WAVE ITERATIVE PROCESS STUDY OF DOUBLE DIPOLE SCATTERING

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Abstract: The paper presents an application of the Wave Iterative Process (WIP) in the case of the study of the scattering of an electromagnetic plane wave by a system composed of two arbitrary placed metallic dipoles. The case of one dipole is studied at first, a second one being introduced into the scattering system later. Mutual influence is also taken into account, current density results are presented, and the possibilities of enlarging the number of dipoles in the scattering structure are also examined.

Keywords: *Wave Concept, Iterative Process, metallic dipole, scattering.*

I. INTRODUCTION

Recent years have witnessed a great development of the necessary tools able to numerically model, simulate the performance of, and design complex electromagnetic systems. An important class of competing computational electromagnetics approaches is based on the use of iterative methods in solving scattering problems [1] [2], due to the simplicity and high efficiency of these methods. Either stationary methods issued from the need of mathematical matrix manipulation [3]-[6], or issued from classic integral formulations [7]-[10], either non-stationary methods that have evolving formulation during the process [11] [12], iterative methods are more and more often employed in the study of electromagnetic scattering problems.

However, the major disadvantages of the classic methods, mainly the restricted geometrical dimension range of applicability, remain unchanged for their iterative versions. As an example, we can mention the Iterative Physical Optics [13] [14] that is limited to large scattering objects in terms of wavelength.

Among the most recent and the most efficient iterative methods, the Wave Iterative Process (WIP) was developed at first as an instrument for the study of in-guide and planar circuits scattering problems [15].

Due to its combined spectral – real domain formulation issued from the description of boundary conditions and modal behavior and due to its applicability to all range of geometrical dimensions of the scattering obstacle, the WIP is highly recommended for its extension to free space electromagnetic diffraction studies.

Already successfully tested in many circuit designs [16]-[19], the WIP has a great potential of development to solving complex shaped structures and multiple obstacle electromagnetic problems.

The paper presents a brief review of the principles of the WIP, wave definitions and scattering operators. The WIP is then applied in the case of an arbitrary plane wave incidence upon a metallic dipole, the study being extended to a scattering system composed of two arbitrarily oriented dipoles. The ways of extension of the study to several spangles are examined in the final part.

II. THEORY

Considered the incidence of a wave on the surface *S* of an obstacle as depicted in Fig 1, the basic principle of the WIP is the concept of waves, the incident one *A*, and the reflected one *B*, defined from the tangent electric field *E*, and the incident magnetic tangent field *H*.

For simplicity reasons, instead of *H*, the "current density" vector *J* is preferred. The following relation defines the *J* vector:

$$\vec{J} = \vec{H} \times \hat{n} \tag{1}$$

where $\hat{n}$ is the normal versor to the scattering surface, defined in each point.

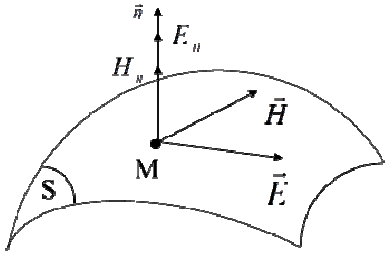

Figure 1. Scattering surface with vector definitions.

The waves *A* and *B* are defined by (2):

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$$\begin{cases} \vec{A} = \dfrac{1}{2\sqrt{Z_0}}(\vec{E} + Z_0\vec{J}) \\ \vec{B} = \dfrac{1}{2\sqrt{Z_0}}(\vec{E} - Z_0\vec{J}) \end{cases} \quad (2)$$

where Z_0 is an arbitrary parameter.

The analytic expression of the iterative process is issued from the continuity and border conditions on the electromagnetic field. In the case of free space scattering, the iterative process is described by (3):

$$\begin{cases} \vec{B}_n + \vec{B}_0 = \hat{S}(\vec{A}_{n-1} + \vec{A}_0) \\ \vec{A}_n = \hat{\Gamma}\vec{B}_n \end{cases} \quad (3)$$

where n is the number of the iteration, $\hat{S}$ is the real domain scattering operator, and $\hat{\Gamma}$ is the modal domain scattering operator.

A schematic of the current density calculation on the scatterer's surface using the WIP is presented in Fig 2.

