Volume 51, Number 4, 2010

ACTA TECHNICA NAPOCENSIS
Electronics and Telecommunications

NONLINEAR SYSTEM IDENTIFICATION USING ADAPTIVE VOLTERRA FILTERS FOR ECHO CANCELLING

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Abstract: **Adaptive nonlinear filtering plays an important role in audio signal processing and echo control. In this contribution a nonlinear system identification method is proposed. The setup is built using adaptive Volterra filters of second and third order with the same memory length. The adaptation is achieved using a Normalized Least Mean Square algorithm with a proper selected step-size parameter for convergence and stability. Functions with nonlinear characteristics are chosen to test the method. Performances are evaluated in terms of Echo Return Loss Enhancement and Mean Square Error. Results show that Volterra filters perform better than linear filters in weakly nonlinear structures.**
Keywords: *Adaptive algorithms, Volterra filters, nonlinear identification, error reduction*

I. INTRODUCTION

An important issue in communication systems is the identification and cancellation of undesired nonlinearities and delays that occur along the echo path [1]. A large variety of methods are used to reduce the consequences of these environmental causes. In order to cancel the echo from an acoustic path a system identification procedure must be made [2, 3].

For such signal processing applications, linear filters played an important role due to their simplicity in design and implementation. However, there are several practical situations where the considered system has a nonlinear behavior. The use of linear filters is inappropriate in this case, making adaptive nonlinear approaches desirable. An adaptive nonlinear filter is a signal processing device with a nonlinear output function of the input signal that adjusts its properties at each iteration using an adaptation algorithm [4].

In this paper a nonlinear identification method is presented using a second and a third order adaptive Volterra filter. Adaptation of the kernels is made using a Normalized Least Mean Square (NLMS) algorithm.

Volterra filters are often used for modeling weakly nonlinear systems like acoustic enclosures [5]. Nonlinear distortions are especially caused by a small loudspeaker operated at its power limit or an overdriven amplifier.

In real-life system identification models, it is presumed that the unknown system is ”sealed” and we refer to it as being a black-box system. Only the dependence of the output signal on the input signal is on record. Knowing this information, with the help of adaptive filters, we can determine the proposed system’s characteristics.

The objective of the proposed structure, based on polynomial filters, is to minimize the error signal so that we can gain a proper replica of the nonlinear system’s output. An adaptive linear NLMS filter is selected as reference and according to several simulation results we compare the performances of the proposed Volterra filters with the ones of the linear filter.

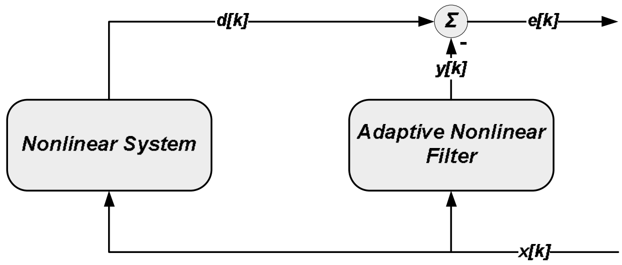

Figure 1. Nonlinear system identification

For simulations, the unknown nonlinear system is conceived using different nonlinear functions that resemble as close as possible, regarding their diagram, the real nonlinear model (ex: saturation characteristic of a speaker [6]).

The setup for nonlinear system identification is shown in *Figure 1*. The system presented seeks to minimize the power of the error signal *e[k]* by substracting the output of the Volterra adaptive filter, which is an estimation of the processed signal from *d[k]* (the nonlinear processing of the input signal *x[k]*).

The proposed structure is tested for a set of nonlinear functions and the comparison between the methods is achieved in terms of Echo Return Loss Enhancement (ERLE) and Mean Square Error (MSE).

This paper is organized as follows: In Section II, the Volterra model is presented, while in Section III, the NLMS adaptive algorithm for Volterra kernels update is described. The proposed structure and simulation results are discussed in Section IV and conclusions are drawn in Section V.

