

## A MATLAB GUI FOR SIMULATING THE PROPAGATION OF THE ELECTROMAGNETIC FIELD IN A 2-D INFINITE SPACE

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**Abstract:** This paper presents a Matlab-based graphical user interface (GUI) that allows the simulation of the propagation of an electromagnetic wave in a 2-D infinite space. The homogeneous, isotropic medium is modeled by using the Finite-Difference Time-Domain technique and its infinity is modeled with the Perfectly Matched Layer. The GUI provides different ways of displaying the graphical results: 3-D animation (or snapshots from the animation) of the wave propagation, field variations in time recorded by probes placed in the medium, the average power spectrum and also calculates the reflection coefficients. The capability of the proposed GUI is demonstrated with a numerical example: the simulation of the wave propagation in a field with cylindrical symmetry.

**Keywords:** Finite-Difference Time-Domain, Perfectly Matched Layer, Matlab GUI, electromagnetic field.

### I. INTRODUCTION

The *Finite-Difference Time-Domain* (FDTD) technique can be used in modeling of the space where the electromagnetic wave propagates. This method was developed by Kane Yee in 1966 [1] and consists in dividing the simulated space into identical cells (*Yee cells*) followed by the discretization of the differential forms of Maxwell's equations, both in time and space.

In order to simulate the propagation of the electromagnetic field in an infinite space, J.P. Berenger proposed in 1994 the *Perfectly Matched Layer* (PML) technique [2]. This special nonphysical layer called PML is placed adjacent to the edges of the FDTD grid and is defined in a way that attenuates by absorption all the waves that enter into it. There are several approaches in the existing literature that works with the PML technique both in 2-D and 3-D coordinate space [3]-[10].

In this paper we present a virtual instrument developed in Matlab Guide, called *PML\_2D\_VI*, which consists in a graphical user interface (GUI) that allows the simulation of the electromagnetic wave propagation through a 2-D infinite space, modeled with PML and FDTD techniques. This GUI is meant as an interactive tool for those who are interested in study the wave propagation in infinite media under user's specified conditions. By varying various parameters of the source or the medium and placing obstacles, the propagation phenomenon can be visualized as 3-D animated plots, time variations of the electric field or average power spectrum.

*Section II* of this paper explains the Matlab modeling of the wave propagation through an infinite space of 2-D

coordinates. *Section III* summarizes the main features of the developed GUI. In *Section IV* an example of using the GUI is presented, including snapshots from the 3-D animation of the wave propagation, the spectrum of the average power and the time variations of the field recorded by three probes placed in the medium. Finally, *Section V* presents the conclusions of this paper.

### II. THE MODELING OF THE WAVE PROPAGATION IN AN INFINITE 2-D SPACE

The 2-D modeling of the electromagnetic field is possible in certain cases. For example, an infinite long wire, current carrying, generates an electromagnetic field in the surroundings. Field distribution is identical in any plane perpendicular to the wire, so it can be studied in one such plane. If one chooses the wire along the  $z$ -axis, then the electric field will have one single component –  $E_z$  – and the magnetic field will generally have two components –  $H_y$  and  $H_z$ .

Another example is the plane-parallel field. If the situation allows, one can choose one of the axes on the direction of propagation and the other two on the radial electric / magnetic field. This is an easier approach, each field having one single component along an axis. However, if the direction of propagation is perpendicular to one axis but makes an acute angle with the others, we can choose one of the axes (e.g.  $z$ -axis) on the direction of the electric field; in this case the magnetic field will be represented by the two projections on the other axis.

If a plane electromagnetic wave propagates in a 2-D homogeneous, isotropic, infinite space ( $\epsilon$ ,  $\mu$ ) along the  $z$ -

axis, then  $H_z=0$ ,  $E_x=E_y=0$  and the Maxwell's equations in this case are:

$$\frac{\partial H_x}{\partial \tau} = -\frac{1}{Z} \cdot \frac{\partial E_z}{\partial y} \quad (1)$$

$$\frac{\partial H_y}{\partial \tau} = \frac{1}{Z} \cdot \frac{\partial E_z}{\partial x} \quad (2)$$

$$\frac{\partial E_z}{\partial \tau} = Z \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - J_z \right) \quad (3)$$

where  $Z = \sqrt{\mu/\epsilon}$  is the wave impedance and we denoted  $\tau = t/\sqrt{\epsilon\mu}$ .

