COMPARISON OF SENSITIVITY ANALYSIS METHODS IN HIGH-DIMENSIONAL VERIFICATION SPACES

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Abstract: The modern electronic systems grow in complexity as the number of potential factors that affect the systems behavior is increasing. The analysis of such systems can be simplified if the most important factors are identified upfront. Sensitivity analysis addresses this problem. The paper describes and compares six sensitivity analysis methods based on: variance decomposition, One-Factor-at–a-Time screening and regression. The methods are applied on a set of custom functions and a real system from the field of automotive electrical systems: a battery control model. The best performances were obtained by the following methods: the variance-based methods (Jansen and EFAST) and the regression based method.

Keywords: *Sensitivity analysis, design verification, factor prioritization, variance decomposition, regression*

I. INTRODUCTION

As the electronic systems are becoming very complex, *the number of factors* that have potential impact on the system response is also high. By factors we refer to any of the following: system parameters, operating conditions, production process parameters, while an example of system response is the power consumption. Moreover, the number of responses that need to be verified is also growing, which leads to time consuming and costly verification. In order to shorten the verification time and reduce the analysis, one should identify the factors that have the greatest impact on the system responses. Despite the high number of factors, usually only some of them have impact on the output response (1-5 factors). The Sensitivity Analysis (SA) aims to tackle this problem [1-2].

When choosing an SA method, one needs to consider the objective of the study, the types of factors involved and the execution cost implied. Here, by execution cost we refer to the number of system evaluations, i.e. simulations or measurements.

Other statistical procedures for SA found in literature are Analysis of Variance (ANOVA) [3] and statistical regression [4-5]. However, for systems with a high number of factors, the ANOVA test has higher computational complexity. The statistical linear regression links the responses and the factors of a system and this relationship is often called a metamodel. The difficulty of the metamodels lies in finding the best fitting model, for which the difference between the predicted and measured responses (residuals) is below a threshold.

SA methods found successful application in various domains such as atmospheric chemistry [6], transport emission [7] or fish population dynamics [8], including a reduced number of factors (<12), but there is *almost no study* of SA applied on electronic systems in a high dimensional space (>20).

In this paper, we provide a comparative assessment of the performance of several SA techniques on a set of custom-defined test functions and a battery model from the field of automotive electrical systems, both including a high number of factors. In the first study, we select a-priori the set of important factors and test the performance of the methods by comparing the set of selected and returned important factors. This study aims to give a direct assessment of the performance of the methods which may be used as a guideline for the study of any real system for which the set of important factors is unknown.

Six SA approaches are compared from the point of view of factor ranking, including also the execution cost implied. SAs were carried out using four variance-based methods [9-11] (Fourier Amplitude Sensitivity Test (FAST), Extended Fourier Amplitude Sensitivity Test (EFAST), Sobol' indices and the Jansen method), a One-Factor-at-a-Time (OAT) method (Morris [1]) and the metamodeling technique based on regression analysis [4], [9]. According to the knowledge of the authors, a direct comparison of these methods has not been done so far. A comparison of other SA methods applied on different systems and several fields is provided in [12-14]. Nevertheless, these studies include a reduced number of factors and compare only a few SA methods.

The paper is structured as follows: Section II describes the current approaches of SA and the theoretical framework of the methods selected for the study, followed by a brief summary of the applications along with the obtained results in Section III. Section IV discusses the limitations and execution costs implied by the studied SA methods. Conclusions are drawn in Section V.

II. SENSITIVITY ANALYSIS METHODS. CURRENT APPROACHES

We are going to present briefly six SA methods that we have evaluated and compared in our study. Four of them are

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based on variance decomposition (Sobol, Jansen, FAST and EFAST), one on OAT approach (Morris) and one on statistical regression (metamodeling). For each method we show the underlying mechanism that reveals the factor's influence on the system response.

