, Cluj Napoca, Romania
[8] D.W. Gu, P. Hr. Petrov, M.M. Konstantinov, *Robust control design with Matlab*, Springer Verlag, London
[9] H. Toivonen, Robust control methods, Abo Akademi University, Turku, 1998 Finland