For implementation in Matlab, let us consider a 2-D discretized structure divided into identical rectangular Yee cells. *Figure 1* illustrates a Yee cell ( $\Delta x \times \Delta y$ ) and the corresponding field components: the electric component  $E_z$  is positioned in the corners of the cell, and the magnetic components ( $H_x$ ,  $H_y$ ) are in the middle of the edges.

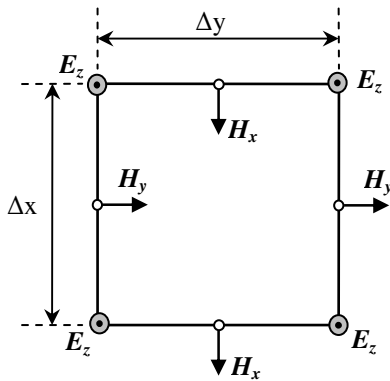


Figure 1. A rectangular Yee cell.

Considering that the medium is dielectric ( $J=0$ ), the corresponding finite-difference equations that result from the discretization of eqs. (1) – (3) are respectively:

$$\begin{aligned} H_x^{n+1}(i,j) &= H_x^n(i,j) - \frac{1}{Z} \frac{\Delta \tau}{\Delta y} [E_z^n(i,j+1) - E_z^n(i,j)] \\ H_y^{n+1}(i,j) &= H_y^n(i,j) + \frac{1}{Z} \frac{\Delta \tau}{\Delta x} [E_z^n(i,j) - E_z^n(i-1,j)] \\ E_z^{n+1}(i,j) &= E_z^n(i,j) + Z \frac{\Delta \tau}{\Delta x} [H_y^{n+1}(i+1,j) - H_y^{n+1}(i,j)] - \\ &\quad - Z \frac{\Delta \tau}{\Delta y} [H_x^{n+1}(i,j) - H_x^{n+1}(i,j-1)] \end{aligned} \quad (4)$$

where we denoted by  $F^n(i,j)$  a field component as a function of the spatial coordinates  $i$  and  $j$  and the time coordinate  $n$  respectively. We mention here that for

reason of implementation, integer indices were used instead of Yee's fractional indices, as we detailed in [9].

In order to simulate the infinity of the space, the artificial layer PML was modeled and placed adjacent to the boundary of the FDTD grid. The waves that are normal to PML will be attenuated according to the absorption function [9]:

$$a(x) = 1 - (1 - a_{min}) \left( \frac{x}{g} \right)^n \quad (5)$$

where  $x$  represents the depth in PML,  $g$  is the thickness of the PML,  $a_{min}$  is the minimum value of the attenuation in PML ( $0 < a_{min} < 1$ ) and  $n$  is the order of the polynomial variation. Tests have shown that best results are obtained for  $n=2$ .

The equations describing the field propagation vary by type of the medium (normal or PML) and by the area of the modeled space. As illustrated in *Figure 2*, the total computational space was divided into 9 areas: area 1 (in the middle) represents the isotropic medium; areas 2 - 9 (on the edges) are defined as PMLs.

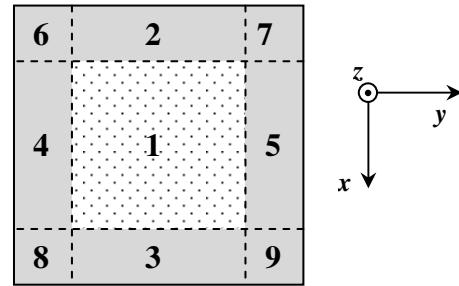


Figure 2. The modeled space was divided into 9 areas: 1 is normal, the rest are PMLs.

