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CONVERGENCE RATE AND STEADY-STATE ERROR IMPROVEMENT IN ACOUSTIC SYSTEM IDENTIFICATION USING THE COMBINATION OF LINEAR NLMS ADAPTIVE FILTERS

Ingrid Maria KOVACS, Cristian CONȚAN, Marina ȚOPA
Basis of Electronics Department, Technical University of Cluj-Napoca, Romania
ingridmariakovacs@gmail.com, {cristian.contan, marina.topa}@bel.utcluj.ro

Abstract: The paper proposes the combination method applied on adaptive linear filters in acoustic system identification. The filters are based on the Normalized Least-Mean-Square algorithm and the combination is applied on two filters of different step sizes and different filter lengths. To evaluate the proposed technique, two types of source signals are used: a white Gaussian noise and a non-stationary audio signal. The performance of the adaptive scheme is analyzed in terms of Echo Return Loss Enhancement. The efficiency of the proposed concept was compared with the one of the two sub-band decomposition model. The advantages of the proposed method were outlined in terms of convergence rate and steady-state error.

Keywords: *Acoustic system identification, adaptive filtering, adaptive combination of filters, sub-band decomposition*

I. INTRODUCTION

The typical acoustic system identification (ASI) block scheme is presented in *Figure 1* [1] [2], where a discrete source signal *x(n)* is applied to the input of the adaptive filter [3] and also to the unknown system.

The adaptation is carried out using a proper adaptive linear model ($\hat{w}(n)$) for the unknown system ($w(n)$), which roughly simulates the reverberation [4]. The aim of the ASI setup is to make a replica of the unknown system by minimizing the error signal *e(n).* This is achieved by subtracting the output of the adaptive filter *y(n),* from the output of the unknown system *d(n):*

$$e(n) = d(n) - y(n) = d(n) - \hat{\mathbf{w}}^T(n)\mathbf{x}(n), \qquad (1)$$

where the following column vectors are involved:

$$\mathbf{x}(n) = [x(n) \quad x(n\text{-}1) \quad x(n\text{-}2) \ldots x(n\text{-}i) \;\ldots\; x(n\text{-}N+1)]^T, \qquad (2)$$

$$\hat{\mathbf{w}}(n) = [\hat{w}_0(n) \quad \hat{w}_1(n) \;\ldots \hat{w}_i(n) \;\ldots\; \hat{w}_{N-1}(n)]^T, \qquad (3)$$

i= $\overline{0, N-1}$, N denotes the vector length and $(\cdot)^T$ is the transposing operation.

Usually, the Normalized Least-Mean-Square (NLMS) algorithm is used in linear ASI configurations, due to its robustness and reduced computational cost [5] [6].

The choice of the step size of the NLMS algorithm implies a tradeoff between a fast convergence rate

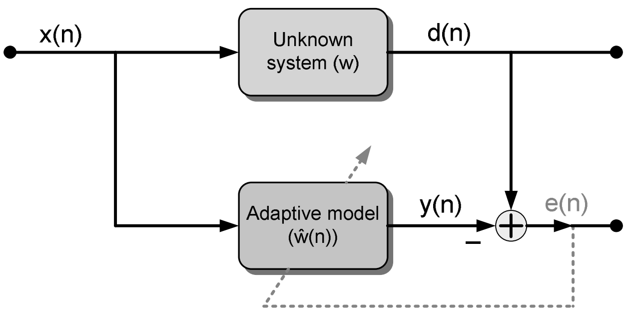

Figure 1. Acoustic system identification setup.

and a small steady-state error. In order to treat this compromise, some adaptive algorithms with variable step-size functions have been proposed in literature, like the Weighted Step Size Projection (WSSP) algorithm in [7], the Normalized Least-Mean-Fourth (NLMF) algorithm in [8] or the sub-band decomposition technique in [9].

The combination of two or more adaptive filters with complementary properties, e.g. different step sizes [10] or different filter lengths [11] is a simple and flexible solution to avoid the compromises hindering the operation of adaptive filters, since the resulting combined filter behaves better or at least as well as the best component.

The paper is organized as follows: Section II presents the behavior of the conventional adaptive model with different filter parameters applied in ASI scenarios, Section III describes the design and implementation of the proposed

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adaptive combination of linear filters, while Section IV discusses the performance of the adaptive combination versus the sub-band decomposition method. In Section V some conclusions are drawn.

