# BROADBAND POWER AMPLIFIER NONLINEARITY CANCELLATION IN OFDM SYSTEMS

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**Abstract: One drawback of OFDM systems is the high peak-to-average power ratio, which imposes strong requirements on the linearity of power amplifiers (PAs). Such linearity requirements translate into high back-off that results in low power efficiency. In order to improve power efficiency, a PA nonlinearity cancellation (PANC) technique is introduced which reduces the nonlinear distortion effects on the received signal. In order to deal with broadband high-power PA (HPA) distortion, the PANC technique employs a Wiener-Hammerstein model. Simulation results show that good levels of distortion cancellation are possible for a system with a nonlinear HPA with memory.**

***Keywords:*** *Orthogonal frequency-division multiplexing (OFDM), nonlinear distortion, power amplifier (PA) nonlinearities cancellation, Wiener-Hammerstein model.*

## I. INTRODUCTION

The principal drawback of an OFDM system is the high peak-to-average power ratio (PAPR). Real power amplifiers (PAs) have a nonlinear transfer function causing signal compression and clipping that result in signal waveform distortion and adjacent channel interference. Power back-off and PAPR reduction techniques reduce the nonlinear distortion effects but result in low power efficiency. Furthermore, broadband PAs introduce memory which gives rise to intersymbol interference (ISI) [1, 2].

From a performance point-of-view (considering both BER and power efficiency) it may be preferable to mitigate the nonlinear distortion at the transmitter, e.g., by applying a predistorter that includes PA memory effects [1]. However, limited power budget and hardware complexity can motivate a receiver cancellation technique. In addition, receiver techniques can be used in combination with a predistorter [3].

A power amplifier nonlinearity cancellation (PANC) technique was proposed in [3, 4] for mitigating the nonlinear distortion effects from a memoryless PA. With an initial estimate of the transmitted OFDM symbols, the distortion effects can be estimated *if the PA model is known*. After that, the nonlinear distortion can be removed from the received signal and new and improved symbol estimates can be obtained. This procedure can be repeated in an iterative manner to obtain almost undistorted estimates in two or three iterations. This concept was used in [5] in a single-user wireline system using adaptive OFDM with a large number of carriers. More recently, similar ideas have been presented in [6].

Considering the more general case of broadband PAs with memory, the above techniques will not yield an adequate performance. This paper generalizes the ideas in [3, 4, 5] and proposes a PANC technique that not only takes into account the nonlinear transfer function of the PA, but also its memory effects. As a consequence, we need to deal with a composite Wiener-Hammerstein (WH) model to describe the corresponding dynamics of the parameter estimation problem at hand. The Wiener-Hammerstein model consists of a static nonlinearity with a linear filter preceding its input and another linear filter at its output. The Wiener-Hammerstein model has been found useful for describing many practical nonlinear system, see, e.g., the exhaustive bibliography on nonlinear system identification [7]. The proposed technique can serve as an alternative or in combination with a transmitter based mitigation technique, e.g., a predistorter.

The paper is organized as follows. Section II introduces the OFDM system and broadband PA model to be used in the remaining parts of the article. The new PANC technique for the broadband PA case is introduced in Section III. Section IV presents numerical simulations followed by conclusions in Section VII.

## II. SYSTEM MODEL

The OFDM transmission model used in this paper is illustrated in Fig. 1. The system under consideration has $N$ subcarriers. Let $\{X(k)\}_{k=0}^{N-1} \in \mathbb{C}$ denote the modulated data symbols associated with carrier $k$. The transmitted time-domain OFDM symbols $\{x(n)\}_{k=0}^{N-1}$ are then obtained via the inverse discrete Fourier transform (IDFT)

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi k \frac{n}{N}},\ n = 0, 1, \ldots, N-1 \qquad (1)$$

After the IDFT, a cyclic prefix is added to the transmitted block $[x(0)\, x(1) \cdots x(N-1)]$ to avoid intersymbol interference (ISI) and simplify equalization.

