Volume 50, Number 3, 2009

ACTA TECHNICA NAPOCENSIS
Electronics and Telecommunications

IMAGE ENHANCEMENT USING A NEW SHOCK FILTER FORMALISM

Cosmin LUDUŞAN[1, 2], Olivier LAVIALLE[2], Sorin POP[1], Romulus TEREBEŞ[1], Monica BORDA[1]

[1]*Technical University of Cluj-Napoca, 26-28 Bariţiu Street, 400027, Cluj-Napoca, Romania*
[2]*Université de Bordeaux, UMR IMS 5218, 351 cours de la Libération, 33405, Talence Cedex, France*

Abstract: We present a new approach based on Partial Differential Equations (PDEs) and shock filter theory for deblurring 2D Gaussian blurred images. The inherent problems of stability posed by the reverse heat equation and the discretization of shock filters are overcome by the use of only the initial conditions. This approach is not well suited for diffusion-oriented techniques since in this case the problem is ill-posed, but when dealing with multiple inputs (the case of fusion methods), it proves to provide sound results. The proposed deblurring model takes into account only the blur problem without having a noise removal component, this being a future step of our work.

Key words: *partial differential equation, image fusion, shock filter, deblurring, image enhancement.*

I. INTRODUCTION

The basic idea behind shock filters is the process of applying either erosion or dilation in a very localized manner, in order to create a "shock" between two influence zones, one belonging to a maximum and the other to a minimum of the signal. By iterating this process (modeled using a PDE framework) according to a small time increment *dt* we can ultimately obtain a piecewise constant segmentation of the input image, thus leading to a deblurred output.

The use of shock filters as a mean of image enhancement is recommended by the advantages this particular method offers: they create strong discontinuities at image edges and furthermore, the filtered signal within a region delineated by those edges becomes flat. In other words, shock filters create segmentation. Due to their discrete mathematical definition they are inherently unstable, meaning that they require special discretization schemes in order to preserve the total variation of the signal. Another property of shock filters underlined in [4] is that they satisfy the maximum-minimum principle which states that the range of the filtered image remains within the range of the original image. Another advantage of shock filters over other image enhancement methods, such as Fourier or wavelet-based ones is that phenomena like the Gibbs phenomenon cannot appear.

Another way of enhancing a blurred image is by a more straightforward approach that of reverse diffusion, more specifically the reverse heat equation. This approach was discussed in [1, 11] and although the results look promising, a few remarks need to be made concerning this method: the reverse diffusion by itself causes instabilities, which eventually lead to the image "blowing up". Therefore the time evolution of this filter needs a predefined stopping criterion which is strongly correlated to the particularities of each image. For example, an image that presents a strongly oriented characteristic will require a smaller number of time iterations before "blowing up" than an image which is strongly homogeneous. This particular problem can be also found in shock filter models that do not explicitly implement the *minmod* numerical scheme described in [10], thus leading to breaking the previously stated maximum-minimum principle.

II. IMAGE ENHANCEMENT THROUGH SHOCK FILTERING

The first definition of the shock filter can be traced back to 1975 when Kramer and Bruckner have defined the first concepts regarding shock filter theory [3]. The Kramer and Bruckner definition can be expressed using the following PDE as demonstrated in [2]:

$$\frac{\partial I}{\partial t} = -sign(I_{dd})\left|I_d\right| \tag{1}$$

where I represents the image and I_d and I_{dd} represent the first, respectively the second directional derivatives of the image I. Eq. (1) represents a generic definition since the direction d is not properly defined. The first term of the equation represents the edge detector (in this case the *Canny* edge detector) used for shock filter steering.

The actual term of *shock filter* was first introduced in 1990 by Osher and Rudin [10] when they proposed this new class of filters based on PDEs and defined the *minmod* numerical scheme for successfully avoiding any instabilities of the algorithm, since the shock filter theory is defined only on a discrete domain. The PDE equation expressing the shock filter defined in [10] uses the zero-crossings of the Laplacian as edge detector:

Manuscript received September 9, 2009; revised October 21, 2009

$$\frac{\partial I}{\partial t} = -sign(\Delta I)\left|I_d\right| \quad (2)$$

Although the only difference between eq. (1) and eq. (2) is the edge detector used, it was proven in [5] that the Kramer-Bruckner and Osher-Rudin filters share the same asymptotic behavior for a 1D regular signal.

