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THE IMPROVEMENT OF THE WAVELET ANALYSIS TECHNIQUES BY USING B-SPLINE FUNCTIONS FAMILY

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Abstract: The goal of this paper is to justify the use of fractional B-spline bases in several Wavelet Analysis Techniques. We are interested in better controlling the differentiation behavior of the wavelet. For that purpose, we propose an edge detector algorithm by using the wavelet transform local maxima, based on B-spline functions. Another task is to search the connection available between energetic analysis and the chosen type of wavelet family and to use the results for Glottal Closure Instants Estimation in Voiced Speech.

Key words: *Wavelets, fractional B-splines, differentiator, energetic analysis*

I. INTRODUCTION

The theory behind wavelets has been developed during the last thirty years independently by mathematicians, scientists and engineers. Researchers are now faced with an ever increasing variety of wavelet bases to choose from. As more and more wavelet solution are proposed, the selection of a particular wavelet function should always be motivated by the problem itself.

Thus, comparative studies are needed more then ever. From the very beginning of wavelets, it has been recognized the strong connection between wavelets and differential operators. So, transient features such as discontinuities are characterized by wavelet coefficients in their neighborhood only, the same way as a derivative acts locally too. In this paper, we propose to investigate how it can be further improved on the wavelet's behavior as differentiator using the B-spline wavelet transform.

The spectogram and scalogram allow the examination of the energy distribution in the time-frequency respectively scale-frequency plane. The energetic analysis is also connected to the chosen type of wavelet family. We suggest a comparison among B-spline wavelets energetic results and the ones acquired by using some wavelets already implemented in Matlab.

In addition, a multiresolution framework for analysis of glottal closure instants is proposed.

Section 2 recalls the necessary background information, section 3 contains a description of applications and finally some conclusions are summarized in section 4.

II. THEORETICAL BACKGROUND

a. The derivative-like behavior of fractional B-spline wavelets

An intense activity in wavelet design has led to the construction of a large variety of wavelet bases; the most important requirements are orthogonality and compact support, high number of vanishing moments, symmetry and regularity, explicit analytical form, an optimal time-frequency localization, etc.

One of the key mathematical properties of wavelets is that they behave like multiscale differentiators. Thus, there is a correspondence between a wavelet with vanishing moments and the differentiation operator. The derivative-like behavior of wavelets was investigated by several researchers including T. Blu, M. Unser, D. Van De Ville.

Fractional B-spline Functions were proposed in 2000, by Thierry Blu and Michael Unser[3]. The primary motivation for considering fractional B-splines instead of conventional ones was that the enlarged family happens to be closed under fractional differentiation.

The uniform *first* order causal B-spline function is defined as:

$$\beta_+^0(x) = x_+^0 - (x-1)_+^0 = \Delta_+^1 x_+^0 \tag{1}$$

$$\hat{\beta}_+^0(w) = \frac{1-e^{-jw}}{jw} \quad \begin{matrix} -\,finite & difference \\ -\,derivative & operator \end{matrix} \tag{2}$$

where $x_+^\alpha = \max\{0, x\}^\alpha$ is the one-sided power function, $\Delta_+ = \delta(x) - \delta(x-1) \overset{F}{\leftrightarrow} 1 - e^{-jw}$ is the finite

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difference operator and $\partial \overset{F}{\leftrightarrow} jw$ is the derivative operator.

