DOUBLY ITERATIVE DECODING OF SPACE-TIME TURBO CODES USING LOG-APP ALGORITHM

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Abstract: In this paper are examined the performances of a low complexity doubly iterative decoder for space-time turbo codes on a quasi-static fading channel, using a Log-APP (logarithmic a posteriori probability) compared to a Max-Log-APP (maximum Log-APP) algorithm decoding algorithm for turbo codes. The decoder uses preliminary soft values of the coded symbols, obtained after a limited number of turbo iterations, in order to reduce the spatial interference of the received signal. In the coding scheme, the core of the iterative decoding structure is a soft-input soft-output APP module. The obtained frame error rate (FER) and bit error rate (BER) performances are compared to those achieved by applying a simple scaling coefficient to the extrinsic information, when a Max-Log-APP algorithm is used.

Keywords: *space-time turbo codes, doubly iterative decoder, Log-APP algorithm, Max-Log-APP algorithm, BER/FER performances.*

I. INTRODUCTION

The spectral efficiency of wireless systems is increased by using multiple antennas transmission techniques [1]. Since the complexity of the optimal decoder is increased exponentially with the modulation size and the antenna number, it is important to design receiver interfaces which obtain near-optimal performance with a moderate complexity.

Among the error correcting codes, turbo codes with iterative decoding have recently become an area of maximum interest. These codes have been discovered by Berrou, Glavieux and Thitimajshima in 1993 [2]. It was shown that with an iterative demodulation-decoding approach, for quasi-static fading channels and perfect channel state information available at the receiver, a performance of 2 to 3 dB away from channel outage probability is obtained. Since the large numbers of antennas increases the receiver complexity, a spatial interference canceling scheme is used, ensuring a good compromise between complexity and performance [3]. Biglieri presents a block scheme for a doubly iterative receiver with minimum mean square error (MMSE) algorithm where the turbo decoder uses the Max-Log-APP algorithm [4].

The concatenated coding scheme admits a suboptimum decoding scheme based on the iterative use of an APP algorithm applied to each constituent code. In [5] was shown that the Log-MAP algorithm brings a 0.2 to 0.4 dB improvement compared to the Max-Log-MAP algorithm. That's why, in this article is described a soft-input soft-output module that implements the APP algorithm in simple logarithmic form, used for the doubly iterative decoding of a concatenated coding scheme, in order to see the performances improvement for scheme from [4].

As we will see in Section III, the Log-APP algorithm brings an improvement in the FER and BER performances, but the complexity of the turbo decoder increases. The models for the transmitter and the receiver are presented in Section II, and Section III presents the Log-APP algorithm for turbo decoder leading to Max-Log-APP algorithm. Simulation results are given in Section IV and Section V concludes the paper.

II. SYSTEM MODEL

It is considered a mobile communication system with N_t transmit antennas and N_r receive antennas. The information bits are turbo-coded with coding rate R_c and block size of $N_t \cdot N$ where N is the number of successive transmissions from the transmit antennas corresponding to a codeword. The encoded vector is obtained from the interleaved bits which are serial to parallel converted and mapped into a given signal constellation.

The signal at the modulator output, $x_{i,t}$, is transmitted by antenna i, $1 \le i \le N_t$, at each time instant, has the average energy of $1/N_t$ and is chosen from a bidimensional constellation of size 2^M. In this paper is used the QPSK (quaternary phase-shift keying) modulation, i.e. M=2. All signals are simultaneously transmitted from a different transmission antenna and have the same transmission period, T. The modulated signals sent on the channel are grouped in the space-time codeword matrix $\boldsymbol{X} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N)$.

