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ON REPLICATION IN DESIGN OF EXPERIMENTS

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Abstract: Modern electronic technology brings systems of higher complexity leading to the need of new analysis techniques. Using Design of Experiment (DoE) and one of its main principles, i.e. replication, we applied controlled variations in input factors, and replicate the measurements sufficient amount of times, in order to assess the impact of noise on the measurement results, as compared to the impact of the factors. The paper illustrates the replication approach with the measurements performed on a low-pass filter. The method proposed in this paper consists of 4 main steps, which assess the impact of noise on the measurement results as compared to the impact of the factors. Of great interest is determining how noise coming from testing the system under real stimuli, interferes with the results obtained after performing controlled experiments in a simulation environment.

Keywords: *Design of Experiments, Replication, Low-Pass Filter, Real-Time Load Emulation*

I. INTRODUCTION

With the evolution of technology, systems of higher complexity are brought to our attention. The number of factors that are involved in the analysis of system characteristics increased. Although there are several ways to generate models that approximate the behavior of the system, there is no certainty that data is accurate enough. Even if the equipment used is calibrated, the data can be affected by internal noise. A prediction or an estimate of the amount of variation encountered is needed.

In order to solve the problems implied by a big amount of data, the concept of DoE (DoE) can be used. By thoroughly planning and executing an experiment, the amount of information obtained regarding the effect on a response variable due to various factors is considerably increased. Using DoE and its main principles i.e. blocking, randomization and replication, any design can be improved and the maximum amount of information can be obtained with the minimum number of simulations [1].

Our work focuses mostly on evaluating if methods applied in pre-silicon verification for design space exploration and system optimization are also applicable in post-silicon verification. We are interested in how measurements/process noise coming from testing the system under real stimuli, interferes with the conclusions we normally draw after performing controlled experiments in a simulation environment. For this, we characterize noise as an ideally independently identically distributed random variable, small enough to draw statistically significant conclusions on the output of computer generated experiments. Replications play an important role to gather enough measurements to draw such conclusions.

Our paper contains a description of the work done so far in this research field in Section II. Basic concepts and strategies related to DoE are presented in Section III. Section IV describes the method to build up metamodels using the replication approach and its implementation in MATLAB. An illustrative example is given in section V and conclusions and future work is presented in the last section.

II. STATE OF THE ART

DoE is a concept approached in several papers from literature. It is successfully used in various domains such as psychology, medicine, biology, and, nowadays, engineering. Several definitions and experimental implementations have been discussed by many known researchers. D. Montgomery, A. Fisher, G. Taguchi, R. Kirk are few of the names worth to mention.

The most significant description of experimental design in engineering is offered by D. Montgomery [1]. In his work, DoE is defined as the process of planning an experiment in such a way that the data is collected in order to obtain accurate results. Montgomery also states that principles like replication, blocking and randomization should be used to obtain a better design and should be a part of every experiment. He explains that replication should be understood as a repetition of the basic experiment and gives a series of examples through which the differences between replication and repeated measurements are highlighted. He insists in using a randomized design to eliminate the averaging effect that commonly appears on factor and he advises to use design techniques like blocking to improve data precision.

In [2], Roger E Kirk defines DoE as a set of inter-related activities consisting of: identifying the variables involved in the process (independent variables, dependent variables and nuisance variables), specifying the way in which replication, randomization and statistical aspects of the experiment will be treated. He uses randomization to provide the distribution of the subjects over the treatment levels without selectively

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biasing the results of the experiment. Also, he states that replication consists of the analysis of two or more experimental units under the same assumptions. In this way, one can obtain an accurate estimate of the error and treatment effects. Blocking is used in one of his papers to isolate the variation attributable to the nuisance variables.

A more practical definition was given by Tom Donnelly in [3]. He states that DoE consists of controlling the input factors in such a manner that the researcher could find out their relationship with the output responses. It is efficient in solving problems with fewer resources by running efficient subsets of every possible combination. Also, in order to extract the information regarding the response's variation, if replication is used, one runs only the trials that are normally needed. A. Fisher [4] emphasized the fact that in order to avoid most of the problems encountered in analysis, it is absolutely necessary to plan a design and conduct an experiment.