The *k*-th order function is defined by convolving the *k-1* order function with the *first* one:

$$\beta_+{}^k(x) = \beta_+{}^{k-1} * \beta^0(x) \tag{3}$$

Analogically with the classical B-splines, the new family is constructed using linear combinations of the integer shifts of the one-sided power functions:

$$\beta_+^\alpha(x) = \frac{\Delta_\alpha^{\alpha+1} x_+^\alpha}{\Gamma(\alpha+1)} \quad \xleftrightarrow{F} \quad \hat{\beta}_+^\alpha(w) = \left(\frac{1-e^{-jw}}{jw}\right)^{\alpha+1} \tag{4}$$

where α is the fractional degree of the B-splines function and $\Gamma(\alpha)$ is the gamma function. As we mentioned before, one of the primary reasons for the success of B-splines in applications is that they can be differentiated very simply by taking finite differences. This property generalizes nicely to the fractional case:

$$\begin{aligned}\partial^\gamma \beta_+^\alpha(x) &\xleftrightarrow{F} (jw)^\gamma \left(\frac{1-e^{-jw}}{jw}\right)^{\alpha+1} \\ &= \left(1-e^{-jw}\right)\left(\frac{1-e^{-jw}}{jw}\right)^{\alpha+1-\gamma}\end{aligned} \tag{5}$$

$$\partial^\gamma \beta_+^\alpha(x) = \Delta_+^\gamma \beta_+^{\alpha-\gamma}(x) \tag{6}$$

From the previous equation, it can be seen that the fractional derivative of order γ of an α fractional B- spline is another $\alpha-\gamma$ fractional B-spline. The key operator in this case is the causal fractional finite difference one:

$$\Delta_+^{\alpha+1} \quad \xleftrightarrow{F} \quad \left(1-e^{-jw}\right)^{\alpha+1} \tag{7}$$

In our study, we also appeal to generalized fractional B-spline functions, which were proposed by Dimitri Van De Ville [4]. His starting point was the fractional B-spline family but he added another parameter τ (the shift). Shifting a function simply means delaying its onset. These type of functions are most conveniently defined in the Fourier domain:

$$\hat{\beta}_\tau^\alpha(w) = \left(\frac{e^{jw}-1}{jw}\right)^{\frac{\alpha+1}{2}-\tau}\left(\frac{1-e^{-jw}}{jw}\right)^{\frac{\alpha+1}{2}+\tau} \tag{8}$$

The fractional derivative of order (α',τ') of a (α,τ) generalized fractional B-spline is another $(\alpha-\alpha',\tau-\tau')$ generalized fractional B-spline:

$$\partial_{\tau'}^{\alpha'} f(t) \; \underbrace{\xleftrightarrow{F} \; (-jw)^{\alpha'/2-\tau'}(jw)^{\alpha'/2+\tau'}}_{\hat{\partial}_{\tau'}^{\alpha'}(w)} \hat{f}(w) \tag{9}$$

$$\partial_{\tau'}^{\alpha'} \beta_\tau^\alpha = \Delta_{\tau'}^{\alpha'} \beta_{\tau-\tau'}^{\alpha-\alpha'} \tag{10}$$

where $\Delta_{\tau'}^{\alpha'} \xleftrightarrow{F} \left(1-e^{jw}\right)^{\frac{\alpha'}{2}-\tau'}\left(1-e^{-jw}\right)^{\frac{\alpha'}{2}+\tau'}$ is the generalized fractional finite difference operator.

Figures 1 and 2 present the discrete wavelet transform coefficients of a piecewise polynomial signal. The first decomposition is obtained by using generalized fractional B-spline functions for $\alpha' = \alpha+1 = 4.5$ and $\tau = 0.5$ and the second one by using Daubechies 3 function as mother wavelet. Signal singularities are compactly characterized by at most two local extrema in each subband, only for the B-spline wavelet coefficients. This feature opens up new ways to analyze a signal using B-spline wavelet bases.