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We denote by $\boldsymbol{x}_t=\boldsymbol{f}(\boldsymbol{c}_t)$ the modulation function which transforms the $M \cdot N_t$ components of the column vector of the coded symbols from the turbo decoder output, at time t, $\boldsymbol{c}_t = (c_{1,t}, \ ..., \ c_{M \cdot N_t, t})^T$ into the column vector $\boldsymbol{x}_t = (x_{1,t}, \ ..., \ x_{N_t, t})^T$. As the modulator can be independent for each of the N_t antennas, this function can also be written as

$$\boldsymbol{x}_{i,t}=f_i(\boldsymbol{c}_{i,t}), \ i \in \{1,\ldots,N_t\}, \ t \in \{1,\ldots,N\} \quad (1)$$

where $f_i(\cdot)$ is the modulation function corresponding to modulator i, and

$$\mathbf{c}_{i,t} = (c_{(i-1)M+1,t}, \ldots, c_{iM,t})^T \quad (2)$$

Furthermore, we can write

$$\boldsymbol{x}_t = \boldsymbol{f}(\boldsymbol{c}_t) = \left(f_1(\boldsymbol{c}_{1,t}), \ldots, f_{N_t}(\boldsymbol{c}_{N_t,t}) \right) \quad (3)$$

The received signal is a noisy superposition of the transmitted signals, corrupted by Rayleigh fading. $\alpha_{i,j}$ is the path gain from the transmit antenna i, $1 \le i \le N_t$, to the receive antenna j, $1 \le j \le N_r$.

The fading is assumed to be block Rayleigh fading and the path gains can be modeled by complex independent Gaussian random variables with zero mean and variance 0.5 for each dimension. The path gains are constant for L symbols corresponding to ηL information bits and independent from one L-symbols block to another. One codeword of the turbo-code contains $F= N_t \cdot N / (R_c \cdot M \cdot L)$ distinct blocks with constant fading.

The signal $y_{t,j}$ received by antenna j at time t is

$$y_{t,j} = \sum_{i=1}^{N_t} \alpha_{i,j} x_{t,i} + z_{t,j} \quad (4)$$

The noise samples, $z_{t,j}$, are modeled as independent realizations of a complex Gaussian random variable with variance $N_0/2$ for each dimension. Equivalently, for $\lambda = 1,\ldots,F$, were F is the number of distinct blocks with constant fading, meaning the number of independent fading states of equal lengths given by a single codeword of length N (it is assumed that F divides N), we have from [4] :

$$\boldsymbol{Y}_\lambda = \boldsymbol{H}_\lambda \cdot \boldsymbol{X}_\lambda + \boldsymbol{Z}_\lambda \quad (5)$$

In Figure 1 is presented the transmitter block scheme, performing a coded modulation with bit interleaving [6] and antenna diversity. The receiver block scheme [4] is given in Figure 2 and uses MMSE iterative algorithm. The used turbo decoding algorithm is the Log-APP described in the next section.

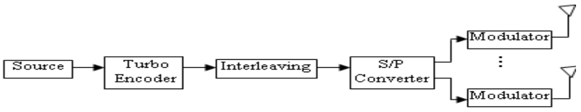

Figure 1. Transmitter block scheme

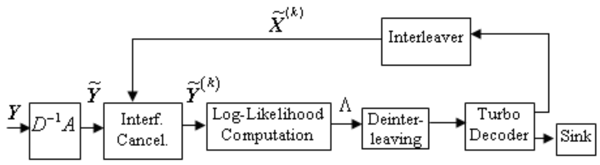

Figure 2. Receiver block scheme

For a real time operation, the spatial interference canceling block must present a delay block of the values received from the channel due to the delay of the soft estimates in the turbo decoder.

The receiver uses a linear MMSE interface, consisting in the linear filter modeled by matrix $\boldsymbol{A}$ which minimizes the mean square error $\mathrm{E}\|\boldsymbol{AY}\text{-}\boldsymbol{X}\|^2$ where $\|\square\|$ means Frobenius norm :

$$\boldsymbol{A} = \left(\boldsymbol{H}^+ \cdot \boldsymbol{H} + \frac{N_0}{E_s} \cdot \boldsymbol{I}_{N_t} \right)^{-1} \cdot \boldsymbol{H}^+ \quad (6)$$

The normalized filtered signal, explained in [4], is

$$\tilde{Y} = D^{-1} A Y = X + LX + GZ \quad (7)$$

where $\boldsymbol{D} = diag(\boldsymbol{AH})$, $\boldsymbol{L} = \boldsymbol{D}^{-1}\boldsymbol{AH} - \boldsymbol{I}_{N_t}$ and $\boldsymbol{G} = \boldsymbol{D}^{-1}\boldsymbol{A}$, so that the matrix $\boldsymbol{L}$ has zeros on its main diagonal. It is required that the matrix D^{-1} exists, meaning that D has no zeros on its main diagonal.

