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MULTY BINARY TURBO CODED WOFDM PERFORMANCE IN FLAT RAYLEIGH FADING CHANNELS

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Abstract: In this paper, the application of Multi Binary Turbo Codes (MBTCs) to two multicarrier modulation techniques used for transmission in the flat radio fading channel is studied. Thus, the performance of the coded versions of Orthogonal Frequency Division Multiplexing (OFDM) and of its wavelets based version (WOFDM) is evaluated under several simulation scenarios. These scenarios take into account the time variability of the radio channel, by the mean of the Doppler shift parameter. An important conclusion is that, under the assumptions made in this paper, WOFDM outperforms OFDM. By the other hand, this technique shows little influence on the choice of the wavelets mother used in the modulator.

Key words: *Duo binary Turbo Codes, WOFDM, OFDM, Flat Fading.*

I. INTRODUCTION

Wavelets represent a successful story of the last decade in signal processing applications. Thus, these signals, with some highly desirable properties, are widely used in domains as compression, denoising, segmentation, in-painting or classification. By the other hand, in data communications, the same successful story can be assigned to multi-carrier modulation techniques. The principle of the OFDM is employed in a large number of transmission standards, over wired and wireless channels: WiFi, WiMAX, DAVB or ADSL.

The wavelet based OFDM (WOFDM), sometimes referred to as wavelet modulation is the point where the above concepts meet with each-other. Although they are widely used in signal processing, few wavelets applications are known in data transmission. The idea that gathers the two concepts is to use wavelet signals as carriers in a multi-carrier data transmission.

Recent research has shown that, by associating the multi-carrier concept and the wavelet signals, some of the OFDM's classical drawbacks can be counteracted. There are different forms of multi-carrier wavelet-based transmission [1-3] and most of them use the Inverse Discrete Wavelet Transform (IDWT) for the implementation of the multicarrier transmitter. The BER performance of WOFDM was extensively studied in previous papers, in AWGN and flat fading channels [4]. Briefly, our investigations showed that, while there is no difference between OFDM and its wavelet-based version for AWGN channels, it may be worthwhile to carry a deeper research on the fading channels case. Thus, WOFDM provided significantly better results than OFDM, no meter what was the wavelet use as carrier. Furthermore, the choice of the wavelets mother is meaningful for the BER performance of WOFDM: Haar wavelet provided, by far, the best results [4]. In general, our empirical research led us to the conclusion that, better the time localization of the carrier, higher the resilience to the time-variant character of the fading and lower the BER of the system. All the above investigations were conducted in a simple scenario, without considering any additional issues as synchronization or channel coding. This time, our goal is to associate the WOFDM transmission with a very powerful channel coding technique, namely the turbo-codes.

Turbo codes are a class of high-performance error correction codes developed in 1993, [5] which are finding use in deep space satellite communications and other applications where designers seek to achieve maximal information transfer over a limited-bandwidth communication link in the presence of data-corrupting noise. They are particularly attractive for cellular communication systems and have been included in the specifications for both the WCDMA (Wideband Code Division Multiple Access)/UMTS (Universal Mobile Telecommunications System) and cdma2000 third-generation cellular standards.

The Multi Binary Turbo-Codes (MTBC) are a parallel concatenation of two binary recursive systematic convolutional (RSC) codes based on multiple-input (r-input) linear feedback shift registers (LFSRs) and they provide several advantages versus the classical turbo-codes. MBTC has already been adopted in the digital video broadcasting (DVB) standards for return channel via satellite (DVB-RCS) and the terrestrial distribution system (DVB-RCT), and also in the 802.16 standard for local and metropolitan area networks. Forward Error Correction (FEC) coding, for wireless applications on fading channels, is an important

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tool for improving communications reliability. MBTCs have been shown to perform near the capacity limit on the additive white Gaussian noise (AWGN) channel, [6].

An association between WOFDM and turbo-codes is not extensively treated in the literature. When presented [7], the results are however very promising, and they show that it is worthwhile to carry a research on this topic. The authors of this paper will associate the two concepts, encoding the data bits with a MBTC, before passing them to the WOFDM modulator. The transmission chain obtained will be called MBTC-WOFDM, and its performance is studied in flat Rayleigh fading channels.

In section 2, the multi-binary turbo-codes and the wavelet modulation principles are briefly presented. Next, the transmission chain used for simulations is described. Simulation results are presented in section IV, whereas the final conclusions are drawn in the last section.

