

RADIAL STUBS DESIGN METHOD FOR MICROSTRIP FILTERS BASED ON NONLINEAR EQUATION SYSTEM

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Abstract: The theoretical background behind this paper is focused on radial stubs design for microstrip filters. A radial stub may be seen as a planar component characterized by its radius and angle. Depending on its geometrical parameters, a radial stub may have an inductive or capacitive behavior depending on frequency. Thus, a radial stub is equivalent with a series inductor-capacitor circuit. The actual paper proposes a method of radial stub design using as input parameters the equivalent capacity, inductance and the dielectric material properties. Equivalent inductance and capacity are expressed by a non-linear equation system that requires a numerical solving method.

Keywords: Radial stub, impedance, non-linear system, numeric method, microstrip, dielectric.

I. INTRODUCTION

The paper presents a numerical method for solving non-linear equation systems when designing microwave circuitry with radial stubs. A nonlinear system can be defined as a system in which the output change is not proportional with the change of the input. In order to validate the method, an elliptical low pass filter design will be further presented as an example. The application field for this method covers the microwave amplifiers, matching networks, antenna, various microwave filters and nowadays is facing a continuous research and development.

II. MOTIVATION

Based on the mathematical model that will generate the nonlinear equation system, the research work is pointed to the implementing process a software tool that can calculate the geometric dimensions of a radial stub. The numerical method for solving such systems has a particular property related to errors caused by truncation, also known as convergence. For a practical design, the system solution shall fit within an acceptable tolerance range without affecting too much the global cutoff frequency of the filter.

The goal of this study is to get an accurate model of the radial stubs as a function of center angle of the disc sector, inner and outer radius, copper layer thickness, core material thickness and its dielectric permittivity constant. This paper is focused mainly on an iterative method for radial stubs design on PCB. At the end, a complex design tool for various filter configurations that generates a complete filter design solution in Gerber format, for example, will be implemented. The work will positively impact the filter design in microwave application field in terms of time reduction for design and automation.

III. MATHEMATICAL MODEL OF THE FILTER

The minimum required data of the filter design are: cutoff frequency, input and output impedance, pass band ripple and at least one indicator of how sharp the frequency response characteristic of the filter shall be, as in (1). Angular frequency for stop and pass band are ω_s, ω_p and A_p, A_s are pass and stop band ripple.

$$k = \frac{\omega_p}{\omega_s} \rightarrow \text{selectivity factor} \quad (1)$$

$$u = \frac{1 - \sqrt[4]{1 - k^2}}{2(1 + \sqrt[4]{1 - k^2})} \quad (2)$$

$$q = u + 2u^5 + 15u^9 + 150u^{13} \rightarrow \text{modular ct.} \quad (3)$$

$$D = \frac{10^{\frac{A_s}{10}} - 1}{10^{\frac{A_p}{10}} - 1} \rightarrow \text{discrimination factor} \quad (4)$$

$$n = \left\lceil \frac{\lg(16D)}{\lg\left(\frac{1}{q}\right)} \right\rceil \rightarrow \text{minimum order} \quad (5)$$

$$V = \frac{1}{2n} \ln \left(\frac{10^{A_p/20} + 1}{10^{A_p/20} - 1} \right) \quad (6)$$

$$p_0 = \left| \frac{q^{\frac{1}{4}} \sum_{m=0}^{\infty} (-1)^m q^{m(m+1)} \sinh[(2m+1)V]}{0.5 + \sum_{m=1}^{\infty} (-1)^m q^{m^2} \cosh(2mV)} \right| \quad (7)$$

$$W = \sqrt{\left(1 + \frac{p_0^2}{k}\right) (1 + kp_0^2)} \quad k = \frac{\omega_p}{\omega_s} \quad (8)$$

$$r = \begin{cases} \frac{n}{2}, n \text{ even} \\ \frac{n-1}{2}, n \text{ odd} \end{cases} \quad u = \begin{cases} \frac{i}{2}, n \text{ even} \\ i - \frac{1}{2}, n \text{ odd} \end{cases} \quad (9)$$

$$X_i = \frac{2q^{1/4} \sum_{m=0}^{\infty} (-1)^m q^{m(m+1)} \sin \left[\frac{(2m+1)\mu\pi}{n} \right]}{1 + 2 \sum_{m=1}^{\infty} (-1)^m q^{m^2} \cos \left(2m \frac{\mu\pi}{n} \right)} \quad (10)$$