The soft estimates $\hat{x}_{i,t}$ provided back to the interference canceling block are obtained considering the assumption above concerning the energy of a transmitted symbol. The estimates can be expressed as a function of the coded bits probability or of their logarithm likelihood ratios

$$\hat{x}_{i,t} = \frac{1}{\sqrt{2 \cdot N_t}} \left[\left(2 \cdot P(c_{2i-1,t} = 1) - 1 \right) + j \cdot \left(2 \cdot P(c_{2i,t} = 1) - 1 \right) \right] =$$

$$= \frac{1}{\sqrt{2 \cdot N_t}} \left[\tanh\left(\frac{L(c_{2i-1,t})}{2} \right) + j \cdot \tanh\left(\frac{L(c_{2i,t})}{2} \right) \right] \quad (8)$$

The filtered signal is sent to the interference cancellation block which outputs a txN matrix $\tilde{Y}^{(k)}$. At iteration k=0, is set $\tilde{Y}^{(0)} = \tilde{Y}$ and for k>0

$$\tilde{Y}^{(k)} = \tilde{Y} - L\hat{X}^{(k-1)} \quad (9)$$

where $\hat{X}^{(k-1)}$ comprises the extrinsic soft estimates of the transmitted bits.

After interference cancellation, at iteration k, the normalized filtered signal can be written as :

$$\tilde{Y}^{(k)} = X + L\left(X - \hat{X}^{(k-1)}\right) + GZ = X + Q^{(k)} \quad (10)$$

where $Q^{(k)} \cong L\left(X - \hat{X}^{(k-1)}\right) + GZ$ accounts for the noise and residual spatial interference at iteration k.

At every iteration is set a covariance matrix which yields the suboptimal log-likelihood ratios :

$$K^{(k)} \cong \frac{1}{N} \mathrm{E}\left[Q^{(k)} Q^{(k)+} \middle| H \right]. \quad (11)$$

The suboptimal simplified log-likelihood ratios of the coded bits are

$$\Lambda\left(c_{i,t}\right)=\ln\frac{\sum_{x=f_a(c_a):c_{ai}=1}\exp\left\{-\frac{\left|\tilde{y}_{a,t}^{(k)}-x\right|^2}{\left(K^{(k)}\right)_{aa}}\right\}}{\sum_{x=f_a(c_a):c_{ai}=0}\exp\left\{-\frac{\left|\tilde{y}_{a,t}^{(k)}-x\right|^2}{\left(K^{(k)}\right)_{aa}}\right\}} \quad (12)$$

where $a=1+\left\lfloor\frac{i-1}{M}\right\rfloor$ and matrix $\mathbf{K}^{(k)}$ can be approximated :

$$K^{(k)}\cong\tilde{K}^{(k)}=\frac{1}{N}\tilde{Y}^{(k)}\tilde{Y}^{(k)H}-E_S\cdot I_{N_t} \quad (13)$$

This matrix and its approximation must be nonnegative definite. Otherwise, the matrix is set to the value assumed in the absence of spatial interferences, namely $\boldsymbol{GG}^{+}N_0$ (this can occur when the algorithm is close to convergence and the interferences are almost eliminated). The initial matrix value, $\boldsymbol{K}^{(0)}$, is

$$\boldsymbol{K}^{(0)}=\boldsymbol{LL}^{+}E_s+\boldsymbol{GG}^{+}N_0 \quad (14)$$

III. LOG-APP DECODING ALGORITHM

The used turbo decoding algorithm is Log-APP described in [7]. The decoding algorithm is particularized for a recursive convolutional systematic component code with rate 1/2.