The evolution of shock filter theory is marked by several improvements over the original Osher-Rudin model. One such improvement refers to making the filters more robust to small scale details an was suggested in [7] by Alvarez and Mazora and consists in introducing a smoothed version of I, that is: $I^{\sigma} = K_{\sigma} * I$, where K_{σ} is a Gaussian kernel with standard deviation σ. Another improvement that was first suggested in [10] is the use of the *Canny* edge detector instead of the *zero-crossing of the Laplacian* one, where the directional derivative is computed along the $\eta \| \nabla I$ direction. These improvements led to the following equation:

$$\frac{\partial I}{\partial t} = -sign(I^{\sigma}_{\eta\eta})\left|\nabla I\right| \quad (3)$$

Another notable improvement was presented in [4] where the edge detector is once again redefined. The directional derivative is computed this time according to a normalized dominant eigenvector w of the structure tensor $J_{\rho}(\nabla I)$:

$$J_{\rho}(\nabla I) = K_{\rho} * (\nabla I \cdot \nabla I^{T}) \quad (4)$$

The new definition of the shock filter according to [4] is:

$$\frac{\partial I}{\partial t} = -sign(I^{\sigma}_{ww})\left|\nabla I\right| \quad (5)$$

This particular model uses a different scale for each of the Gaussian kernels it employs: the *structure scale σ* that determines the size of the flow-like patterns and the *integration scale ρ* which has the role of averaging the orientation information in order to have a more robust orientation estimator.

Another interesting improvement is the one defined in [9] where the proposed model performs image enhancement and denoising at the same time. The shock filter component of the model defined in [9] is the following:

$$\frac{\partial I}{\partial t} = -\alpha_e(1 - h_{\tau}(I^{\sigma}))sign(I^{\sigma}_{\eta\eta})\left|\nabla I\right| \quad (6)$$

where $h_{\tau}(x) = \begin{cases} 1 & if \quad x < \tau \\ 0 & elsewhere \end{cases}$

Even if the original model has been improved over the years, being made more robust to small scale structures or flow-like patterns, it still presents a series of shortcomings like its susceptibility to noise or the fact that, as stated in [1] , the shock filter is a texture killer. Also [1] advocates the fact that the concept of sock filter cannot be truly regarded as a reverse heat equation since it is basically performing a very local and simple operation on a neighborhood, thus being no more than an enhancement operator. The PDE-based non-local filtering method described in [1] is not directly linked to the notion of shock filter to its strictest of senses since it uses the PDE formalism to describe a non-local time evolving reverse heat equation.

III. NUMERICAL MODELS FOR SHOCK FILTER IMPLEMENTATION

As already discussed in the previous paragraphs, all shock filter models have a specific discrete implementation in order to comply with the maximum-minimum principle.

III.1. Existing stable numerical schemes

The maximum-minimum principle allows the development of stable algorithms that do not require a specific stopping criterion or for that matter a stopping time. The evolution of these models is independent from this point of view since the solution tends to converge and stabilize itself after a sufficiently large number of iterations. The classical numerical scheme that complies with the previously stated principle is the *minmod* scheme which was also defined by Osher and Rudin in [10]:

$$m(x, y) = \begin{cases} sign(x)\min(|x|, |y|) & if \;\; xy > 0 \\ 0 & if \quad xy \le 0 \end{cases} \quad (7)$$

Based on the *minmod* operator (eq. 7) the discretization of the gradient norm $\left|\nabla I\right|$ is performed as follows:

$$\left|\nabla I^{n}_{i,j}\right| = \sqrt{\left\{m\left[D^{+}_{x}(I^{n}_{i,j}), D^{-}_{x}(I^{n}_{i,j})\right]\right\}^{2} + \left\{m\left[D^{+}_{y}(I^{n}_{i,j}), D^{-}_{y}(I^{n}_{i,j})\right]\right\}^{2}} \quad (8)$$

where $D^{+}_{x}, D^{-}_{x}, D^{+}_{y}$ *and* D^{-}_{y} represent the numerical approximations of the first order derivatives with respect to the x and y directions based on the forward and backward finite differences scheme.

An alternative stable numerical scheme is the one proposed in [8] where in order to abide by the maximum-minimum principle and avoid the divergence of the final solution, the input image is normalized between [0;1] before applying the shock filter. The filtered result is then set back to its initial dynamic range. This scheme proves to work slightly better than the one described by eq. (7) and (8) mainly in preserving small scale details around joints and corners belonging to different image structures.

III.2. Our new naturally unstable numerical schemes

Although it may sound a little odd, the use of such numerical schemes proves in some cases to yield more realistic results. Since the shock filters are not truly reverse heat equations we cannot assume that by evolving a stable numerical scheme of a shock filter onto a Gaussian blurred image we will be able to obtain the initial solution of that equation (that cannot be achieved even with an ideal reverse heat equation, since the blurring process is isotropic in nature).