A. The Variance-based methods

The major objective of the variance-based methods is to quantify the amount of variance each input factor X_i contributes with on the unconditional output variance, *V(Y)*. The unconditional variance, *V(Y)*, can be decomposed into conditional variances as described in (1), where $E[Y \mid X_i]$ is the expectation value over the whole variation interval of the input X_i and $V[Y \mid X_i]$ is the conditional variance of response Y and is obtained by taking the variance over all factors, except X_i [9], [15].

$$V(Y) = V(E[Y \mid X_i]) + E[V(Y \mid X_i)] \tag{1}$$

The variance-based methods are able to determine main effects, as well as higher order effects (interactions and nonlinear effects) of the factors. For this, the so-called first order effect indices (2) and total order effect indices (3) are used:

$$S_i = \frac{V\left(E\left[Y \mid X_i\right]\right)}{V(Y)} \tag{2}$$

$$S_{Ti} = \frac{E\left[V\left(Y \mid X_{\sim i}\right)\right]}{V(Y)} = \frac{V(Y) - V\left(E\left[Y \mid X_{\sim i}\right]\right)}{V(Y)} = 1 - \frac{V\left(E\left[Y \mid X_{\sim i}\right]\right)}{V(Y)} \tag{3}$$

where $X_{\sim i}$ denotes all factors except X_i.

Sobol indices

The method is based on the variance decomposition [9], [11]. Sobol suggested decomposing the model function *f* into summands of increasing dimensionality as in (4):

$$f(X_1,...,X_k) = f_0 + \sum_{i=1}^{k} f_i(X_i) + \sum_{i=1}^{k}\sum_{j=i+1}^{k} f_{ij}(X_i, X_j) + ... + f_{i...k}(X_i,...,X_k) \tag{4}$$

In order to compute the total sensitivity indices, Homma and Saltelli proposed the decomposition of $f(X)$ according to two subsets of input factors: X_i and the other containing the complementary set $X_{\sim i}$, as described in (5):

$$f(\mathbf{X}) = f_0 + f_i(X_i) + f_{\sim i}(X_{\sim i}) + f_{i,\sim i}(X_i, X_{\sim i}) \tag{5}$$

where f_0 denotes the expectation value of the output. Based on (5), the first and total order sensitivity indices are obtained as indicated in [9], [11].

Jansen (Winding Stairs) method

The Winding Stairs (WS) method proposed in [10] introduces a new sampling scheme, the winding stairs WS-matrix, and evaluates *Y* after each drawing of a new value from the marginal distribution of an individual factor in a random manner.

The WS sample estimate of *V(Y)* has the form of (6), where *k* denotes the number of factors, *N* is the WS-matrix sample size and *y(m,i)* is the *(m, i)*th element in the WS matrix.

$$V^{WS}(Y) = \frac{1}{k(N-1)} \sum_{i=1}^{k} \left[\sum_{m=1}^{N} y^2(m,i) - \left[\frac{1}{N} \sum_{m=1}^{N} y(m,i) \right]^2 \right] \tag{6}$$

Theory on how to build up the WS-matrix cyclically can be found in [10], while the approach of computing the sensitivity indices is described in [9].

FAST method

The FAST method [9] is an elegant estimation procedure for the first order indices. The method relates each uncertain input factor to a frequency ω_i and transforms it by:

$$X_i(s) = G_i(\sin(\omega_i s)) \ , \tag{7}$$

where G_i is a suitably defined parametric equation to get the desired probability function for X_i and allowing each factor to be varied as $\omega_i s$ is varied in $(-\pi; \pi)$; $\{\omega_1, \ldots, \omega_k\}$ denotes a set of linearly independent integer frequencies, i.e. they are free of interferences up to a given order *M* (usually 4 or 6). For example, (7) is selected as [9]:

$$X_i(s) = G_i(\sin(\omega_i s)) = \frac{1}{2} + \frac{1}{\pi}\arcsin(\sin(\omega_i s)) \tag{8}$$

to get a uniform distribution of X_i in (0;1). The selection of ω_i is made using a table. The output variance is approximated by performing a Fourier analysis.

