A STUDY OF THE PERMUTATION SCHEMES USED IN THE MOBILE WIMAX

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Abstract: With the goal of decreasing the interference at sub-carrier level, the allocation of sub-carrier frequencies in an OFDMA based system is selective. This selective allocation can be seen like a permutation of a frequency "sequence". This paper proposes a new analyzing tool for the frequencies permutation used in WiMAX mobile communications: the distances spectrum (DS). This tool can offer a meaningful measure for these permutations, with respect to the most performing permutations used in turbo-codes, which, given their major importance, are strongly studied and developed.

Key words: *permutations, sub-carrier frequency, interleaving distances spectrum, spreading degree.*

I. INTRODUCTION

There are few applications of the permutations in the field of communications: security, turbo-coding, radio communications. In security for example, the symmetric data encryption algorithms use permutations. One of the components of each turbo-coding system is the interleaver, which is also a permutation device. The allocation of the sub-carrier frequencies to different users (described in the standard 802.16.e) is made in an intelligent manner and , in fact, it is carried-out by a permutation. A permutation system is very attractive from an electronic point of view because it has a very simple and cost effective implementation, requiring only a few wires.

There are two objectives of a permutation system: the decorrelation of a sequence and the spreading of the neighbors from the initial sequence.

Why decorrelation? The most eloquent answer is the shuffle of playing cards, required for destroying the "playing cards groups" formed in a previous game. In communications, the correlation of the groups of bits is unwanted when we desire to extract from each bit its own information, independent of the information carried by its neighboring bits. If the bits from a sequence are correlated then all of them will give the same information like the first bit of the considered sequence. Consequently, the second bit, the third one and so on will not give any additional information. By the other hand, the permutations used in the symmetric data encryption algorithms were conceived to maximize the decorrelation of the symbols composing the current block.

Why spreading? In the case of the mobile WiMAX system the Doppler Effect occurs. Due to the mobility of the devices the value of the frequency of a sub-carrier can vary between the input and the output of a communication channel, hence producing interference with the sub-carriers having neighbor frequencies (if for example these frequencies were already allocated to a fix user). By spreading, the distance between the initial neighbor frequencies is increased and the interference can be reduced or avoided. So, our assumption is that the permutations included in the WiMAX sub-carrier frequencies allocation system are conceived to maximize the spreading of sub-carrier frequencies.

In the case of an interleaver both effects are required. It is possible to maximize simultaneously the decorrelation and the spreading? This question did not receive an answer yet. The goal of this paper is to analyze the spreading of the permutations which implement the sub-carriers frequencies allocation system, described in the standard 802.16e. The tool used for this purpose is the distances spectrum proposed in [1], in connection with the turbo-coding interleavers analysis. The structure of the paper is as follows:. in section II the distances spectrum is defined. The WiMAX permutation schemes are explained in section III. Some results are reported in section IV. The last section is dedicated to some concluding remarks.

II. DISTANCES SPECTRUM

In this paper we propose a qualitative and quantitative method for the analysis of the spreading realized by a given permutation system, based on the computation of the Distances Spectrum (DS). This measure will be used to analyze the quality of the PUSC permutation system.

In [1], the function DS, $s: J \rightarrow \mathbf{N}$ was defined as:

$$s(k) = no[d(i, j) = k], \quad k \in J, \tag{1}$$

where $J = \{2, 3, \dots, 2N\text{-}2\}$, and $no[condition]$ represents the number of cases in which the *condition* is true, $\mathbf{N}$ denotes the natural numbers set and $d(i,j)$ is the distance between the positions i and j defined as:

$$d(i, j) = j - i + |\pi(i) - \pi(j)|, \quad \forall i, j \in I, j > i, \tag{2}$$

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where π is the permutation map:

$$\pi : I \rightarrow I, \quad I = \{1,2,... N\}. \quad (3)$$

In the last years, a significant effort was made on the study and on the design of the turbo codes (TCs) owing to their huge performance. Implicitly, the best interleaving methods were searched and investigated. One of the best interleavers is the S-interleaver, [2]. The theoretical DS for an *S*-interleaver with the length *N*=1000 bits is shown in Fig.1. To assess it in a quantitative manner, we define the spreading degree as:

$$G = supp\{s(k)\} \,/\, \max_{k \in J} s(k), \quad (4)$$

i.e. the ratio between the support of the function *s* (the number of distances *d* in which *s*(*d*) > 0) and the maximum value of DS.

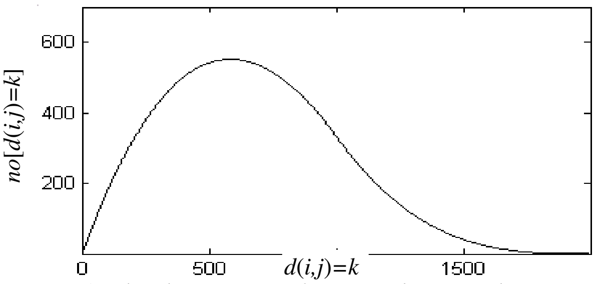


Figure 1. The theoretic DS for a random interleaver with N=1000.

