DECISIONS TOOLS IN FINANCIAL TIME SERIES PROCESSING: A STOCHASTIC DOMINANCE PERSPECTIVE

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Abstract: The aim of this paper is to provide an implementation of a stochastic dominance algorithm that establish which of the two distributions is preferred by individuals who have an aversive risk profile. Although, this topic related to decision making theory could be applied in almost every filed where one has to choose between empirical distributions, this study is performed on financial markets across Europe. The focus is to emphasis the impact of an important event (or shock) which occurs in the evolution of a time series. In the case of financial market such an event has been considered the latest financial crisis. The relevance of the study consist in that, it offers a different perspective for investors when choosing between the different financial assets. Our approach indicates that using a generalized stochastic dominance concept, the effects of an important event such the financial crisis is caught also in the attitude of individuals in regards to risk. This approach together with the Meyer algorithm for generalized stochastic dominance is a useful tool in risk aversion analysis when choosing among distributions. Its applicability could be also extended to other fields where a distinction between empirical distributions of time series (signals) is necessary.

Keywords: *stochastic dominance, utility function, risk aversion, empirical distributions*

I. INTRODUCTION

One of today's most important things is the possibility to choose – we all have to make choices and we all have to decide – what is the best for us. Theoretically speaking, the best decision is the one that bears the least amount of risk. From the statistical viewpoint, stochastic dominance resolves risky decisions while making the weakest possible assumptions. Generally, stochastic dominance assumes that an individual is an expected utility maximizer. Additionally, concerning the definition and the main idea of stochastic dominance, further assumptions relative to preference for wealth and risk aversion should be taken into consideration.

The topic presented in this approach is a very challenging one since it allows choosing between empirical distributions associated with a two data sets. In practice, there are many fields where measuring systems are recording the data without doing some processing based on the fact that the characteristics or properties of data are often unknown. The issue which could occur is related to the principles and the approaches which can arise in some necessary further processing of data. If these further procedures involve a selection between two signals (or time series) one possible good approach is the stochastic dominance. The idea is to make a proper choice based on a minimal set of assumption for the analysed data set [1]. Therefore the decision is relaying on the structure of empirical distribution of the used signal (time series). The reason why the choosing between distributions is not an easy task is related to the random behaviour of the analyzed signals. Thus, when an individual has to choose between two random variables having quite similar distributions then he or she has to deal with a specific amount of uncertainty [1]. Due to the fact that this uncertainty can be evaluated with complex measures, the decision making process is sometimes rather difficult. In practice are known many measures for risk and uncertainly, including the econometrical and statistical approaches. The stochastic dominance is one good tool at hand, in the sense that it can be used also for classifying the distribution of random variables [1]. It is also related to utility function concepts, which involves that each individual is defined by its utility function and risk aversion function. Accordingly to each risk profile, a decision could be taken. These concepts are the core of several stochastic dominance application and they are used also in our implementation.

In this study, we analyze the effect of generalized (first and second order) stochastic dominance changes in a returns distribution of some financial time series. For the sake of simplicity, there is not stressed a clear distinction between signals, time series and random variables, since they are used for the same concept: the price (or return) evolution of a stock index. As experimental data, we chose eight important European Stock Indexes as the input random variables for the proposed approach. Four of these indexes are representative for West Europe and the rest of them are representative for European emerging markets.

Therefore, the current study is focused on the main stock index for each analyzed country in order to capture the effect of financial crisis over investor decisions. Since a stock index is considered to be a global and a representative

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measure of a country's economy, the behavior of investors will be correlated to the evolution (also in terms of preferences) of these indexes. The idea of using stochastic dominance in general form is not so old and it was firstly presented by Meyer [17]. Hence, the implemented measure for stochastic dominance is considered to be an interesting tool for a good analysis of the recent financial crisis impact on investors' preferences and decisions.

Stochastic dominance is a subject which captures the interest of researches since long time ago in past. Nevertheless, the majority of the researches were done in the last three decades. The first contribution regarding the optimal behaviour of risk averse following stochastic dominance changes in returns distribution was realized by some researchers in 1970 [18]. The research was continued by Meyer and others mathematicians interested in classifying different distribution without knowing their theoretical laws as stated in [17], [9] and [10]. One small weakness of these studies is related to the fact they were not able to analyze the effects of first and second-degree stochastic dominance on panel data with more than two distributions.