II. THE LINEAR ADAPTIVE MODEL

A. Theoretical Aspects

In this work, we consider the linear NLMS filter as a proper adaptive model. The traditional NLMS algorithm follows a stochastic implementation of the steepest-descent algorithm [12]. It replaces the cost function by its instantaneous estimate. Thus, the cost function of the NLMS algorithm can be defined as:

$$J_{NLMS}(n) = \frac{1}{2} E\left[\left(\frac{e(n)}{\|\mathbf{x}(n)\|}\right)^2\right] \approx \frac{1}{2}\left(\frac{e(n)}{\|\mathbf{x}(n)\|}\right)^2, \quad (4)$$

where $E\{\cdot\}$ denotes statistical expectation and $\|\cdot\|$ represents the Euclidian norm.

The steepest-descent recursion in the NLMS case using the cost function as in (4) is written as:

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) - \mu \nabla J_{NLMS}(n), \quad (5)$$

where μ denotes the step-size parameter and ∇ is the gradient operator described as in the following column vector:

$$\nabla = \left[\frac{\partial}{\partial \hat{w}_0} \frac{\partial}{\partial \hat{w}_1} \frac{\partial}{\partial \hat{w}_2} \ldots \frac{\partial}{\partial \hat{w}_i} \ldots \frac{\partial}{\partial \hat{w}_{N-1}}\right]^T. \quad (6)$$

By using (1) and (6) we compute the cost function's gradient:

$$\nabla J_{NLMS}(n) = -\frac{1}{\|\mathbf{x}(n)\|^2} \cdot \mathbf{x}(n) e(n). \quad (7)$$

By substituting (7) in (5) we obtain the tap-weight vector update equation for the linear NLMS algorithm:

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \frac{\mu}{\|\mathbf{x}(n)\|^2} \mathbf{x}(n) e(n). \quad (8)$$

In the following, we will deduce a range for the values of the step size for which the adaptive algorithm remains stable in the mean error sense. We consider the length of the filter to be equal to the length of the system.

By changing the sign in both sides of equation (8) and by adding $\mathbf{w}$, it results that:

$$\mathbf{w} - \hat{\mathbf{w}}(n+1) = \mathbf{w} - \hat{\mathbf{w}}(n) - \frac{\mu}{\|\mathbf{x}(n)\|^2} \mathbf{x}(n) e(n), \quad (9)$$

where

$$\mathbf{w} = [w_0 \quad w_1 \ldots w_i \ldots w_{N-1}]^T \quad (10)$$

includes the acoustic system's coefficients.

By denoting

$$\mathbf{w} - \hat{\mathbf{w}}(n+1) = \mathbf{v}(n+1), \quad (11)$$

equation (9) becomes:

$$\mathbf{v}(n+1) = \mathbf{v}(n) - \frac{\mu}{\|\mathbf{x}(n)\|^2} \mathbf{x}(n) e(n). \quad (12)$$

By taking into account that

$$d(n) = \mathbf{w}^T \mathbf{x}(n), \quad (13)$$

the error signal may be written as:

$$\begin{aligned} e(n) &= \mathbf{w}^T \mathbf{x}(n) - \hat{\mathbf{w}}^T(n) \mathbf{x}(n) \\ &= \mathbf{v}^T(n) \mathbf{x}(n) = \mathbf{x}^T(n) \mathbf{v}(n). \end{aligned} \quad (14)$$

Finally, by substituting (14) in (12) we obtain:

$$\mathbf{v}(n+1) = \mathbf{v}(n) - \frac{\mu}{\|\mathbf{x}(n)\|^2} \mathbf{x}(n) \mathbf{x}^T(n) \mathbf{v}(n). \quad (15)$$

By taking expectations in both sides of (15), we get:

$$\mathrm{E}[\mathbf{v}(n+1)] = \mathrm{E}\left[I - \frac{\mu}{\|\mathbf{x}(n)\|^2} \mathbf{x}(n) \mathbf{x}^T(n)\right] \mathrm{E}[\mathbf{v}(n)], \quad (16)$$

which is equivalent to:

$$\mathrm{E}[\mathbf{v}(n+1)] = \left(I - \frac{\mu}{\|\mathbf{x}(n)\|^2} \mathbf{R}\right) \mathrm{E}[\mathbf{v}(n)], \quad (17)$$

where **R** is the autocorrelation matrix of $\boldsymbol{x}(n)$.