III. PERMUTATION SCHEMES USED IN WIMAX

The WiMAX technology uses at its physical level a multi-carrier modulation technique, called Orthogonal Frequency Division Multiplexing (OFDM).

In fact, a modified version of OFDM is defined in [3], which transforms OFDM from a simple transmission technique into a multiple access method. Thus, at a certain moment of time (for a certain OFDM symbol), the total number of available carriers (N_c) are shared between several users. According to [4], N_c ranges from 128 to 2048. Note that not all the carriers can be employed for data transmission. At the edge of the dedicated frequency band, some of them are used as guard subcarriers. There are also pilot subcarriers, used for channel estimation, and the so called DC subcarrier, which indicates the middle of the frequency band, and it is not modulated by the data. The way that the physical resources (carriers) are mapped to the logical ones (called subchannels) is described by a permutation scheme. Basically, one subchannel represents a set of physical subcarriers dedicated to a certain user, for downlink (DL) or uplink (UL) transmission. There are several permutation schemes defined in the standard. In the following, only those based on frequency diversity will be analyzed.

When employed in DL, Partially Used Sub-Channelization (PUSC) is based on the following strategy: after separating the guard subcarriers, the remaining resource is split into up to three sub-sets (groups) of carriers, called segments. These segments do not occupy contiguous regions of the available spectrum, because the subcarriers which compose a segment are spread using an algorithm described in [3]. Next, within each segment, the fixed pilot frequencies are allocated. Finally, the data subcarriers are mapped to subchannels using a permutation formula:

$$\begin{aligned} &subcarrier(k,s) = N_{sbchs} \cdot n_k + \\ &[p_s(n_k \bmod N_{sbchs}) + DLPermBase] \bmod N_{sbchs}, \end{aligned} \quad (5)$$

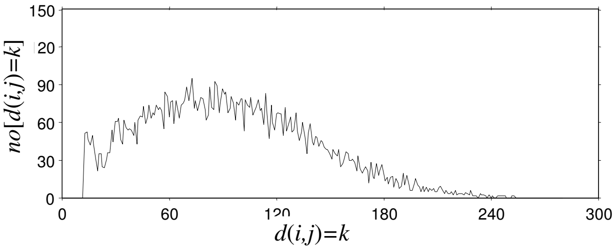
where *k* is the logical subcarrier *k* from the logical subchannel *s*, N_{sbchs} is the total number of subchannels, n_k is an integer that depends on *k* and *s*, p_s is a permutation sequence, cyclically shifted *s*-times to the left, and *DLPermBase* is a random integer within a certain range. Note that the later parameter is the only one giving randomness to the way that the available subcarriers are mapped to sub-channels. This formula answers the following question: what is the physical subcarrier index, corresponding to the logical subcarrier *k*, from the logical subchannel *s*?

Another diversity scheme used in DL is called Fully Used Sub-Channelization (FUSC). FUSC is quite similar to PUSC, only that the a-priori splitting into segments is not performed any more. Consequently, the available data subcarriers are seen as a whole, and they are used in the entire cell. The strategy used for defining the subchannels is basically the same.

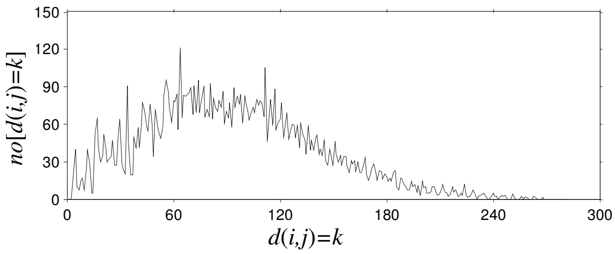
In the UL direction, the allocation algorithm relies on a different basis. There is a minimal granularity resource, which is called tile, and which is composed of a small number of contiguous subcarriers. Next, a tile permutation formula is applied, to spread the tiles within the physical spectrum.