Many mathematicians and economists studied this concept for classifying different distributions having associated a certain risk. Here are included elements like prospects lotteries [12], assets and some the annual production in agriculture [15]. Moreover, there were some trials for bounds identification of a priori specification of risk aversion coefficient and found that "strongly risk averse" range might not be too high [16]. In another paper, McCarl realized an empirical examination on risk aversion coefficients by using generalized stochastic dominance [15]. His results show that non-dominance regions are composed of smaller dominance regions. Levy [13] discussed in a paper the first, second and third – degree stochastic dominance rules regarding portfolios with and without the riskless asset, nonlinear utility theory, random variables, respectively the relationship between stochastic dominance rules and risk definition.

Recently, it was realized a procedure to estimate the critical values of Kolmogorov-Smirnov test of stochastic dominance arbitrary order in a general prospect case [14]. They obtained, in their research, the asymptotic distribution of mentioned test for stochastic dominance of various types, respectively they demonstrated that the consistency allows generic dependence of prospects having non independent and identical distributed observations.

The Kolmogorov-Smirnov type tests have been used also to analyze an arbitrary degree of stochastic dominance [2]. They also used a lot of simulation and bootstrap methods in order to conduct inference for different degrees of stochastic dominance beyond the first order.

There are also several theoretical papers in which are presented key concepts very useful in creation of measures and indictors for stochastic dominances. In his paper, Davidson [8], who presented a theoretical approach, it has been analyzed the relationship between stochastic dominance and welfare, stochastic dominance and poverty, respectively stochastic dominance and inequality. The study results show that new stochastic orders can be derived from others unconditional distributions.

Another topic which involves the usage of stochastic dominance is related to estimation of the Lorenz curves [7]. There are approaches for estimating these curves, which are fundamental tools for stochastic dominance, respectively to combine empirical estimations with a robust estimation of the upper tail distribution by using the household disposable incomes from United Kingdom during 1981 year.

In a recent study [4] which is presenting an interesting approach, the authors where trying to measure the elitism by using stochastic dominance. The study was focused on two fields. The first field answered to the question of what is the most effective way for increase the welfare of a society. This study was focused on the comparison of 17 countries using income date. In the second field, they measured the scientific performance of academics and institutions in terms of research. This application was done on the journals from the Journal Economic Literature by departments. Their results show that the more unequal and the more efficient is the distribution, the higher it is ranked.

Based on the presented ideas and approaches the rest of the paper is organized in three main sections. The next section presents theoretical aspect related to stochastic dominance. Then we presented the methodology used and also the implemented approach. Afterward, there are presented some characteristics of the used data and the experimental results. In the end some conclusion are shown and we proposed some further developments.

II. STOCHASTIC DOMINANCE AND ITS APPLICATIONS

Generally speaking, the distribution of the return's assets are in general quite complex and is often hard to choose between them form a certain risk profile. There are many criteria to classify the dominance of a distribution over another. From this point of view, this study is relaying on a generalized order of dominance criterion. Theoretically, there is possible to have any order of dominance, but in practice, the characteristics of distribution will lead sometimes to an impossibility of stating the dominance order of one asset to another. Thus, there are defined the first order and the second order stochastic dominance, which could be frequently encountered in real applications. Hence, in the following parts there are presented the basic concepts related to these types of dominance.

In stochastic dominance theory, a random variable w_1 dominates stochastically of first-order the random variable w_2 if:

$$P[w_1 > w] \geq P[w_2 > w] \text{, for any } w \in \mathbf{R}. \tag{1}$$

If the distributions for these random variables are taken into consideration, then the last expression is equivalent with: $F_{w_1}(w) \leq F_{w_2}(w)$, for any w. The functions $F(.)$ are the cumulative distributions of the stochastic variables w and the functions $f(.)$ are the distribution functions. ***First-order stochastic dominance*** also implies that:

$$E\{w_1\} \geq E\{w_2\}. \tag{2}$$

The equation (2) is equivalent with:

$$\int_{\Omega} w f_{w1}(w)dw \leq \int_{\Omega} w f_{w2}(w)dw, \quad (3)$$

where $\Omega = [w_{\min}; w_{MAX}]$ is the values range for w_1 and w_2. If this equality is verified for the both expressions of the expectations, one can state that:

$$E_{w_1} \geq E_{w_2} \Leftrightarrow \int_{\Omega} F_{w_2}(w)dw \leq \int_{\Omega} F_{w_1}(w)dw \quad (4)$$