Manuscript received October 27, 2010; revised December 9, 2010

II. THE VOLTERRA MODEL

A discrete time-invariant and causal nonlinear system with memory can be represented by the Volterra series expansion as related in [4, 5]:

$$y(k)=h_0+\sum_{m_1=0}^{\infty}h_1(m_1)x(k-m_1)+\sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}h_2(m_1,m_2)x(k-m_1)x(k-m_2)+ \\ +...+\sum_{m_1=0}^{\infty}...\sum_{m_p=0}^{\infty}h_p(m_1,...,m_p)x(k-m_1)...x(k-m_p)+..., \quad (1)$$

where $x(k)$ is the discrete input signal and $y(k)$ is the output of the system.

The p-th order discrete Volterra kernel $h_p(m_1,\ldots,m_p)$ has a general symmetry: $h_i(m_1,\ldots,m_i) = h_i(m_i,\ldots,m_1)$ as in [2]. It means that for any $p!$ possible permutations of $m_1,\ldots,m_p$ the generalized p-th order Volterra kernel remains unchanged, i.e. we consider only coefficients with non-decreasing indices m_p: $m_p > m_{p-1}$.

The offset term h_0 is a constant which resembles to a DC component. In simulations no DC component is required.

This paper treats the third order Volterra filter ($p = 3$) and all the Volterra kernels have the same finite memory length M. Relation (1) can be written as:

$$y(k)=\sum_{m_1=0}^{M-1}h_1(m_1)x(k-m_1)+\sum_{m_1=0}^{M-1}\sum_{m_2=0}^{M-1}h_2(m_1,m_2)x(k-m_1)x(k-m_2)+ \\ +\sum_{m_1=0}^{M-1}\sum_{m_2=0}^{M-1}\sum_{m_3=0}^{M-1}h_3(m_1,m_2,m_3)x(k-m_1)x(k-m_2)x(k-m_3). \quad (2)$$

Let us define the vectors as follows as in [7]:

$$\mathbf{x}_1[k]=(x[k],x[k-1],...,x[k-M+1]);$$
$$\hat{\mathbf{h}}_1=(\hat{h}_1[0],\hat{h}_1[1],...\hat{h}_1[M-1]);$$
$$\mathbf{x}_2[k]=(x^2[k],x[k]x[k-1],...,x[k]x[k-M+1],x^2[k-1],x[k-1]x[k-2],... \\ ...,x^2[k-M+1]);$$
$$\hat{\mathbf{h}}_2=(\hat{h}_2[0,0],\hat{h}_2[0,1],...\hat{h}_2[0,M-1],\hat{h}_2[1,1],\hat{h}_2[1,2],... \\ ...\hat{h}_2[M-1,M-1]);$$
$$\mathbf{x}_3[k]=(x^3[k],x[k]^2x[k-1],...,x[k]^2x[k-M+1], \\ ,x^3[k-1],x[k-1]^2x[k-2],...,x^3[k-M+1]);$$
$$\hat{\mathbf{h}}_3=(\hat{h}_3[0,0,0],\hat{h}_3[0,0,1],...\hat{h}_3[0,0,M-1],\hat{h}_3[1,1,1],\hat{h}_3[1,1,2],... \\ ...\hat{h}_3[M-1,M-1,M-1]).$$

Relation (2) can be rewritten:

$$y[k]=\hat{\mathbf{h}}_1[m_1,k]\mathbf{x}_1{}^T[k]+\hat{\mathbf{h}}_2[m_1,m_2,k]\mathbf{x}_2{}^T[k]+\hat{\mathbf{h}}_3[m_1,m_2,m_3,k]\mathbf{x}_3{}^T[k]. \quad (3)$$

The number N_p of coefficients contained in the kernel vectors $\boldsymbol{h}_i$ depends on the order and memory of the filter as follows:

$$N_p=\frac{(M+p-1)!}{(M-1)!\,p!}, \quad (4)$$

where M is the memory length of the filter and p is the filter order. If symmetric kernels are considered of memory M, the first kernel consists of M coefficients, the second order kernel needs $M(M+1)/2$ coefficients, while the third order Volterra kernel needs computation of $M(M+1)(M+2)/6$ coefficients.