The previous equation can be written on each axes as:

$$v'_m(n+1) = \left(1 - \frac{\mu}{\|\mathbf{x}(n)\|^2} \gamma_m\right) v'_m(n), \quad (18)$$

where $v'_m(n)$ are the elements of the column vector from (17) and γ_m are the eigenvalues of **R**.

Equation (18) can be written with respect to the initial coordinates ($v'_m(0)$) as:

$$v'_m(n) = \left(1 - \mu \frac{\gamma_m}{\|\mathbf{x}(n)\|^2}\right)^n v'_m(0) \ . \qquad (19)$$

The convergence in the mean is hold if $v'_m(n) \to 0$, which means that:

$$\left|1 - \mu \frac{\gamma_m}{\|\mathbf{x}(n)\|^2}\right| < 1 \ . \qquad (20)$$

From (20), it results that:

$$-2 < -\mu \frac{\gamma_m}{\|\mathbf{x}(n)\|^2} < 0 \ , \qquad (21)$$

which may be written as:

$$0 < \mu < \frac{2}{\left(\frac{\gamma_m}{\|\mathbf{x}(n)\|^2}\right)_{\max}} \ . \qquad (22)$$

The denominator of the right hand side term in (22) is approximately 1. It results that $\mu \in (0;2)$ for the stability of the NLMS algorithm.

Usually in ASI scenarios the Echo Return Loss Enhancement (ERLE) is used as a performance measure [13]. ERLE is defined as the ratio of the power of the desired signal over the power of the residual error signal. It is a smoothed measure (in dB) of the amount of the echo that has been attenuated:

$$\mathrm{ERLE\,[dB]} = 10\log_{10} \frac{\mathrm{E}\{d^2(n)\}}{\mathrm{E}\{e^2(n)\}} \ , \qquad (23)$$

The efficiency of the adaptive filter is influenced by the probability density function of the input signal, the step-size parameter and by the filter length.

B. Simulation results

For the next simulations, two types of input signals have been considered: a white Gaussian noise (WGN) and a non-stationary audio signal (speech signal). The acoustic system was modeled by a Schroeder reverberator. The ERLE performance was evaluated for different scenarios for varied values of the step sizes and the filter lengths of the involved filters.

From the ERLE characteristics in *Figure 2*, it results that the first NLMS filter ($NLMS_1$) which has a smaller step-size parameter is characterized by a smaller convergence rate than the second filter ($NLMS_2$) which has a higher step-size parameter.

However, *Figure 3* illustrates the ERLE evolution for two NLMS filters with different filter lengths this time and equal step sizes. Analyzing the ERLE curves of the two NLMS filters it can be observed that the first filter ($NLMS_1$) which has a larger filter length settles to a higher steady-state value of ERLE, while the second filter ($NLMS_2$), which has a reduced filter length settles to a smaller value of steady-state ERLE.

The same observations can be made in the case of a non-stationary audio input signal. When choosing the same filter lengths, but different step sizes, one can notice that the two filters settle to the same steady-state ERLE value, but the first filter ($NLMS_1$) which has a smaller step size, has a smaller convergence rate than the second filter ($NLMS_2$). This fact is

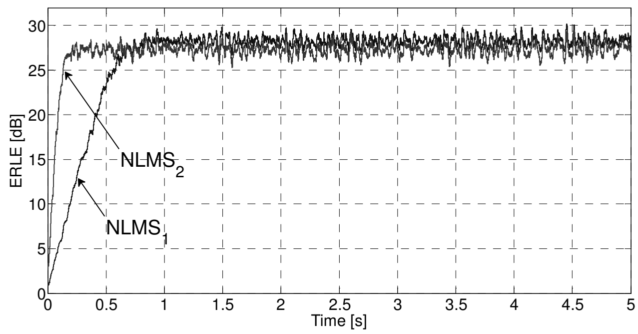

Figure 2. ERLE evolution for two NLMS filters for a WGN as input (N_1= N_2=400; μ_1=0.1 μ_2=0.5).