However, if w_1 stochastically dominates w_2 of first-order, then $F_{w1}(w) \leq F_{w2}(w)$, which is in contradiction with the previous inequality. Therefore, the stochastic dominance of w_1 over w_2, implies that $E_{w_1} \geq E_{w_2}$.

In almost every situation, an individual who prefers more than less, will have an "associated" utility function $U(w)$ having the property $U'(w) > 0$. Furthermore, the utility function $U(.)$ is a monotonic increasing function. A detailed presentation on the utility functions and their properties can be found in [6]. If w_1 first-order stochastically dominates w_2 then:

$$E\{U(w_1)\} > E\{U(w_2)\} \quad (5)$$

For the described random variables w_1 and w_2, which are characterized by the repartition function $F_{w_1}(w)$ and $F_{w_2}(w)$, the graphical representation of first order stochastic dominance is presented in Figure 1.

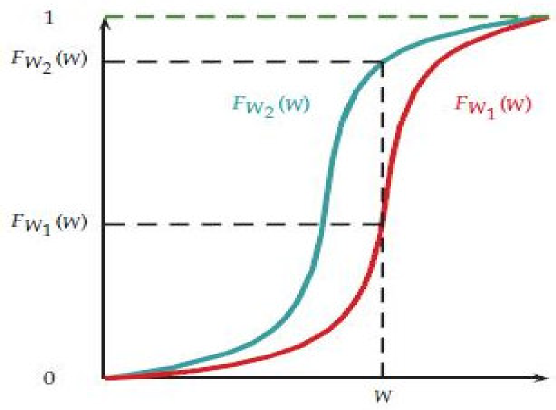

Figure 1. Graphical representation of first order stochastic dominance.

In practice, it is rare the case when a distribution dominates at first order level another distribution. This is applicable especially in financial market filed, but it is also true for signals (times series) from other areas. In order to classify the distributions from the risk point of view a higher order of stochastically dominance is needed. In its abstract form the ***second order stochastic dominance*** could be expressed in the following form, where w_1 dominates stochastically w_2:

$$\int_{w_{\min}}^{a<w_{MAX}} P[w_1 > w]dw \geq \int_{w_{\min}}^{a<w_{MAX}} P[w_2 > w]dw, \quad (6)$$

for any $a \in \Omega$.

The last expression can be written more intuitively, using the cumulative distribution function associated to each random variable (e.g. – an asset return) as:

$$\int_{w_m}^{a} [F_{w_1}(w) - F_{w_2}(w)]dw \leq 0, \text{ for any } a \in \Omega \quad (7)$$

Hence, if w_1 dominates stochastically w_2 at second order level, then w_1 dominates stochastically w_2 at first order level too.

When the utility functions are implied, the previous statement is equivalent with:

$$E\{U(w_1)\} \geq E\{U(w_2)\} \quad (8)$$

where $U(.)$ is any increasing utility function. Going forward with this approach, the usage of utility function implies also the risk aversion concept. More details about risk aversion topic could be found in [9], but in order to have second order dominance between two variables, then $U'(w) > 0$ and $U''(w) < 0$.

The stochastic dominance can easily be spread over more sub-profiles of the repartition functions, since it can be justified the fact that the successive derivatives of the utility alternate: $(-1)^{i+1}U^{(i)}(w) \geq 0$ for $i = 1,2,...$. Therefore it would be possible to state for each interval what degree of dominance exists between the distributions.

Similar with the graphical representation for first order stochastic dominance, we present a figure which illustrate a situation for the two repartition function $F_{w_1}(w)$ and $F_{w_2}(w)$, related to second order stochastic dominance.