The algorithm is based on the soft-input soft-output component decoder model. The SISO module is a four-port device, with two inputs and two outputs. The inputs are the probability distributions of the information and code symbols, and forms as outputs an update of these distributions based upon the code constrains.

The output probability distributions, given in [8], at time k for check parity and systematic bits are determined as

$$\pi_k\left(c_p;O\right)=\ln\left[\sum_{e:c(e)=c}\exp\left\{\alpha_{k-1}\left[s^S(e)\right]+\pi_k\left[u(e);I\right]+\beta_k\left[s^E(e)\right]\right\}\right]+h_c \quad (15)$$

$$\pi_k\left(u;O\right)=\ln\left[\sum_{e:u(e)=u}\exp\left\{\alpha_{k-1}\left[s^S(e)\right]+\pi_k\left[c_p(e);I\right]+\beta_k\left[s^E(e)\right]\right\}\right]+h_u \quad (16)$$

where $h_c=\ln H_c$, $h_u=\ln H_u$. H_c and H_u are normalization constants for the output probability distributions whose natural logarithms are denoted with π_k.

$s^S(e)$ and $s^E(e)$ in the above relations represent the initial and final state on branch *e* in the trellis section. $u(e)$ represents the systematic bit and $c(e)$ the coded one from branch *e*.

Also, $\alpha_k(\cdot)$ and $\beta_k(\cdot)$ are obtained by forward and backward recursions, respectively:

$$\alpha_k(s)=\ln\left[\sum_{e:s^E(e)=s}\exp\left\{\alpha_{k-1}\left[s^S(e)\right]+\pi_k\left[u(e);I\right]+\pi_k\left[c_s(e);I\right]+\pi_k\left[c_p(e);I\right]\right\}\right],\quad k=1,\ldots,n \quad (17)$$

$$\beta_k(s)=\ln\left[\sum_{e:s^S(e)=s}\exp\left\{\beta_{k+1}\left[s^E(e)\right]+\pi_{k+1}\left[u(e);I\right]+\pi_{k+1}\left[c_s(e);I\right]+\pi_{k+1}\left[c_p(e);I\right]\right\}\right],\ k=n-1,\ldots,0 \quad (18)$$

I and O stand for input and output, respectively.

The initial values for terminated trellis are

$$\alpha_0(s)=\begin{cases}0, & s=S_0\\ -\infty, & otherwise\end{cases} \quad (19)$$

$$\beta_n(s)=\begin{cases}0, & s=S_0\\ -\infty, & otherwise\end{cases} \quad (20)$$

For unterminated trellis we have

$$\beta_n(s)=0,\ s=S_0,\ldots,S_{N_S-1}. \quad (21)$$

where N_S is the number of states in the code trellis.

The main difficulty of the previous recursions consists in evaluating the logarithm of a sum of exponentials as

$$a=\ln\left[\sum_{i=1}^{L}\exp\{a_i\}\right] \quad (22)$$

If we approximate (22) as

$$a=\ln\left[\sum_{i=1}^{L}\exp\{a_i\}\right]\cong a_M \quad (23)$$

with $$a_M=\max_i a_i,\ i=1,\ldots,L \quad (24)$$

we obtain the Max-Log-APP algorithm. Recursions in (17) and (18) become

$$\alpha_k(s)=\max_{e:s^E(e)=s}\{\alpha_{k-1}\left[s^s(e)\right]+\pi_k\left[u(e);I\right]+\pi_k\left[c_s(e);I\right]+\pi_k\left[c_p(e);I\right]\} \quad (25)$$

$$\beta_k(s)=\max_{e:s^S(e)=s}\{\beta_{k+1}\left[s^E(e)\right]+\pi_{k+1}\left[u(e);I\right]+\pi_{k+1}\left[c_s(e);I\right]+\pi_{k+1}\left[c_p(e);I\right]\} \quad (26)$$

and

$$\pi_k\left(c_p;O\right)=\max_{e:c(e)=v}\{\alpha_{k-1}\left[s^s(e)\right]+\pi_k\left[u(e);I\right]+\beta_k\left[s^E(e)\right]\}+h_c \quad (27)$$

$$\pi_k\left(u;O\right)=\max_{e:u(e)=u}\{\alpha_{k-1}\left[s^s(e)\right]+\pi_k\left[c_p(e);I\right]+\beta_k\left[s^E(e)\right]\}+h_u \quad (28)$$