III. DOE CONCEPTS

DoE is a set of experiments performed to obtain statistical information about the manner in which factors and their interactions can affect the output response. The factors of a system are considered to be the input variables and can be divided in two main categories: design/controllable factors and the nuisance/uncontrollable factors.

An experiment is planned to get the maximum information for minimum expenditure and in the minimum possible time. The experiment can be planned also with the purpose of eliminating measurements errors and avoiding any systematic errors. The flow of an experiment includes certain steps as: defining the problem, selecting the process factors and the response, choosing the experimental design, performing the experiment, analyzing the obtained results and drawing the necessary conclusions [1].

The experiments are done according to a previously created design. Depending on the application, the designs can be central composite designs (CCD), Latin hypercube samplings (LHS), full-factorial designs, fractional factorial designs, etc. Full factorial design contains all possible combinations from a set of factors, performing an experimental run at every combination of the factor levels. Central Composites consist of sets of experimental runs like factorial design, a set of center points and a set of axial points. Although it can be used to build quadratic models of the response, without needing to use three-level factorial experiments; it is not always a feasible design because regions of interest at particular points cannot be represented. LHS is used for uncertainty analysis and can be applied to multiple variables.

No matter what design is created at first, principles like *randomization, replication or blocking* can be used for improvement [1].

Randomization is used for random allocation of the experimental units across the treatment groups. The main advantage of using randomization is that a series of issues can be avoided or eliminated: system bias, selection bias and cheating experimenters.

Blocking is used to avoid obscuring the main effects by isolating a systematic effect, attributable to a nuisance factor. In this way, comparisons among factors of interest have an increase precision, similar factors being arranged in groups in order to have a successful analysis.

Replication consists in the repetition of an experiment by using the same methods, needed for determining the experimental error. This principle is used because it identifies the variation and uncertainty in measurements. All experiments contain variations induced by experiments which are not identical. For a better understanding, consider a continuous process which produces items in a batch mode. By choosing ten items, from two batches and testing those two - three times, an analysis can be done upon the data obtaining: estimates of the batch-to-batch variations, estimates of the mean effects and measurements of the testing error [1].

Several papers found in literature prove that by using these three principles, the obtained results are accurate and reliable. These concepts can be easily applied in real life situations.

IV. IMPLEMENTATION

Being a concept successfully used in domains such as psychology, medicine and biology, DoE and its principles are nowadays used also in engineering. Our focus is on replication which represents the repetition of the basic experiment. In every experiment, variations appear because of the fact that experimental units are physically identical. By performing the experiment more than once, variation can be estimated and eliminated. More attention was given to this principle because of its advantages i.e. estimating the experimental error, decreasing it and thereby increasing the precision; obtain precise estimates of the mean effects.

The goal is to apply controlled variations of the factors, and replicate the measurements sufficient number of times, in order to assess the impact of noise on the measurement results, as compared to the impact of the factors. To draw conclusions on how the noise is distributed, relative to the response and the factors, it is necessary to have replications on the experiment. By replications we will have not one single response value for each point in the factor space, but rather a distribution for each sample as in Figure 1 [1].

Since the regression which extracts the coefficients of the metamodel can only be done with unique sample points, we must approximate the distribution with its average and decide if the metamodel accuracy is good enough to be used further for predicting new points, given that any response measurement can be affected by a noise, comparable to what we have seen.

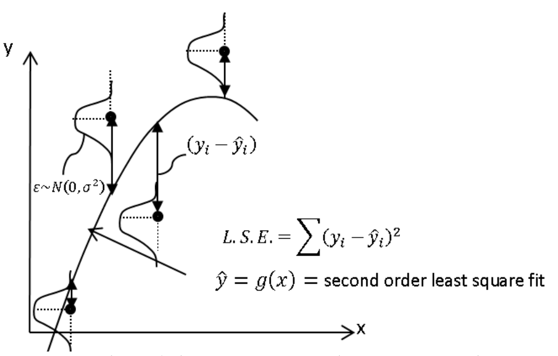

Figure 1: Plot of the output y with respect to the input x

We propose a method to build a metamodel using the replication approach. These steps should be performed:

1. Clean up the data

Sometimes the measurement fails because the test program misbehaves or the instruments do not provide the result as expected, so the result is either a Not-a-Number value or a sample with a different, wrong order of magnitude. These samples are called outliers and must first be removed from the available data. There are more ways to do this, such as using the inter-quantile range of several samples [10]. We decided on using a binning algorithm, which first computes the intervals for the samples and the number of samples and afterwards, removes the samples separated by big gaps or which have a small amount compared to the others.