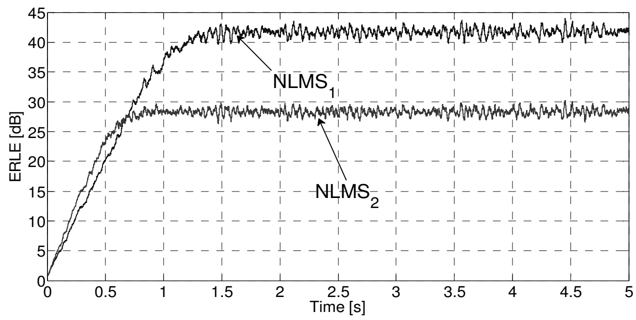

Figure 3. ERLE evolution for two NLMS filters for a WGN as input (N_1=500 N_2=400; μ_1=μ_2=0.1).

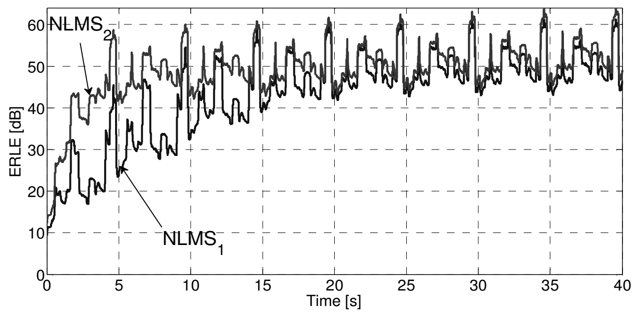

Figure 4. ERLE evolution for two NLMS filters for a non-stationary audio input signal (N_1= N_2=800; μ_1=0.1 μ_2=0.5).

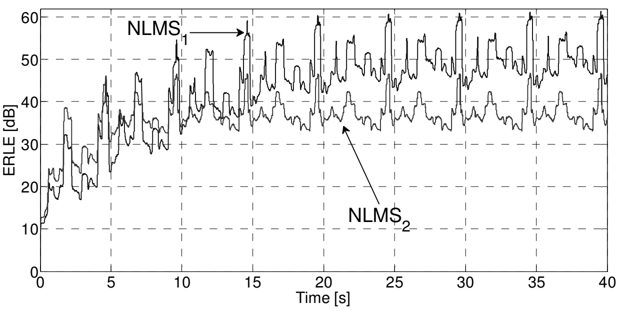

Figure 5. ERLE evolution for two NLMS filters for a non-stationary audio input (N_1=800 N_2=400; μ_1=μ_2=0.1).

illustrated in the simulation results from *Figure 4*.

For the case of equal step-size parameters and different filter lengths (*Figure 5*), the filter with the higher length ($NLMS_1$). has a higher steady-state ERLE value than the filter with a smaller length ($NLMS_2$).

III. THE PROPOSED COMBINATION METHOD

A. Theoretical Aspects

The performance of adaptive filters can be boosted by employing the combination of filters with complementary parameters [14]. The paper proposes to determine an optimum system identification model by combining two filters with different step sizes and filter lengths in order to minimize the tradeoff between convergence and steady-state error inherent to the operation of individual filters as shown in Section II.

Considering two NLMS filters with different step sizes μ_1 and μ_2, ($\mu_1 < \mu_2$) and different filter lengths N_1 and N_2, ($N_1 > N_2$), experiments proved that the filter with the step size μ_2 and length N_2 has a higher convergence rate than the filter with the step size μ_1, and length N_1, but with the disadvantage of a smaller steady-state value of ERLE. The solution employed in this paper to treat this compromise is to combine two adaptive filters in order to obtain a combined filter, which will act as well as the best of the two filter components (both in stability and convergence stage).

Figure 6 depicts the proposed structure for this method as presented in [14]. The structure combines two linear adaptive filters that model the distortions caused by the unknown system. Equation (25) expresses the overall output using the combination of the mentioned filters:

$$y_{comb}(n) = \lambda(n) y_1(n) + [1-\lambda(n)] y_2(n) , \qquad (24)$$

where $y_1(n)$ and $y_2(n)$ are the outputs of the component filters and $\lambda(n)$ is the mixing parameter. The mixing parameter $\lambda(n)$ is computed as a sigmoidal activation function which keeps the mixing parameter in the range [0;1] as follows:

$$\lambda(n) = sgm[a(n)] = \frac{1}{1+e^{-a(n)}} . \qquad (25)$$

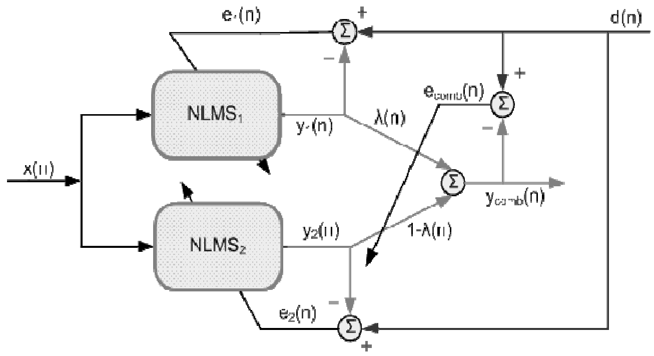

Figure 6. Combination of two linear NLMS adaptive filters.