The OFDM symbols $x(n)$ are then passed through a nonlinear PA with memory, here modeled using a Wiener-Hammerstein structure [1]. The Wiener-Hammerstein model is frequently used to model broadband PAs and, as seen in Fig. 1, it is formed by a cascade of a linear filter $A(z)$, a nonlinear static function $g[\cdot]$, and another linear filter $B(z)$. The filters, $A(z)$ and $B(z)$, are here modeled as

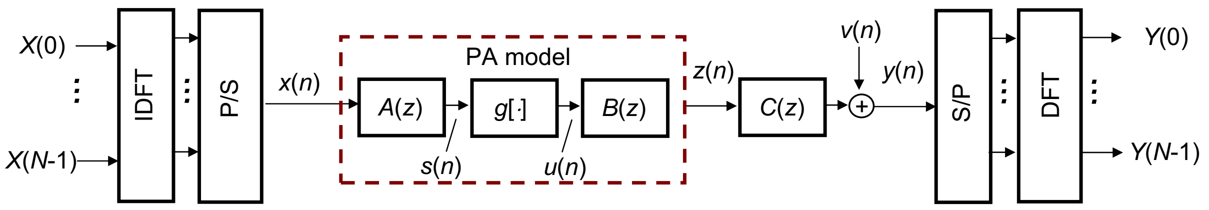


*Figure 1. OFDM system model employed in the paper.*

FIR filters of orders $L_a$ and $L_b$, respectively. The transmitted signal $z(n)$ can now be expressed as

$$z(n) = \sum_{m=0}^{L_b} b_m g[s(n-m)] \tag{2}$$

where

$$s(n) = \sum_{m=0}^{L_a} a_m x(n-m). \tag{3}$$

The static nonlinearity is modeled as a polynomial with coefficients $\{g_{2k+1}\}_{k=0}^{K}$. We consider here an implementation that uses orthogonal (Hermite-based) polynomial basis functions. This basis is particularly useful with OFDM signals which are approximately complex Gaussian distributed [12]. The are many other advantages of employing orthogonal polynomials, e.g., improved numerical stability and robustness to quantization noise and finite-precision errors. When using Hermite-based polynomials, model output $g[s(n)]$ is given by

$$u(n) = g[s(n)] = \sum_{k=0}^{K} g_{2k+1}\psi_{2k+1}[s(n)]$$
$$\psi_{2k+1}[s(n)] = \sum_{l=0}^{k} (-1)^{k-l} \frac{\sqrt{k+1}}{(l+1)!} \binom{k}{l} \phi_{2l+1}[s(n)] \tag{4}$$

Under the assumption that the transmission channel $C(z)$ remains constant over at least one OFDM block (block fading model) and can be modeled as an FIR filter of order $L_c$, the received signal can be written as

$$y(n) = \sum_{m=0}^{L_c} c_m z(n-m) + \nu(n). \tag{5}$$

where $\nu(n)$ is additive noise, assumed here to be circular complex Gaussian with variance $\sigma_\nu^2$. The frequency-domain signal of (5) is obtained via the discrete Fourier transform (DFT), i.e.,

$$Y(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(n) e^{-j2\pi n \frac{k}{N}},\ k = 0, 1, \ldots, N-1 \tag{6}$$

In order to detect the data symbols $X(k)$ and express the effects of the nonlinear distortion, it is common and useful to represent the output of the nonlinear static block $g[\cdot]$ as a sum of two uncorrelated components [13]

$$g[s(n)] = K_L s(n) + d(n)$$
$$\mathrm{E}[s^*(n)d(n)] = 0 \tag{7}$$

The first term in (7) is just a scaled version of the input signal ($K_L \leq 1$), while $d(n)$ is an additive distortion term.

Substituting (7) in (2) and assuming the effective channel length $L = L_a + L_b + L_c$ falls within the cyclic prefix, (6) reduces to

$$Y(k) = C(k)B(k)\left[K_L A(k)X(k) + D(k)\right] + \Psi(k) \tag{8}$$

where for subcarrier $k$, $\Psi(k)$ is the additive noise, $D(k)$ is the nonlinear distortion (DFT of $\{d(n)\}_{n=0}^{N-1}$), and $A(k)$, $B(k)$ and $C(k)$ denote the responses of the linear filters of the PA model and the wireless channel.

## III. POWER AMPLIFIER NONLINEARITY CANCELLATION TECHNIQUE

This section presents an iterative approach for removing the nonlinear PA effects at the receiver. It is an extension of the iterative ML-based technique in [5] to the case of nonlinear PAs with memory and time-varying transmission channels.