2. Characterize noise

The remaining samples are replications of different samples of the factor space. To compute the noise, a first assumption is made: the noise should be white noise, i.e. have zero mean. Then, for each set of replications, the following rule should be used:

Noise = Measurement–Average (Measurements)/Replication (1)

a. Noise distribution should be normal with mean zero.
b. The standard deviation of the noise should be small enough, as compared to the differences between replications. To perform this operation, the average replication vector can be easily compared with the average noise.
c. The noise should be independently distributed with the respect to the response and factors values. If not, then a source of systematic variation is included in what was considered to be pure noise. If that is the case, then we should extract the term from the noise which is depending on the factors.

3. Average the replications

The average and standard deviation for the response and for the noise is computed using basic functions.

4. Build the metamodel

The resulted data can be used to build a metamodel i.e. an approximation model of the response. The fitness of the metamodel should be evaluated and the residuals (errors in metamodel prediction) must be compared to the noise. To use the metamodel for prediction, a correction must be applied to the confidence interval corresponding to the fitting errors.

These four steps are all implemented in MATLAB [11], because of the capabilities for fast vector operation, statistical computations and plotting possibilities. The outlier detection, noise characterization and metamodelling are realized with functions from the statistical toolbox [11].

V. RESULTS

The system under test is a low-pass filter with the following parameters: the resistance *R*, capacitance *C* and voltage supply *V*. A full-factorial experiment on a 3-factor set was replicated to draw conclusions on how the noise is distributed, relative to the response and the factors. The experiment contained 27(3^3) runs and each combination was replicated 100 times.

The filter network was simulated in real-time, based on the given supply voltage, with an FPGA connected to a dynamically acting power amplifier circuitry. This concept is a novel technique to consider application variances during post-silicon verification of automotive power micro-electronics [12].This way, a complete exploration of the factor space, for real-life experiments during lab measurements is also possible. The maximum value of the current through the filter is measured and the noise impact as related to the factors, on this response, will be analyzed.

No. run	*V* (V)	*R*(Ω)	*C*(uF)	No Outliers	Meas.Mean	Noise Mean (1.0e-015*)	Noise Std.
1.	8.0000	9.5000	0.0900	2	0.4581	0.1858	0.0045
2.	8.0000	9.5000	0.1000	3	0.6682	0.1178	0.0123
3.	8.0000	9.5000	0.1100	5	0.9070	-0.6318	0.0067
4.	8.0000	10.0000	0.0900	7	0.4643	-0.1356	0.0048
5.	8.0000	10.0000	0.1000	9	0.6979	-0.0729	0.0115
6.	8.0000	10.0000	0.1100	11	0.9412	-0.1920	0.0115
7.	8.0000	10.5000	0.0900	13	0.4872	0.1034	0.0026
8.	8.0000	10.5000	0.1000	15	0.7331	-0.1004	0.0105
9.	8.0000	10.5000	0.1100	16	0.9959	0.0720	0.0157
10.	12.0000	9.5000	0.0900	1	0.4424	-0.0967	0.0026
11.	12.0000	9.5000	0.1000	4	0.6506	0.1593	0.0026
12.	12.0000	9.5000	0.1100	6	0.8644	-0.0929	0.0037
13.	12.0000	10.0000	0.0900	8	0.4587	0.0616	0.0024
14.	12.0000	10.0000	0.1000	10	0.6859	-0.0641	0.0154
15.	12.0000	10.0000	0.1100	12	0.8986	-0.2171	0.0104
16.	12.0000	10.5000	0.0900	14	0.4762	-0.0069	0.0035
17.	12.0000	10.5000	0.1000	16	0.7223	-0.0593	0.0119
18.	12.0000	10.5000	0.1100	18	0.9457	-0.0641	0.0347
19.	16.0000	9.5000	0.0900	3	0.4216	0.0230	0.0026
20.	16.0000	9.5000	0.1000	4	0.6749	-0.1720	0.0165
21.	16.0000	9.5000	0.1100	6	0.8218	0.2896	0.0550
22.	16.0000	10.0000	0.0900	8	0.4357	0.0555	0.0022
23.	16.0000	10.0000	0.1000	10	0.6506	-0.0370	0.0039
24.	16.0000	10.0000	0.1100	10	0.8643	0.1718	0.0070
25.	16.0000	10.5000	0.0900	3	0.4580	-0.0423	0.0021
26.	16.0000	10.5000	0.1000	16	0.6843	-0.1923	0.0082
27.	16.0000	10.5000	0.1100	19	0.9060	-0.0918	0.0026