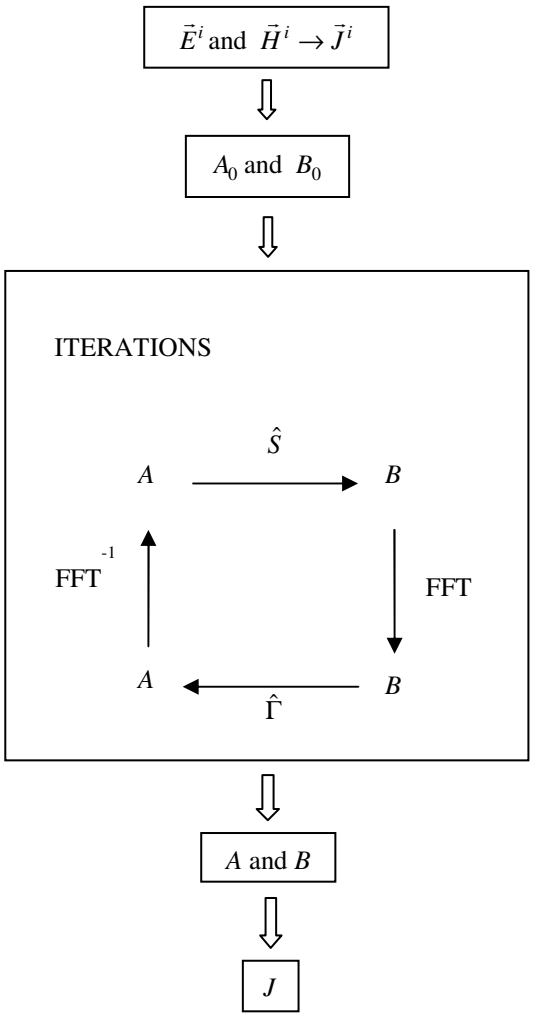

Figure 2. Schematic of the iterative process

The real domain scattering operator is given by (4):

$$\hat{S} = I_S - I_M \quad (4)$$

where I_S and I_M are the dielectric and respectively metallic domain definition functions.

The modal domain scattering operator is given by (5):

$$\hat{\Gamma} = \sum_m |f_m\rangle \frac{1 - Z_0 Y_m}{1 + Z_0 Y_m} \langle f_m| \quad (5)$$

where $\{f_m\}$ is a complete orthonormal modal base and Y_m is the modal admittance of the m^{th} mode.

A_0 and B_0 are the waves corresponding to the incident electromagnetic field on the studied structure, defined as:

$$\begin{cases} \vec{A}_0 = \dfrac{1}{2\sqrt{Z_0}}(\vec{E}^i + Z_0\vec{J}^i) \\ \vec{B}_0 = \dfrac{1}{2\sqrt{Z_0}}(\vec{E}^i - Z_0\vec{J}^i) \end{cases} \quad (6)$$

Once the iterative process accomplished, the electric field and the current density can be calculated using the inverse relations of (2), that is:

$$\begin{cases} \vec{E} = \sqrt{Z_0}(\vec{A} + \vec{B}) \\ \vec{J} = \dfrac{1}{\sqrt{Z_0}}(\vec{A} - \vec{B}) \end{cases} \quad (7)$$

The first equation of (3) is written in the real domain, the second one being written in the spectral domain, and therefore a direct and inverse fast modal (*Fourier*) transformations (FMT and FMT^{-1}) are used to pass from one to the other.

Let us consider a metallic dipole, placed along the z axis, illuminated by a plane wave with a normal incidence, as shown in Fig. 3.

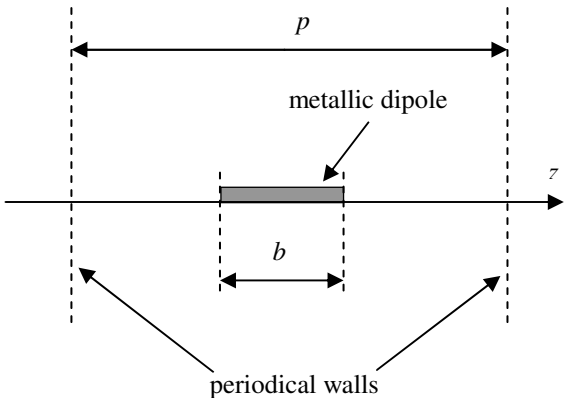

Figure 3. Metallic dipole between periodical walls.

The study of the dipole scattering would theoretically involve an infinity of points following the z axis, which is numerically impossible.