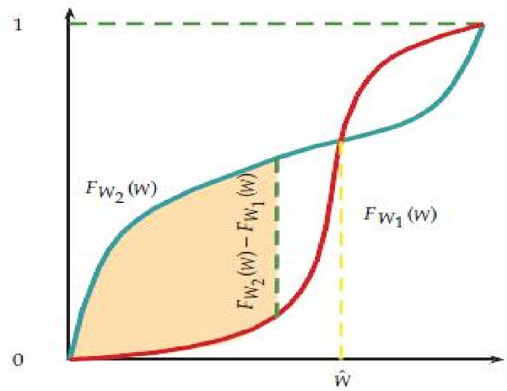

Figure 2. Graphical representation for second order stochastic dominance.

Even if it is more difficult in terms of interpretation,

higher-order dominances have an important role in the analysis of risky decisions. First of all, some researchers [3] proved that in general the stochastic dominance can be presented by the "*generalized criterion of Fishburn*" (GCF), also known as "the rule for partial variance of the mean" [11]. If we use the same variables w_1 and w_2 for which the distributions functions are *f*(*w*) and *g*(*w*), when integrating twice by parts and having the notation $F(w_{\min}) = 0$, one can get:

$$GCF_f(w,2) = \int_{w_{\min}}^{w} (s-w)^2 dF(s) = 2\int_{w_{\min}}^{w_1}\int_{w_{\min}}^{w_2} F(s)dsdy \quad (9)$$

Then: $CGF_f(w,2) - CGF_g(w,2) \geq 0$. (10)

This result allows for stating the other definition for *third-order stochastic dominance*, which would be equivalent with the fulfillment of the following conditions:

$$E\{g(w)\} - E\{f(w)\} \geq 0 \text{ and } CGF_f(w,2) - CGF_g(w,2) \geq 0 \quad (11)$$

Reading again these fundamental concepts, the conclusion is straightforward: if there is *n*-order stochastic dominance, there is also subsequent-order stochastic dominance (*n*+1, *n*+2, ...). Moreover, we can also note that the higher the order of the dominance is, the more we have to evaluate an important number of integrals. The process of evaluating the integrals is assuring that a certain order of dominance is achievable. Hence, if a higher order for integrals could be evaluated, then a higher order of dominance could be established.

III. METHODOLOGY

In order to demonstrate the usefulness of the stochastic dominance in the standard corpus of decision theory, here it will be described one standard application. For many other applications, one can read [19]. An important application of previous concepts could be found in signal processing and especially in time series analysis. The presented application is related to investment decisions in stock markets from Europe. Hence, the application presented here is strongly related to stochastic dominance Meyer's Algorithm applied in stock markets. Usually, an investor has to decide between two prospects (financial assets), *X* and *Y*, whose revenues or returns are randomly distributed. If the price (value of a stock index) for a specific day (e.g. – let's say day *t*) is noted P_t, then the return is defined as: $R_t = \ln(P_t / P_{t-1})$. The investor will choose or will prefer the asset *X* instead of *Y* if (the inequality (8) is fulfilled) :

$$\int_0^A U(w)dF_X(w) > \int_0^A U(w)dF_Y(w), \quad (12)$$

where *X* and *Y* are considered random variables defined on the interval [0; A]. Based on the utility function approaches, it is not very difficult to demonstrate from a risk prospective that $U'(w) > 0$, which simply means that any individual prefers more than less. Basically, if this property is verified, it is obtained the equivalent form of (12):

$$E\{U(X)\} > E\{U(Y)\} \Leftrightarrow \int_0^A U'(w)[F_Y(w) - F_X(w)]dw > 0 \quad (13)$$

It is known also [15] that from an economical point of view the utility curve is characterized by its risk aversion function defined as:

$$-\frac{U''(w)}{U'(X)} = R(w) \quad (14)$$

Often in the literature it is used the concept of risk aversion coefficient, due to the simplification $R(w) = c$.