For increased precision, which introduces a higher complexity, we can use the following recursion algorithm for the quantity in (22):

$$a^{(1)}=a_1$$

$$a^{(i)}=\max\left(a^{(i-1)},a_i\right)+\ln\left[1+\exp\left(-\left|a^{(i-1)}-a_i\right|\right)\right],\ i=2,\ldots,L$$

$$a=a^{(L)}.$$

In this case we obtain the Log-APP algorithm. We will use the identity

$$a = \ln\left[\sum_{i=1}^{L} \exp\{a_i\}\right] = \max_i \{a_i\} + \delta(a_1,\dots,a_L) = \max_i{}^*\{a_i\} \quad (29)$$

where $\delta(a_1,\dots,a_L)$ is a correction term which can be determined using a look-up table (i.e. the values are obtained through approximations of the correction term, for example like in [9]). This can be neglected in the Max-Log-APP algorithm. In our simulations we use the true value of the correction term without any other approximation.

Noting the LLR ratio of bit Z at time k by

$$L_k[Z;\cdot] = \ln\frac{P_k[Z=1]}{P_k[Z=0]} = \pi_k[Z=1] - \pi_k[Z=0], \quad (30)$$

the extrinsic bit information for the uncoded bit u and for the coded bit c_p bits are :

$$L_k(c_p;O) = \max^*_{e:c_p(e)=1}\left\{\alpha_{k-1}\left[s^S(e)\right] + u(e)L_k\left[u(e);I\right] + \beta_k\left[s^E(e)\right]\right\} - \max^*_{e:c_p(e)=0}\left\{\alpha_{k-1}\left[s^S(e)\right] + u(e)L_k\left[u(e);I\right] + \beta_k\left[s^E(e)\right]\right\} \quad (31)$$

$$L_k(u;O) = \max^*_{e:u(e)=1}\left\{\alpha_{k-1}\left[s^S(e)\right] + c_p(e)L_k\left[c_p(e);I\right] + \beta_k\left[s^E(e)\right]\right\} - \max^*_{e:u(e)=0}\left\{\alpha_{k-1}\left[s^S(e)\right] + c_p(e)L_k\left[c_p(e);I\right] + \beta_k\left[s^E(e)\right]\right\} \quad (32)$$

Values given by (31) and (32), scaled with a sub-unitary coefficient [8], are used to obtain the input of the spatial interference canceling block from Figure 2.

IV. SIMULATION RESULTS

The simulations were performed considering the same scenario as in [4], using QPSK modulation (M=2), a random interleaver of length 2080 in the turbo code of global rate 1/2, forward and feedback generator polynomials of (5, 7) in octal form, and a random interleaver between the turbo encoder and the serial to parallel convertor.

A spectral efficiency of $\eta = 16$ bits/sec/Hz, for 16 transmit and 16 receive antennas were also considered. The space-time codeword is a matrix of 130 columns.

The FER and BER performances obtained with Log-APP decoding algorithm are compared to the best FER and BER performances from the case where Max-Log-APP with scaling of extrinsic information [8] is used. In both cases, the turbo decoder performs 10 iterations with a genie stopper type stop criterion.

k=0, k=1 or k=4 iterations were used to cancel spatial interferences. For k=0 we considered the scaling coefficient s=0.9, for k=1, s=0.8 and for k=4 , s=0.75, as were been determined in [8].

Also, the space time codeword is a matrix of 130 columns and F, the number of distinct blocks with constant fading, is equal to 1, 5 and 26.

Simulation results showing FER and BER performances versus signal-to-noise ratio per bit (E_b/N_0) are given in Figure 3 and Figure 4 for F=1, using k=0, 1 and 4. In Figure 5 and Figure 6 are given the FER and BER performances for F=5, and for F=26 the performances are given in Figure 7 and Figure 8.