The error of the combined filter takes the form:

$$e_{comb}(n) = d(n) - y_{comb}(n) , \qquad (26)$$

The parameter *a(n)* adapts as follows:

$$a(n+1) = a(n) + \frac{\mu_a}{r(n)}[e_2(n) - e_1(n)] e_{comb}(n) \lambda(n)[1-\lambda(n)] \qquad (27)$$

where μ_a is a step-size parameter and $e_1(n)$ respectively $e_2(n)$ are the individual error signals delivered at the outputs of the two NLMS filter blocks.

The normalized adaptation of *a(n)* is obtained by using:

$$r(n) = \beta r(n-1) + [1-\beta][e_2(n) - e_1(n)]^2 , \qquad (28)$$

with a proper selected forgetting factor β. In simulations, these parameters are set to the following values: $\beta = 0.9$ and μ_a=1.

B. Simulation results

The compromise inherent to the choice of the filter parameters can be best illustrated by employing filters with complementary parameters (different step sizes and different filter lengths). *Figure 7* depicts the evolution of ERLE for two NLMS filters with WGN as input, having $\mu_1 < \mu_2$ and $N_1 > N_2$. Employing a smaller step size, the first filter ($NLMS_1$) converges slower than $NLMS_2$, but to a higher steady-state ERLE value due to the larger filter length.

The same approach has been carried out for a non-stationary audio signal as input. *Figure 8* illustrates the evolution of ERLE for two NLMS filters with assorted features ($\mu_1 < \mu_2$ and $N_1 > N_2$). The compromise between a high convergence speed and a small steady-state ERLE value can be noticed also in this case.

Figure 9 presents the results for the proposed method of filter combination. By applying the combination on the two

NLMS filters from *Figure 7*, a global filter is obtained which converges as fast as filter $NLMS_1$ but to the steady-state ERLE of filter $NLMS_2$. The mixing parameter decides among the two filters by taking values in the range [0; 1].

Figure 10 illustrates the performance of the proposed method of combination in the case of non-stationary audio input signals and the evolution of the mixing parameter. The advantage of the proposed method consists again in the fact that the filter obtained from the combination performs as good as the best component in each stage of the filter. The mixing parameter's evolution is also presented.

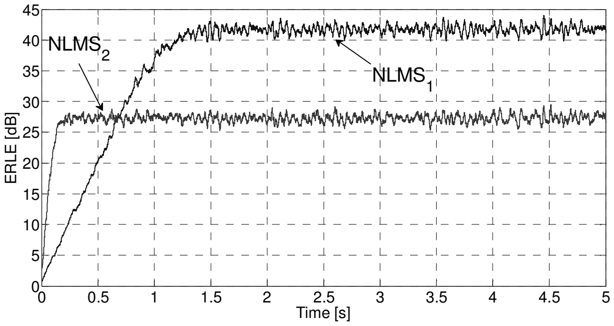

Figure 7. ERLE evolution for two NLMS filters with complementary parameters for WGN as input.

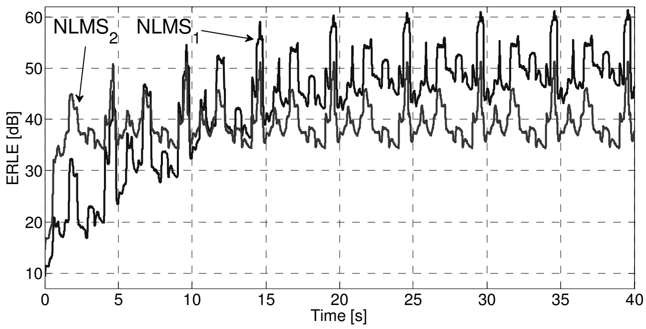

Figure 8. ERLE evolution for two NLMS filters with complementary parameters for a non-stationary audio input signal.

