

# DECOUPLED SECOND-ORDER VOLTERRA FILTER STRUCTURE USING THE NLMF ADAPTIVE ALGORITHM FOR NONLINEAR ACOUSTIC ECHO CANCELLATION

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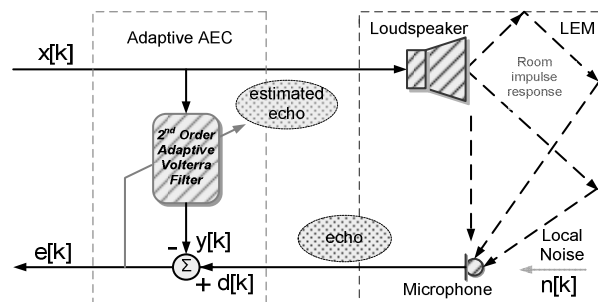
**Abstract:** The paper proposes a nonlinear acoustic echo cancellation setup based on a decoupled scheme of the second-order adaptive Volterra filter. The acoustic distortions that take place in the Loudspeaker-Enclosure-Microphone setup are modeled using measured linear and quadratic Volterra kernels. The adaptation of the filter weights is achieved by exploiting the Normalized Least-Mean-Fourth algorithm. The proposed method is compared with the usual direct adaptation procedure of the second-order Volterra filter using the same algorithm and the same step size values. Comparisons between these techniques regard the evolution of the Echo Return Loss Enhancement in terms of convergence rate and steady-state error. Experimental results for Laplacian distributed excitation show that the new approach increases the speed of convergence in comparison to the one given by the coupled version without enhancing the steady-state error.

**Keywords:** Acoustic echo cancellation, Volterra filters, adaptive algorithms, nonlinear distortions.

## I. INTRODUCTION

Volterra filters are a favorable tool, often used for nonlinear acoustic echo cancellation scenarios [1, 2]. Concurrently they represent a real model to identify the characteristics of a Loudspeaker-Enclosure-Microphone (LEM) setup that works better in effective applications than the linear approach. The major advantage of these filters consists in considering the linear reflections generated by the loudspeaker-microphone enclosure and also the nonlinear distortions produced by the small-sized components used in sound transmission operated at high signal level. *Figure 1* depicts the general setup for nonlinear echo cancellation using a second-order adaptive Volterra filter. The main idea is to shape as accurate as possible the features of the LEM structure with the help of the adaptive filter as noted by [3]. The mentioned linear and nonlinear distortions are applied to the input signal  $x[k]$  so that by subtracting the output of the adaptive filter  $y[k]$  from the signal intercepted by the microphone  $d[k]$ , the error signal  $e[k]$  should become minimum in order to reproduce the local noise ( $e[k] \approx n[k]$ ).

If we look closer at the acoustic echo path one can find the main sources of nonlinearities concentrated along the propagation chain of the audio signal as shown in *Figure 2* where the loudspeaker and the amplifier are operated at maximum capacity. The noticed physical components exhibit different types of nonlinearities: with memory for the loudspeaker as in [4] and memoryless for the overdriven amplifier as in [5]. The nonlinearities can be adequately modeled by an adaptive Volterra filter. The indicated features of the LEM setup are gradually shaped in form of Volterra kernels using a proper adaptive algorithm. Taking into account the robustness and stability, the Normalized



*Figure 1. Acoustic echo cancellation setup using the second-order adaptive Volterra filter.*

Least-Mean-Square (NLMS) algorithm is one of the most used algorithms for acoustic echo cancellation involving Volterra filters [3]. However in terms of convergence rate, one can deduce from the comparison achieved in [6] that the Normalized Least-Mean-Fourth (NLMF) algorithm performs better than the NLMS algorithm used with Volterra structures for various nonlinear acoustic scenarios.