IV. RESULTS

We will compare in the following (from the DS point of view) a practical *S*-interleaver with the length *N*=140 bits with the 'interleaver' defined by the frequencies permutation of a segment in the DLPUSC case, with N_{sc}=512. We consider only the ordering relation between the 140 frequencies composing one segment. Thus, we obtain a sequence of positions from 1 to 140, which is permuted. Following now the place conquered by the positions in the "arrangement" given by the standard 802.16e, the Table I (computed using eq. (4)), we can assess the permutation quality using the DS measure (fig. 2). The two diagrams in Fig. 2 show a similitude between the PUSC frequency permutation and the permutation realized by the *S*-interleaver. In Table II ,for both permutation systems (in the columns 2 and 3) the minimum and the maximum interleaving distances (d_{min}, D_{max}) are indicated. Other measures in this table are the maximum DS value (s_{max}), the support of IDS (*supp*(*s*)), and the spreading degree (*G*). The parameters from Table II prove the similitude between the two permutation systems. The very good spreading degree realized by the spreading of the frequencies positions is remarkable.



a)



b)

Figure 2. The DS for: a) the S-interleaver with S=13 and N=140; b) the permutation system defined by the positions sequence of the 140 sub-carrier frequencies of BS segment 0.

Table I
THE SUB-CARRIERS COMPOSITION

	1	2	3	4	5
1	94	207	101	290	114
2	98	287	112	295	146
3	104	291	115	347	157
4	108	298	150	351	174
5	113	343	154	371	182
6	151	348	158	375	185
7	155	372	183	380	189
8	175	376	186	460	200
9	179	453	193	464	203
10	184	456	197	90	211
11	194	462	201	95	214
12	198	91	212	105	288
13	204	99	285	109	341
14	208	102	292	116	344
15	213	106	296	147	352
16	294	145	342	152	369
17	297	148	353	176	374
18	346	156	370	180	454
19	350	173	378	187	458
20	354	178	381	190	465
21	379	188	455	196	89
22	382	192	466	206	100
23	459	199	92	210	103
24	463	202	96	286	110
25	93	149	191	289	373
26	97	153	195	293	377
27	107	177	205	345	457
28	111	181	209	349	461

The parameter d_{min} shows the minimum distance between any two positions. This parameter is equal to 3 in the case of permutation of the frequency positions; which is a small value. The reason is that there is a pair of frequencies with positions, before and after permutation, quite neighbors. In practice, the situation is a little bit different because we considered only the order of frequencies ignoring the real distance between the neighbor frequencies.

TABLE II
THE DS PARAMETERS

Parameter	*S*-interleaver	Frequencies' positions permutation system DLPUSC	Frequencies' values permutation system DLPUSC
d_{min}	13	3	4
D_{max}	253	268	506
s_{max}	95	121	56
supp	233	249	478
G	2.4526	2.0579	8.5357
N	140	140	140

To see the impact of this simplification, the DS for the permutation of the frequencies is presented in Fig. 2, considering their real values (not only their position in an increasing order). The parameters of this DS are presented in the 4-th column from Table II. Thus, in reality, d_{min} is four times higher then the deviation between frequencies and the spreading degree is much better.

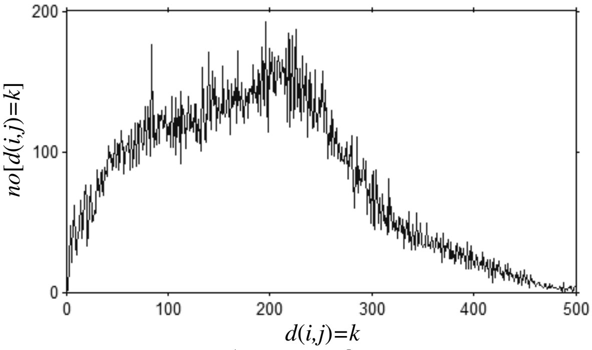
In the DLPUSC-1024 case, the 3 sets of 280 frequencies corresponding to the 3 segments were analyzed from the spreading point of view (fig. 3). We considered just the frequencies order and not their values. How it was expected, due to the bigger length (280 elements/frequencies now, against 140 in the previous case), the DS functions reproduce better the theoretical case from Fig. 1. The spreading degree confirms this remark: G=2.75 for the segment 0, G=2.546 for the segment 1, and G=2.22 for the segment 2. For these results, we considered the value of DLPermBase (eq.(5)) equal to 0.

A similar analysis was made for the DLFUSC and ULPUSC cases. The results illustrated in Fig. 4 indicate a weaker spreading in the DLFUSC case compared with the DLPUSC one (the spreading degree is 1.513 for DLFUSC1024 and 1.316 for DLFUSC512, respectively). Practically the ULPUSC doesn't realize spreading (the spreading degree is 0.232 for 1024 subcarriers and).

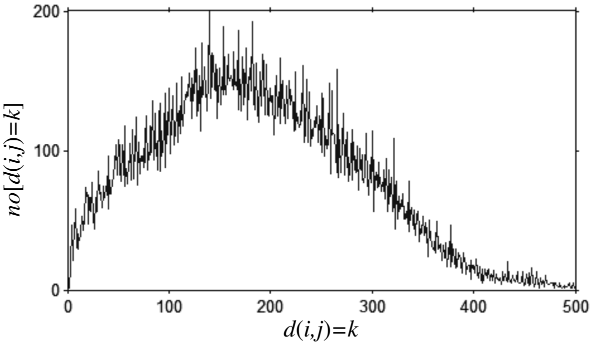
However, the later result, for the UL case may be not fully relevant. This is because, according to [3, 4] the data points are not mapped sequentially to the logical subcarriers composing a subchannel, a permutation formula being used instead. This permutation is not taken into account in our analysis.