When using a third order Volterra adaptive filter in the system identification procedure, the main objective is to estimate the Volterra kernels as accurate as possible, in order to reconstruct the features of the nonlinear model. For nonlinear system identification using an adaptive Volterra filter the setup presented in *Figure 1* is used.

An adaptation algorithm minimizes at each iteration the expected value of the mean square error:

$$MSE = E[\{d(k)-y(k)\}^2]. \quad (5)$$

Adaptive filters can be used with the Volterra model for kernel estimation, due to the liniarity of the input-output relation according to the kernels. The nonlinearity is represented only by several multiplications of the delayed input signal.

For updating the Volterra kernels, two main classes of adaptation algorithms are used: Gradient Descent Algorithms (Least Mean Square, Normalized Least Mean Square, Projection Algorithm) and Least Square Algorithms (Recursive Least Square, Fast-Recursive Least Square, Fast Kalman) [2]. Different types of these algorithms are used for their convergence properties.

In our simulations we chose the NLMS adaptive algorithm because of its stability and fast convergence.

III. THE NLMS ALGORITHM USED FOR VOLTERRA SYSTEM IDENTIFICATION

The NLMS algorithm takes into account the variation of the input signal and selects a normalized step-size parameter.

To apply the NLMS algorithm for the Volterra kernel update, the input vector $\boldsymbol{x(k)}$ and the desired output of the nonlinear system $d(k)$ need to be known. Also an initialization of the Volterra kernel vectors is made.

The adaptation of the kernels using NLMS consists of three steps according to [8, 9]:

1) Compute the output of the filter:

$$y_i(k)=\sum_{i=1}^{p}\mathbf{h}_i^T\mathbf{x}_i(k); \quad (6)$$

2) Estimate the error:

$$e_i(k)=d(k)-y_i(k); \quad (7)$$

3) Update the Volterra kernels:

$$\mathbf{h}_i(k+1)=\mathbf{h}_i(k)+\mu_i e_i(k)\mathbf{x}_i(k), \qquad i = 1, 2,..., p. \quad (8)$$

The Volterra kernels depend on the sample's index k. This means that the kernels are time varying so the update is performed in every iteration. In this way we can keep trace of the signal's variation.

The error signal $e(k)$ is minimized in the mean-square sense as in relation (7).

The NLMS algorithm uses a normalized step-size μ_i:

$$\mu_i = \frac{\alpha_i}{\mathbf{x}_i^T(k)\mathbf{x}_i(k) + \varphi}. \qquad (9)$$

The constants α_i and φ are positive and should be selected appropriately. The value φ was introduced to prevent division by zero or a very small value when the norm $\boldsymbol{x}_i^T(k)\boldsymbol{x}_i(k)$ is small. The step-size parameter α_i controls the rate of convergence and stability. It should be selected properly in the range $0 < \alpha_i < 2$.

IV. PROPOSED STRUCTURE AND SIMULATION RESULTS

The proposed structure for nonlinear system identification is presented in *Figure 2*. Its aim is to compare the results of various adaptation filters with different orders. For experiments, the comparison is made starting from a linear filter with the tap-weight vector $\boldsymbol{h_1}$ and advancing to a second order Volterra filter that updates the kernel vectors $\boldsymbol{h_1}'$, $\boldsymbol{h_2}$ and a third order Volterra filter updating $\boldsymbol{h_1}''$, $\boldsymbol{h_2}'$ and $\boldsymbol{h_3}$. The adaptation of these kernels is made using the NLMS adaptation algorithm with different step-sizes.

As it can be noticed from the block structure, the Volterra kernel vectors are updated at each sample of the input audio signal.

After updating the linear, the second and the third order Volterra kernels, the output for every adaptive filter $y_1[k]$, $y_2[k]$, $y_3[k]$ is rated. The error signals $e_1[k]$, $e_2[k]$, $e_3[k]$ are computed for the proposed filters as being a difference between the output of the unknown nonlinear system and the output of the adaptation filters. At next iteration the Volterra kernels are updated again with respect to the previous kernel vectors, the resultant error, step-sizes and the current samples of the input signal.