$$Y_i = \sqrt{\left(1 - \frac{X_i^2}{k}\right) (1 - kX_i^2)}; \quad i = \overline{1, 2, \dots, r} \quad (11)$$

$$a_i = \frac{1}{X_i^2}; \quad b_i = \frac{2p_0 Y_i}{1 + p_0^2 X_i^2}; \quad (12)$$

$$c_i = \frac{(p_0 Y_i)^2 + (X_i W)^2}{(1 + p_0^2 X_i^2)^2} \quad (13)$$

$$p_i = \frac{-b_i \pm \sqrt{b_i^2 - 4c_i}}{2}; \quad z_i = \pm j\sqrt{a_i}; \quad (14)$$

$$K = \begin{cases} \frac{H_0 \alpha}{s + \alpha p_0}, & n \text{ odd} \\ H_0, & n \text{ even} \end{cases} \quad \alpha = \sqrt{\omega_p * \omega_s} \quad (15)$$

Table 1. Filter coefficients [1, 2]

| i | X_i | Y_i | a_i | b_i | c_i |
|-----|-------|-------|-------|-------|-------|
| 1 | 0.258 | 0.933 | 15 | 0.864 | 0.283 |
| 2 | 0.972 | 0.046 | 1.058 | 0.036 | 0.964 |
| 3 | 0.867 | 0.246 | 1.328 | 0.198 | 0.835 |
| 4 | 1.031 | 0.055 | 0.934 | 0.042 | 1.04 |

$$K = \frac{H_0 \alpha}{s + \alpha p_0} = \frac{1.19 * 10^8}{s + 4.785 * 10^9} \quad (16)$$

$$H(s) = K \prod_{i=1}^r \frac{s^2 + \alpha^2 a_i}{s^2 + \alpha b_i s + \alpha^2 c_i} \quad (17)$$

Transfer function of the low pass filter is expressed with the equations (2) to (17). In Figure 1 and Table 1 can be observed the filter schematic and its normalized converted values.

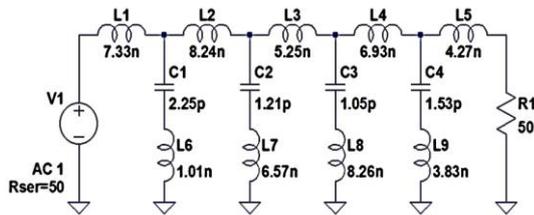


Figure 1. Elliptical Low Pass Filter Schematic

IV. THEORETICAL BACKGROUND OF THE RADIAL STUBS

Radial stubs are widely used in microwave circuitry as amplifier bias lines, ground connected filter branches etc. A radial stub will replace a short circuit connection to ground at its resonant frequency. Thus, all the inductor-capacitor sections of the filter in Figure 1 can be implemented with tuned radial stubs. There are four series sections having 1.716GHz, 1.785GHz, 2.07GHz and 3.34GHz resonant frequencies. The input impedance of a radial stub has multiple times debated and approximated. S.L. March [3], [4] defines an approximated input impedance expression as in (18) to (19).

$$Z_{in} = \frac{-j120\pi h Y_0(\beta r_i) J_1(\beta r_0) - J_0(\beta r_i) Y_1(\beta r_0)}{r_i \theta \sqrt{\epsilon_{eff}} J_1(\beta r_i) Y_1(\beta r_0) - Y_1(\beta r_i) J_1(\beta r_0)} \quad (18)$$

$$\beta = \frac{2\pi \sqrt{\epsilon_{eff}}}{\lambda_0}; \quad W_{ech} = (r_i + r_0) \sin\left(\frac{\theta}{2}\right) \quad (19)$$

$$L_{rs} = \frac{120\pi h}{\theta c} \left[\ln\left(\frac{r_0}{r_i}\right) - 0.5 \right] [H] \quad (20)$$

