NONLINEAR ACOUSTIC SYSTEM IDENTIFICATION USING ADAPTIVE COMBINATIONS OF VOLterra FILTERS

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Abstract: The paper proposes two nonlinear system identification methods in order to reduce the compromise between convergence speed and steady-state that occurs when using third order adaptive Volterra filters. Combination of filters and combination of kernels schemes are outlined. Adaptation of the kernels is achieved using a Normalized Least Mean Square algorithm. The tradeoff between speed and stability is emphasized by selecting different step-size parameters for each kernel. Both nonlinear approaches are applied in acoustic Loudspeaker-Enclosure-Microphone setups where several sources of nonlinearities exist: the overdriven amplifier’s or small loudspeakers’ operated at high volume. A third order polynomial with distinct Linear-to-Nonlinear Ratios is chosen to model these distortions. The evaluation is made in terms of Echo Return Loss Enhancement in experiments conducted for various input signals. Results show that the overall adaptive combinations perform each time better or at least as well as the best component filter.

Keywords: Nonlinear system identification, Volterra filters, adaptive combination, adaptive algorithms

1. INTRODUCTION

Hands-free communication systems and video telephony are becoming increasingly important in many of today’s communication scenarios. However due to free sound propagation from the loudspeaker to the microphone acoustic echo arises in the local room and it is being fed back to the far end speaker [1]. Such setups usually require an acoustic echo cancellation which is a typical problem of system identification. This is a signal processing application of identifying an unknown system using a suitable model.

In standard acoustic system identification approaches [2] it is assumed that the echo path is linear and time-variant so it can be modeled by a linear filter. But since we are dealing with small audio equipment operated at the highest allowed signal level, the loudspeaker-enclosure-microphone (LEM) setup contains not only linear echo, but also nonlinear distortions caused by the hardware used in the transmission path. In this case linear adaptive filters are no longer appropriate to model the echo path. Nonlinear adaptive filters are required instead. The typical setup for nonlinear acoustic system identification (ASI) is shown in Figure 1. It seeks to minimize the power of the error signal \( e[k] \) by subtracting the output of the nonlinear adaptive filter \( \hat{y}[k] \), which is an estimation of the processed signal from \( d[k] \) (the nonlinear processing of the input signal \( x[k] \)).

The nonlinearities encountered in the echo path are indicated in Figure 2. The main sources of nonlinearities are the loudspeaker and the power amplifier functioning at maximum capacity. Nonlinear distortions caused by the overdriven amplifier are usually memoryless and can be represented by a saturation curve [3]. Another kind of nonlinearity but with memory is caused by the loudspeaker operated at its power limit [4].

![Figure 1. Nonlinear ASI scenario with adaptive filter.](image)

In order to enclose all the mentioned distortions from the LEM setup various structures for nonlinear system identification were analyzed: polynomial structures [5] or adaptive Wiener and Volterra models [2]. In this paper a nonlinear system identification method is described using different adaptive combination techniques of Volterra filters. The adaptation of individual Volterra filters included in combinations is achieved by updating the associated kernels using the Normalized Least Mean Square (NLMS) algorithm.

The proposed combination structures are tested for different nonlinear scenarios in terms of Echo Return Loss Enhancement (ERLE).

The paper is organized as follows: in Section II the key elements regarding Volterra models are presented while Section III reviews the adaptive combination approaches applied on third order Volterra filters. The performances of both nonlinear system identification techniques are evaluated in Section IV. Section V contains the main conclusions of the
The length of each Volterra kernel $h_p$ depends on the order and the memory length of the filter as follows:

$$\text{length}(h_p) = (M + p - 1)! / (M - 1)! p!.$$  

(4)

The main objective when using Volterra structures in nonlinear system identification setups is to gain a proper replica of the unknown system’s parameters depicted by the Volterra kernels. In real acoustic scenarios no a priori knowledge on the nature of the nonlinear system is available. We only have information on how the system affects the input signal. So by updating the Volterra kernels using an adaptive algorithm, the nonlinear system’s features are evaluated at each iteration of the input signal. In simulations the NLMS adaptive algorithm is employed. The kernel update rule is defined as in [7]:

$$h_p[k+1] = h_p[k] + \alpha_p e_p[k] x_p[k] x_p^T[k] + \varphi,$$  

(5)

where $e_p[k] = d[k] - y_p[k]$ is the estimated error minimized at each iteration. The constants $\alpha_p$ and $\varphi$ are positive and should be selected appropriately. The value $\varphi$ was introduced to prevent division by zero or a very low value when the norm $x_p^T[k] x_p[k]$ is small. The step-size parameter $\alpha_p$ controls the rate of convergence and stability and should be in the range $0 < \alpha_p < 2$. However the selection of the step-size parameter is subject to several compromises regarding the performances of the filter. For example if we select two distinct values for $\alpha_p$ in the range mentioned $\alpha_A > \alpha_B$, the evolution of the same Volterra filter that uses in turns the chosen step-sizes is different. The Volterra filter will converge faster using $\alpha_A$, but will settle to a higher residual error value than the case in which $\alpha_B$ is used.