The performance of the proposed setup is estimated in terms of ERLE and MSE:

- ERLE is defined as the attenuation of the echo signal as it passes through the signal path or i.e. the amount of echo discarded using echo cancellation. No nonlinear processing of the output signal is required for extra echo reduction. ERLE value is calculated in decibels at each iteration.
- MSE is a performance criterion that shapes the difference between an estimator and the quantity that is estimated. In our case the error is the amount by which the desired response of the unknown system differs from the output of the adaptive nonlinear filter. In this manner MSE measures the quality of the identification system in terms of variation of the estimated signal in comparison to the output of the nonlinear system.

Both parameters are expressed as:

$$ERLE = 10\log\frac{E\{y^2[k]\}}{E\{e^2[k]\}};$$
$$MSE = E\{e^2[k]\}, \qquad (10)$$

where $E\{z[k]\}$ denotes statistical expectation of the discrete variable $z[k]$:

$$E\{z^2[k]\} = \frac{1}{N}\sum_{k=1}^{N} z^2[k], \qquad (11)$$

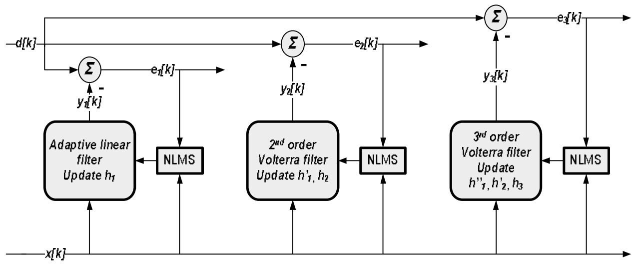

Figure 2. Proposed structure for nonlinear system identification

To compare the proposed methods for system identification, different mathematical functions are selected to describe the nonlinear behavior of the system. These functions have a nonlinear dependence on the input signal and present a weakly nonlinear characteristic so that the Volterra adaptive filter can be applied [10, 11].

In Matlab simulations, for all cases, the same filter memory is chosen, M is *25* samples and the step-size parameter is selected in the range $(0, 2)$.

There is no standard in selecting a suitable step-size parameter for the NLMS adaptive algorithm. The step-size parameters are selected by comparing different results for several values. We are looking for a fair convergence and stability of the filter. From experimental results a dependence between the step-size parameters is deduced. The ratio of consecutive index step-size parameters should be set to an order of magnitude.

The excitation $x[k]$ is an audio signal, sampled at *11.025 kHz*, with amplitude in the range of $[-1, 1]$.

Evolution of ERLE and MSE is studied for the following nonlinear dependence of the desired signal on the input signal:

a) $$d(x[k]) = \frac{ax[k]}{\sqrt{a^2 + x^2[k]}} \qquad (12)$$

This function models a soft clipping nonlinear saturation characteristic specific to a loudspeaker [6], with horizontal asymptotes at $-a$ and a. The parameter a is set to *5* and the graph is presented in *Figure 3*.

The performances of the proposed model are depicted in *Figures 4* and *5*.

As it can be seen, the obtained ERLE in *Figure 4* reaches a steady state of almost 40 dB for the third order Volterra adaptive filter. Also in terms of ERLE we can observe a gain of approximately 20 dB for the third order adaptive Volterra filter in comparison to the linear filter and a difference of 10 dB between the third and the second order Volterra filters. Yet, Volterra filters converge faster to a steady state than the linear filter.

In *Figure 5* an important reduction of the MSE value computed for the third order Volterra filter can be noticed, while the MSE of the linear adaptive filter has a peak at $3.25 \cdot 10^{-4}$.