EFAST method

The Extended FAST method [9] can compute both the first and total effect sensitivity indices by estimation of the variance in the complementary set, as in the Sobol method. This method introduces a random phase-shift, φ_i, in order to get a more flexible sampling scheme. Equation (8) becomes:

$$X_i(s) = G_i(\sin(\omega_i s)) = \frac{1}{2} + \frac{1}{\pi}\arcsin(\sin(\omega_i s + \varphi_i)) \tag{9}$$

The output variance is approximated by performing a Fourier analysis.

B. The OAT Morris method

The experimental plan of the Morris method [1], [9] consists of individually randomized OAT designs. The impact of changing one factor at a time is evaluated in turn, which constitutes the bases of the so called '*elementary effects*' data analysis computed as in (10):

$$ee_i(x_i) = \frac{f(x_1, x_2, ..., x_{i-1}, x_i + \Delta, x_{i+1}, ...x_k) - f(x)}{\Delta} \tag{10}$$

for any value x_i of input X_i between [0; 1-Δ]. Δ is called the variation size and is a predetermined multiple of 1/(*p*-1) and *p* denotes the number of levels of a factor. Theory on how to produce the Morris mean μ and its related standard deviation σ for each factor can be found in [1].

C. The Regression-based method (metamodeling)

Regression methods are used to build an approximation of a model, i.e. a metamodel [4-5] to fit the input data to the simulation data. The main types of designs used for an experiment are: central composite design (CCD), Latin

hypercube sampling (LHS), D-optimal and orthogonal arrays (OA) [4]. The performance of regression analysis depends on making assumptions about the model. Low-order polynomial functions are often employed. The generalized form of a second-order regression equation is:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i \cdot x_i + \sum_{i=1}^{k} \beta_{ii} \cdot x_i^2 + \sum_{i<j} \sum \beta_{ij} x_i x_j + \varepsilon \quad (11)$$

where β_0 is the free term coefficient, β_i β_{ii} and β_{ij} are the main effect, the quadratic effect and the interaction effect regression coefficients, respectively; ε is a random error. Based on the magnitude of the regression coefficients, one can perform a sensitivity ranking of the regression coefficients. However, these magnitudes are relative and a standardized regression approach is often suggested [14].

The quality of regression can be judged by comparing the residuals, i.e. the difference between the simulations (measurements) and the predictions, to a predefined threshold. For complex high order effects, this threshold is often exceeded, so the regression analysis cannot be used for the assessment of SA in such cases.

Table 1 describes the sensitivity metrics of each method, along with the execution cost which reflects the number of system evaluations required for SA.

Table 1. Comparison between sensitivity analysis methods

Class of methods based on	Method	Sensitivity measures		Execution cost(*C*)
		Symbol	***Significance***	
Variance	Sobol	S_i, S_{Ti}	S_i- 1st order effect sensitivity index S_{Ti}- total effect sensitivity index	$N \cdot (2k+1)$
	Jansen	S_i, S_{Ti}		$N \cdot k$
	FAST	S_i		$2 \cdot M \cdot OM(k)+1$
	EFAST	S_i, S_{Ti}		$k \cdot (2M\omega_{max}+1)/Nr$
OAT	Morris	μ,σ	mean and standard deviation of elementary effects	$r \cdot (k+1)$
Regression	Meta-model	β	regression coefficient	$min(size(\beta))$

where: k - no. factors, N - sample size, M - no. Fourier coefficients, OM - set of predefined frequencies, ω_{max}= max$\{\omega_1,\ldots,\omega_k\}$, r- no. elementary effects, , Nr-no. search curves, $\beta = [\beta_0\ \beta_i\ \beta_{ii}\ \beta_{ij}]$and $size(\beta)$ depends on the design;

III. RESULTS

A. Evaluation of SA methods with custom test functions

As a first step of analysis, the methods were tested via a simple mathematical example, which had a possibly self-evident sensitivity pattern. This enabled a proper comparison between the prediction of the methods and the expectation of the experimenter.