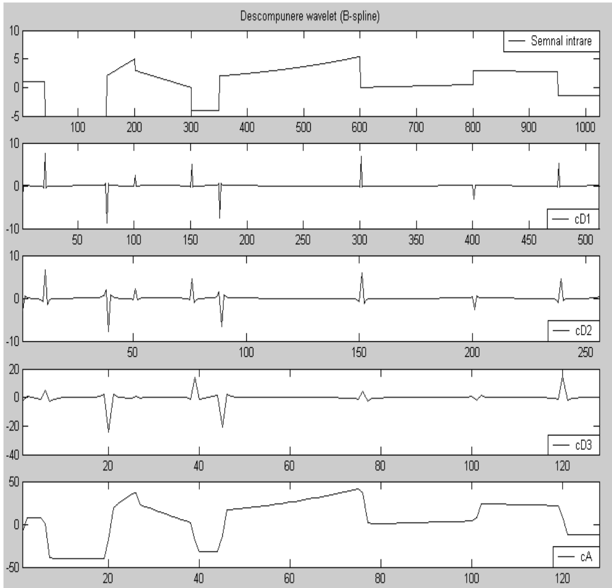

Figure 1. Generalized B-spline wavelet decomposition

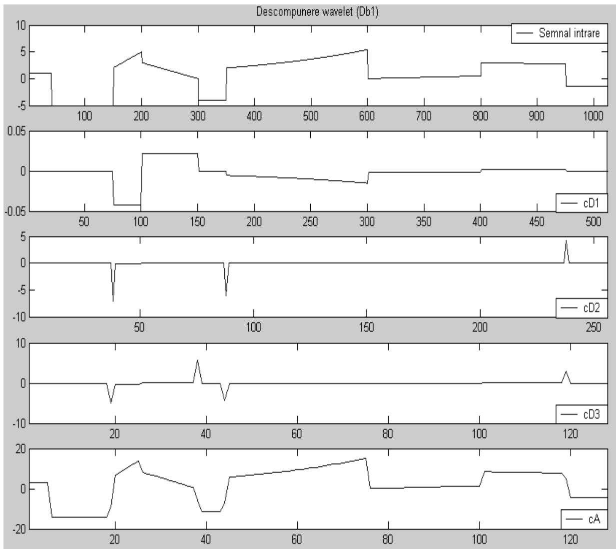

Figure 2. Daubechies 3 wavelet decomposition

b. The energetic analysis of the signals is connected to the chosen type of wavelet function.

Short Time Fourier Transform (STFT) gives a description of both the time and frequency characteristics of the signal. The Fourier spectrogram is defined as the square modulus of the STFT. The common method of analysis of signals has a limited time-frequency resolution, and therefore limited accuracy. If the length of the window is small, the spectrogram will be well localized in time, but it will have poor frequency resolution. If the window is long, the frequency resolution will be good, but the localization in time will be poor. This is the fundamental weakness of the spectrogram.

The wavelet transform has become a valuable analysis tool due to its ability to elucidate simultaneously both the spectral and the temporal information within the signal. Wavelet spectrogram, called scalogram, communicates the time frequency localization property of the wavelet transform. The result of wavelet transform relates to the mother wavelet. In our experiments, we have mainly worked with fractional B-splines, Daubechies and Shannon wavelets. Figures 4 and 5 show the spectrogram which resulted_from the entry signal which represents the vowel “o” (Figure 3). The first bi-dimensional representation (Figure 4) is resulted by using the fractional B-spline function of 0.4 order. Figure 5 illustrates the resulted scalogram using the Shannon function as mother wavelet.