$$C_{rs} = \frac{\theta r_0^2 \epsilon_{eff}}{240\pi h c} [F] \quad (21)$$

$$Z_{LCser} = -j \frac{120h}{\theta} \left\{ \frac{c}{f r_0^2 \epsilon_{eff}} - \frac{2\pi^2 f}{c} \left[\ln\left(\frac{r_0}{r_i}\right) - 0.5 \right] \right\} \quad (22)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\frac{r_0}{c} \sqrt{\frac{\epsilon_{eff}}{2} \left[\ln\left(\frac{r_0}{r_i}\right) - 0.5 \right]}} \quad (23)$$

$$\begin{cases} \frac{180 * 120\pi h}{\theta \pi c} \left[\ln\left(\frac{r_0}{r_i}\right) - 0.5 \right] = L_{rs} \\ \frac{\theta \pi r_0^2 \epsilon_{eff}}{180 * 240\pi h c} = C_{rs} \end{cases} \quad (24)$$

It can be observed that the equivalent inductance (20) and capacity (21) are nonlinear dependent on geometric dimensions of the radial stub, dielectric permittivity and thickness. For a better symmetry of the filter, the radial stubs can be distributed following a bowtie. In this case the LC resonant sections have to be doubled. The impedance expression in (22) and (23) leads to system (24).

V. ITERATIVE METHOD FOR SOLVING NONLINEAR EQUATION SYSTEM

Let a nonlinear equation system [5] be defined as in equation (22). The nonlinear function f_i is defined as in $f_i: \mathbb{R}^n \rightarrow \mathbb{R}$ where $i = 1, 2, 3, \dots, n$ and $(x_1, x_2, x_3, \dots, x_n) \in \mathbb{R}^n$.

$$\begin{cases} f_1(x_1, x_2, \dots, x_n) = 0 \\ f_2(x_1, x_2, \dots, x_n) = 0 \\ \dots \\ f_n(x_1, x_2, \dots, x_n) = 0 \end{cases} \quad (25)$$

One solution of a system of equations having n variables is represented by a point $(p_1, p_2, \dots, p_n) \in \mathbb{R}^n$ for which is valid the identity (23).

$$f_1(a_1, \dots, a_n) = \dots = f_n(a_1, \dots, a_n) = 0 \quad (26)$$

Due to the fact that a nonlinear system of equation is unlikely solvable with analytical methods, an iterative procedure may be used. The iterative method is an algorithm that generates a sequence of approximate solutions for the systems. There are two main concepts that have to be stated: convergence and consistency.

In figure 2 are presented the approximate solutions for each iterative step related to the target p_0 . The solution convergence within the error circles ϵ_n , leads to none or multiple possible solutions for the system.

The error is defined as in (27). Excluding even the rounding errors of the computational systems that performs the calculus, the iteration will never get the exact solution. It does make sense to invoke the termination criteria that is used limit the number of iterations for the sake of efficiency and computational time.

$$\varepsilon = |p_i - p_0| \quad (27)$$

Based on this iterative solving method, a LabVIEW program has been implemented. In the equations (20) and (21) can be seen the number of variables that inductance and capacity are depending on. They are: inner radius r_i or disc sector base width, outer radius r_0 and center angle θ , see figure 3.

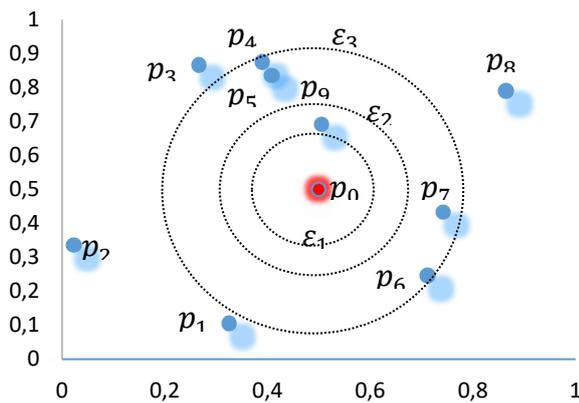


Figure 2. Approximate solutions

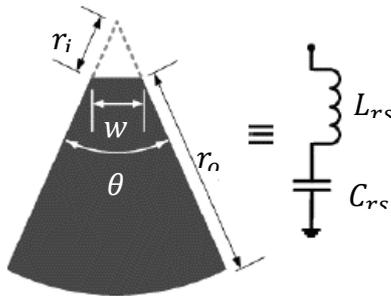


Figure 3. Radial stub geometry

The LabVIEW program defines three data vectors corresponding to the variables and an individual iteration step. The structure of the virtual instrument is made out of three main parts: the parameter iteration loops, solution error comparator and a user interface that displays the solution and plots an image with the calculated radial stub. The comparator checks the system solution and generates an event each time the inductance and capacity are within the tolerance range. After the first iteration cycle it is possible that more than one solutions to be found. If more than one solution have been found, the program decreases the tolerance and restart until one solution with minimum tolerance is returned.