Starting from the presumption made in [6] that the NLMF Volterra filter converges faster than the NLMS Volterra filter for a given set of input signals, in this paper we propose a decoupled version of the second-order Volterra filter for the nonlinear acoustic echo cancellation procedure to speed up even more the convergence of the NLMF Volterra-based adaptive filters. The basis of this proposal is sustained in [7], where the formulation for the partially decoupled Volterra filters is applied on higher order structures using the NLMS adaptation algorithm. In this case the adaptation of the Volterra kernels is done independently; each kernel uses its own error signal for

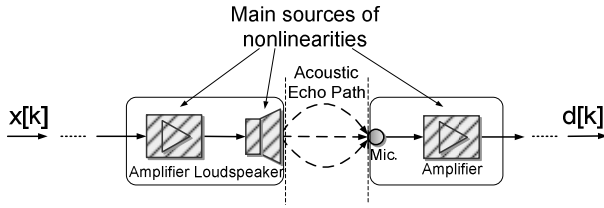


Figure 2. Nonlinear acoustic echo path.

the update equation. We presume that this method might work better than the direct computation of the Volterra kernels structure because the values of the independently computed errors are larger than the ones of the overall error of the coupled setup at least at the beginning of the adaptation. This will eventually speed up the convergence rate. To make use even more of the raised initial stages of the individual error signals we use the NLMF adaptation algorithm [8] to update the Volterra kernels in both direct computation and decoupled version of the filter. This will boost the step size of the usual NLMS algorithm by a factor of  $2e^2[k]$ .

The dissimilarities between the direct estimation procedure of the second-order Volterra kernels that employ the NLMF algorithm for adaptation and its partially decoupled version are tested for a real acoustic enclosure that exhibits linear and second-order distortions with disposed characteristics. The achieved echo reduction of the methods is evaluated in a representative way for acoustic nonlinearities in terms of Echo Return Loss Enhancement (ERLE).

The paper is organized as follows: Section II describes the key particularities of the two implied Volterra structures together with the NLMF adaptation algorithm while in Section III the simulation results for both cases are compared. Conclusions are drawn in Section IV.

## II. DECOUPLED SECOND ORDER NLMF VOLTERRA FILTERS

One of the best suited mathematical models that can shape weak nonlinearities is the Volterra filter. The general form of a  $N$ -th order Volterra filter is depicted in [2] as:

$$y[k] = \sum_{p=1}^N \sum_{m_1=0}^{M_p-1} \dots \sum_{m_p=m_{p-1}}^{M_p-1} h_p[m_1, \dots, m_p] x[k-m_1] \dots x[k-m_p], \quad (1)$$

where  $x[k]$  is the discrete input signal,  $y[k]$  is the output signal of the Volterra filter and  $M_p$  represents the kernels' memory lengths. One can take into account only Volterra kernels  $h_p[m_1, \dots, m_p]$  with non-decreasing indices ( $m_p \geq m_{p-1}$ ) for the sake of simplicity because the kernels are considered to be symmetric [9].

As the order of the Volterra filter increases the identification of real nonlinear distortions may be more precise, although the computational complexity becomes very demanding. This is the reason why the second-order Volterra filter is sufficient to incorporate a large class of

nonlinearities encountered in real-life signal processing scenarios like acoustic echo cancellation setups. The second-order Volterra filter is described as follows:

$$y[k] = \sum_{m_1=0}^{M_1-1} \hat{h}_1[m_1] x[k-m_1] + \sum_{m_1=0}^{M_1-1} \sum_{m_2=m_1}^{M_2-1} \hat{h}_2[m_1, m_2] x[k-m_1] x[k-m_2] \quad (2)$$

that can be rewritten in vector notation as:

$$y[k] = \underbrace{\hat{\mathbf{h}}_1[m_1, k] \mathbf{x}_1^T[k]}_{y_1[k] \rightarrow \text{linear}} + \underbrace{\hat{\mathbf{h}}_2[m_1, m_2, k] \mathbf{x}_2^T[k]}_{y_2[k] \rightarrow \text{nonlinear}}, \quad (3)$$

using the defined vectors:

$$\mathbf{x}_1[k] = (x[k], x[k-1], \dots, x[k-M_1+1]);$$

$$\hat{\mathbf{h}}_1[k] = (\hat{h}_1[0], \hat{h}_1[1], \dots, \hat{h}_1[M_1-1]);$$

$$\mathbf{x}_2[k] = (x^2[k], x[k]x[k-1], \dots, x[k]x[k-M_2+1],$$

$$x^2[k-1], x[k-1]x[k-2], \dots, x^2[k-M_2+1]);$$

$$\hat{\mathbf{h}}_2[k] = (\hat{h}_2[0,0], \hat{h}_2[0,1], \dots, \hat{h}_2[0, M_2-1], \hat{h}_2[1,1], \hat{h}_2[1,2], \dots, \hat{h}_2[M_2-1, M_2-1]).$$

The length of the kernels depends on the filter order and the filter's memory length as follows:

$$\text{length}(\mathbf{h}_p) = \frac{(M_p + p - 1)!}{(M_p - 1)! p!}. \quad (4)$$

Now that we have the general nonlinear model suited for the identification of the LEM features, a proper algorithm is needed to shape the Volterra kernels using both the linear processing and the nonlinear distortions of the input signal. In this work, two particular methods of adapting the second-order Volterra kernels are outlined using the NLMF algorithm.