In the following, two methods based on adaptive combination as in [8] are presented. Combining two adaptive Volterra filters with different step-sizes represents a simple solution to lessen the tradeoff between the speed of convergence and steady-state.

### III. ADAPTIVE COMBINATION

The performances of Volterra filters can be improved by employing the adaptive combination of filters with complementary capabilities since the new combined filter behaves better or at least as well as the best component as related in [9].

In this section two combination approaches are explained in detail: combination of Volterra filters and combination of Volterra kernels.

#### A. Combination of Volterra Filters (CVF)

In the next sequence we are going to evaluate the general form of the convex combination scheme for two adaptive filters.
with assorted performances. The global output is defined as:

\[ y(n) = \lambda(n)y_I(n) + [1 - \lambda(n)]y_{II}(n), \]

where \( y_I(n) \) and \( y_{II}(n) \) are the outputs of the component filters and \( \lambda(n) \) is a mixing parameter. The mixing parameter \( \lambda(n) \) is adjusted at every iteration of the input signal using the sigmoidal activation:

\[ \lambda(n) = \text{sgm}[a(n)] = \frac{1}{1 + e^{-a(n)}}. \]

For adapting the mixing parameter, gradient descent methods are used as in [9]:

\[ a(n + 1) = a(n) + \frac{\mu}{r(n)}[e_{II}(n) - e_I(n)]e(n)\lambda(n)[1 - \lambda(n)], \]

where \( \mu \) is a step-size parameter. The normalized adaptation can be obtained using:

\[ r(n) = \beta r(n - 1) + [1 - \beta][e_{II}(n) - e_I(n)]^2, \]

with a proper selected forgetting factor \( \beta \).

The error signal is estimated as follows:

\[ e_i(n) = d(n) - y_i(n), \quad i = I, II, \]

where \( d(n) \) represents the microphone signal. The advantage of the sigmoidal activation function is to keep the mixing parameter in the range \([0, 1]\).

In this paper we apply the convex combination for two third order Volterra filters with different step-sizes as depicted in Figure 3. In order to minimize the compromise between the speed of convergence versus residual error steady-state we use three different step-size parameters for each kernel, i.e. \( \alpha_A \) and \( \alpha_B \) for the linear kernels (\( \alpha_A > \alpha_B \)), \( \alpha_A' \) and \( \alpha_B' \) for the quadratic kernels (\( \alpha_A' > \alpha_B' \)), \( \alpha_A'' \) and \( \alpha_B'' \) for the third order ones (\( \alpha_A'' > \alpha_B'' \)). In this way the combined filter will be composed by the combination of two different third order Volterra filters, both with distinct kernel step-sizes.

The output of the combined filter is obtained as:

\[ y(k) = \lambda(k)y_A(k) + [1 - \lambda(k)]y_B(k), \]

where \( y_A(k) \) and \( y_B(k) \) are the outputs of the component third order Volterra filters:

\[ \begin{align*}
    y_A(k) &= y_{1A}(k) + y_{2A}(k) + y_{3A}(k), \\
    y_B(k) &= y_{1B}(k) + y_{2B}(k) + y_{3B}(k).
\end{align*} \]

The outputs \( y_{1i}(k), y_{2i}(k) \) and \( y_{3i}(k) \) correspond to the linear, second order and third order kernels respectively when \( i = [A, B] \).

For the linear kernels (\( \alpha_A > \alpha_B \)), \( \alpha_A' > \alpha_B' \), \( \alpha_A'' > \alpha_B'' \)) as depicted in Figure 4.