Altghouh the BER and FER performances are better as k increases, from simulations, we observe that for k=4, the doubly iterations introduce relatively more errors compared to k=0 and k=1, and the coding gain obtained for k=4, is smaller than the one achieved when using k=0, and k=1. We observe a small improvement in FER and BER performances when F is increased, because if the noise affects a block, all the contained bits are affected and if a block is correctly demodulated, none of the contained bits is affected.

For F=1 and k=0, for FER=10^{-2}, a 0.2 dB coding gain is obtained and for BER=10^{-4}, is achieved a 0.3 dB coding gain. For F=5 and k=0, for FER=10^{-2}, was obtained a coding gain of 0.45 dB and for BER=10^{-4}, a 0.4 dB coding gain. For F=26 and k=0, for FER=10^{-2}, a coding gain of 0.45 dB is obtained and for BER=10^{-4}, a 0.4 dB coding gain is achieved.

Also, for F=1 and k=1, for FER=10^{-3}, a coding gain of 0.2 dB is obtained and for BER=10^{-5}, is achieved a 0.2 dB coding gain. For F=5 and k=1, for FER=10^{-3}, a coding gain of 0.3 dB is obtained and for BER=10^{-5}, a 0.4 dB coding gain is achieved. For F=26 and k=1, for FER=10^{-3}, a coding gain of 0.35 dB is obtained and for BER=10^{-5}, is achieved a 0.4 dB coding gain.

For k=4, for F=1, F=5 and F=26 the best improvement of the performances when using Log-APP algorithm compared to Max-Log-APP algorithm is 0.15 dB for FER and 0.2 dB for BER.

V. CONCLUSIONS

Knowing that for turbo codes a Log-APP algorithm brings a 0.2 to 0.4 dB improvement for BER performances compared to the Max-Log-APP algorithm, in this paper was used the Log-APP algorithm in a doubly-iterative decoding receiver, in order to see the improvement of the performances, compared to the Max-Log-APP algorithm. A small disadvantage is that the Log-APP algorithm introduces a higher complexity of the turbo decoder.

The FER and BER performances obtained for Log-APP algorithm are compared to the best FER and BER performances obtained for the same doubly-iterative decoding scheme that uses the Max-Log-APP algorithm [4], where the simulations were performed for a scaling coefficient of extrinsic information of s=0.9 for k=0, s=0.8 for k=1, and s=0.75 for k=4, as is determined in [8], and also for F=1, F=5 and F=26. We observe a small FER and BER improvement when F increases.

For F>1, the scheme must be modified by adding F filters and F interference canceling blocks, which operates in parallel for every fading block, also a serial-parallel converter at input of filters and a parallel-serial converter at