For example, the electric component in area 1 is given by:

$$\begin{aligned} E_z^{n+1}(i,j) &= E_z^n(i,j) + b_e \cdot [H_y^n(i,j) - \\ &\quad - H_y^n(i-1,j) - H_x^n(i,j) + H_x^n(i,j-1)] \end{aligned} \quad (6)$$

where  $b_e = \frac{\Delta t}{\epsilon_0 \cdot \Delta x}$ . As for the PML areas,  $E_z$  was divided into two sub-components,  $E_{zx}$  and  $E_{zy}$ , both collinear with  $E_z$ ; the two sub-components are associated to  $H_x$  and  $H_y$  respectively, in the following sense:  $E_{zx}$  and  $H_x$  results in a wave that propagates in the  $y$ -direction;  $E_{zy}$  and  $H_y$  results in a wave that propagates in the  $x$ -direction.

$$E_z^n(i,j) = E_{zx}^n(i,j) + E_{zy}^n(i,j) \quad (7)$$

where:

$$\begin{aligned} E_{zx}^{n+1}(i, j) &= a_{ex}(i, j) \cdot E_{zx}^n(i, j) - b_e \cdot [H_x^n(i, j) - H_x^n(i, j-1)] \\ E_{zy}^{n+1}(i, j) &= a_{ey}(i, j) \cdot E_{zy}^n(i, j) + b_e \cdot [H_y^n(i, j) - H_y^n(i-1, j)] \end{aligned} \quad (8)$$

As a result of this division, one of the resulted waves will be attenuated and the other not, depending on the position of the PML area, since only the waves which are normal to PML must be attenuated. Consequently, the absorption coefficients  $a_{ex}(i, j)$  and  $a_{ey}(i, j)$  vary by the area as follows:

$$\begin{aligned} a_{ex}(i, j) &= \begin{cases} 1, & \text{for areas 2, 3} \\ 1 - (1 - a_{min})(j/g)^2, & \text{for areas 4 - 9} \end{cases} \\ a_{ey}(i, j) &= \begin{cases} 1, & \text{for areas 4, 5} \\ 1 - (1 - a_{min})(i/g)^2, & \text{for areas 2, 3, 6 - 9} \end{cases} \end{aligned} \quad (9)$$

Similarly one can find the equations describing the magnetic components.

### III. THE DESCRIPTION OF THE GUI

A GUI application called *PML\_2D\_VI* has been created in Matlab and allows the simulation of the wave propagation through a 2-D infinite space, modeled as explained in the previous section of the paper. In the following, we will describe the appearance and use of the GUI.

The GUI is separated in two main areas: on the left the user can set all the characteristics of the simulation; the right side is dedicated to the graphical display of the results.

The left side of the GUI has two main sections: *General Settings* and *Results Settings*.

In *General Settings* section, there are 4 sub-sections where the user can adjust the input parameters of the simulation (see *Figure 3*):

- 1) The *Medium* sub-section contains the properties of the modeled space:
  - the *type* (infinite space / rectangular cavity);
  - the FDTD grid's *resolution*, representing the number of Yee cells in a wavelength;
  - the *length* and the *width*, given in multiples of the resolution;
- 2) The *PML* sub-section includes the two parameters of the artificial layer:
  - the *thickness* of the PML (in grid's cells);
  - the minimum attenuation (*amin*);
- 3) The *Excitation* sub-section gives the characteristics of the excitation:
  - the *type* (infinite wire / plane wave);
  - the *shape* (Gauss pulse / continuous sinus);
  - the *position* in the simulated space (the coordinate on *x*-axis and *y*-axis);
  - the *amplitude* and the *frequency* (for Gauss pulse, this is the inverse of the pulse's duration);
  - the *duration* of the simulation (expressed in periods of excitation);

4) In the *Obstacles* sub-section, one or more objects can be placed in the simulated space. For each object the following characteristics can be set:

- the *material* (dielectric / conductor);
- the relative permittivity (*eps relative*) – enabled only for dielectric;
- the *section's* shape (rectangular / circular / wall);
- the position of the obstacle's *center*;
- the size (the *length* and *width*).

We mention here that the 2-D representation of the simulated space (the default plot in the right side of the GUI) is constantly updated after each adjustment done in the *General Settings* section. In this way, the user can verify / correct the effect of each input parameter.