That is why we introduced a first approximation, considering the structure as periodical, the dipole being placed in the middle of the *p* period.

The metallic (electric wall) domain function is given by:

$$I_M(z)=\begin{cases}1 & if \quad z\in\left[-b/2, b/2\right]\\ 0 & elsewhere\end{cases} \tag{8}$$

I_S is the complementary function, defining the magnetic wall domain:

$$I_S(z)=\begin{cases}0 & if \quad z\in\left[-b/2, b/2\right]\\ 1 & elsewhere\end{cases} \tag{9}$$

Due to the periodicity of the structure, the expressions of the functions of the modal base are given by:

$$\left\langle f_m(z)\right| = a_m e^{-j\frac{2\pi}{p}mz} \tag{10}$$

where a_m is determined by the othonormation condition:

$$\left\langle f_m \middle| f_m \right\rangle = 1 \tag{11}$$

Its value results:

$$a_m = \frac{1}{\sqrt{p}} \tag{12}$$

The modal admittance derives from the expression of the cylindrical modes [20]:

$$Y_m = \frac{\gamma_m}{j\omega\mu}\frac{H_n^{(2)}(\gamma_m a)}{H_n^{(2)'}(\gamma_m a)} \tag{13}$$

where $H_n^{(2)}$ is the second kind *Hankel* function.

Taking into account the very small size of the radius, we can write:

$$Y_m = \frac{\gamma_m}{j\omega\mu}\frac{B_0(\gamma_m a)}{B_0'(\gamma_m a)} \tag{14}$$

where

$$\gamma_m = \sqrt{\left(\frac{2\pi m}{p}\right)^2 - k_0^2} \tag{15}$$

and B_0 traduces:

$$B_0(\gamma_m a)=\begin{cases}H_0^{(2)}(\gamma_m a) & if \quad \gamma_m \in IR\\ \frac{2j}{\pi}K_0(j\gamma_m a) & if \quad \gamma_m \in iIR\end{cases} \tag{16}$$

Considering the incident plane wave as x-polarized, the incident electromagnetic field is given by:

$$\begin{cases}E^i = E_0 e^{-jk_0 z}\\ J^i = H_0 I_M e^{-jk_0 z}\end{cases} \tag{17}$$

where I_M is the dipole definition function (8), E_0 and H_0 are the amplitudes.

The waves used for the initialization of the iterative process are given by:

$$\begin{cases}A_0 = \frac{1}{2\sqrt{Z_0}}\left(E_0 + Z_0 H_0 I_M\right)e^{-jk_0 z}\\ B_0 = \frac{1}{2\sqrt{Z_0}}\left(E_0 - Z_0 H_0 I_M\right)e^{-jk_0 z}\end{cases} \tag{18}$$

Taking into account the small size of the radius of the dipole, we expect a quite large number of iterations to be done in order to achieve the convergence of the method. This issue is investigated in section III. On the other hand, the small geometric dimensions of the structure ensure a very fast modal convergence, determining an overall very good computational cost.

In the following we shall continue the study by placing the dipole in an arbitrary inclination with respect to the incident wave. The situation is depicted in Fig. 4.

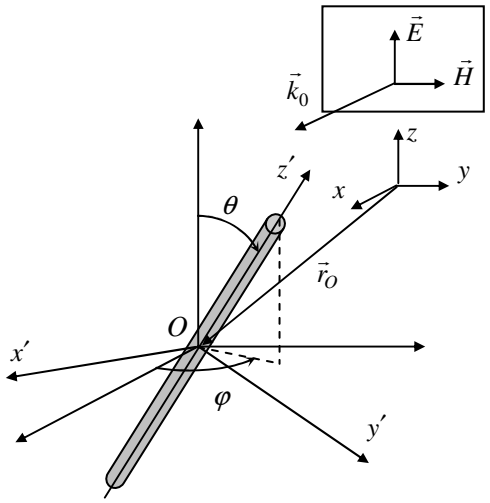

Figure 4. Arbitrarily placed metallic dipole

The rectangular coordinates system was chosen as oriented after the incident wave. The geometrical position of the dipole is described using the coordinates of the center *O* and the two inclination angles θ with respect to the *z* axis and φ with respect to the *x* axis.

