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ADVANCED FLEXIBLE ADAPTIVE FILTERING – TOWARDS STRUCTURE, TIME AND STATISTICAL AGILITY

Colin F.N. COWAN
ECIT, Queen's University Belfast, Northern Ireland Science Park, Queen's Island, Belfast BT3 9DT.
Phone:+44 (0)28 9097 1879, Fax:+44 (0)289097 1702, Email: c.cowan@ecit.qub.ac.uk

Abstract : This paper considers the evolution of modern adaptive filtering in the context of applications in wireless communications. The main contribution is to introduce a common framework which potentially allows the adaptive system to include structure, e.g. number of coefficients, as one of the adaptable parameters. The proposed structure also allows for statistical variations to be taken into account in a highly time variant manor. Some examples are also cited to illustrate these features.

Key Words: Adaptive filters, structure adaptation, Kurtosis, time variation, equalization, echo cancellation.

I. INTRODUCTION

In the context of classical adaptive filtering [1] we normally find that such systems are based on finite impulse response (FIR) filters with adaptation algorithms based on least squares optimization. This introduces restrictions such as assumed Gaussianity of disturbances and the use of 'worst case' design in terms of filter length. This is quite acceptable in many application scenarios such as speech coders, equalizers for line of sight transmission, etc..

However, in the context of modern wireless communications systems these constraints are, generally, unacceptable. For instance, in a mobile communications scenario the length of the channel time dispersion varies considerably dependent on whether the environment is urban or rural. There is also a high degree of time variation and, in modern systems the disturbances are dominated by co-channel users and not Gaussian noise.

This leads us to introduce a new generalized structure [2] which has the potential to include the actual estimation structure as part of the adaptable parameter set. It also allows optimization of statistical performance by including Kurtosis estimation as part of the optimization objective function. Some key examples are then used to illustrate performance in a number of typical application scenarios.

II. GENERALISED FRAMEWORK

Classical adaptive estimators tend to be formulated in the following way:

$$\mathbf{y} = \mathbf{A}\mathbf{w} + \mathbf{n} \qquad (1)$$

Where **A** is some form of fixed observation matrix, **n** is a set of disturbances (noise), and **w** is an unknown weighting matrix.

In the situation of interest here the observation matrix is no longer fixed in form but also contains unknown elements, such as size or nonlinear transformation etc. In other words, **A** is only partially known. We may reformulate this as follows:

$$\mathbf{y} = \mathbf{A}'\mathbf{P}\mathbf{w} + \mathbf{n} \qquad (2)$$

Where **A**' is now the known observation matrix and **P** an unknown permutation, sizing or transform matrix. If we also take into account statistical agility then we must also allow for adaptation of the objective function.

Overall this leads to the generalized structure which is illustrated in figure 1. The core feature within this is that the form of the signal processing applied to the input observations is *a-priori* unknown and forms part of the adaptable parameter set.

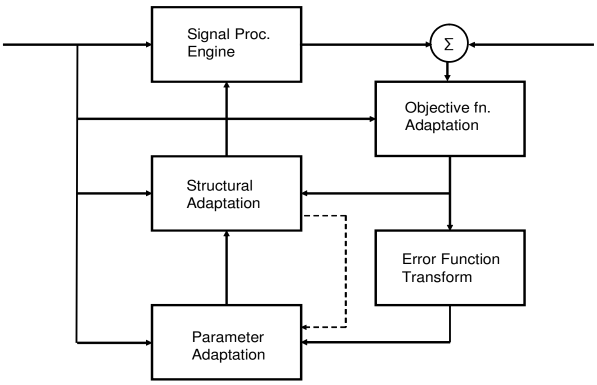

Figure1. A generalized single channel adaptive structure.

At this point we will proceed by introducing a number of key examples which illustrate the behaviour of this generic structure.