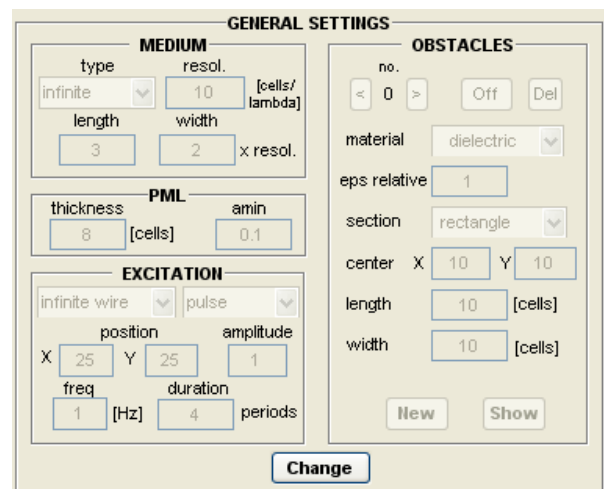


Figure 3. The section for setting the parameters of the simulation.

The *Results Settings* section offers several possibilities for displaying the results (see *Figure 4*):

1) In *Surface* sub-section the user can set the simulation parameters for displaying the 3-D plot of the propagation:

- animation or snapshots from the animation (*dynamic / static*);
- the viewpoint of the graphic representation (given by *elevation* and *azimuth*);
- the *space* and *time steps* for the animation;
- the color appearance of the graph can be frozen by setting *zmin* and *zmax* (the scale on *z*-axis) at values slightly out of the range of values expected for the displayed curve; otherwise, the colors change continuously as the animation runs.

2) The *Spectrum* sub-section allows the displaying of the average power spectrum (Poynting vector), by setting:

- the number of *samples* on which the average power is calculated;
- the resolution of the *grid* (1, 2, 4, 8, 16 or 32).

3) The *Probes* sub-section gives the possibility of placing 1 to 4 probes as desired in the space in order

to visualize the field variations recorded by them. The GUI also calculates the reflection coefficient as the maximum of the reflected wave in terms of percentage of the maximum of the incident wave for each recording probe ( $max2/max1$ ).

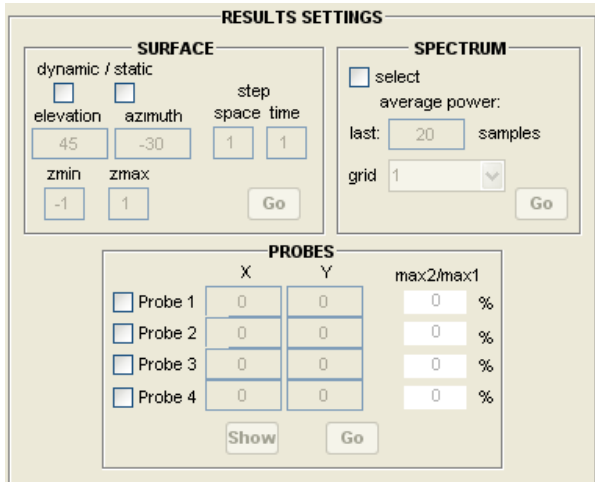


Figure 4. The section for selecting how to display the results.

All the graphical results are displayed in the right part of the GUI and become active depending on the chosen display mode.

#### IV. AN EXAMPLE OF USING THE GUI

In order to illustrate the usefulness of the proposed GUI, this section presents an example in which the propagation of an electromagnetic wave in a 2-D homogeneous infinite space is simulated. A conductor obstacle is placed in the simulated medium.