The dipole is therefore entirely described by:

$$d: \quad \left(\vec{r}_O \begin{cases}x_O\\ y_O\\ z_O\end{cases} \quad \theta \quad \varphi\right) \tag{19}$$

This way of describing the geometrical position of the dipole presents the advantage of the possibility of taking into account a multiple dipole scattering structure.

Let us denote with (x', y', z') the coordinates system proper to the dipole (Fig. 4). The incident electromagnetic field is given in the new coordinates system by:

$$\vec{E}^i = E_0 e^{-jk_0 x_O} e^{-jk'_x x'} e^{-jk'_y y'} e^{-jk'_z z'} \cos\theta \cdot \vec{z}' \quad (20)$$

where the following notations are made:

$$k'_x = k_0 \cos\varphi \cos\theta \quad (21)$$

$$k'_y = k_0 \sin\varphi \cos\theta \quad (22)$$

$$k'_z = k_0 \sin\theta \quad (23)$$

Due to the small size of the radius, we can approximate (20):

$$\vec{E}^i = E_0 e^{-jk_0 x_O} e^{-jk'_z z'} \cos\theta \cdot \vec{z}' \quad (24)$$

Denoting the new amplitude:

$$E'_0 = E_0 e^{-jk_0 x_O} \cos\theta \quad (25)$$

the values of the incident electromagnetic field become similar to (17):

$$\begin{cases} E^i = E'_0 e^{-jk'_z z'} \\ J^i = H'_0 I_M e^{-jk'_z z'} \end{cases} \quad (26)$$

and the expressions of the initialization waves are given by:

$$\begin{cases} A_0 = \dfrac{1}{2\sqrt{Z_0}} \left(E'_0 + Z_0 H'_0 I_M\right) e^{-jk'_z z'} \\ B_0 = \dfrac{1}{2\sqrt{Z_0}} \left(E'_0 - Z_0 H'_0 I_M\right) e^{-jk'_z z'} \end{cases} \quad (27)$$

Once the iterative convergence of the process achieved, the current density on the surface of the dipole is calculated using the second relation of the system (7). Results are presented in section III.

In the following, we are going to enlarge the scattering structure by placing a second metallic dipole and by taking into account the influence of the radiated field of the first dipole on the second one. Let us consider a diffraction structure formed by two metallic dipoles, being arbitrary placed and having arbitrary inclinations with respect to the incident plane wave, as presented in Fig. 5. The geometrical position of the dipoles is described in the same manner as (6), that is:

$$d_1 : \begin{pmatrix} \vec{r}_{O1} \begin{cases} x_{O1} \\ y_{O1} \\ z_{O1} \end{cases} & \theta_1 & \varphi_1 \end{pmatrix}; \; d_2 : \begin{pmatrix} \vec{r}_{O2} \begin{cases} x_{O2} \\ y_{O2} \\ z_{O2} \end{cases} & \theta_2 & \varphi_2 \end{pmatrix} \quad (28)$$

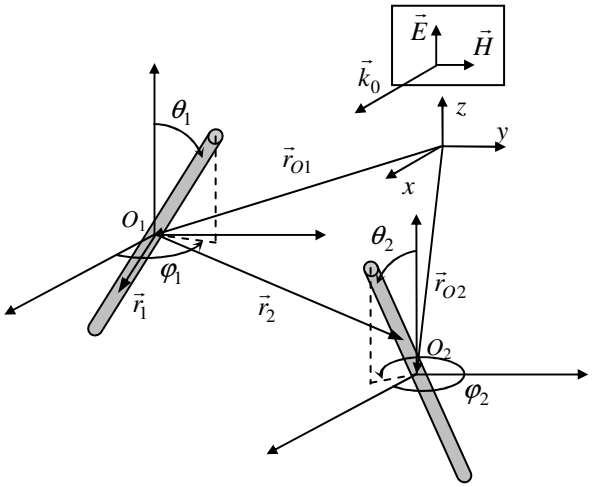

Figure 5. Two arbitrarily placed metallic dipoles

The current densities inducted by the incident plane wave on each dipole are calculated in the same manner as presented in the previous paragraph.

We are now interested in evaluating the influence of the current density on the first dipole on the second one. For this, we need to know the electric field radiated by the first dipole on the second one.