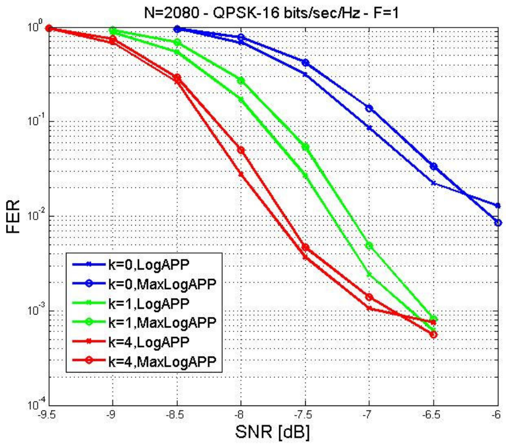

Figure 3. FER performances for the iterative receiver with k=0, 1 and 4 for F=1

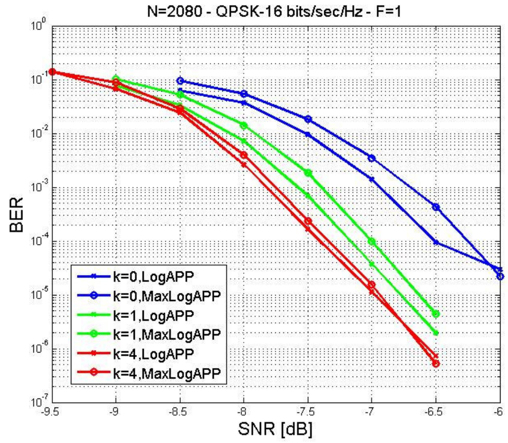

Figure 4. BER performances for the iterative receiver with k=0, 1 and 4 for F=1

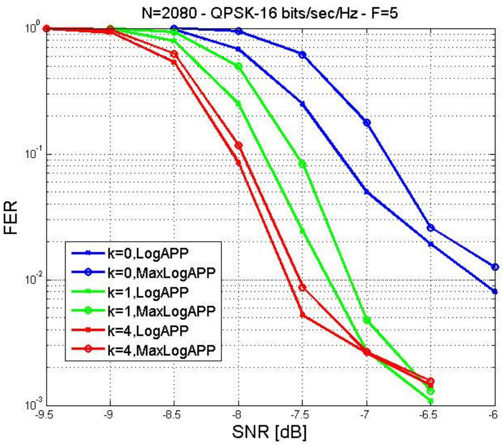

Figure 5. FER performances for the iterative receiver with k=0, 1 and 4 for F=5

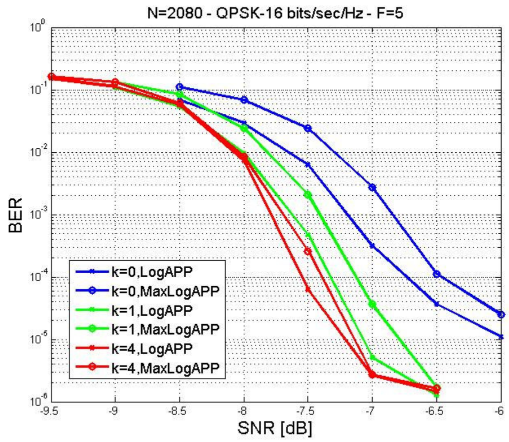

Figure 6. BER performances for the iterative receiver with k=0, 1 and 4 for F =5

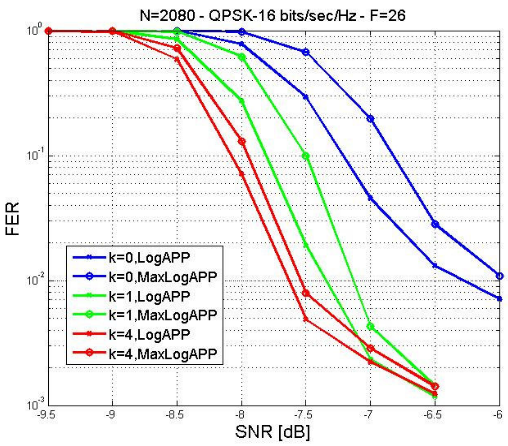

Figure 7. FER performances for the iterative receiver with k=0, 1 and 4 for F=26

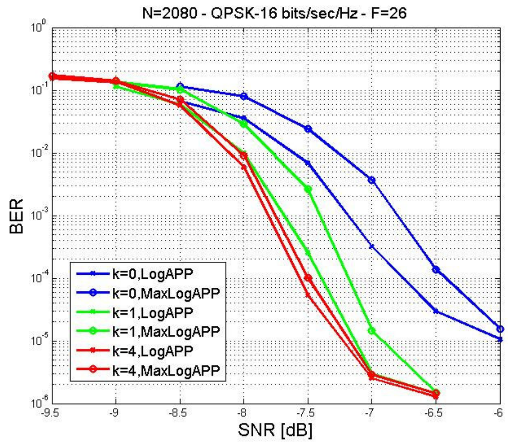

Figure 8. BER performances for the iterative receiver with k=0, 1 and 4 for F =26

output of interference canceling blocks. So, the soft estimates delay does not affect the scheme functioning compared to the case where F=1.

For F=1, F=5 and F=26, and for k=0 and k=1 iteration to cancel the spatial interferences, a 0.45 dB coding gain improvement is obtained for FER. For BER, a 0.4 dB coding gain improvement is obtained. For k=4 the improvement is 0.15 dB for FER and 0.2 dB for BER.

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