Figure 5 illustrates the right side of the GUI, where the default plot is the 2-D representation of the simulated space. On the *General Settings* panel the following parameters of the simulation were set:

- the type of the simulated space: infinite;
- the resolution: 10 cells / wavelength (the grid is with grey dotted line in Figure 5);
- the modeled space: square of side  $4 \times resol. = 40$  cells (the bold dotted line square in Figure 5);
- the PML is of 5 cells thickness and 0.1 minimum attenuation (in Figure 5 it's the area between the dotted-line rectangle and the bold-line rectangle);
- the source: an infinite wire, with the shape of Gauss pulse of amplitude 1 and frequency 1 Hz (the small black square at coordinates (25, 35) in Figure 5);
- the duration of the simulation: 6 periods;
- the obstacle: conductor, square-shape (in Figure 5 it's the grey square) with the center at coordinates (25, 20) and of side 6 cells;

We placed 3 probes in the space, at coordinates (15, 10), (10, 35) and (25, 30) respectively (marked with X's in Figure 5). These 3 probes will record the variations of

the electric field at each iteration. After running the simulation, the reflection factor for each recorded variation will be calculated and displayed in the *Probes* sub-section of the GUI ( $max2/max1$ ).

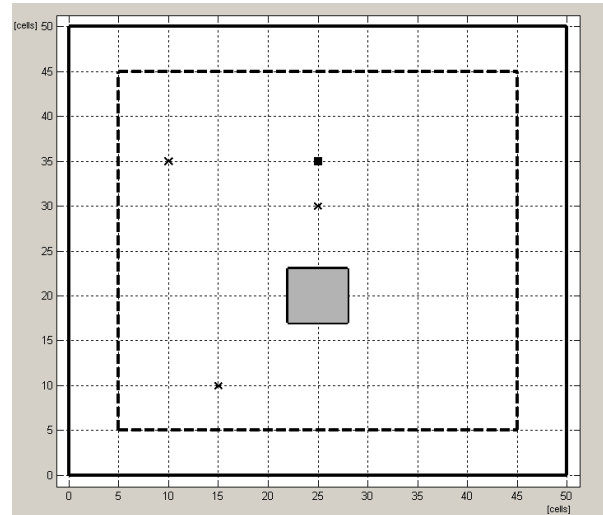


Figure 5. The 2-D simulated space (default plot).

Figure 6 presents a snapshot taken from the beginning of the 3-D animation of the propagation (at iteration  $kc = 11$ , the total number of iterations being 120). One can notice the border of the PML, the obstacle, the 3 probes and the excitation (Gauss pulse).

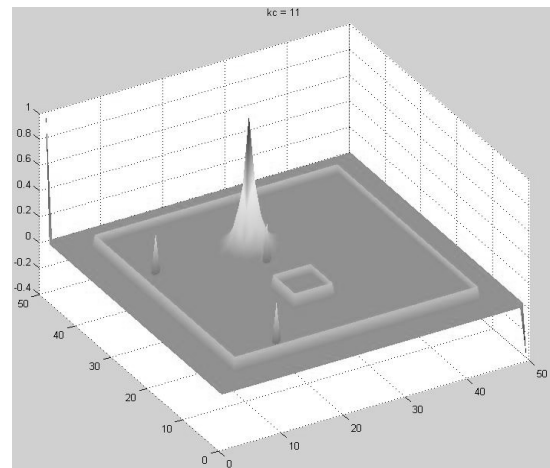


Figure 6. A snapshot from the beginning of the 3-D animation of the propagation.

Further, the snapshots shown in Figure 7 (a-f) illustrate the most significant snapshots of wave propagation. From its origin, the wave starts to propagate uniformly in all directions (Figure 7.a). When a part of it enters the nearest PML (Figure 7.b), it attenuates but no reflections occur; in the same snapshots it can be seen how another part of the wave reflects when it encounters the conductor obstacle in its path. The reflected wave

(with darker gray in *Figure 7.c*) starts to propagate towards away from the obstacle. Next, *Figure 7.d* illustrates the diffraction phenomenon, causing the wave to bend around the conductor obstacle. Further, the front

wave attenuates, as it enters the PML (*Figure 7.e*). Finally, both reflected and incident waves have been absorbed by the PML, as seen in *Figure 7.f*.