The custom defined test functions were polynomial functions which included different types of factor effects (main, quadratic and first order interactions) as in (12) and the coefficients β_i and β_{ij} indicate the importance of factor x_i.

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k} \sum_{j>i}^{k} \beta_{ij} x_i x_j \quad (12)$$

This study served as a guideline about the performance of each method, highlighting the compromise implied by the execution cost and it can be considered as reference for the study of real system applications for which the impact of the factors on the response is not known a-priori.

First, 60 polynomial functions of the form of (12) were implemented in MATLAB and 30 factors were considered for each. Then, we selected a set of important (target) factors by setting 90% of the effects to them, while the rest of 10% was distributed to the rest of the factors. An issue was to explore the robustness of the methods to noise, i.e. how capable a method is to determine an effect even if the system was affected by a random noise. Noise scenarios represent the case where the analysis is made on measurement data and not simulation ones. Noise added on the factors represents the case where the experimenter sets a value of a factor, but the equipment delivers only an approximation of it during a measurement due to limited precision, perturbation etc. A response affected by noise might be due to other factors which have not been included in the analysis. Because the most common noise is the Gaussian one, a random Gaussian noise was superimposed on either the response or the factors.

In order to find out the variance of the noise, the measure of Signal-to-Noise Ratio (SNR) was considered and it was defined as the ratio of the variance of the response (or the factors) and the variance of the noise:

$$SNR_y = \frac{\sigma_y^2}{\sigma_{noise}^2} \quad (13)$$

The factors and responses affected by noise are described in (14)-(15):

$$x_{i\ noise} = x_i + \sigma_{noise} \cdot \eta_i \quad (14)$$

$$y_{i\ noise} = y_i + \sigma_{noise} \cdot \eta_i \quad (15)$$

where η_i are normally distributed pseudorandom numbers.

The methods were tested for seven SNR values SNR_{dB} ={25, 20, 15, 10, 5, 0, -5} dB, where a SNR=25 dB had equivalent performance as if no noise was added. The metamodel was built using a D-optimal design [4].

A subsequent approach was to compare the performance of the methods also for a higher number of factors (50) and higher order effects (cubic effects and higher order interactions). The summary of the setups is shown in Table 2. The execution cost implied is illustrated in Table 3 and it depends on the number of factors of the system.

In order to test the performance of the methods, we defined the *pass rate* criteria. This measure computes the percentage of selected important (target) factors, x_t, which have also been identified in the top of most important factors of a method. Taking into consideration that for each function we selected a number of 2-4 important factors, it was considered a sufficient condition that the factor is returned in the top five most important factors.

Figure 1 illustrates a comparison of the methods' performance for setup I and noise added on the response. The FAST method is capable of detecting only the main effects and it was tested only for this type of effect. The metamodel exceeds the performance of the Jansen and EFAST methods for low SNR values. From the point of view of the execution cost, the metamodel and the Jansen methods were *more efficient* than the EFAST method. The

Morris method had the advantage of the *lowest execution cost*, but this implied also the compromise of a lower performance. The methods' behavior for noise added on the factors was similar.

Table 2. Test functions' setup

Setup	No. factors	Factor effects' types
I	30	main, quadratic, first order interactions
II	50	main, quadratic, first order interactions
III	30	quadratic, cubic, higher order interactions

Table 3. Execution costs

Method	Execution cost (no. runs)	
	30 factors	*50 factors*
Sobol	1830	3030
Jansen	900	1500
FAST	23081	87241
EFAST	1950	3250
Morris	310	510
Metamodel	496	1326