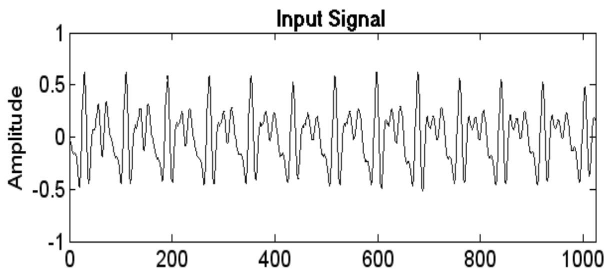

Figure 3. Voice signal

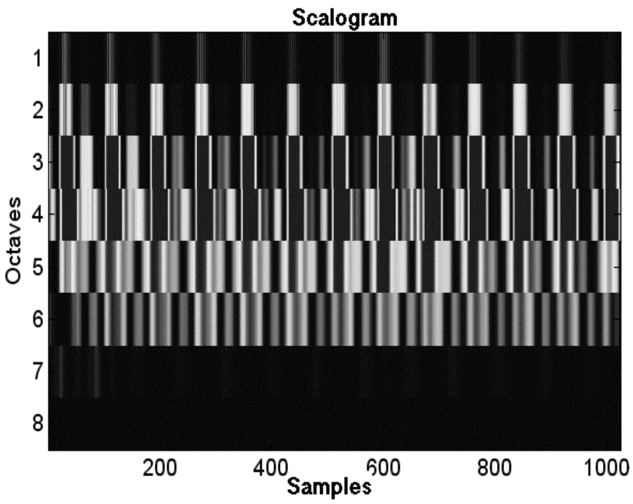

Figure 4. The resulted scalogram obtained using fractional B-splines wavelet of 0.4 order

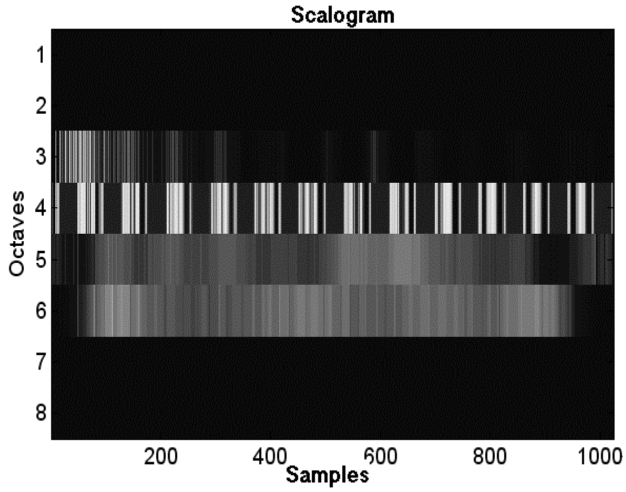

Figure 5. The resulted scalogram obtained using Shannon function as mother wavelet

Beyond the analysis of a number of different signals, using the Daubechies, Shannon and fractional B-splines wavelet families, further observations could be made: In the case of_using B-splines fractional functions there are several energy levels of different intensities which are distributed on several octaves. Many details revealed themselves to us, and even more information was made available besides that provided by other wavelet functions[8].

III. APPLICATIONS

Further, we describe two relevant applications based on the presented theoretical background.

a. A B-spline Wavelet Approach to Edge Detection

Singularities and irregular structures often carry essential information in a signal. Discontinuities in the intensity of an image indicate the presence of edges in the scene. Edge detection is an important task in image processing. It is a main tool in pattern recognition, image segmentation, and scene analysis.

Traditional edge detection methods are using gradients (*e.g.* weighted differences). Gradient methods work quite well for truly smooth images, but they are highly susceptible to the effects of noise. In order to reduce the effects of noise, other methods have been explored, such as the spectral methods and the wavelet methods. Since wavelets provide excellent representations of discontinuous signals, they have been used in modern edge detection.

The objective of this application is to explore the capability of B-spline wavelets to create improved edge detectors. Our idea is based on the fact that these functions act like fractional differentiators

Wavelet Transform Modulus Maxima (WTMM) developed by Mallat, carries the properties of sharp signal transitions and singularities. B-spline Wavelet Transform is written as a multiscale differential operator.