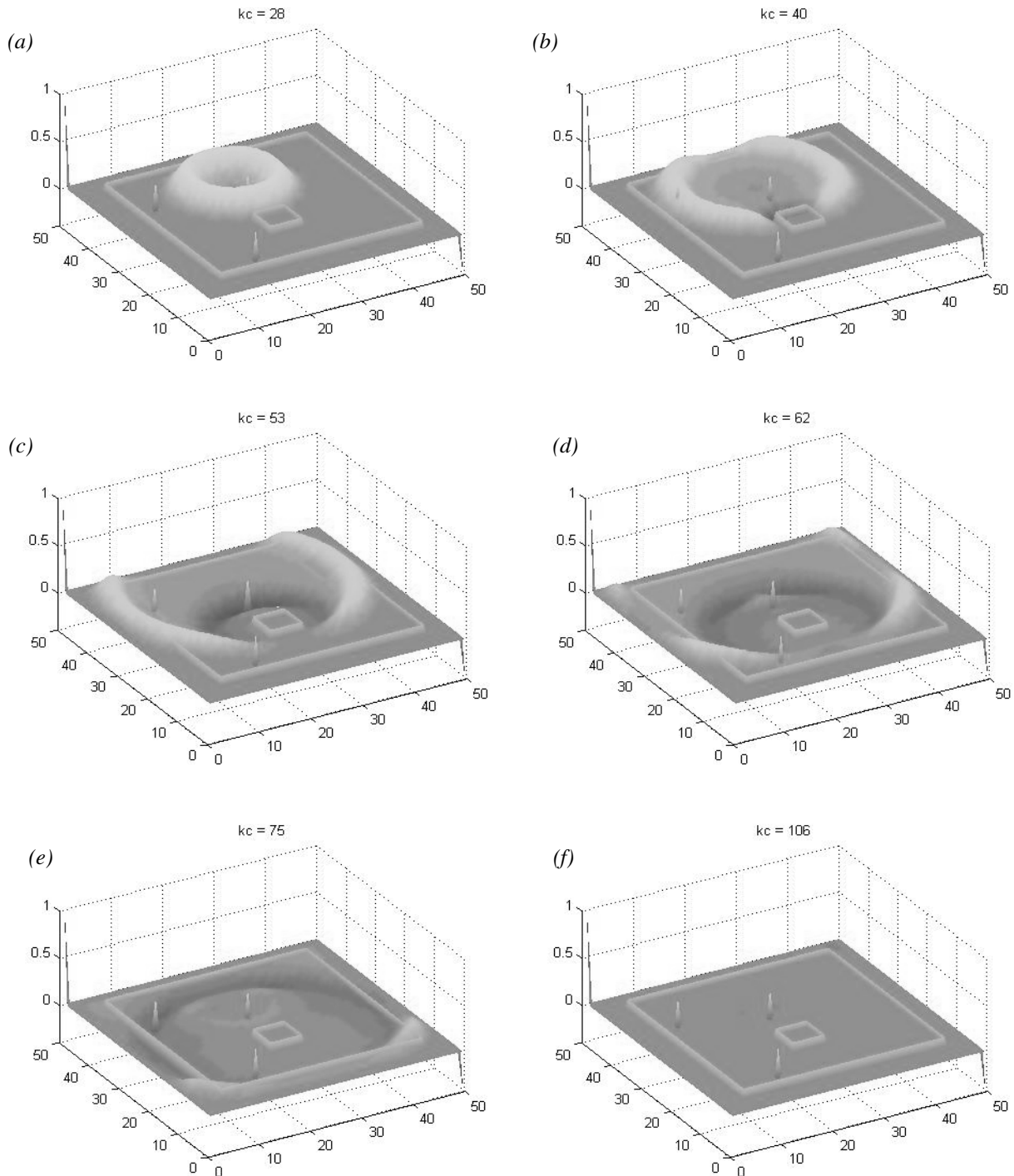


Figure 7. A few snapshots taken from the 3-D animation of the wave propagation.

Figure 8 presents the variations of the field recorded by the probes, plotted versus the iteration number ( $kc$ ). On these plots, the user can measure with the cursor the coordinates in any point of the curves. The inverse waves of small amplitude observed at *Probe 1* and *2* at iterations

$kc=75$  and  $kc=62$  respectively, are due to reflections from the conductor obstacle, and not the PML. If the simulation is done without the obstacle, these inverse waves do not appear anymore, as seen for *Probe 1* in *Figure 9*.