The first method uses the normal coupled version of the second-order Volterra filter together with the NLMF adaptation algorithm as in [6]. The update equations of the two kernel vectors are described as:

$$\begin{cases} \hat{\mathbf{h}}_1[k+1] = \hat{\mathbf{h}}_1[k] + 2 \frac{\mu_1}{\mathbf{x}_1^T[k] \mathbf{x}_1[k] + \varphi} e^3[k] \mathbf{x}_1[k], \\ \hat{\mathbf{h}}_2[k+1] = \hat{\mathbf{h}}_2[k] + 2 \frac{\mu_2}{\mathbf{x}_2^T[k] \mathbf{x}_2[k] + \varphi} e^3[k] \mathbf{x}_2[k], \end{cases} \quad (5)$$

where the global error is defined as  $e[k] = d[k] - y[k]$  and the output of the overall filter is computed as  $y[k] = y_1[k] + y_2[k]$ . The structure for implementing the NLMF adaptation algorithm for second-order Volterra filters is presented in Figure 3. The resulting error is used to adjust both of the filter weights  $\mathbf{h}_1$  and  $\mathbf{h}_2$  and this induces full coupling between the Volterra kernels.

The decoupled second-order Volterra version of the NLMF algorithm is generated by the following set of equations for kernel updates and for computing each of the error signals:

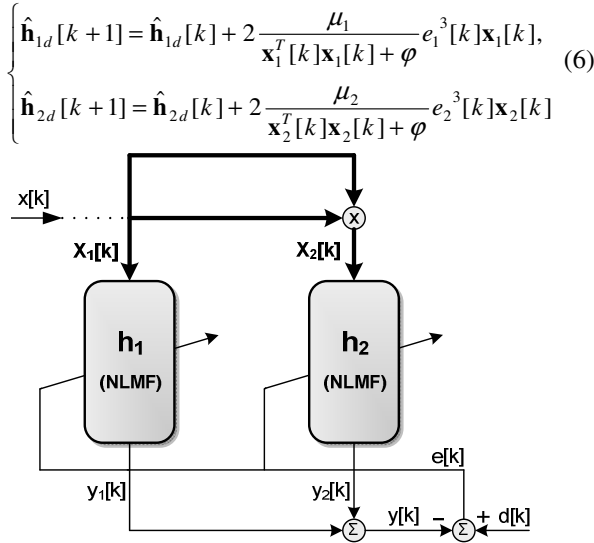


Figure 3. Coupled second-order adaptive Volterra filter structure using the NLMF algorithm.

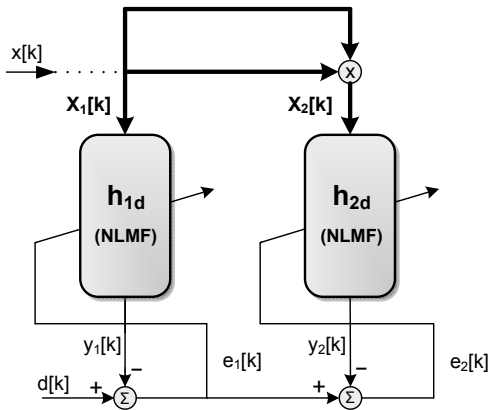


Figure 4. Decoupled second-order adaptive Volterra filter structure using the NLMF algorithm.

and

$$\begin{cases} e_1[k] = d[k] - y_{1d}[k], \\ e_2[k] = d[k] - y_{1d}[k] - y_{2d}[k], \end{cases} \quad (7)$$

where  $y_{1d}[k] = \hat{\mathbf{h}}_{1d}[k]\mathbf{x}_1^T[k]$  and  $y_{2d}[k] = \hat{\mathbf{h}}_{2d}[k]\mathbf{x}_2^T[k]$  represent the outputs of each decoupled adaptive structure.