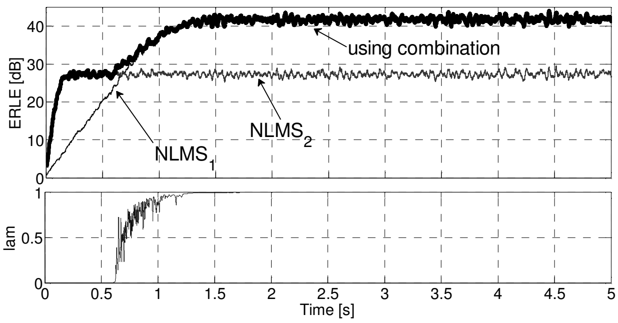

Figure 9. ERLE performance for adaptive filter mixing combination method using a WGN as excitation (top); the parameter's evolution (bottom).

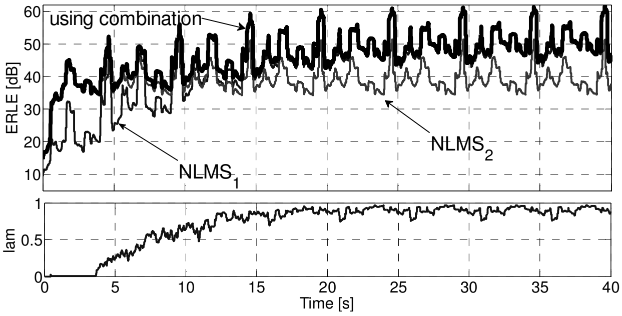

Figure 10. ERLE performance for adaptive filter combination method using speech as excitation (top); the mixing parameter's evolution (bottom).

IV. COMBINATION METHOD VERSUS SUB-BAND DECOMPOSITION

A. Theoretical Aspects

The sub-band decomposition is another method which has been studied for improving the performance of adaptive filters [15] [16]. It is an approach to increase the convergence speed in comparison to the full band solution. *Figure 11* depicts the sub-band adaptive filtering scheme. We use the analysis filter banks *P(z)* to decompose the original signal by subdividing its spectra. A set of independent filters perform the adaptive filtering. The outputs of these filters are subsequently combined using a synthesis filter bank *Q(z).*

B. Simulation results

By using two NLMS filters with the same step size and filter length, the non-stationary audio signal was decomposed in two sub-bands. The evolution of ERLE for the full band and the two sub-band decomposition is depicted in *Figure 12*. The sub-band decomposition has the advantage of a higher convergence speed in comparison to the full band ($NLMS_1$) at approximately the same steady-state error.

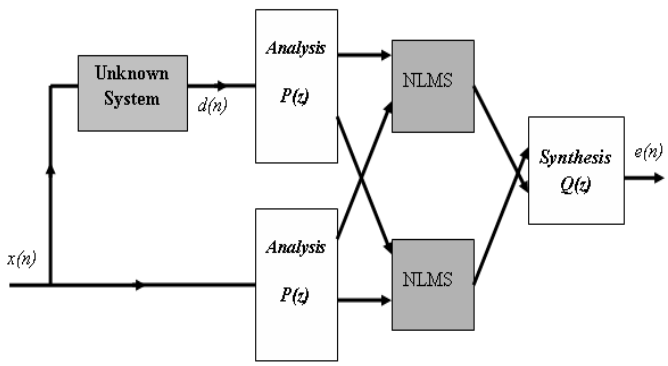

Figure 11. Two sub-band decomposition block scheme used in acoustic system identification application.

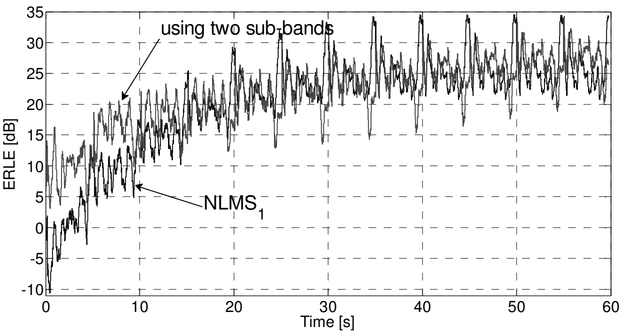

Figure 12. Evolution of ERLE for full band and two sub-bands decomposition of an audio signal.