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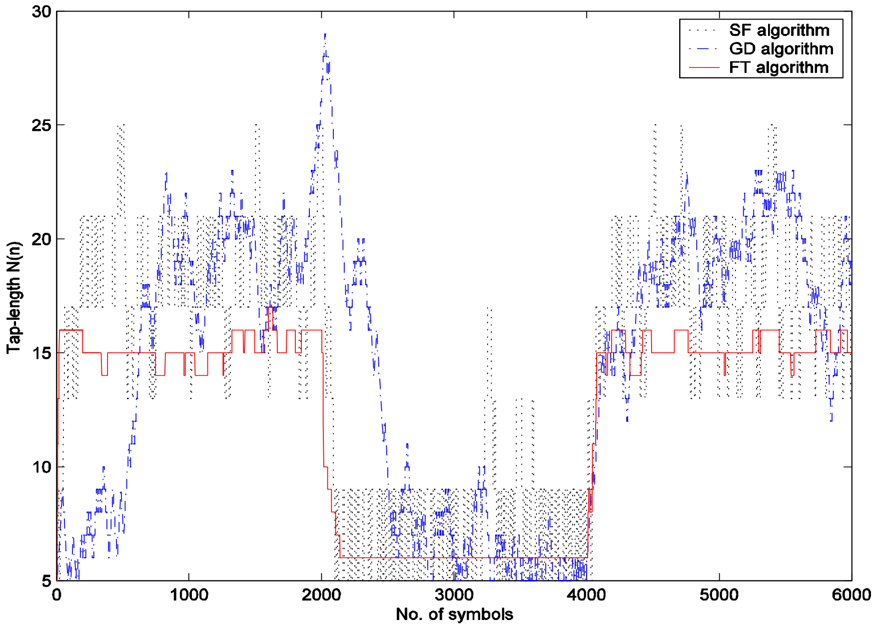

Figure 2. Length adaptation over time for 2 discrete changes in length.

III. STRUCTURE ADAPTATION

In this section we will concentrate on the structural adaptation aspect. Initially we will restrict this to a degree in that we will only consider the single channel finite impulse response structure where the number of coefficients is unknown. This form of adaptation was introduced in [3]. Key to the design of this algorithm are the concepts of fractional time delay and length leakage. The first ensures smooth operation of the stochastic algorithm, while the second prevents overestimation of length (therefore maintaining optimal efficiency).

The algorithm is summarized as follows:

$$n_f(k+1) = [n_f(k)] - \alpha] - \gamma[e^2_{N(k)}(k) - e^2_{N(k)-\Delta}(k)]$$

The true tap length is then : (3)

$$N(k+1) = \begin{cases} \lfloor n_f(k) \rfloor, \left| N(k) - n_f(k) \right| \geq \delta \\ N(k), \text{otherwise} \end{cases}$$

where $n_f(k)$ is the fractional filter length at iteration k, $e^2_{N(k)}(k)$ is the instantaneous mean square error at iteration k for actual filter length $N(k)$, α is the leakage factor (small), γ is the convergence gain, δ is an empirically derived threshold level, and $\lfloor x \rfloor$ denotes the integer part of x.

This corresponds to **P** in equation 2 being an incomplete identity matrix with a number of zeroes in the lower right hand diagonal corner. The exact number of these zeroes is determined by $N(k)$.

Figure 2 shows the result of a system modeling experiment where the modeled system is FIR of length 15, changing to length 6 and back to 15. The result of the length adaptation is shown for three different algorithms. The algorithm in equation 3 is labeled as FT. The SF result is a segmented form introduced in [4], and the GD is a gradient algorithm introduced in [5]. It is clear that the FT algorithm has much superior tracking and stability, which is due to the use of fractional delay and leakage concepts.

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4)$$

This algorithm was further developed in [6] to include lag estimation in the application of equalization and then applied to optimize the structure of a decision feedback equalizer (d.f.e.) [7]. These are really quite small modifications to the example cited here where the matrix **P** is still diagonal but now also has a number of zeroes in the upper left hand diagonal corner.