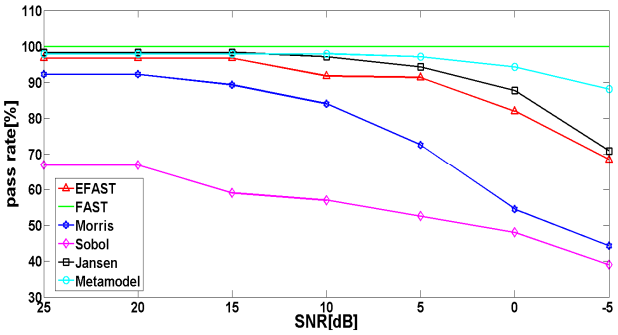

Figure 1. Performance comparison of methods; noise added on y

Figure 2 shows a comparison of the methods' performance for setup I against setup II. Some methods have slightly poorer performance when increasing the number of factors (except for Sobol). By this we proved the assumption that the important factors can be determined easier from a lower number of factors.

Another study was to test whether the methods determine factors with higher effects (cubic and higher order interaction) with the same accuracy as the ones with lower effects (the execution cost remained the same as in setup I). For this, we considered polynomial functions as:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i^2 + \sum_{i=1}^{k} \sum_{j \geq i}^{k} \beta_{ij} x_i^2 x_j \qquad (16)$$

Figure 3 illustrates a comparison of the methods' performance for setup I against setup III. The factors with higher order effects are detected with a lower accuracy, proving that the factors with lower effects are easier determined than the factors with higher order effects.

The Sobol method has the poorest performance in all setups. It needs to be mentioned that the sampling matrix size was selected to have execution costs of similar magnitudes for all methods. By increasing the execution cost, the Sobol method achieves higher performance (see Figure 4). An increase of the execution cost for the Morris, Jansen and the metamodel approach does not bring much improvement on the performance.

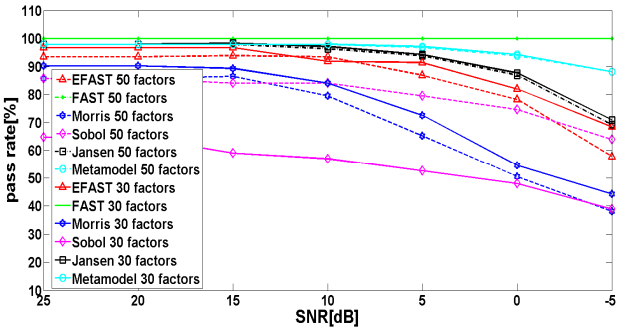

Figure 2. Performance comparison of methods; 30 factors vs. 50 factors; noise added on y

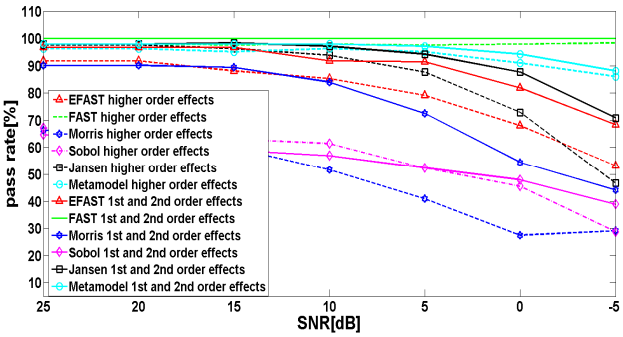

Figure 3. Performance comparison of methods; 1st and 2nd order effects vs. higher order effects; noise added on y

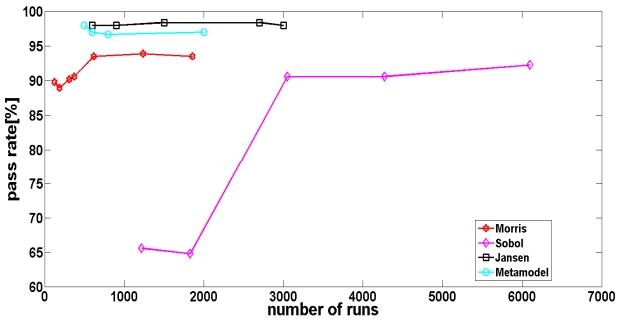

Figure 4. Performance comparison of the Morris, Sobol,Jansen and Metamodel methods for different execution costs; noise added on y; SNR=20 dB

The polynomial functions were a great tool to explore the performance of the SA methods and the accuracy of detecting factors even if the system was affected by noise. The conclusion of this study was that the Jansen and metamodel based methods achieved the highest performances, along with the FAST method, but this has the limitation that it identifies only the main effects.