The choice for a preferred asset could be made by an investor for whom the utility function $U(w) = R(w)$ verifies the following constraint [15]:

$$R_1(w) \leq R(w) \leq R_2(w) \quad (15)$$

Therefore, the integral presented in equation (13) has its maximum value if the following expression states true:

$$\int_0^A U'(w)[F_Y(w) - F_X(w)]dw < 0 \quad (16)$$

Thus, any investor for whom the utility function verifies constraint (15) will choose the prospect *Y* rather than *X*. Hence, $E\{U(Y)\} > E\{U(X)\}$ means that Y dominates *X*, as stated also in equation (8).

In order to write the algorithm used to take the correct decision, it is important to notice that the risk aversion coefficient describes an ordinary differential equation of second-order , as expressed in equation (14). Thus, for this kind of equation the initial condition – i.e.: $U'(0)$ needs to be known. On the other hand, a utility function is only defined by an infinite continuously and derivable transformation (function). In other words, the two functions $U(.)$ and $\tilde{U}(.) = aU(.) + b$ describe the same investor's preference. Since $\tilde{U}'(w) = aU'(w)$, it is possible to normalize the derivatives in a such a way that $U'(0) = 1$. Thus, the notation $V(w) = U'(w)$ is used.

Therefore, the described algorithm consists of two steps, as presented below. This algorithm is presented in a similar way also in [15]:

The first step consists in evaluation for the expression of *J**, by knowing that $U'(0) = 1$:

$$\max_{R_1(w) \leq R(w) \leq R_2(w)} \left\{ \int_0^A U'(w)[F_Y(w) - F_X(w)]dw \middle| U''(w) = -R(w)U'(w) \right\} \quad (17)$$

The second step establishes which prospect (asset) is preferred accordingly with the value of *J**. Thus if *J** < 0 one will choose *Y* as a preferred asset (prospect).

The integral mentioned above, in the first step, does not appear to be an integral of optimal control. Therefore is needed another form this integral and also a resort to make a change of variable (i. e. – $U'(w) = V(w)$. Consequently, the integral will become:

$$\int_0^A V(w)[F_Y(w) - F_X(w)]dw \Big| V'(w) = -R(w)V(w) \quad (18)$$

In order to maximize the integral describe in equation (17), there are needed the optimality conditions. The optimality conditions will lead to an achievement of the result, which could show which prospect is preferable in the detriment of the other one. The algorithm that finds the optimality condition is based on the Hamiltonian operator:

$$\begin{aligned} \mathrm{H} = V(x)[F_Y(x) - F_X(x)] - \psi(x)[R(x)V(x)](x) \\ + \lambda_1(x)[R(x) - R_1(x)] - \lambda_2(x)[R(x) - R_2(x)] \end{aligned} \quad (19)$$

Therefore, this transformation applied to the equation (17) is leading to a rewriting of the integrals as it follows:

$$R(w) = \begin{cases} R_1(w), \text{ if } \int_w^A [F_Y(s) - F_X(s)]U'(s)ds > 0 \\ R_2(w), \text{ if } \int_w^A [F_Y(s) - F_X(s)]U'(s)ds \le 0 \end{cases} \quad (20)$$

Hence, if the function $R(w)$ is computed in an optimal way, then the rest of the algorithm consists only in evaluation of J^*. Depending on its value, the dominance of one asset (distribution) over another is determined.

We implemented the described algorithm in C# .NET programming language. Since the used time series are grouped in arrays and matrices, the software's usefulness is evident. Before presenting a full description of the algorithm, we mention that each prices series for each analyzed index has been transformed in returns. The way how the returns are computed was already presented at the beginning of this section. Further, the return series has been transformed in histograms (distributions) in order to build up the probability repartition functions. Since the length of each data set is sufficient for computing the probability distribution function, we implemented an algorithm for automatic scaling of each data set accordingly to a predefined number of histograms bins. These computed repartition functions are then applied as inputs to the Meyer algorithm.

The difficulty in the implementation of Meyer's algorithm lies in the fact that the function is defined by a forward integral and not by a backward integral as the usual integrals. For a better comprehension of implementing Meyer's algorithm, starting from empirical data, that we have $F_X(.)$ and $F_Y(.)$, we defined two constant functions in each discrete time interval. The functions are defined over one partition such as: $0 = w_0, \ldots, w_i, \ldots, w_N = A$ and $w_{i+1} - w_i = h$, where *h* is a small constant and *N* is the size of analyzed data. This parameter, needs to have an acceptable value from the computational point of view, which can lead to achieve a good accuracy for the approximation of the integral obtained using a step with this (specified) value as it is described in a recent work related to optimal control [A4].