II. BASIC CONCEPTS OF MBTC-WOFDM

A. Multi-binary turbo codes

The general scheme of a $r/(r+1)$-rate recursive systematic convolutional (RSC) multi binary codes is presented in [8], where r represents the number of inputs for RSC encoders. In this work we consider the value of r set to 2. Therefore, a scheme of the 8-state duo binary encoder is represented in Figure 1, [9].

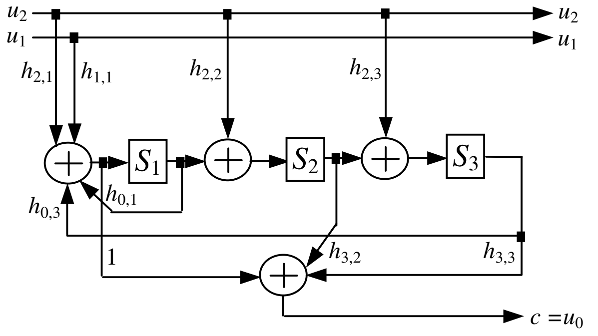

Figure 1: Duo binary recursive systematic convolutional code with m=3 memory units (8-state), which is the building block for duo binary turbo encoding, with polynomials 15 (feedback) and 13 (redundancy) in octal form (DVB-RCS constituent encoder).

Denoted by $S_t = \left[s_3^t \quad s_2^t \quad s_1^t\right]^T$ and $U_t = \left[u_2^t \quad u_1^t\right]^T$ are related to the encoder state. The full generator matrix of the 3 – memory multi input encoder has the following form:

$$H = \begin{bmatrix} h_{3,3} & h_{2,3} & h_{1,3} & h_{0,3} \\ h_{3,2} & h_{2,2} & h_{1,2} & h_{0,2} \\ h_{3,1} & h_{2,1} & h_{1,1} & h_{0,1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 7 & 1 & 5 \end{bmatrix}_{10} \tag{1}$$

By skipping the weights for the redundant bit and the weights for the recursive part, we obtain the matrix H_0 as follows:

$$H_0 = \begin{bmatrix} h_{2,3} & h_{1,3} \\ h_{2,2} & h_{1,2} \\ h_{2,1} & h_{1,1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{2}$$

The feedback vector and the output vector have the form:

$$\begin{aligned} H_F &= \left[h_{0,3} \quad h_{0,2} \quad h_{0,1}\right]^T = \left[1 \quad 0 \quad 1\right]^T \\ H_{out} &= \left[h_{3,3} \quad h_{3,2} \quad h_{3,1}\right]^T = \left[1 \quad 1 \quad 0\right]^T \end{aligned} \tag{3}$$

where the operator $(.)^T$ denotes the transpose of a vector. The main equation of the circuit in Figure 1 is:

$$(S_{t+1})_{3\times1} = (H_0)_{3\times2} \cdot (U_t)_{2\times1} + (T)_{3\times3} \cdot (S_t)_{3\times1} \tag{4}$$

where the matrix T is defined as:

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0_{2\times1} & I_2 \\ \multicolumn{2}{c}{H_F} \end{bmatrix} \tag{5}$$

The redundant output is equal to:

$$c^t = H_{out} \cdot S_t + W \cdot S_{t+1} \tag{6}$$

where the vector W is to [0 0 1].

A parallel concatenation of two identical r-ary RSC encoders with an interleaver (∏) is presented in Figure 2.

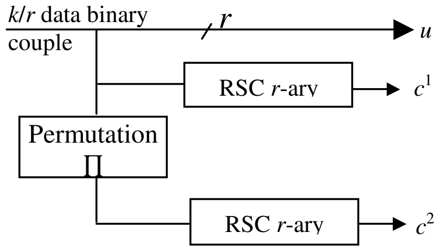

Figure 2. The r-ary turbo-encoder.

Blocks of k bits (k being a multiple of r) are encoded twice by the bi-dimensional code, whose rate is $r/(r+2)$ (for r=2, it results 1/2 rate), to obtain the multi binary turbo encoder scheme from figure 2.

In this paper we used the S-interleaver, ∏, [10]. The S-interleaver is a random type interleaver. Therefore, unlike the pure random interleaver, a minimum interleaving distance equal with S is forced by construction. The interleaving mapping construction algorithm will be next presented. We select a possible future position for the current bit. This position is compared to the last S positions already selected.