VI. EXPERIMENTAL RESULTS

The filter has been realized on a standard FR4 double layered printed circuit board of 1.6mm thickness and 4.66 dielectric permittivity. The series inductors of the filter were designed as transmission lines which connect the radial stubs orthogonally. In the figure 4 one can see two design methods. On the upper side of the test board is the symmetrical design followed by its asymmetric variant form. The LC resonant branches were split into LC pairs having half of the capacity but double the inductance. In some cases this method reduces the needed surface of a radial stub based on the equations (28), (29), (30) presented in article [6].

$$Z = j \left(\omega L - \frac{1}{\omega C} \right) \quad (28)$$

$$Z_a = \frac{j}{2} \left(\omega L_a - \frac{1}{\omega C_a} \right) \quad (29)$$

$$L_a = 2L; C_a = \frac{C}{2}; \quad (30)$$

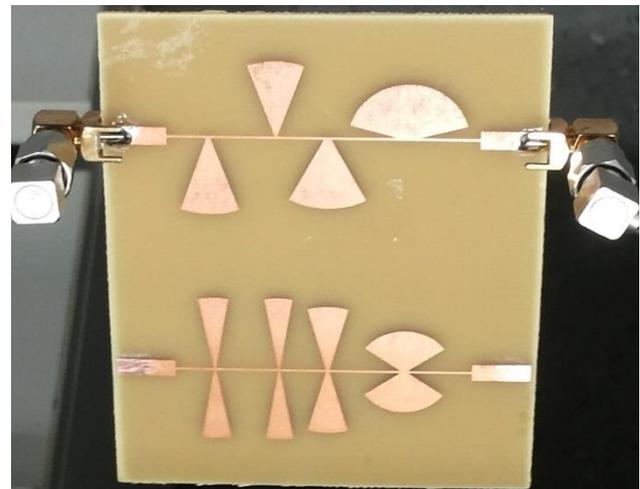


Figure 4. Low pass filter top stencil

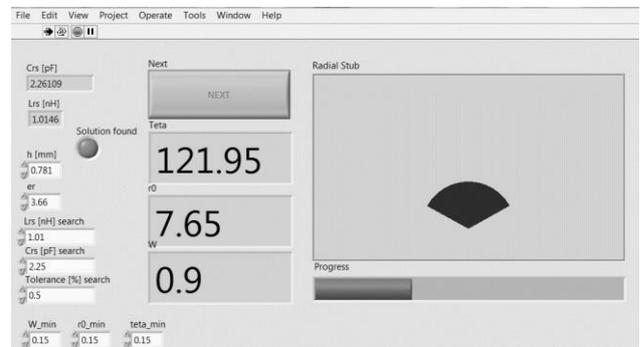


Figure 5. Front panel of the LabVIEW application

The application front panel in figure 5 allows to set the starting values of the radial stub, the mechanical and electrical properties of the printed circuit board and the tolerance. In figure 6 can be observed the block diagram of the LabVIEW program following a multiple loop structure.