In order to evaluate the radiated electric field by the current density due to the incident wave on the first dipole, at the level of the second dipole, we shall use the vector potential $\vec{A}(\vec{r}_2)$:

$$\vec{A}(\vec{r}_2) = \frac{1}{4\pi} \int_{-b/2}^{b/2} \vec{J}_1(\vec{r}_1) \frac{e^{-jk_0|\vec{r}_2 - \vec{r}_1|}}{|\vec{r}_2 - \vec{r}_1|} dl_1 \quad (29)$$

where $\vec{J}_1$ is the current density induced by the incident wave on the length l_1 of the first dipole.

The electric field radiated by the first dipole on the second one is given by:

$$\vec{E}_{12}(\vec{r}_2) = -j\omega\mu_0 \cdot \vec{A}(\vec{r}_2) + \frac{1}{j\omega\varepsilon_0} \cdot \nabla_2 \cdot \left[\nabla_2 \cdot \vec{A}(\vec{r}_2)\right] \quad (30)$$

where the subscript 2 of the ∇ operator designates the fact that all derivation operations are made with respect to the second dipole own coordinates system.

The generated magnetic field by the current distribution on the second dipole is given by:

$$\vec{H}_{12}(\vec{r}_2) = \nabla_2 \times \vec{A}(\vec{r}_2) \quad (31)$$

Since the current density on the first dipole is the result of the iterative process, the integral operation in (29), and the derivation operations in (30) and (31) must be numerically evaluated.

Section III contains the results obtained for the initial current densities on the two dipoles, as well as for the mutual influence of dipole 1 on dipole 2, for a wavelength distance between them.

III. NUMERICAL RESULTS

Let us consider the incidence of a 10 GHz plane wave on the surface of a dipole having the following dimensions: The radius is $a = 12{,}5$ μm and the length is $b = 0{,}015$ m.

We will examine at first the iterative convergence of the method, for the case of the normal wave incidence.
Fig. 6 presents the evolution of the *A* wave with the number of iterations. As we expected, due to the very small size of the radius, a big number of iterations is necessary in order to achieve the convergence. For an accurate result, at least 250 iterations must be made.

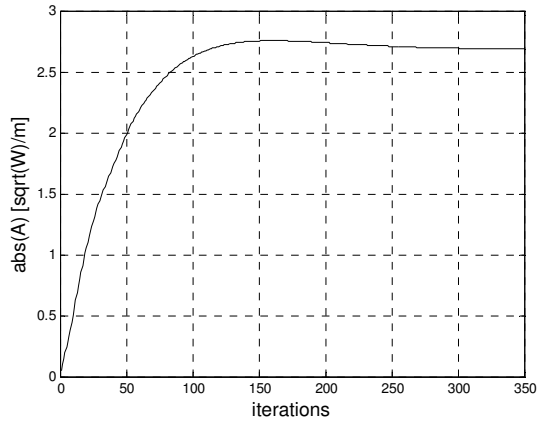

Figure 6. Iterative convergence of the A wave – normal incidence

Fig. 7 presents the study of the size of the period *p* that must be considered between two walls. The current density is evaluated in the center of the dipole, at the point $z = 0$. A period of 30 wavelengths is sufficient for a better than 1% precision.

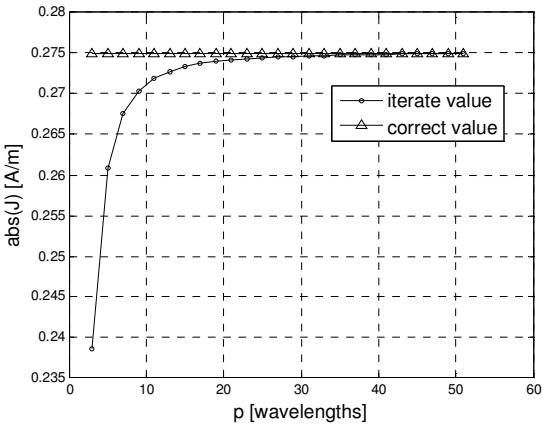

Figure 7. The influence of the walls period on the result precision – normal incidence

Fig. 8 presents the current density on the dipole length, for a normal incidence of the wave, calculated using both WIP and the Moment Method [21]. We can notice both methods give very close results.

Fig. 9 also presents the results for the case of an inclined dipole, having the following position:

$$d: \quad \left(\vec{r}_O \begin{cases} \lambda \\ \lambda \\ \lambda \end{cases} \quad 30° \quad 45° \right) \tag{32}$$

where λ is the wavelength.