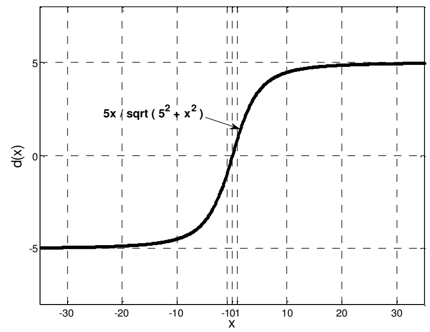

Figure 3. The chart of function (12)

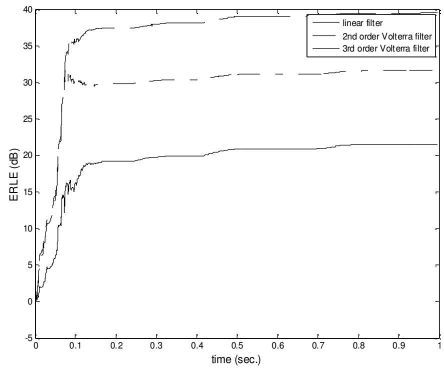

Figure 4. Estimated ERLE for function (12)

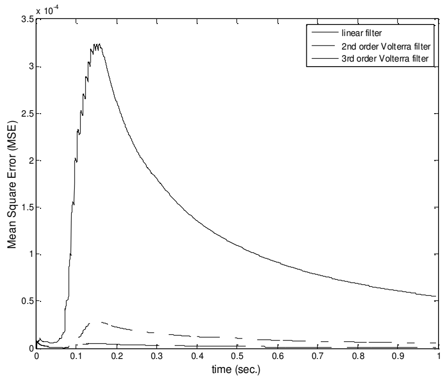

Figure 5. Evolution of MSE for function (12)

b) $d(x[k]) = 1 - e^{-x[k]}$ (13)

The second function depicted in *Figure 6* presents a nonlinearity similar to the first function, but the horizontal asymptote is set to 1. The performances of the proposed model for this type of nonlinearity are depicted in *Figure 7* and *Figure 8*.

ERLE reaches a steady state of 30 dB for the third order adaptive Volterra filter, while the linear filter converges to a steady state of only 15 dB. MSE values in this case are also acceptable for the adaptive Volterra filters in comparison to the MSE values of the linear NLMS adaptive filter.

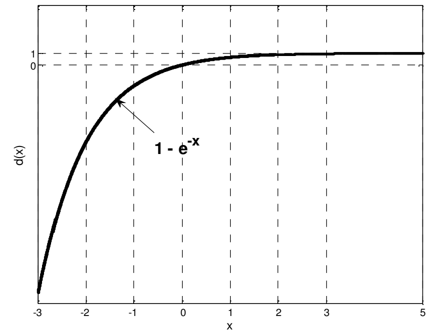

Figure 6. The chart of function (13)

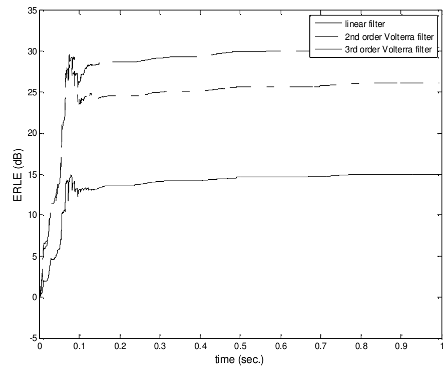

Figure 7. Estimated ERLE for function (13)

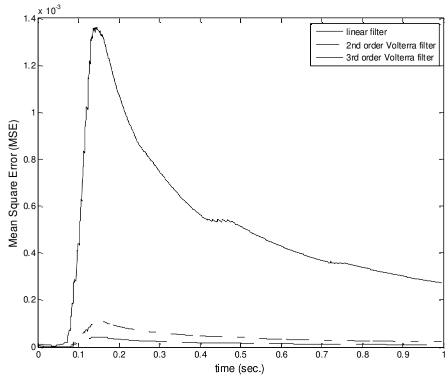

Figure 8. Evolution of MSE for function (13)

In the next simulations, two third order polynomial functions with different coefficients are studied. We are looking for a relationship between the variation of the polynomial factors and the evolution of ERLE and MSE.

In *Figure 9* we can see the following two functions drawn on the same graph:

c) $d(x[k]) = x[k] + x^2[k] + x^3[k]$ (14)

d) $d(x[k]) = 6x[k] + 3x^2[k] + x^3[k]$ (15)

Equation (14) stands for a third order polynomial function with all coefficients set to *1*. *Figure 10* compares the ERLE values of the adaptive filters obtained for the desired polynomial function (14). Also the MSE in this case is indicated in *Figure 11*. The estimated ERLE decreases in contrast with the previous simulations, reaching an echo attenuation of near 24 dB for the third order Volterra filter. A relevant reduction of MSE can be noticed for the second and the third order Volterra adaptive filter against the MSE values of the NLMS linear filter.