Further on, we propose to make a comparison between the performance of the sub-band decomposition and the proposed combination method. Thus, a second NLMS filter was proposed with complementary properties of the filter applied for the full band case. In *Figure 13* the ERLE performance of full band and two sub-band decomposition is illustrated, along with the ERLE characteristic of the proposed (complementary) NLMS filter. The second filter ($NLMS_2$) has a higher convergence speed than the full band ($NLMS_1$) and the two sub-band decomposition method, but settles to a smaller steady-state ERLE value. By applying the combination method on the two NLMS filters from *Figure 13*, a global ERLE is obtained. *Figure 14* illustrates the optimized ERLE performance of the overall combination of the two filters and also the evolution of the mixing parameter. *Figure 15* depicts the ERLE performance of the sub-band decomposition and for the proposed combination method for a clearer view. The overall output filter obtained after applying the combination on filters $NLMS_1$ and $NLMS_2$ has the advantage of a larger convergence rate than the sub-band approach applied on the full band adaptive approach ($NLMS_1$). Both characteristics reach approximately the same steady-state ERLE, with a slightly higher stabilization value for the sub-band decomposition. From these observations we can conclude that the proposed combination method has better performance than the sub-band decomposition model.

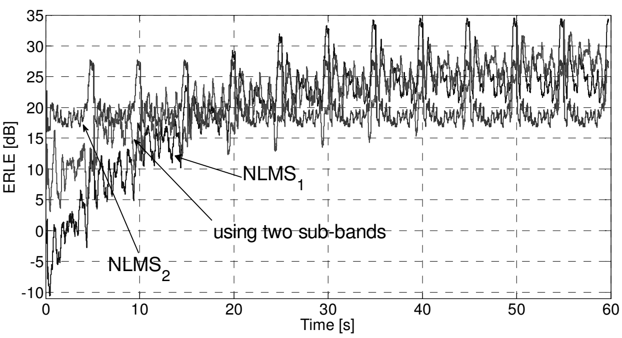

Figure 13. Evaluation of ERLE for the reference full band NLMS filter, for two sub-band decomposition and for the complementary NLMS filter.

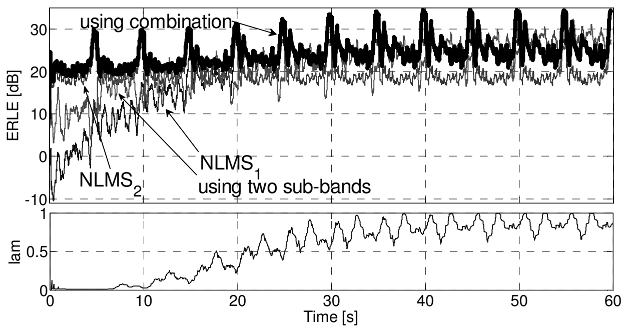

Figure 14. ERLE performance of two NLMS filters, of the sub-band decomposition and of the proposed combination of filters (top); the mixing parameter (bottom).

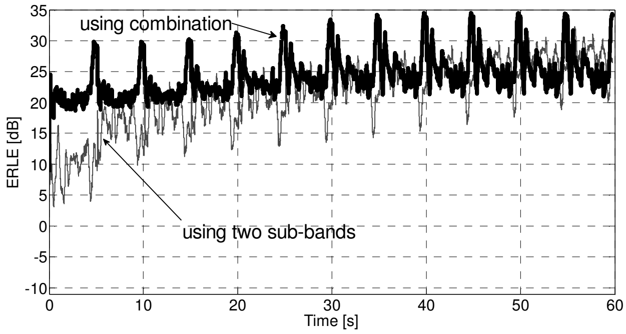

Figure 15. Evaluation of ERLE for two sub-band decomposition and for the combination method.

V. CONCLUSIONS

In this paper, a system identification method for combination of NLMS filters was proposed. The approach was presented as a flexible solution to deal with the compromise between the speed of convergence and steady-state value of the error. The performance of the adaptive scheme is analyzed in terms of ERLE, when inputs are either a white Gaussian noise or a non-stationary audio signal.

First the theoretical aspects regarding the NLMS algorithm were presented and then the influence of the step size and the length of the filter was studied. The proposed combination method was applied on two adaptive filters with different step sizes and different filter lengths. The method was compared to the sub-band decomposition method, which is known as an approach to increase the convergence speed. The sub-band decomposition method has a small advantage in the steady-state stage, but the proposed combination method has a much higher convergence speed for the simulated occurrences.

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