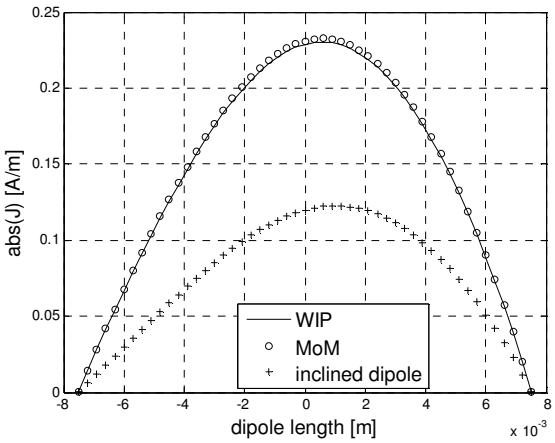

Figure 8. Current density on the dipole length

Let us now consider two metallic dipoles, given by:

$$d_1: \quad \left(\vec{r}_{O1} \begin{cases} x_{O1} = \lambda \\ y_{O1} = \lambda \\ z_{O1} = \lambda \end{cases} \quad \theta_1 = 30° \quad \varphi_1 = 45° \right) \tag{33}$$

$$d_2: \quad \left(\vec{r}_{O2} \begin{cases} x_{O2} = \left(1+1/\sqrt{3}\right)\lambda \\ y_{O2} = \left(1+1/\sqrt{3}\right)\lambda \\ z_{O2} = \left(1+1/\sqrt{3}\right)\lambda \end{cases} \quad \theta_2 = 60° \quad \varphi_2 = 30° \right) \tag{34}$$

The positions of the two dipoles were chosen so that between the two canters we have one wavelength λ.

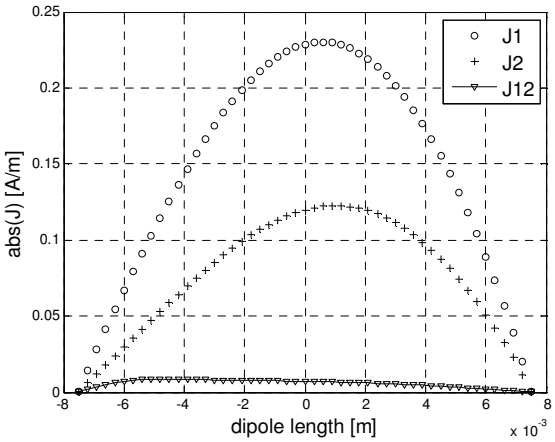

Figure 9. Current densities on two inclined dipoles and mutual influence of the first on the second one

Figure 9 presents the current densities on the two dipoles

due to the incident plane wave, as well as the influence of the first dipole on the second one.

In order to visualize the order of the influence of the first dipole on the second one, we represent in Fig. 10 the current density on the second dipole, with and without the radiation

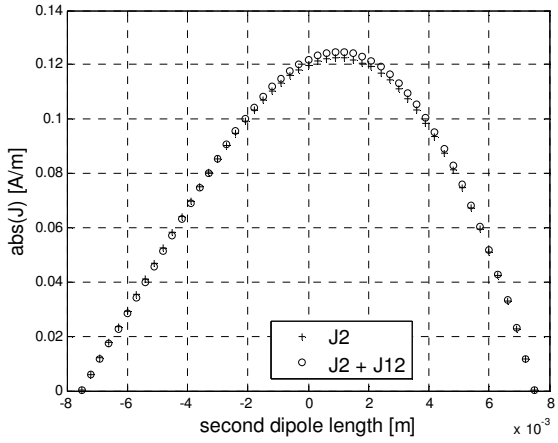

Figure 10. Current density on the second dipole with and without the mutual influence of the first one

due to the first one.

One can notice that although the mutual influence has a low level, it cannot be neglected, especially if a multiple dipole scattering structure is to be taken into study. If the distance between the dipoles is increased, the influence of the radiated field of a dipole on the other one decreases, and it can be neglected for the values greater than 5λ .

IV. CONCLUSION

An application of the WIP was realised in the case of the study of the scattering of an electromagnetic plane wave by a system composed of two arbitrary placed metallic dipoles. The case of one dipole was studied at first, a second one being introduced into the scattering system later; mutual influence was also taken into account. In the future we shall concentrate our efforts on enlarging the number of dipoles of the scattering system, as well as on the optimisation of the computational process.

ACKNOWLEDGMENT

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