In order to detect the uncoded $X(k)$ from the received signal in (8), we can consider the following ML decision rule for subcarrier $k$

$$\hat{X}(k) = \arg\min_{X(k)} \left\{ H_L(k) \left[ X(k) - \frac{Y(k)}{H_L(k)} + \frac{D(k)}{A(k)} \right] \right\} \tag{9}$$

where

$$H_L(k) = K_L A(k)B(k)C(k) \tag{10}$$

Assuming that $H_L(k)$, $A(k)$, and $D(k)$ are all known at the receiver,[1] the ML estimate is given by the symbol $X(k)$ with the minimum distance to $Y(k)/H_L(k) - D(k)/A(k)$. Knowing the static nonlinear function $g[\cdot]$ allows us to estimate $D(k)$ through (7) as

$$d(n) = g[s(n)] - K_L s(n),\ n = 0, \ldots, N-1$$
$$D(k) = \frac{1}{N} \sum_{n=0}^{N-1} d(n) e^{-j2\pi n \frac{k}{N}},\ k = 0, 1, \ldots, N-1 \tag{11}$$

We see from (3) that $s(n)$ depends on both $\{X(k)\}_{k=0}^{N-1}$ and $A(z)$. Following the approach in [5], we can use an iterative technique that employs tentative decisions $\hat{X}(k)$ for mitigating the nonlinear distortion effects. The technique is depicted in Fig. 2 and summarized in Table 1.

[1] For example using an available Wiener-Hammerstein model identification technique [8, 9, 10, 11].

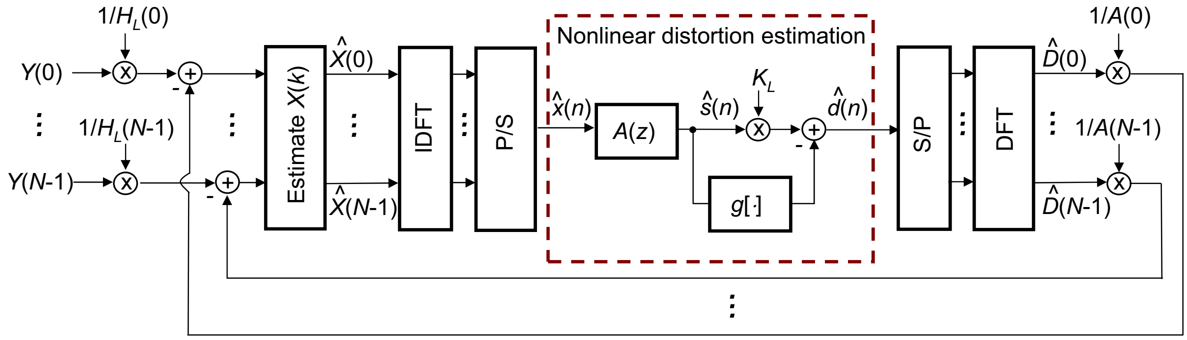


*Figure 2. Power amplifier nonlinearity cancellation (PANC).*

*Table 1. Power amplifier nonlinearity cancellation (PANC)*

Initialization:
$\hat{D}^{(0)}(k) = 0,\ k = 0, \ldots, N-1$
for $m = 1$ to $I_{\max}$
{
Symbol decoding:
$\hat{X}^{(m)}(k) = \arg\min_{X(k)} \left\{ H_L(k) \left[ X(k) - \frac{Y(k)}{H_L(k)} + \frac{\hat{D}^{(m-1)}(k)}{A(k)} \right] \right\}$
Time domain:
$\hat{x}^{(m)}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X^{(m)}(k) e^{j2\pi k \frac{n}{N}}$
Estimate $s(n)$:
$\hat{s}^{(m)}(n) = \sum_{l=0}^{L_a} a_l \hat{x}^{(m)}(n-l)$
Estimate distortion $d(n)$:
$\hat{d}^{(m)}(n) = g[\hat{s}^{(m)}(n)] - K_L \hat{s}^{(m)}(n)$
Distortion in frequency domain $D(k)$
$\hat{D}^{(m)}(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} d^{(m)}(n) e^{-j2\pi n \frac{k}{N}}$
}

## IV. SIMULATIONS

The performance of the general PANC technique was evaluated in an OFDM system with 16-QAM modulation on $N = 512$ subcarriers. The carrier frequency is $f_0 = 5$GHz and the bandwidth is $B = 20$MHz. The length of the cyclic prefix is equal to 64. The channel is Rayleigh fading with independent propagation paths each generated according to a Jake's Doppler spectrum. The power loss and delay profiles of the channel are: $[0, -1, -3, -9]$ [dB] and $[0, 1, 2, 3]$ $[\mu s]$ corresponding to an Urban scenario. The terminal speed is set to 2 km/h.