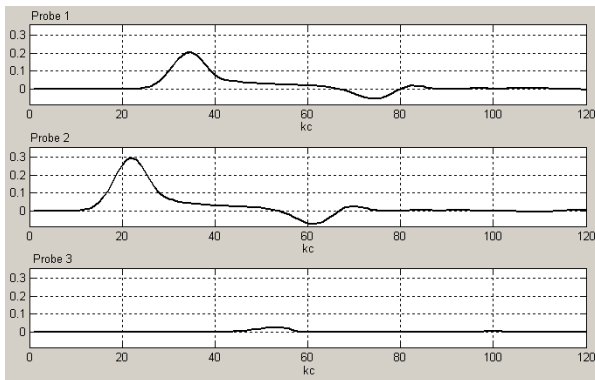


Figure 8. The variation of the E field recorded by Probe 1 at (10, 35), Probe 2 at (25, 30) and Probe 3 at (15, 10).

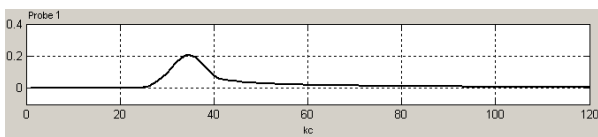


Figure 9. The variation of the E field recorded by Probe 1 when there is no obstacle in the simulated space.

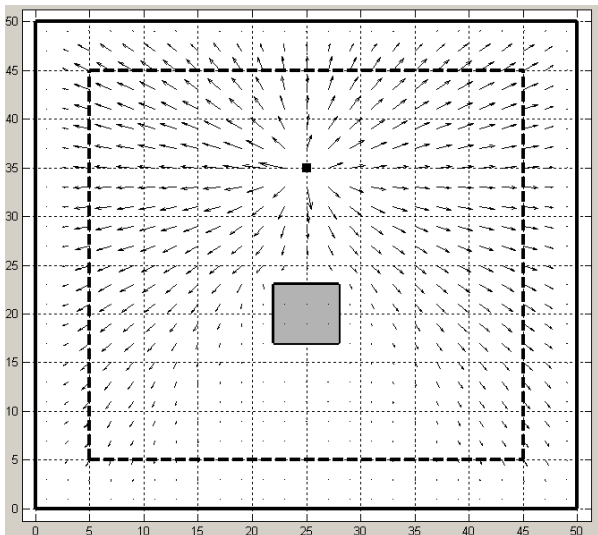


Figure 10. The power spectrum.

Figure 10 illustrates the power spectrum, computed on the last 20 samples of the field. About this representations, we have to make some clarifications here: i) the Poynting vector is calculated for a harmonic excitation (and not for a pulse), as the average over last period; ii) theoretically, vector intensity decreases proportionally with the distance to the source; therefore, the graphical representation undergoes because vectors are very long near the source and tend to zero away from the source; in order to obtain a representation easier to interpret, instead of power density we actually plotted here *the intensity of radiation*, which is Poynting vector multiplied by the distance from the source; thus, in those

areas of space where the Poynting vector decreases with distance, the vectors have equal lengths.

## V. CONCLUSIONS

In this paper, we developed and presented a Matlab GUI based on the modeling of the electromagnetic wave propagation in a 2-D homogeneous, isotropic free-space medium. The space was modeled with the FDTD technique and the infinity of the space was simulated by surrounding all the sides of the computational domain with artificial absorbing layers called PML. The GUI allows the user to set the characteristics of the medium and of the PML, to choose the excitations, to place obstacles of different sizes and shapes in the space, to visualize the results as 3-D animations of the field propagation, or time variations of 1-4 probes placed as desired in the space, or as the spectrum of the average power (the Poynting vector). The paper also presents an example of using the GUI, which illustrates the effectiveness of using the PML in attenuating the entering waves. Further work includes the extension of the GUI to the 3-D case and also the study of the propagation in PML for different angles of incidence.

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