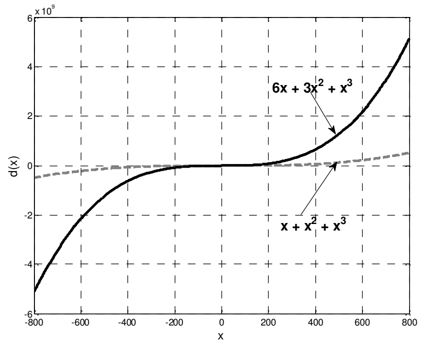

Figure 9. The chart of polynomial functions (14) and (15)

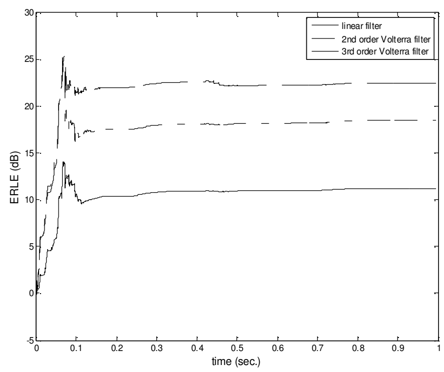

Figure 10. Estimated ERLE for function (14)

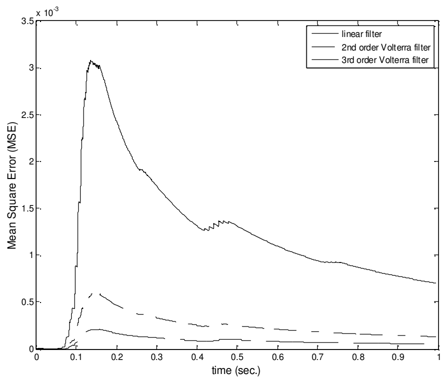

Figure 11. Evolution of MSE for function (14)

In relation (15) the third order polynomial function's coefficients are changed. We have an interest in how the new parameters affect the performances of the adaptive filters.The performances of the system identifier for the proposed polynomial function (15) are depicted in *Figures 12* and *13.* As a remark, an ERLE of near 30 dB is obtained for a third order adaptive Volterra filter in comparison to the previous function for which an ERLE of 23 dB was obtained for the same filter properties. In terms of MSE it can be noticed that the second and the third order adaptive Volterra filters perform better than the NLMS linear filter.

In the case of polynomial functions, a slight dependence of ERLE on the polynomial coefficients is marked. The ERLE quantity increases as the coefficient of the linear member has a higher value in comparison to the nonlinear coefficients, thus the slope enhances. This remark strengthens the usability of adaptive Volterra filters in weakly nonlinear systems.

Figure 14 depicts the output of the nonlinear system *d[n]* and the contribution of the linear and the Volterra adaptive filters to identify *d[n]*. The nonlinear system has a third order polynomial dependence on the input signal as presented in equation (15). We mark a superior replica of the nonlinear system's output created by the Volterra filters in comparison to the replica produced by the linear NLMS adaptive filter.

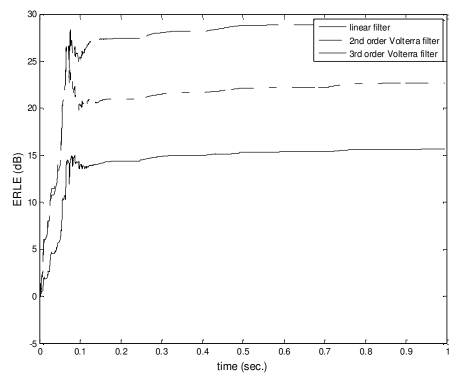

Figure 12. Estimated ERLE for function (15)