The broadband PA is modeled as a Wiener-Hammerstein system, where the linear filters, taken from [14], are given by

$$A(z) = \frac{1 + 0.1z^{-2}}{1 - 0.1z^{-1}} \text{ and } B(z) = \frac{1 - 0.1z^{-1}}{1 - 0.2z^{-1}} \tag{12}$$

and the static nonlinearity is modeled using a Solid State Power Amplifier (SSPA) model, i.e.,

$$g[x(n)] = \frac{|x(n)|}{\left[1 + \left(\frac{|x(n)|}{A_s}\right)^{2p}\right]^{1/p}} \exp(j\angle x(n)) \tag{13}$$

where the parameter $p$ adjusts the smoothness of the transition from the linear region to the saturation region, and $A_s$ is the amplifier input saturation. The results are evaluated for different clipping levels $\gamma$ defined as

$$\gamma = \frac{A_s}{\sqrt{E_n\{|x(n)|^2\}}} \tag{14}$$

where $\sqrt{E_n\{|x(n)|^2\}}$ is the RMS value of the OFDM signal.

For the implementation of PANC in Table 1, we assumed FIR estimates of $H_L(z)$, $A(z)$ and a polynomial estimate of $g[\cdot]$ were available at the receiver, see, e.g., [8, 9, 10, 11]. The number of taps in the FIR filter $\hat{A}(z)$ was $L_a + 1 = 5$ and the static nonlinearity $\hat{g}[\cdot]$ was a conventional polynomial order of $K = 3$.

Figures 3 and 4 show the BER versus SNR for two different clipping levels, $\gamma = 3$ dB and $\gamma = 4$ dB. Results obtained with linear PA and nonlinear (NL) PA without PANC are included for reference. We see that the PANC technique can reduce the nonlinear distortion effects, resulting in a performance close to that of a linear PA.

## V. CONCLUSIONS

A power amplifier (PA) nonlinearity cancellation (PANC) technique was proposed that reduces the harmful effects of broadband PAs in OFDM systems. The proposed technique estimates the nonlinear distortion by passing tentative decisions through a broadband PA model. The memory effects of the PA are taken into account by modeling the PA as a Wiener-Hammerstein system. The PANC technique is suitable for systems where reduced implementation complexity is required in the transmitter and can be combined with other distortion mitigation techniques, e.g., predistortion. Simulation results confirm a substantial reduction of nonlinear distortion is obtained by PANC.

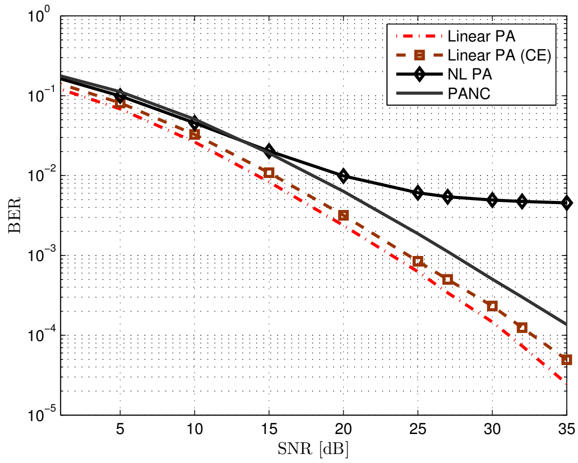


*Figure 3. BER versus SNR for PANC in an OFDM system with 16-QAM modulation on $N = 512$ subcarriers, for the case of linear PA and nonlinear PA. The mobile speed is set to $v = 2$ km/h. Wiener-Hammerstein type nonlinear PA was considered where $g[\cdot]$ was an SSPA model with $p = 2$. Clipping level $\gamma$ was set to 4 dB. Results obtained with linear PA and NL PA without PANC are included for reference.*

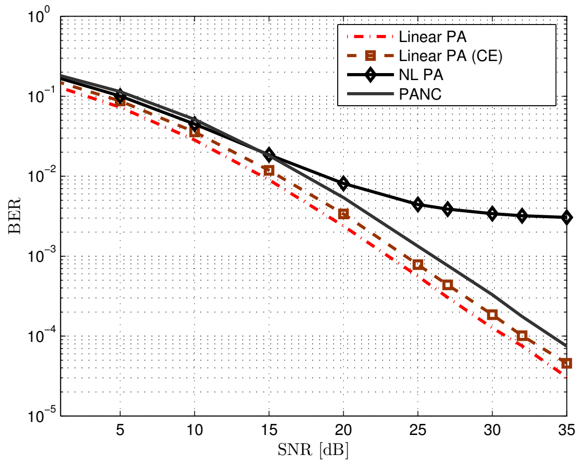


*Figure 4. BER versus SNR for PANC in an OFDM system with 16-QAM modulation on $N = 512$ subcarriers, for the case of linear PA and nonlinear PA. The mobile speed is set to $v = 2$ km/h. Wiener-Hammerstein type nonlinear PA was considered where $g[\cdot]$ was an SSPA model with $p = 2$. Clipping level $\gamma$ was set to 5 dB. Results obtained with linear PA and NL PA without PANC are included for reference.*

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