IV. HIGHER ORDER METRICS

In this section we turn to examine a case where the disturbances on the observation are non-Gaussian in nature. This occurs naturally in the case of co-channel interference in the cellular mobile environment. In this particular example the dominant interference is sub-Gaussian in nature and least squares is therefore a sub-optimal error criterion. In fact, it is well known that the 4th order metric is vastly superior in this case. However, such a metric leads to a highly non-linear optimization which can be very difficult to stabilize in it's stochastic form. In [8] we introduced a robust form of the least mean fourth algorithm which exhibits very stable performance characteristics. This may be summarized as follows:

$$\mathbf{w}(n+1)=\mathbf{w}(n)+\frac{\gamma_{xe}e^{3}(n)\mathbf{x}(n)}{\delta+\lambda_x(\mathbf{x}^{\mathbf{T}}(n)\mathbf{x}(n))+\lambda_e(\mathbf{e}^{\mathbf{T}}(n)\mathbf{e}(n))} \quad (5)$$

Where here $\mathbf{x}(n)$ is the vector of observed channel outputs, *e(n)* is the observation error, γ_{xe} is the adaptation gain, δ is a small number to avoid numerical instability, and γ_x and γ_e are constants between 0 and 1 which balance the normalisation between signal and error power.

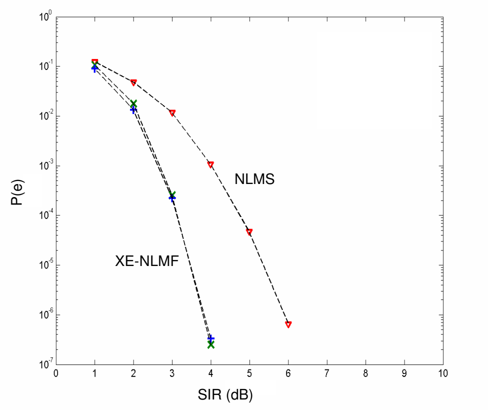

Figure 3. Bit error rate performance of the higher order equalizer in a co-channel interference situation.

Figure 3 shows the bit error rate performance of the XE-NLMF algorithm (equation 5) used to adapt a simple FIR equalizer compared to one adapted using a standard normallised least-mean-squares algorithm [1]. Clearly, using the fouth order metric provides an approximately 2dB gain relative to symbol-to-interference ratio (SIR).

In certain scenarios it can be difficult to pre-select the best possible cost function as the statistical properties are *a priori* unknown. In such a situation it is necessary to invoke some form of adaptation strategy, which is done in a relatively straight-forward way in [9] and further developed in [10] in the context of a data echo canceller application environment.

V. TIME VARIATION

In this section we focus on the effect of time variation. Specifically, the situation to be examined is a channel equalization scenario where the channel introduces inter-symbol interference (ISI) which varies rapidly with time relative to the sampling rate. Here we take the particular case of base-band binary signaling.

In this particular instance there is a high degree of correlation between the channel impulse response and the instantaneous amplitude of the received signal *a(k)* [11]. Therefore we can use this to enhance performance by the application of a non-linear transformation to the received signal vector which has the effect of instantaneously permuting an array of equalizer coefficients which are closest to the accurate equalizer. We refer to this as the amplitude banded (ABF) equalizer, which is given in equation 6 [12].

$$y(k)=\mathbf{a}^{\mathbf{T}}(k)\mathbf{h}(k)$$

where

$$\mathbf{a}^{\mathbf{T}}(k)=[a(k) \quad a(k-1) \quad ..... \quad a(k-N+1)]$$

$$\mathbf{h}^{\mathbf{T}}(k)=[h_1(k) \quad h_2(k) \quad ..... \quad h_K(k)]$$

$$h_p(k)=\sum_{q=1}^{L}x_{pq}(k)w_{qp}(k)$$

$$\mathbf{W}(k)=\begin{bmatrix} w_{11}(k) & w_{12}(k) & . & w_{1N}(k)\\ w_{21}(k) & w_{22}(k) & . & .\\ . & . & . & .\\ w_{L1}(k) & . & . & w_{LN}(k)\end{bmatrix}$$

$$\mathbf{X}(k)=\begin{bmatrix} x_{11}(k) & x_{12}(k) & . & x_{1L}(k)\\ x_{21}(k) & x_{22}(k) & . & .\\ . & . & . & .\\ x_{N1}(k) & . & . & x_{NL}(k)\end{bmatrix} \quad (6)$$

$$x_{ij}(k)=\begin{cases} 1 \text{ if } T_{L-1}<|a(k-i+1)| \\ 1 \text{ if } T_{j-1}<|a(k-i+1)|\le T_j \\ 0 \text{ otherwise}\end{cases}$$

$$j=1,2,....L-1 \text{ and } i=0,1,....N-1$$

Where L is the number of threshold levels and T_j is the predetermined amplitude range. The product of **W**_(k)_ and **X**_(k)_ has the effect of selecting one of the sub-coefficients from each column of **W**_(k)_ dependent on the instantaneous amplitude of the input sample *a(k-i)* applied to that weight. During the adaptation phase, only the N selected weights are updated using the NLMS algorithm.