B. Evaluation of the SA methods on the battery model

The knowledge provided by the tests from the previous section served as reference for the study of a real system,

which was a battery model provided by MATHWORKS [17], [18]. It included 24 factors (summarized in Table 4) and consisted in six equivalent circuit cells as shown in Figure 5, the alternator and the load. In the evaluation survey that follows, the factors are simply referenced as x_1, x_2,…, etc., because we are only focused on how these methods give common or different results. We selected six responses of interest for the battery study: the maximum battery current (y_1), the maximum battery voltage (y_2), the battery SOC (State of Charge) at the end of the analysis (y_3), the battery temperature (its integration over time which is related to its lifetime) (y_4), the maximum generated current (y_5) and the maximum generated voltage (y_6).

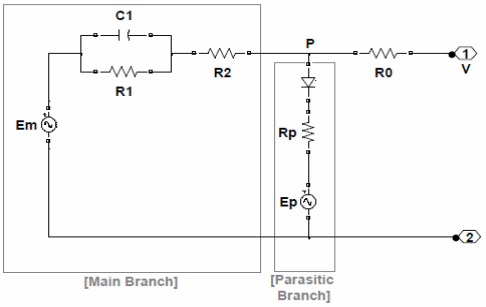

Figure 5. The battery cell equivalent circuit

Table 4.The factors of the battery model

Type of model factor	#factors	Factor name
Battery Model Factors	1	*NominalCap* (x_1)
Initial Conditions	1	*theta_init* (x_2)
Thermal Factors	3	*Ctheta* (x_3), *Area exposed to air* (x_4), *Rtheta* (x_5)
Capacity Factors	5	K_c (x_6), *Co** (x_7), *delta* (x_8), *I** (x_9), *theta_f* (x_{10})
Parasitic Branch Factors	5	*Ep* (x_{11}), *Gpo* (x_{12}), *Vpo* (x_{13}), *Ap* (x_{14}), *Taup* (x_{15})
Main Branch Factors	9	*Emo* (x_{16}),K_e (x_{17}), A_o (x_{18}), R_{oo} (x_{19}), R_{10} (x_{20}), A_{21} (x_{21}), A_{22} (x_{22}), R_{20} (x_{23}), Tau_1(x_{24})

The purpose of the experiments is to assess and to compare the accuracy of each method. We applied each method for all the responses and the results showed that in some cases the methods identified the same factors as being important, but in other cases they identified different factors. In order to check the consistency of the methods we used the cross-validation approach.

The cross-validation was applied on the EFAST, Morris, Jansen and Sobol methods because only these methods generate a different design of experimental runs for the same number of factors.

We wanted to test whether these methods return the same set of important factors and if there is a difference in the sensitivity indices' values for two different setups. Table 5 and Table 6 present the results for two of the methods in a decreasing order of the factor importance. For any setup, after analyzing the sensitivity indices' magnitudes, it can be noticed that there is a clear boundary between the most important factors (first and second factor in this case) and the rest of the factors. When analyzing the results of the Morris screening method, it is important to analyze μ and σ at the same time. The values of these measures differ in some extent for the two setups, but the ranking of the most important factors is the same: factor x_{16} is the most important factor, followed by factor x_{18} and factor x_{19} which has a non-linear or interaction effect. Note that x_{16} represents the open-circuit voltage source of the battery, which from an expert's view could have high impact on the battery model responses; x_{19} represents the value of the terminal resistance R_0, while A_0 (denoted with x_{17}) is a factor impacting the terminal resistance R_0.