Figure 6. Original Image

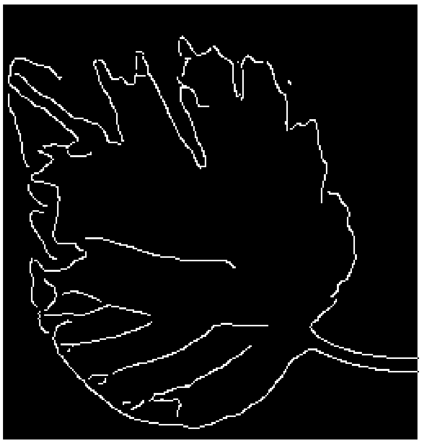

Figure 7. Classical Edge Detection

Figure 8. WTMM Edge Detection

Figure 8 displays the wavelet maxima where the modulus value is above a threshold. They suggest the location of edges where the image has large amplitude variations.

The magnitude of the wavelet transform modulus at the corresponding locations indicates the strength of the edges caused by sharp transitions.

The results are compared with the ones obtained by using a classical method (Figure 7).

b. Estimation of Glottal Closure Instants in Voiced Speech

The glottal source waveform is an important characteristic used in voice analysis, speaker emotional state identification, naturally sounding, speech synthesis and others related applications. It is known that the amplitude changes in the speech signal can be related with the glottal wave's phases.

From an anatomical point of view, the voice signal is generated by the glottis (the area where the vocal folds are situated) while the resonating part is formed by the vocal tract and the lips play the radiating role. The glottal wave generated by the voice source is quasi-periodic for the vocalized sounds. It has more abrupt closing slope and more slanting opening slope. The main excitation occurs at the instant of glottal closure.

In this paper, we consider the closed glottis instants detection problem within the framework of obtaining a energetically voice signal representation and searching for maxima in successive scales.

The difficulties of the task lie not only in the flexibility of vibration period widely varying, but also in the complexity of acoustic modification by the vocal track to the speech wave respond of the vibration.

We present some results obtained by applying our fractional B-spline wavelet filter bank and by applying Frobenius norm. The Frobenius norm of a complex-valued matrix is defined as the square root of the sum of the absolute squares of the element, and is also equal to the square root of the sum of the matrix singular values.

To verify the applicability of the octave-band decomposition mentioned above we have carried out a number of experiments with synthesized vocalized voice signals. The maxima can be retraced through the octaves starting from the octave containing the pitch frequency.

Figure 9.a. shows one of our test waveform. The sampling frequency is Fs=11025Hz and it represents the vowel "o". The signal is analyzed on 1024 samples frames in a time-scale distribution.

Figure 9.b. represents the 4^{th} octave of the wavelet decomposition computed by using the fractional B-spline wavelet function of 0.4 order. Figure 9.c. represents the Frobenius measure for Glottal Closure Instants detection.

The input signal is also mixed with a white noise - SNR=10dB (Figure 10.a) and is analyzed by using the same methods[10].

There have been also observed some advantages of using fractional B-splines wavelets, comparing to other common wavelets such as Daubechies or Shannon for this area of application.

It seems that the multiresolution structure is promising for our task: GCI detection. It can be seen that the Frobenius measure fails completely in noisy conditions and the multiresolution detector remains stable[6].

Further, we can apply this algorithm for more accurate spectrum analysis of speech signals and for more precise control of the excitation source for high-quality speech synthesis. It can also be applied to the improvement of speech detection or voiced/ unvoiced/ mixed/ silent classification algorithms.