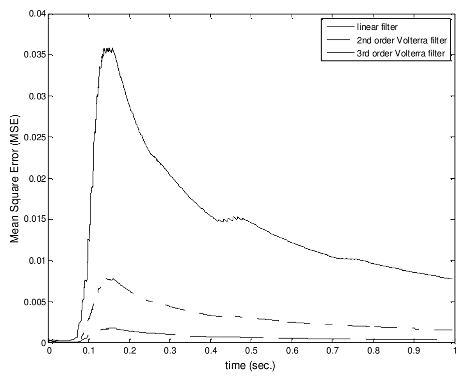

Figure 13. Evolution of MSE for function (15)

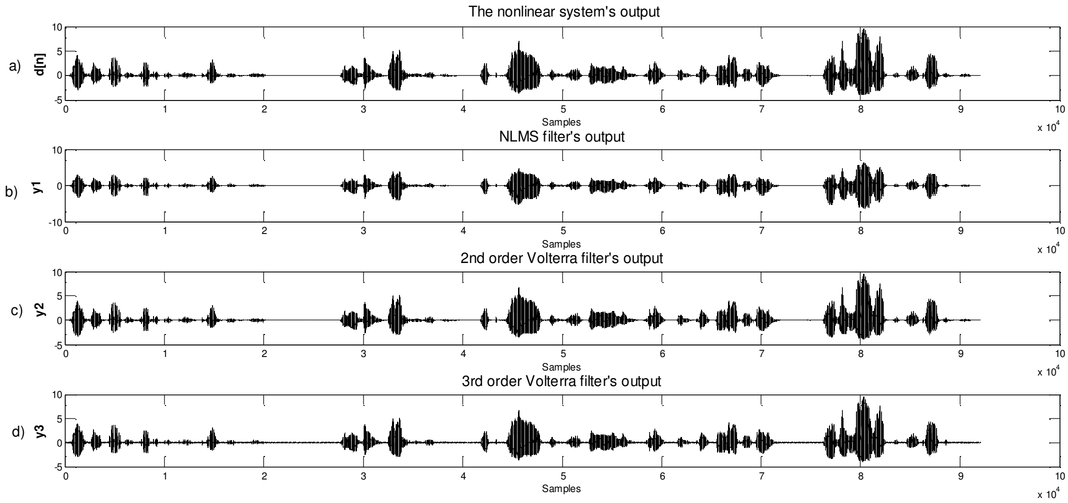

Figure 14. a) The nonlinear system output, b) the linear filter output, c) the second order Volterra filter output, d) the third order Volterra filter output

V. CONCLUSION

The paper proposes a nonlinear system identification method using the second and third order adaptive Volterra filters with the same memory length. The adaptation was achieved with the help of an NLMS algorithm; the step-size parameter was selected properly in the range (0, 2).

The nonlinear adaptive filters' performances were evaluated in terms of ERLE and MSE and compared with the performances of a linear NLMS filter. The experimental results showed that the third order adaptive Volterra filter performed better than the linear filter and the second order adaptive Volterra filter for each simulation.

Four functions were proposed to describe the nonlinear system. It was noticed that if the nonlinear level of the unknown system grows, the computed values of ERLE decrease, though for the second and third order adaptive Volterra filters, a reliable replica of the nonlinear system's output can be obtained. The first two functions describe weak nonlinearities. In simulations using Volterra filters, the gained ERLE has an acceptable value of approximately 40 dB. In the case of acoustic systems, echoes with ERLE of 40 dB have a small probability of being perceived by the listener. If the nonlinearity order changes and we use polynomial models like the last two functions, the value of ERLE decreases below 40 dB. However, the computed ERLE strongly depends on the coefficients of the polynomial input-output function. As the value of smaller degree coefficients grows, we get a higher steady state value for ERLE.

The disadvantage of the adaptive Volterra filters performances consists in the computational complexity required to implement these filters.

Further work will be dedicated to the identification of real audio systems and echo cancellation using audio signals measured in a Loudspeaker-Enclosure-Microphone setup.

ACKNOWLEDGMENT

This paper was supported by the project "Doctoral studies in engineering sciences for developing the knowledge based society-SIDOC" contract no. POSDRU/88/1.5/S/60078, project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013 and the Romanian National University Research Council under Grant ID 1057.

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