If the condition:

$$|\pi_s(i) - \pi_s(j)| > S \text{ for } i \text{ and } j \text{ with } |i - j| < S \qquad (7)$$

is satisfied we go further. If the condition is not true, we select another position for the current bit, which will also be tested. The process will be repeated up to the moment when all the positions for the N_s bits will have been found. The simulations demonstrated that, if $S < \sqrt{N_s/2}$, then the process will converge.

B. Wavelet modulation and the multicarrier concept

The wavelet modulation, referred to as WOFDM relies on a multicarrier modulation concept, exactly as OFDM. Despite its excellent behavior in unfriendly channel environments, OFDM raises some practical problems which are difficult to overcome. Thus, the bandwidth is increased by the use of a cyclic prefix and the transmission is highly sensitive to the Doppler spread introduced by the time variant channels. Next, the OFDM systems are very sensitive to the narrow band interferences. Furthermore, the synchronization in time and frequency is critical for the system performance and the peak-to-average ratio of the signal is very large due to the non-constant nature of the envelope. Finally, the out-of-band rejection of such a signal is not satisfactory, since the OFDM spectrum is made of sinc functions, whom sidelobes contain an important amount of energy. Some of these shortcomings can be eliminated by using wavelet carriers instead of the OFDM's complex exponentials.

In any multi-carrier approach, data is transmitted using several parallel substreams. In OFDM, every such a stream modulates a different complex exponential subcarrier, the used subcarriers being orthogonal to each other. The orthogonality is the key point that allows subcarrier separation at receiver. This modulation is implemented computing an Inverse Discrete Fourier Transform, (IDFT). The crucial idea which links the classical OFDM and the wavelet-based technique is that, in the same manner that the complex exponentials define an orthonormal basis for any periodic signal, the wavelet family as defined in (8) forms a complete orthonormal basis for $L^2(\Re)$:

$$<\psi_{j,k}(t), \Psi_{m,n}(t)> = \begin{cases} 1, \text{ if } \; j = m \text{ and } k = n \\ 0, \text{ otherwise} \end{cases} \qquad (8)$$

Equation (8) indicates that all the members of the wavelet family $\{\Psi_{j,k}(t) = 2^{-j/2}\Psi(2^{-j}t - k)\}_{k\in Z}$ (we considered s0=2 and τ0=1) are orthogonal to each other. Consequently, if instead of complex exponential waveforms we use wavelet carriers, we will still be able to separate these subcarriers at receiver, due to their orthogonality. As for the classical OFDM, the WOFDM symbol can be generated by digital signal processing techniques, such as the Inverse Discrete Wavelet Transform (IDWT). In this case, the transmitted signal is synthesized from the wavelet coefficients $w_{j,k} =< s(t), \psi_{j,k}(t) >$ located at the k-th position from scale j (j=1,..., J) and approximation coefficients $a_{J,k} =< s(t), \varphi_{J,k}(t) >$, located at the k-th position from the coarsest scale J:

$$s(t) = \sum_{j=1}^{J}\sum_{k=-\infty}^{\infty} w_{j,k}\psi_{j,k}(t) + \sum_{k=-\infty}^{\infty} a_{J,k}\varphi_{J,k}(t) \qquad (9)$$

Note that, in practice, a sampled version of the signal s(t) is generated by means of Malat's Fast Wavelet Transform (FWT) algorithm [11]. In this case, the input data vector can be seen as follows:

$$data = [\{a_{J,k}\}, \{w_{J,k}\}, \{w_{J-1,k}\}, ..., \{w_1, k\}] \qquad (10)$$

This input data sequence is modulated onto a contiguous finite set of dyadic frequency bands and onto a finite number of time positions k within each scale. The implementation using IDWT is shown in figure 3:

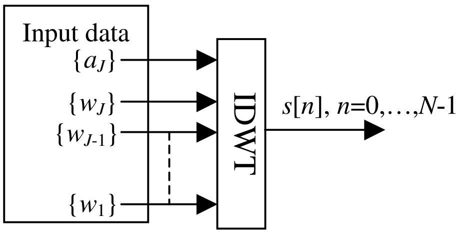

Figure 3: Synthesizing the WOFDM symbol using IDWT: practical implementation.

Note that the output in figure 3 is represented by a sequence of samples, and not by an analog signal. This is because the Mallat's algorithm is based on digital filter banks and operates with discrete-time inputs and outputs.

III. SIMULATION SCENARIO

The transmission chain used for simulation purposes is shown in figure 4.

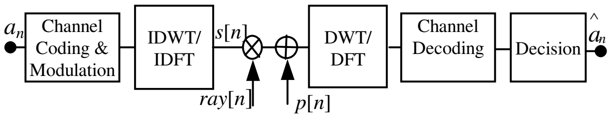

Figure 4: The baseband simulation chain for MBTC-WOFDM.