Using kernel combination, the output of the new combined filter reads:

\[ y(k) = \sum_{p=1}^{3} y_p(k) = \sum_{p=1}^{3} \lambda_p(k) y_{pA}(k) + [1 - \lambda_p(k)] y_{pB}(k), \]

where \( y_p(k) \) represents the combination of two kernels of order \( p \) (\( y_{pA}(k) \) and \( y_{pB}(k) \)) and \( \lambda_p(k) \) is the corresponding mixing parameter defined as:
of each kernel involved in the combination is reduced minimizing their own associated error signal. The error signal implying its own output and also the outputs of all distinct combined kernels is given by

\[ e_p(k) = d(k) - [y_p(k) + \sum_{p \neq p'} y_{p'}(k)], p = [1,2,3], \]

in order to reduce the global error: \( e(k) = d(k) - y(k) \).

All component kernels are updated independently minimizing their own associated error signal. The error signal of each kernel involved in the combination is reduced implying its own output and also the outputs of all distinct combined kernels \( y_p(k) \):

\[
\begin{align*}
\lambda_p(k) &= \frac{1}{1 + e^{-\alpha_p(k)}}. \\
\end{align*}
\]

The parameters \( \alpha_p(k) \) adapt as follows:

\[
\left\{ \begin{array}{l}
\alpha_p(k+1) = \alpha_p(k) + \frac{\mu}{r_p(k)} [e_{pB}(k) - e_{pA}(k)] (y_p(k)[1 - \lambda_p(k)] - 1), \\
r_p(k) = \beta_p(k-1) + [1 - \beta] [e_{pB}(k) - e_{pA}(k)]^2, p = [1,2,3], 
\end{array} \right.
\]

(17)

IV. EXPERIMENTS

In the following section both combination methods are tested in nonlinear system identification setups in order to illustrate their performances regarding speed of convergence and steady-state control. To point out the compromise between convergence and steady-state, two third order Volterra filters are chosen. Each Volterra kernel is updated with the NLMS algorithm using individual step-size parameters and taking into account the conditions imposed in Section III.

The unknown echo path is modeled as a third order polynomial of the audio input samples. This nonlinear dependency is illustrated in the representation of the microphone signal by a third order Volterra model:

\[
d(k) = h_{00}(k)x_0^T(k) + \gamma(k)[h_{01}(k)x_1^T(k) + h_{02}(k)x_2^T(k)],
\]

(19)

where \( h_{00}(k), h_{01}(k) \) and \( h_{02}(k) \) depict the Volterra kernels. They are appropriately generated to resemble as close as possible the ones identified in [9], where a real nonlinear acoustic system setup made of a small low-cost loudspeaker inside a low-reverberant room is detected. The parameter \( \gamma(k) \) is introduced to control the linear-to-nonlinear ratio (LNLR) of involved signal powers.

To evaluate the proposed techniques, several experiments are conducted using white Gaussian noise and speech as excitations \( x[k] \). The adaptive Volterra filter based schemes are compared for different LNLR values in terms of Echo Return Loss Enhancement (ERLE):

\[
ERLE = 10 \log_{10} \frac{E[d^2[k]]}{E[e^2[k]]},
\]

(20)

where \( E[\cdot] \) denotes statistical expectation.

1) Simulations using Gaussian noise as the filter input: In the first set of experiments a white Gaussian noise with zero mean and amplitude in the range \((-1; 1)\) is employed as the input signal. As for the nonlinear echo path design, different lengths for each kernel vector are chosen following equation (4). Thereby for the linear kernel \( M_1 \) equals 320 taps, \( M_2 \) is 50 taps for the second order kernel and \( M_3 \) equals 25 taps for the third order kernel vector. Parameter \( \gamma(k) \) is selected properly so that the LNLR value can reach 20dB.

First of all we test the performance of a third order adaptive Volterra filter in comparison with the ones of inferior order Volterra filters for the imposed nonlinear acoustic scenario. Different step-sizes are selected for each kernel: \( \alpha_A = 1 \) for the linear kernel, \( \alpha_A^* = 0.052 \) for the quadratic and \( \alpha_A'' = 0.0052 \) for the third order kernel. Also the memory lengths of kernels are the ones used to describe the nonlinear echo path. The input signal and ERLE evolution for each filter are depicted in Figure 5. As it can be seen the third order Volterra filter outperforms the linear and the second order Volterra filter. All three patterns converge fast but to distinct steady-states. The speed of convergence depends especially on the linear step-size in each case. By using the third order Volterra filter we get a steady-state of 30 dB in contrast to the steady-state of the linear filter which is limited by the LNLR value.

Figure 4. The setup for CK with different step-sizes.