Table 5. Top five sensitivity indices; Morris method; y_1

Morris							
Experiment 1				***Experiment 2***			
factor	μ	factor	σ	factor	μ	factor	σ
x_{16}	2.7249	x_{16}	0.0034	x_{16}	2.7234	x_{16}	0.0042
x_{18}	0.0132	x_{19}	0.0029	x_{18}	0.0130	x_{19}	0.0024
x_{20}	0.0006	x_{18}	0.0005	x_{20}	0.0007	x_{18}	0.0004
x_{21}	0.0003	x_{17}	0.0004	x_{21}	0.0004	x_2	0.0002
x_5	0.0002	x_{20}	0.0002	x_4	0.0002	x_{20}	0.0002

Table 6. Top five sensitivity indices; EFAST method; y_1

EFAST							
Experiment 1				***Experiment 2***			
factor	*Si*	factor	*STi*	factor	*Si*	factor	*STi*
x_{16}	0.9957	x_{16}	0.9980	x_{16}	0.9960	x_{16}	0.0094
x_{19}	0.0013	x_{19}	0.0037	x_{19}	0.013	x_{19}	0.0038
x_{17}	0.0002	x_{12}	0.0025	x_{17}	0.0002	x_3	0.0027
x_{18}	0.00002	x_{17}	0.0025	x_{18}	0.0002	x_8	0.0026
x_2	0.00001	x_1	0.0025	x_2	0.0001	x_{17}	0.0026

When analyzing the results of the variance-based methods, it is more accurate to consider the total effect sensitivity indices [9].

From Table 6, it can be observed that the sensitivity indices' values change insignificantly from one experimental design to another. The rank of the most important factors is again limited from the rest of the factors which are unimportant: x_{16} and x_{19} have the greatest impact on the output, while x_{12}, x_{17} and x_1 (from experiment 1) and x_3, x_8 and x_{17} appear by chance. The conclusions are similar for the Sobol and Jansen methods.

Based on the cross-validation approach, we also computed the similarity rate in terms of common target (important) factors of the two setups of a method.

Let M_1={x_{t1}, x_{t2}… x_{tm}} be the set of the m target factors determined by the first experimental design and M_2={x_{t1}, x_{t2}… x_{tn}} be the set of the n target factors determined by the second experimental design. The *similarity rate* (SR) of the factors returned by two different designs of the same method was computed as in (17) considering the cardinality of the intersection of M_1 and M_2 and their union; the results are summarized in Table 7.

$$SR = \frac{card\left(M_1 \cap M_2\right)}{card\left(M_1 \cup M_2\right)} \tag{17}$$

Table 7. Cross-validation using the similarity rates

Method	**Similarity rate [%]**						
	Response						
	y_1	y_2	y_3	y_4	y_5	y_6	On average
Morris	100	100	100	66.6	100	100	**94.4**
EFAST	100	100	100	100	100	100	**100**
Jansen	100	100	100	100	100	100	**100**
Sobol	50	50	50	33.3	0	50	**38.8**

For the metamodeling approach, prior to analyzing the regression coefficients, it is important to test whether the metamodel is fit. If this condition is not held, the conclusions about the most important factors could be erroneous. We used the analysis of residuals as the measure of fitness [19].

The metamodel was built using the D- optimal design and the validation was made using a Monte Carlo design. For all responses, except the battery temperature, the maximum normed residuals did not exceed the threshold. Drawing conclusions regardless the test of fit could lead to erroneous decisions in terms of factor prioritization.

Table 8 shows a comparison of the most important factors according to the EFAST and Jansen methods (which achieved the best performance on the test functions from the previous section) for the battery SOC, for which the metamodel was fit. Analyzing also the sensitivity indices' values, the metamodel returns the same set of important factor as the EFAST and Jansen methods. If we analyze the results for the battery temperature (Table 9), where the metamodel was not fit, the coefficients' values are very high and the rank of the important factors is less similar with the EFAST and Jansen methods. This proves that the regression coefficients are unreliable if the metamodel is not fit.