Most of the parameters used in simulations are displayed in Table 1. We will now detail the most important features of the simulations described in this paper.

A. The transmitter

The data source a_n is a sequence of binary data symbols. This sequence is encoded using a MTBC and next transformed into a bipolar sequence, by the BPSK baseband modulator. The decoding algorithm used in this paper is the MaxLogMAP algorithm, [12]. We consider the extrinsic information with a scaling factor set to 0.75 [13]. The influence of the number of iterations and of the length of the interleaver on the performance of TCs over Rayleigh fading channels is already studied in [7]. The multi-carrier symbol synthesizer is either an IDWT or an IDFT processor, for WOFDM and OFDM respectively. For the wavelet based approach, we tested two different wavelets: Haar and Daubechies-12. Previous simulations made for the not-coded system showed that Haar wavelets mother provide the best results. We tried to check if this conclusion remains valid for the coded case, and to what extent. The number of carriers for both systems is N=1024.

Table 1: The most important simulation parameters.

Parameter	**Used variant**
The turbo code configuration	Parallel
The component code	RSC code with: 3 memory (H=[6 7 1 5])
Turbo coding rate	1/2
Punctured	no
Modulation	BPSK
Systems	OFDM WOFDM, with wavelets mother: Haar and Daubechies12 N=1024 carriers
Channel	Flat fading + AWGN fm=0.05 and 0.1
Interleaver	S-interleaver, S=9
Data block length	2 x 256 = 512 bits
Decoding algorithms	MaxLogMAP
Iteration number	15 iterations with a stop criterion iteration based on APP (A Posteriori Probability) distribution.
Simulated block number	Inversely proportional with the logarithm of the BER.

B. The channel

We simulate the behavior of a radio channel, an environment where the multicarrier approach suites the best. Such a channel exhibit small scale fading, leading to two independent characteristics: time variance and frequency selectivity of the channel [14]. The variance in time of the radio channel's behavior can be expressed by the mean of the Doppler shift parameter, which depends on the relative motion between transmitter and receiver (v) and on the transmission wavelength (λ). The maximum value of this parameter is:

$$f_d = v/\lambda \tag{11}$$

The authors use in this paper a normalized version of the Doppler shift:

$$f_m = f_d \cdot T_S \tag{12}$$

where T_S is the time dedicated to the transmission of one data symbol, or the symbol time. We considered two values for the f_m parameter, as detailed in Table 1. In slow fading scenarios, T_S must be much smaller than the coherence time of the channel expressed as:

$$T_C = \frac{0.423}{f_d} \tag{13}$$

Taking into account (12,13), our best case scenario (f_m=0.05) leads to a coherence time T_C which is approximately 8 times higher than T_S. In the other case the coherence time is only 4 times longer than the symbol duration.

Thus, the channel stays unchanged for the duration of a few serial symbols. Though, when evaluating the channel behavior, one should take into account that in the multi-carrier communications the transmitted symbol is longer. Usually, since the whole N samples data vector is required at demodulator to identify the transmitted symbols, we can consider that the multicarrier symbol duration (an OFDM or a WOFDM block) is N times longer than the serial symbols brought at modulator's input. Note that under these conditions, the channel response changes during the transmission of one symbol, or block, and can be considered as a fast fading channel.

From the frequency selectivity point of view, the scenario taken into account refers to flat fading model, where the frequency response of the channel is considered approximately constant in the transmission band. This means flat frequency response of the channel, which can be implemented as a one-tap filter.

The small scale fading envelope can be modeled with a Rayleigh distribution, if there is no line of sight in transmission. The impact of the Rayleigh flat fading is given by the multiplicative $ray[n]$ (see figure 4). A white noise $p[n]$ is then added to the signal above, obtaining the sequence $r[n]$ to be processed by the receiver:

$$r[n] = s[n] \cdot ray[n] + p[n] \tag{14}$$

C. The receiver

The main block of the receiver is composed of a DWT/DFT demodulator. The data is next decoded by the turbo-decoder. Note that the turbo-decoder operates with a soft input. This means that the data samples at decoder input are neither binary nor bipolar symbols on which a hard decision has been taken a-priori. Instead, the decoder operates with signal samples having continuous values, resulted from the channel impact (multiplicative Rayleigh and additive white noise) on the bipolar transmitted samples. Practically, the decoding algorithm leads to a hard decision on the transmitted symbol, incorporating this way the final detector. By the other hand, neither synchronization nor equalization issues were considered.