By comparison with equation 2, it is easily seen that here the **P** matrix is a dynamic permutation of the weight matrix such that **P** takes the following form (note that this is an arbitrary example):

$$\mathbf{P}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\end{bmatrix}$$

The structure may be more easily understood by reference to figure 4, which shows an interpretation in block diagram form for 3 amplitude bands.

The simulation result shown in figure 5 is for the situation where our channel is modeled as a 2 coefficient FIR filter with coefficients which are generated by a second order Markov process (auto-regressive generator). The sampling rate is 2.4 kHz. And the band-width of the coefficient time variation is 0.5 Hz.. The equalizer were of length 8 samples and the ABF equalizer employed 8 amplitude bands. It may be clearly seen from figure 5 that the converged mean square error of the ABF equalizer exceeds the linear LMS equalizer by a margin of at least 15 dB.

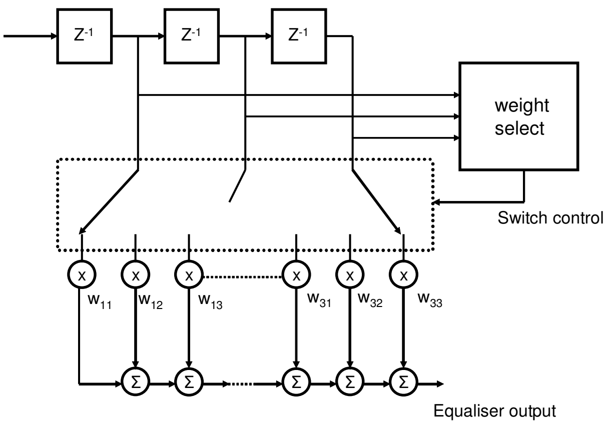

Figure 4. Simplified block diagram of the amplitude banded equalizer.

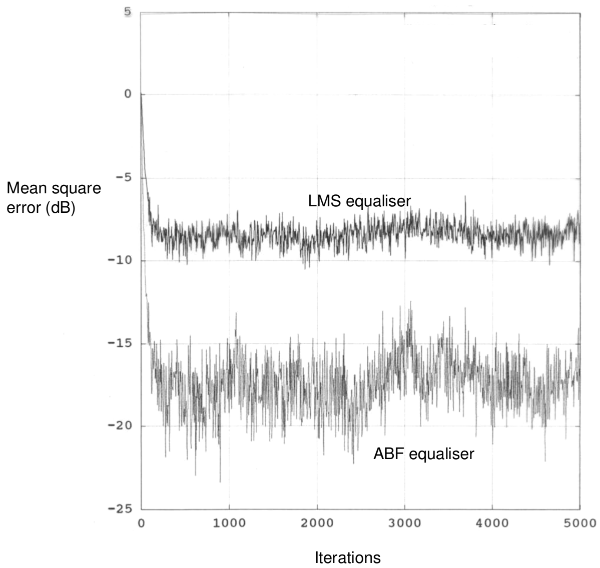

Figure 5. Comparison of the linear LMS equalizer with the amplitude banded equalizer in terms of mean square error, for a fade rate of 0.5 Hz.

VI. CONCLUSIONS

This paper has considered some generic issues relating to the application of adaptive filters in modern wireless communications systems. This has focused on the problem areas of situations were the disturbances present are non-Gaussian in nature, the filter order is unknown or the channel in question is highly time variant. Novel structures have been presented to tackle each of these cases. These have been presented in a unified formulation which provides a generalized adaptive filter structure.

It is, of course, not possible to fully automate this structure at present as there are too many unknowns in a completely unconstrained model. However, the structure presented here has the effect of considerably loosening the design constraints.

Further work in this area will be concentrated on the introduction of non-linear kernels and the application to multi-channel multiple input, multiple otput (MIMO) systems [13].

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