As a final analysis of accuracy, we compare the similarity among the methods, i.e. the rate in which they identify the same set of important factors. Table 10 summarizes the similarity rates of the SA methods on average, taking into consideration all responses.

Table 8. Most important factors; Y_3

EFAST		**Jansen**		**Metamodel**	
factor	S_{Ti}	factor	S_{Ti}	factor	$\Sigma\|\beta\|$
x_{16}	0.5856	x_{16}	0.9574	x_{16}	0.0107
x_2	0.2293	x_2	0.2532	x_2	0.0062
x_3	0.0689	x_3	0.0165	x_3	0.0040
x_{20}	0.0222	x_5	0.0144	x_{12}	0.0039

Table 9. Most important factors; Y_4

EFAST		**Jansen**		**Metamodel**	
factor	S_{Ti}	factor	S_{Ti}	factor	$\Sigma\|\beta\|$
x_2	0.7483	x_2	1.0353	x_{16}	3303.445
x_{16}	0.6985	x_6	0.5923	x_2	3109.005
x_{17}	0.6561	x_4	0.5245	x_{10}	2035.991
x_{23}	0.5367	x_{14}	0.4855	x_{12}	1925.231

Table 10. Cross-validation using the similarity rates

Method	**Similarity rate [%]**					
	On average					
	Morris	EFAST	Jansen	Sobol	FAST	Metamodel
Morris	100	53	45	37	46	44
EFAST	53	100	75	40	93	90
Jansen	45	75	100	36	86	90
Sobol	37	40	36	100	38	40
FAST	46	93	86	38	100	86
Metamodel	44	90	90	30	86	100

The Morris method does not achieve similarity rates higher than approximately 50%, meaning that only the half of the returned important factors are considered important by the rest of the SA methods. The EFAST, FAST, Jansen and metamodeling achieve the highest number of common important factors. This result was expected because the analysis with custom test functions also concluded that these methods are the most performant ones. The Sobol method has the lowest similarity rates. Only a percentage of 40% of the returned important factors are returned to be important also by the EFAST method, while with the metamodeling approach, it agrees only on 30% of the important factors.

IV. DISCUSSIONS

In order to choose the suitable SA method, one needs to consider several aspects such as the execution cost implied for the analysis, the number of factors and responses of the system and also the type of factors involved. A first step of analysis for systems with a high number of factors could be to apply the Morris method, because it has the advantage of a low execution cost and the ability to determine the most influential factors on the output response. All the variance-based methods, except FAST, compute both the main effect and total effect indices. The total effect indices are a more accurate sensitivity measure, since it takes into account all types of effects involving that factor.

A possible approach for reliable conclusions would be to compare the results of several SA methods. However, this would mean an increased execution cost because most of the SA methods studied impose their own experiment plan. This means that for the variance-based and the Morris OAT methods perform different sampling of the factors' space and require the system evaluation in those setups. The advantage of comparing several SA methods is that of more robust conclusions. The metamodeling approach does not have any limitations regarding the design. The D-optimal design which was used in this paper had the advantage of a low execution cost, compared to other commonly used designs: central composite design, Latin hypercube sampling and factorial designs.

V. CONCLUSIONS

Six SA methods were applied on a set of custom-defined functions and a battery model, both including a high number of factors. The reason for using the test functions was to do a comparison in a controlled manner of the performance of the methods in terms of factor prioritization for different effect types and to determine the execution cost implied by each method. The Jansen, EFAST and metamodeling approach had the best performance in identifying the most important factors. The metamodel and the Morris method had the lowest execution costs. The metamodel has the limitation that it provides conclusive information about the systems only if the maximum residual error is below a predefined threshold. The Morris method implied a compromise between the capability of determining the most important factors and a low execution cost.

The methods were also evaluated on a real life application, a battery model. We compared the rankings provided by each method and quantified at what degree different methods agreed on the most important factors. By such an analysis we were able to determine the most influential factors on the battery's responses of interest.

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