IV. SIMULATION RESULTS AND DISCUSSIONS

For the performance evaluation, BER and FER statistics are computed. The results are synthesized in figures 5 and 6. Several conclusions can be drawn. The most important remark is that previous results obtained for the un-coded case [4] are confirmed. Thus, all the WOFDM transmissions have smaller BER and FER than in the OFDM for both Doppler shifts taken into account.

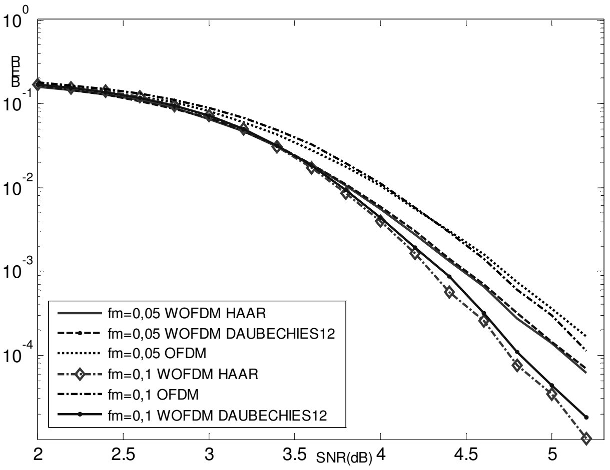

Figure 5: BER performance of MBTC-WOFDM and OFDM in the Rayleigh flat fading channel.

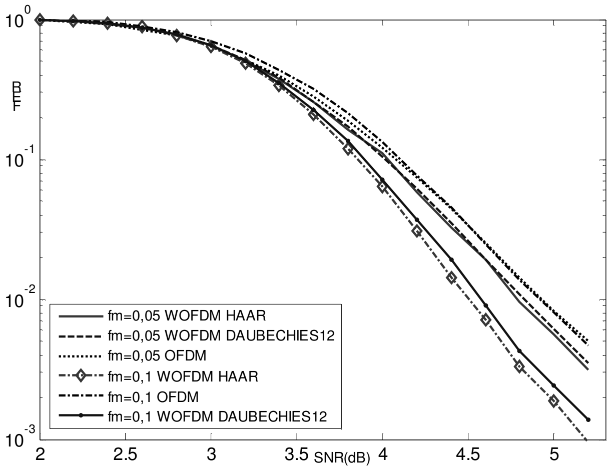

Figure 6: FER performance of MBTC-WOFDM and OFDM in the Rayleigh flat fading channel.

The OFDM curves represent the worst case plots from the figures above. The WOFDM performance improvement is higher for f_m=0.1, where for a BER of 0.0001, this technique brings a gain of approximately 0.4dB. The remark is valid for the FER measure too, with approximately the same gain for a FER of 0.005. The gain brought by WOFDM is not as spectacular as in the un-coded case, due to the errors correction power given by the turbo-code.

Regarding the WOFDM methods, the only difference in the performance of the system issued by the wavelet mother used is a small gain of Haar-based WOFDM compared to Daubechies-12, only noticeable at f_m=0.1.

Surprisingly, while there is no Doppler-issued difference for the OFDM transmission, in the case of WOFDM, the performance is better at higher Doppler shifts. Our guess is that this behavior is generated by the way that the erroneous bits are distributed over the data blocks, before the decoding algorithm is applied. Though, a conclusion in this direction shall be drawn only after carrying out a study of the error distribution.

V. CONCLUSIONS AND FURTHER WORK

In this paper we evaluate the BER performance of MBTC associated with two multicarrier modulation techniques: OFDM and its wavelet based version, in a flat fading channel. The WOFDM provides better results than OFDM, especially for high values of the Doppler shift parameter, which quantifies the time variability of the channel. Furthermore, the coded WOFDM performance shows no significant dependency on the wavelets mother used for DWT computation. A surprising result of our simulations is that, unlike for the un-coded system, the BER and FER performance for the MBTC-WOFDM is better for higher values of the Doppler shift.

In order to rigorously explain the later result, a detailed research will be conducted in the future, aiming to study how the erroneous bits are distributed over the transmitted blocks. By the other hand, as noticeable from figures 5 and 6, the gap between OFDM and WOFDM enlarges with increasing SNR values. An extended range for this parameter will allow to better highlight this effect in future works. We will also consider for our further research the second very important characteristic of the radio channels, besides their time variability: the frequency selectivity.

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