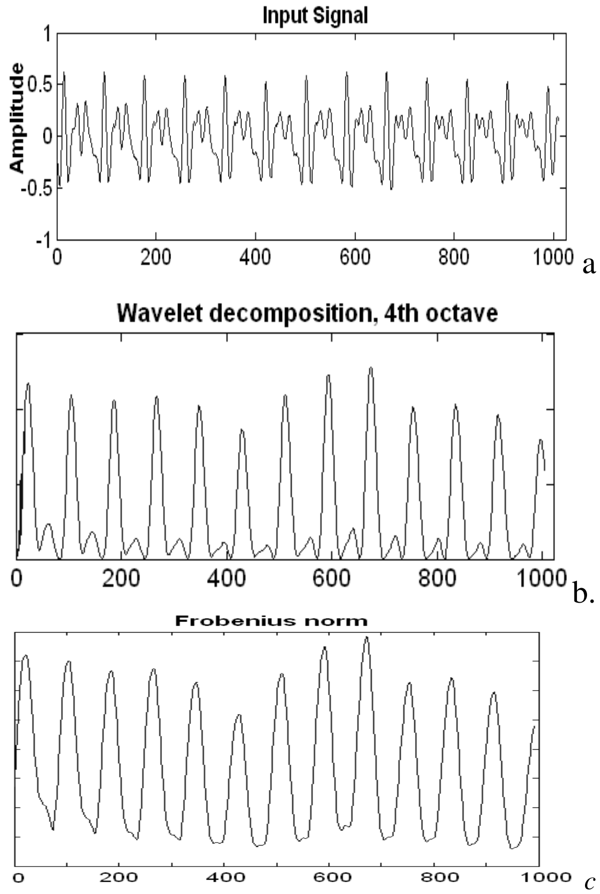

Figure 9. a. Voice signal;
b. WT, 4^{th} octave, obtained from 0.4 B-spline;
c. The Frobenius norm.

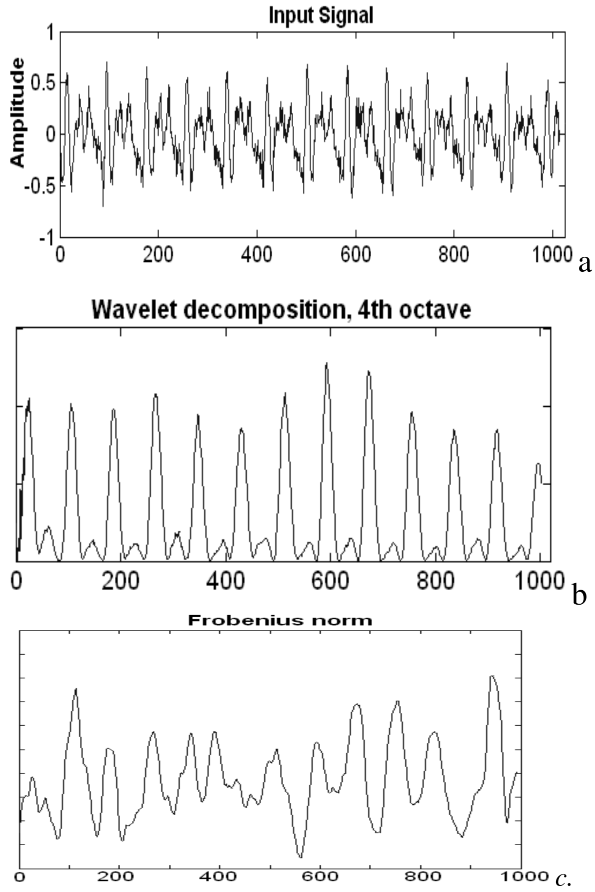

Figure 10. a. Voice signal + white noise;
b. WT, 4^{th} octave, obtained from 0.4 B-spline;
c. The Frobenius norm.

IV. CONCLUSIONS

Wavelets provide a unified framework for a number of techniques, which had been developed independently for various signal and image processing applications. Our purpose was to improve some of these techniques by using the B-spline functions.

We showed that the corresponding fractional and generalized B-spline wavelets behave like multiscale differentiation operators of fractional order. This is in contrast with classical wavelets whose differentiation order is constrained to be an integer. This property is well suited for image edge detection. Singularities detection can be carried out finding the local maxima of the fractional B-spline wavelet transform.

Determination of the instants of glottal closure from speech wave using wavelet transform is equivalent to finding a particular local modulus maxima pattern across several scales in the time-scale plane. When detecting the Glottal Closure Instants, all octave band decompositions showed superior performance in comparison with the covariance methods.

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