V. CONCLUSIONS

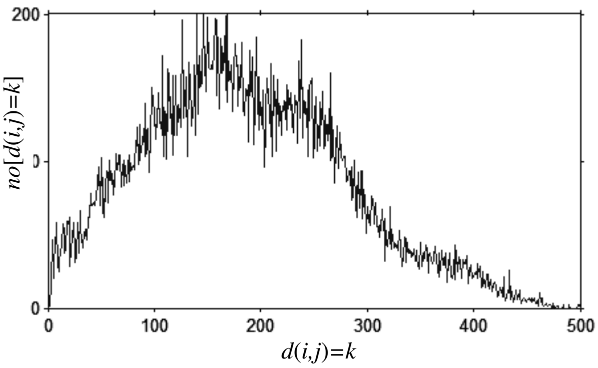
The frequency spreading in the WiMAX systems is studied in this paper. Our analysis uses a measure imported from the channel coding area, namely an interleaver distance spectrum. In the turbo-coding context, the IDS shows what is the spreading degree achieved by a certain type of interleaver. In our case, this tool is employed in order to



a) segment 0

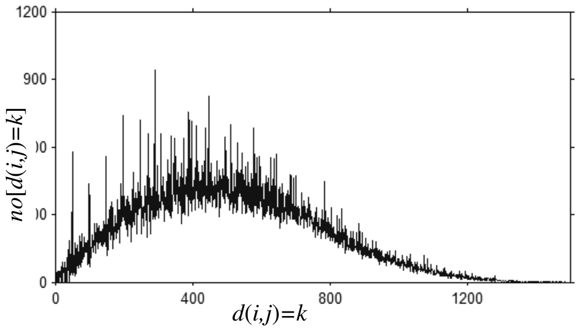


b) segment 1

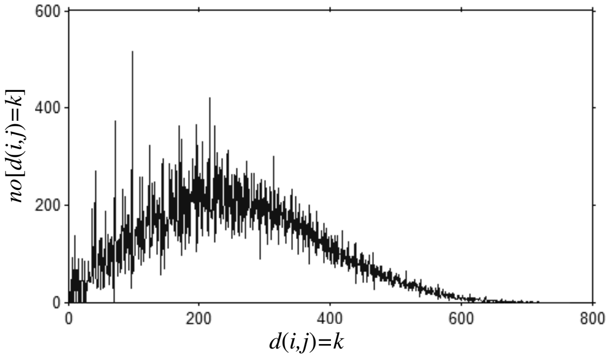


c) segment 2

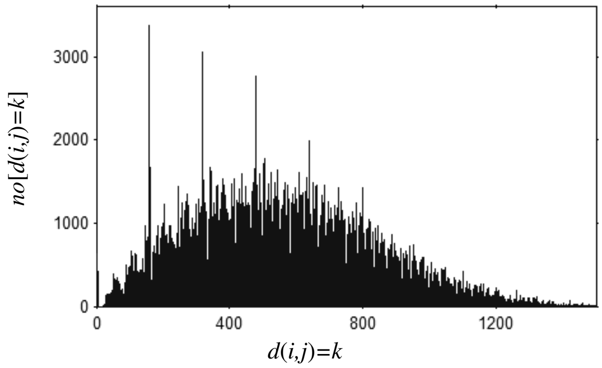
Figure 3. The DS for the sets of 280 frequencies which constitute the resource allocated to the 3 segments in the DLPUSC 1024 case.



a) DLFUSC1024



b) DLFUSC512



c) ULPUSC1024

Figure 4. The DS for the sets of the frequencies which constitute the resource allocated in the DLFUSC and ULPUSC cases.

assess the performance of the frequency permutations defined in the WiMAX standards.

Our study shows that, among all the considered permutations, the DLPUSC case provides the best results from the frequency spreading point of view. It minimizes the sub-carriers interference, spreading their frequencies at an important distance. This conclusion supports the conclusion of a study having the same subject, but using a different approach based on link level simulations, [5]. Next, DLFUSC, shows poorer results (slightly better than one half the spreading degree observed in the PUSC case). By the other hand, the fact that there are permutations with a higher spreading degree, lead us to the idea that this permutation can be improved. No real spreading is achieved by the UL schemes, where the algorithm used to map the subcarriers relies on a different basis. A more detailed analysis must be performed in order to confirm these results.

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