Table 1: Measurements done on a low pass filter

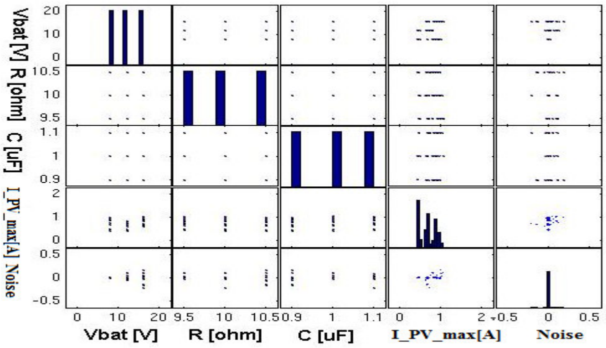

Figure 2. Matrix plots of factors, response and noise.

The steps described in the previous section were performed as follows:

1. Measurements are done on the available system according to the created design represented in Table 1, column 2-4. The results, outliers and noise characteristics are comprised in column 5-8 of the same table.
2. Checks on the noise distribution are performed. For a better visualization, matrix plots are used to see the pairwise relationships between the involved variables. Figure 2 shows the matrix plot for our system. One can notice how the factors, response and noise depend on each other. Correlations are seen as clusters of points which are grouped linearly on the diagonal. The main diagonal of the plot shows distributions of sample points. It can be noticed that the initial assumption i.e. normal distributed noise with mean zero was fulfilled. The computed standard deviation of the noise was small enough, as compared to the differences between replications. In Figure 2 no systematic variation between factors and noise is showed, which means that noise is independently distributed.
3. The average and standard deviation for the response and for the noise was computed using MATLAB functions like *mean* and *std*. Figure 3 shows the plot matrix of the average and standard deviation for the response, average and standard deviation for the noise, as depending on each other, for the replications. It can be seen that the noise has an average approximately zero, and a small enough maximum value; the noise does not depend on the response or on the factors. Therefore we can average the measurements and move on to the metamodelling.

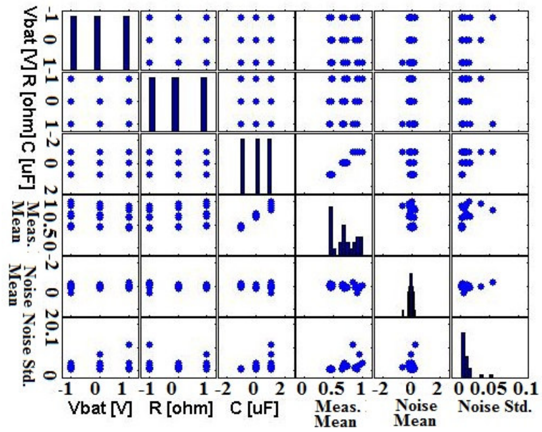

Figure 3. Plot matrix of factors, mean measurements, mean and standard deviation of the noise.