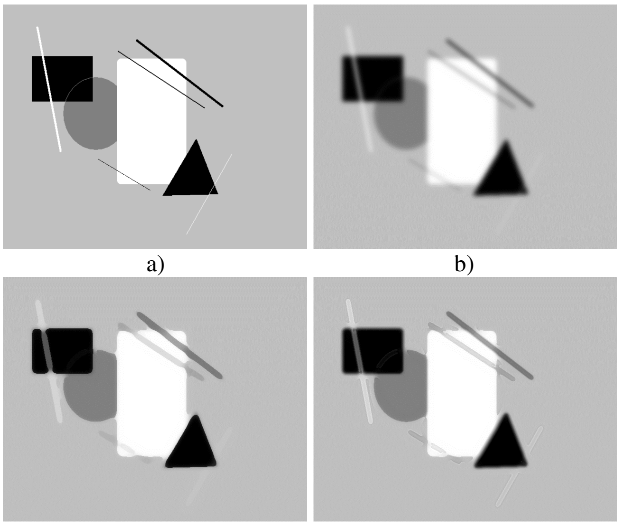


Figure 1 a) Original synthetic image; b) Gaussian blurred image: σ=5
c) Deblurred image - classic scheme; d) Deblurred image - proposed scheme eq. (9);

Figure 1 illustrates the main advantages of using an unstable discretization scheme for implementing a shock filter (Fig.1 d): better contour preservation, better contrast since the PDE's evolution is not restricted by the *minmod* operator and, as it can be seen from the above example, better detail preservation (e.g. the thin white line in the bottom right corner of the black triangle which is almost invisible in Fig. 1 c) is preserved reasonably well in Fig. 1 d)). For the above example the same evolution time was used $t=5$ with $dt=0.1$ time increments. The filtering in Fig. 1 c) was done using eq. (3) with $\sigma=3$. The filtering in Fig. 1 d) was done using the first of our two proposed schemes:

$$\frac{\partial I}{\partial t} = -sign(I^0_{\eta\eta}) \cdot \exp(-\frac{|\nabla I|}{K_s}) \cdot |\nabla I| \qquad (9)$$

Eq. (9) is implemented using a classical finite differences scheme for computing its second order directional derivative $I_{\eta\eta}$ and its gradient norm. In order to avoid oscillations due to the classical implementation, we use only the initial second order directional derivative $I^0_{\eta\eta}$ extracted from the initial blurred image in computing the *Canny* edge detector. The shock filter's speed is controlled by employing an exponential term, similar to the one used in the Perona and Malik anisotropic diffusion equation.

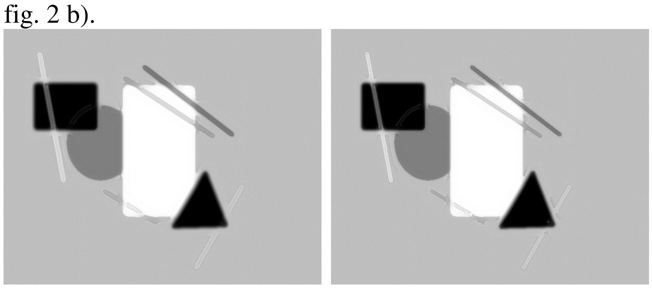
Due to the promising results obtained using the shock filter described by eq. (9) our work led us to experimenting even further with the concept of unstable numerical schemes and the use of only the initial directional derivative in employing the edge detector. The well-posedness of the problem when using only the initial second order directional derivative was mathematically proven in [8], thus the use of only the initial directional derivative in the edge detecting process is mathematically sound and allows us to successfully implement unstable numerical schemes that, as long as they are properly parameterized, can yield better performances than classical *minmod* models.

The edge detector envisioned using only the initial geometry retrieved through $I^0_{\eta\eta}$ would ideally capture the undistorted second order directional derivative identical to the one obtained by extracting it from the initial non-blurred image. Since this assumption would make our problem ill-posed, the goal is to try to retrieve from the blurred image a second order directional derivative as similar as possible to the one from the initial non-blurred image. Since the classical shock filter is a very localized edge enhancement operator, we propose to use it to enhance the second order directional derivative and with the resulting information as initial condition to determine the edge detector's response:

$$I^{0,\tau}_{\eta\eta} = I^{0,\tau-1}_{\eta\eta} + d\tau \cdot dI^{0,\tau-1}_{\eta\eta}, \quad where \quad \frac{\partial I^0_{\eta\eta}}{\partial \tau} = -sign(I^{0,\eta\eta}_{\eta\eta}) |\nabla I^0_{\eta\eta}| \qquad (10)$$
$$\frac{\partial I}{\partial t} = -sign(I^{0,\tau}_{\eta\eta}) \cdot \exp(\frac{|\nabla I|}{K_S}) \cdot |\nabla I|$$

$\partial\tau$ and ∂t are the time increment parameters that describe the time evolution concept used in the PDE formalism and can have distinct or identical values.