The parallel structure of a second-order Volterra filter using the decoupled NLMF algorithm is depicted in Figure 4. Each weight vector is adapted independently of the rest of the higher order weights, i.e we presume that the effectiveness of the usual NLMF second-order Volterra filter can be improved by adapting each Volterra kernel with its own error signal. In this way the lower order weights remain unaffected by the statistics of higher order kernels.

### III. SIMULATION RESULTS

The proposed structures: the coupled second-order NLMF Volterra filter and the partially decoupled second-order NLMF Volterra filter are evaluated in this section in terms of convergence rate and steady-state error after the stabilization of the filters.

In order to shape the nonlinear output of the LEM setup a second-order polynomial structure is proposed, written in Volterra form by separating the linear and the nonlinear arguments of the reference signal  $d[k]$ . Besides the nonlinear distortions of the input signal, local noise is added to the microphone intercepted signal. The output of the LEM system follows the model:

$$d[k] = \underbrace{\mathbf{h}_{01}[k]\mathbf{x}_1^T[k]}_{\text{linear}} + \alpha[k] \underbrace{\mathbf{h}_{02}[k]\mathbf{x}_2^T[k]}_{\text{nonlinear}} + \beta[k] \underbrace{n[k]}_{\text{local noise}}, \quad (8)$$

where  $\mathbf{h}_{01}[k]$  and  $\mathbf{h}_{02}[k]$  represent the linear and the quadratic measured kernels of a real acoustic enclosure setup. These weights are depicted in Figures 5 and 6 that have been obtained from measurements at a sampling rate of 8KHz. The memory lengths of the kernels have been set to 320 and 64x64 taps respectively, so that the coefficients with significant values are included. Also the parameters  $\alpha[k]$  and  $\beta[k]$  have the role to set the values of the Linear-to-Nonlinear Ratio (LNLr) and the ones of the Signal-to-Noise Ratio (SNR) to the desired amounts. Throughout simulations, the LNLr quantity is kept constant at 10 dB and  $\beta[k]$  is also chosen to set the SNR at 30 dB. The input is a Laplacian distributed noise. Additive white Gaussian noise is used as local noise.

The efficiency of the two Volterra setups is compared in terms of ERLE defined as in (9), using in every case the corresponding errors of each particular structure:

$$\text{ERLE} = 10 \log \frac{E\{d[k]^2\}}{E\{e[k]^2\}}. \quad (9)$$

The memory lengths of the filter kernels are selected as the ones that characterize the nonlinear model.

In simulations, the following step size parameters are employed for both the coupled and the decoupled Volterra

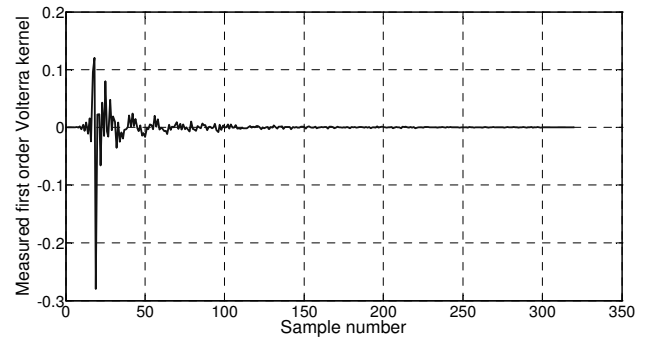


Figure 5. Linear measured Volterra kernel.