4. From the available sample points a quadratic model with interactions can be extracted. Its coefficients can be then used to evaluate the response in any point. The residuals can be computed altogether, i.e. with noise included:

$$Residuals = Predictions - Measurements \qquad (2)$$

The residuals are plotted and compared to the noise, in a normplot. This plot rescales its axis to compare the sample points to the ones of a normal distribution placed on the dotted line. The green line represents the noise, while blue is for the residuals. Both are approximately normally distributed with mean zero and small enough maximum values. Predictions in new points will always be affected by the noise and the residuals and must be under controlled limits. The metamodel can be visualized in Figure 5 as 1D plot, with confidence intervals marked with dashed red lines around the predicted line (marked with blue). The confidence is considered to be good enough over the complete scale. Normed values were used to build and plot the metamodel because they allow a comparison between the effects of each of the factors upon the response. It can be noticed in Figure 5, that the voltage coming from the battery has a significant effect on the mean value of the studied current.

This four-step analysis showed how the replication of an experiment can be used to characterize the noise. As mentioned before, this analysis used 100 replicates of each sample point of the full-factorial experimental design. This number of replicates is rather big. There is no reason to use more replicates that needed because the process might become time consuming and expensive. Further work will involve finding the necessary number of replications and the specific points in the factor space which are needed to draw the same conclusions i.e. if the distribution is good enough and if the metamodel is good enough for prediction, i.e. if the final confidence intervals are small enough.

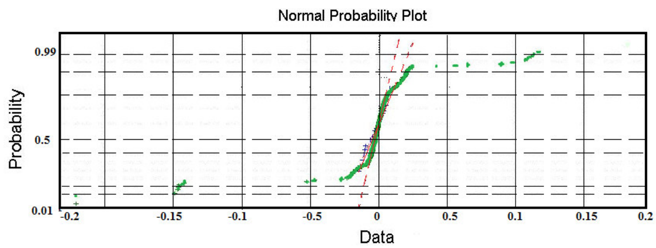

Figure 4. Normal distribution plot.

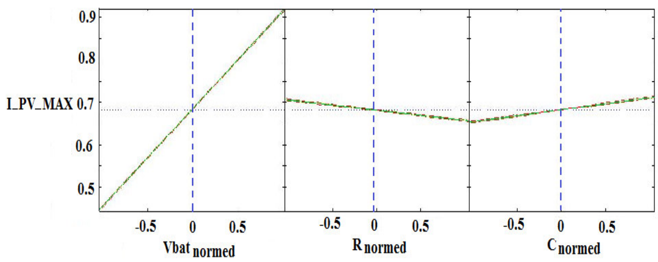

Figure 5. Built metamodel with normed values of three factors (R, C, and V)

VI. CONCLUSIONS

Replication is the repetition of the basic experiment. By performing the experiment more than once, variation can be estimated and eliminated. The paper is focused on showing the importance of replication in DoE to characterize the noise coming from the test and measurement equipment, the building up of a metamodel using the replication approach.

Our interest was in determining in which way the noise obtained by testing a system under real stimuli interferes with the results obtained by performing a controlled experiment in a simulation environment. Thus, noise was characterized as an ideally independently identically distributed variable, small enough to draw statistically significant conclusions on the output of the computer generated experiments. In order to be able to draw such conclusions, replication was used. We applied controlled variations in the factors and replicated the measurements a sufficient amount of times.

The necessary steps in building up a metamodel using replication are the following: remove the samples with small amount of data compared to the others, characterize the noise, average the replications and build the metamodel using those samples.

The system under test was a low pass filter with 3-factor set. We used a full-factorial experiment i.e. 27 runs, which was replicated 100 times, measured the maximum value of the current through the filter and analyzed the noise impact as related to the factors. From the plots obtained we observed that the noise had a mean close to zero and did not depend on the response or on the factors. Therefore, the measurements were averaged and the metamodel was built. By plotting the residuals and comparing them to the noise, we observed that both were approximately normally distributed with zero mean and small enough maximum values. Also we obtained a confidence good enough over the complete scale for the metamodel.

Further work will be devoted to finding the necessary number of replications and the specific points in the factor space which are needed to draw the same conclusions i.e. if the distribution is good enough and if the metamodel is good enough for prediction, i.e. if the final confidence intervals are small enough.

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