Figure 5. Evolution of ERLE for Volterra filters with different orders (top) and the input signal (bottom).
Next the performances of CVF and CK schemes with different step-size are evaluated. The two independent third order Volterra filters employed in the mentioned adaptive combinations are the one used in experiments related in Figure 5 and one with the following step-sizes: \( \alpha_\beta = 0.05 \) for the linear kernel, \( \alpha_\beta' = 0.05 \) for the quadratic kernel and \( \alpha_\beta'' = 0.001 \) for the cubic kernel. As for the parameters involved in combinations, the step-size is selected \( \mu = 1 \) and the forgetting factor \( \beta = 0.9 \) is chosen.

Figures 6 and 7 show the performance of each single Volterra filter and also the estimated ERLE for CVF and CK approaches together with the evolution of the mixing parameters.

Figure 6 indicates the evolution of CVF in terms of ERLE when the outputs of both filters are involved in the combination. As we can see, filter (B) has a lower convergence speed but tends to reach a higher steady-state than the third order Volterra filter (A). However by using CVF, a new filter is obtained that converges as fast as filter (A) and yet stabilizes to an ERLE value imposed by filter (B).

The CK strategy is depicted in Figure 7. In this case we combine kernels of the same order instead of filter outputs. Again it is noticeable that the new Volterra filter always behaves better or at least as well as the best constituent.

The same conclusion can be drawn if we monitor the mixing parameters’ \( \lambda(k) \) and \( \bar{\lambda}(k) \) characteristic. At the beginning their values are close to 1 meaning that the combinations follow filter (A) which converges faster and then settle to 0 when the new filter keeps trace of higher steady-state values obtained for filter (B).

In Figure 8 the evolution of both CVF and CK methods are depicted. The same performances are noticed regarding convergence rate while the steady-state value of the CK approach is slightly superior.

2) Simulations using speech as the filter input: The next set of experiments conducted for CVF and CK evaluation use audio signal as excitation, digitized at 8 kHz and with amplitude in the range (−1; 1). The LNLR value is set to 15 dB in this case and regarding combination parameters, \( \mu \) is changed to 0.1.

The same hierarchy is kept for different orders of adaptive Volterra filters as in the case when Gaussian noise was used as input. In Figure 9 we can conclude that for the input signal depicted in the lower side of the chart, the third order Volterra filter is again superior to the second order Volterra filter and to the linear filter. The same filters were used as the ones compared in Figure 5.

To point out the resemblance between the two combination techniques, the parameters of Volterra filter (A) are kept unchanged as in the white Gaussian noise input case while for Volterra filter (B) with lower convergence rate but with higher steady-state the following step-size parameters are selected: \( \alpha_\beta = 0.2 \), \( \alpha_\beta' = 0.05 \) and \( \alpha_\beta'' = 0.005 \).
In Figures 10 and 11 it can be noticed that CVF and CK structures always outperform the two involved third order Volterra filters and they follow each time the ERLE characteristic of the best constituent filter. Even if it is not as obvious as in the evolution of the first pair, the graph of the new mixing parameters involved in combinations follows the same rule: in the first seconds their value is close to 1 but eventually settle towards 0.

The evolutions of the two adaptive combination approaches in terms of ERLE are almost identical as depicted in Figure 12.

V. CONCLUSION

In this paper, starting from the concepts proved in [9], two nonlinear system identification methods were proposed to reduce the compromise between fast convergence and low steady-state facing third order adaptive Volterra filters behavior. The particular Volterra kernels are updated with the NLMS adaptive algorithm using different step-size parameters. The performances of the CVF and CK approaches were evaluated in terms of ERLE and taking into account the evolution of the mixing parameters. Convergence speed versus steady-state was monitored by comparing the estimated ERLE evolution for the new combined filters with the ones of each single adaptive third order Volterra filter counted in the combinations. The conducted experiments using several input signals for a third order acoustic nonlinear system with different LNLR values show that in weakly nonlinear systems the two proposed approaches are equally effective, behaving each time better or at least as well as the best component.

The disadvantages of the proposed models consist in computational complexity required to implement the adaptive third order Volterra filters. Also the combination schemes increase the complexity of the algorithm. The computational demands are reduced in the combination of kernels case in contrast to the CVF setup where different Volterra outputs are combined directly.

Further work will be dedicated to nonlinear audio system identification using different adaptation techniques applied on measured acoustic nonlinearities.

ACKNOWLEDGMENT

This paper was supported by the project “Doctoral studies in engineering sciences for developing the knowledge based society-SIDOC” contract no. POSDRU/88/1.5/S/60078, project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

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