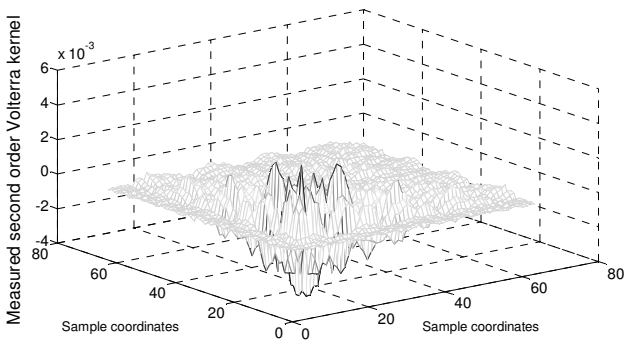


Figure 6. Quadratic measured Volterra kernel

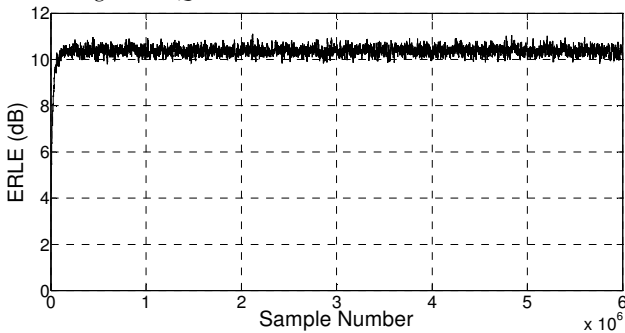


Figure 7. ERLE evolution for linear filters

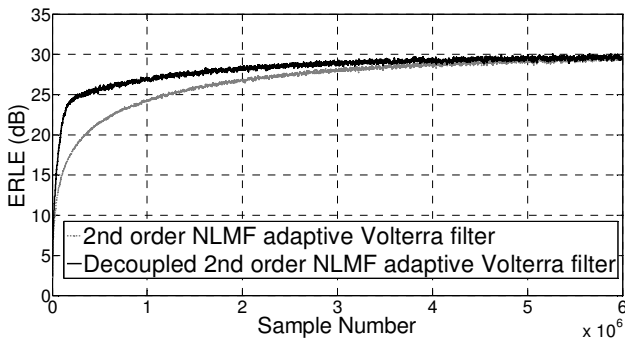


Figure 8. ERLE evolution for the coupled and decoupled second-order Volterra filters

setups:  $\mu_1 = 0.1$  and  $\mu_2 = 0.05$  for updating the linear and the quadratic kernels.

If one uses only the linear filter to identify the acoustic properties of the LEM in both approaches the same evolution of the estimated ERLE is obtained. In Figure 7 it can be seen that the behavior of the two proposed linear filters are identical due to the fact that for the adaptation of the weights the same residual error is used. A high rate of convergence can be noticed and the settlement value of the ERLE reaches 10 dB equal to the value of the provided NLNR.

The evolution of ERLE changes if second-order Volterra filters are involved. Figure 8 shows that the partially decoupled setup of the adaptive Volterra filter presents a better convergence speed compared to the coupled second-order Volterra version. This is due to the larger error signal associated to the adaptation of the second-order kernel of

the partially decoupled Volterra filter at the beginning of the adaptation which offers a so called new step size for adaptation, higher than in the case of the coupled second-order Volterra kernel. The values of ERLE tend to stabilize at a value close to the one of the SNR at 30 dB.

#### IV. CONCLUSIONS

Taking into consideration the results handed in the papers [6] and [7], in this work a comparison between two nonlinear acoustic echo cancellation procedures based on the general second-order Volterra structure was proposed. The suggested methods use distinct adaptation techniques for updating the Volterra kernels regarding the dependency on the global or the individual error signal. The direct computation of the Volterra coefficients procedure and the decoupled one have been tested for a real LEM setup that includes linear and second-order acoustic distortions. The adaptation of the kernels was achieved in both cases using the NLMF algorithm. The comparison between these nonlinear identification techniques was evaluated in terms of ERLE's evolution outlining the similarities and differences regarding the convergence rate of the linear and quadratic filters associated with their own adaptation procedure.

From the experimental results one can relate that when we are dealing with linear filters a rapid convergence and a steady-state ERLE equal to the one of the LNLNR can be noticed. The efficiency of the decoupled version takes place only in the case of the second-order Volterra filter which depicts a higher convergence speed than the fully coupled version of the second-order NLMF Volterra filter. Besides the highly convergence rate of the decoupled Volterra filter, in both cases the steady-state ERLE reaches the value of the SNR. This shows that the adaptation of the decoupled second-order Volterra kernels has the advantage of a better convergence rate without affecting the steady-state error.

The NLMF's major dependency on the error and also by using measured Volterra kernels lowers the range of the probability density functions that describe the input signals for which the proposed techniques can be applied successfully.

Further work will be dedicated on other adaptation methods that can be able to increase the convergence rate of the Volterra filters used for nonlinear acoustic echo cancellation.

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