Figure 2 illustrates the improvements brought by the model described by eq. (10) compared to the model described by eq. (9). The overall parameters are the same: $t=5$ with $dt=0.1$ and $\tau=100$ with $d\tau=0.1$. It can be seen that by using eq. (10) we can further improve the edge detection thus increasing the accuracy of the enhancement, making it almost close to the ideal concept of precisely localized edge enhancement. All the other benefits of the unstable scheme illustrated in fig. 1 d) are preserved in the filtered results of fig. 2 b).



a) b)

Figure 2 a) Deblurred image – based on eq. (9) b) Deblurred image – based on eq. (10)

So far we have analyzed the behavior of our shock filter models in an inverse-diffusion frame, showing that the proposed approach is well-posed. This assumption holds as long as a stopping criterion is defined and the use of only the initial geometry retrieved by the second order directional derivative is mathematically sound and feasible (as long as we do not assume ideal conditions for extracting $I_{\eta\eta}$). We will see in the next paragraph that in the case of image enhancement by means of image fusion we can make this assumption under specific conditions, thus making even the ideal case a well-posed problem.

IV. SHOCK FILTERS AND IMAGE FUSION

Using as a starting point the model described by eq. (10) we can imagine the following fusion scenario: we have 2 inputs of the same scene, but each blurred in different regions by a Gaussian blur. In this case we can assume that it is possible to retrieve the ideal second order directional derivative from each region of the image that is not affected by blur, provided that the blurring was performed in a complementary manner.

Figure 3 a) Original Lena image; b) Gaussian blur (σ=5) left side;
c) Gaussian blur (σ=5) right side; d) Fusion result (RMSE=14.06, PSNR=25.17 dB with respect to original image Lena);

The concept of using the discretization schemes (eq. 9 and 10) for shock filter modeling in a fusion-like framework is still in its emergent phase, since there are still certain details that need to be further studied and improved.

V. CONCLUSIONS

The alternative approach of defining and implementing shock filters proposed in this work has proven to reveal interesting new ways of looking and working with shock filters as image enhancement tools. Although using unstable numerical schemes is more complicated than using the stable ones, we have demonstrated that there are situations that could benefit from such approaches.

We believe that there are still new ways of working with shock filters that have not yet been discovered, and this encourages us to continue our work in researching new possibilities of expanding the PDE-based image processing formalism.

Acknowledgement

This work was funded by the Romanian Agency UEFISCSU within the PN 2 IDEI no.908 72/01.10.2007 research grant.

REFERENCES

[1] A. Buades, B. Coll and J-M. Morel, “Image enhancement by non-local reverse heat equation”, *Technical Report Preprint CMLA 2006-22*, 2006.

[2] F. Guichard and J.M. Morel, ”A Note on Two Classical Enhancement Filters and Their Associated PDE’s”, *International Journal of Computer Vision*, Vol. 52 (2) pp.153-160, 2003.

[3] H.P. Kramer and J.P. Bruckner, “Iterations of a non-linear transformations for enhancement of digital images”, *Pattern Recognition*, Vol.7 pp. 53-58, 1975.

[4] J. Weickert, “Coherence-Enhancing Shock Filters” *Lecture Notes in Computer Science: Pattern Recognition*, Springer, Berlin, Vol. 2781 pp. 1-8, 2003.

[5] J.G.M. Schavemaker, M.J.T. Reinders, J.J. Gerbrands and E. Backer, “Image sharpening by morphological filtering”, *Pattern recognition*, Vol. 33 (6) pp. 997-1012, 2000.

[6] J.M. Bioucas-Dias, M.A.T. Figueiredo and J.P. Oliveira, “Total Variation-Based Image Deconvolution: A Majorization-Minimization Approach”, *International Conference on Acoustics, Speech and Signal Processing (ICASSP ’06)*, Vol.2 pp. II-II, 2006.

[7] L. Alvarez and L. Mazora, “Signal and image restoration using shock filters and anisotropic diffusion”, *SIAM Journal of Numerical Analysis*, Vol. 31 pp. 590-605, 1994.

[8] L. Remaki and M. Cheriet, “Numerical Schemes of Shock Filter Models for Image Enhancement and Restoration”, *Journal of Mathematical Imaging and Vision*, Kluwer Academic Publishers, The Netherlands, Vol. 18 pp. 129-143, 2003.

[9] P. Kornprobst, R. Deriche and G. Aubert, “Image coupling, restoration and enhancement via PDEs”, *International Conference on Image Processing (ICIP ‘97)*, Vol. 2 pp. 458-461, 1997.

[10] S. Osher and L.I. Rudin, “Feature-oriented image enhancement using shock filters”, *SIAM Journal of Numerical Analysis*, Vol. 27 pp. 919-940, 1990.

[11] V.P. Namboodiri and S. Chaudhuri, “Image Restoration using Geometrically Stabilized Reverse Heat Equation”, *International Conference on Image Processing (ICIP ‘07)*, Vol. 4 pp. 413-416, 2007.