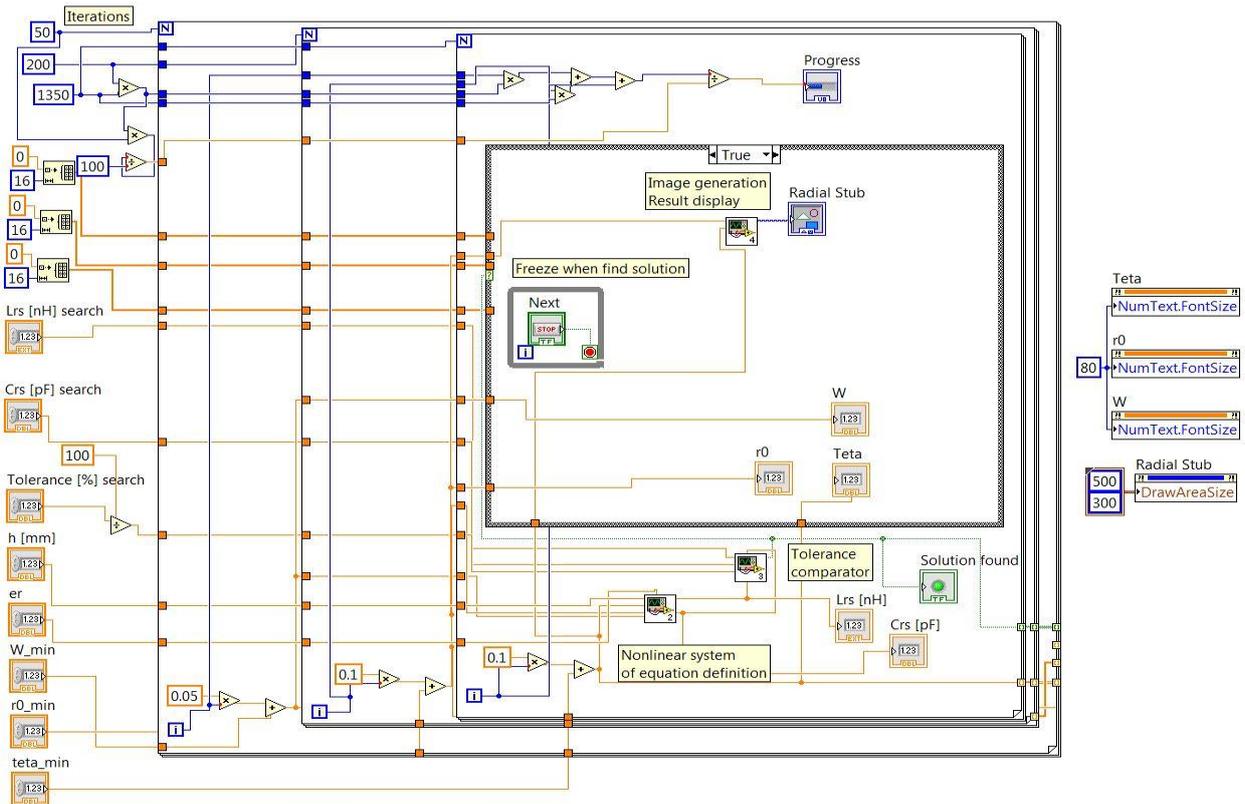


Figure 6. Block diagram of the LabVIEW application

VII. CONCLUSIONS

The experimental LabVIEW tool relies on the described iterative method for solving the nonlinear system of equations. The simplicity of implementation is an advantage and offers the possibility to adjust easily the input parameters.

The low pass filter variant presented in this paper was tested with a network analyzer in order to see the difference between their cutoff frequency and the expected one. In figure 8 are shown the frequency responses up to 3GHz. It was noticed that the cutoff frequencies were few hundreds MHz different. The main reason is the variation of the FR4 dielectric permittivity with frequency.

For a future design, a special PCB material for radiofrequency applications as Rogers 4350B will be used. On the other hand, the tolerance of the LabVIEW application when generates the stub dimensions may affect the filter performance. In table 2 are presented the errors introduced by different changes in width, angle and thickness.

Table 2. Parameter sensitivity

| Parameter | Variation | Impedance change |
|------------------|-----------|------------------|
| Core thickness | 20% | 12% |
| Width | 20% | 10% |
| Copper thickness | 20% | 3% |

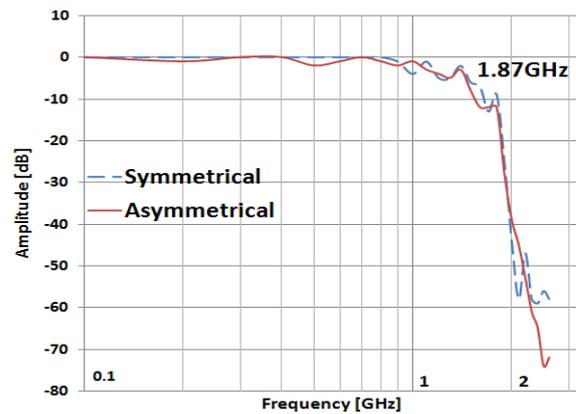


Figure 7. Frequency response of the filters

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