Then, the expression $F_Y(w) - F_X(w)$ has to be evaluated. Considering that $F_X - F_Y \ge 0$ in the interval $[w_{N-1}; w_N]$ and knowing that $U'(.) > 0$, then the following integral is positive:

$$\int_{w_{N-1}}^{w_N = A} [F_Y(w) - F_X(w)]U'(w)dw > 0 \quad (21)$$

On this interval, $U'(w)$ verifies the differential equation $U''(w) = -R_1(w)U'(w)$, whose final solution for $w \in [w_{N-1}; w_N]$ is:

$$U'(w) = U'(w_N)e^{\int_w^{w_N} R_1(s)ds} \quad (22)$$

Although $U'(w_N)$ was not known from the beginning of algorithm, it is not very importance and it can be evaluate it arbitrarily. The contribution of the interval $w \in [w_{N-1}; w_N]$ for the optimal value of the target objective function (J^*) is given by :

$$J_1^* = \int_{w_{N-1}}^{w_N = A} [F_Y(w) - F_X(w)]U'(w_N)e^{\int_w^{w_N} R_1(s)ds} dw \quad (23)$$

The next step in the algorithm is w_{N-2}, where it is also possible to calculate $U'(w_N)$ by using the discretized form of differential equation which defines $U'(.)$, as it follows:

$$U'(w_{N-1}) = U'(w_{N-2})(1 - R_j(w_{N-2})h) \Rightarrow$$
$$U'(w_{N-2}) = \frac{U'(w_{N-1})}{1 - R_j(w_{N-2})h} = \frac{U'(w_N)e^{\int_{w_{N-1}}^{I_N} R_1(s)ds}}{1 - R_j(w_{N-2})h} \quad (24)$$

This allows the evaluation of the new integral:

$$J_2^* = [F_Y(w_{N-2}) - F_X(w_{N-2})] = \frac{U'(w_N)e^{\int_{x_{wN-1}}^{I_N} R_1(s)ds}}{1 - R_j(w_{N-2})h} + J_1^* \quad (25)$$

At this point, one can make a choice for $R(w)$ based on the value of J_2^*. Thus, if $J_2^* > 0$ then $R(w_{N-2}) = R_1(w_{N-2})$, else if $J_2^* \le 0$, then $R(w_{N-2}) = R_2(w_{N-2})$. Moreover, it is possible to calculate $U'(w_{N-3})$ and reiterate the operation until it is reached the step 0, for $U'(0)$. This allows deciding whether *X* or *Y* is dominant, for the two vectors data set, according to the presented approach. Then, for each possible combination of two distributions from the complete set, the proposed approach has been used.

The presented methodology can be resumed in a few algorithmic steps, which can be implemented in any programming language in order to test the performances of the proposed approach on different data sets too. Therefore, the main inputs of the algorithm are the raw data w (for each index it is used a data set and the computation are using at one time only two series) and the bounds for risk aversion coefficients R_i, where i=1,2. Then, the following sequences are:

Set the *number of bins* or points (N) in order to build the histograms for the analyzed series;
Set the *initial value* for $J^*=J_1$;
For i=1;N
evaluate $U'(w_{N-i})$;
compute J_i^* by using equation (25) and based on its sign compute R_{N-1} ;
End for;
Decide if exists dominance for the analyzed random variables based on the sign of J_N^*.

The presented algorithm is "packed" in some specific routines in order to be applied for all pairs of data sets which have to be analyzed. In the end a triangular "matrix" is summarizing the results obtained by these computations.

IV. USED DATA

In order to test the change in preferences for a certain stock market, we use daily closing data of eight indices from developed stock markets [CAC40 (France), DAX (Germany), FTSE 100 (England), and SWISS (Switzerland)], respectively from emerging markets [BET (Romania), PX (Czech Republic), SOFIX (Bulgaria) and WIG (Poland)]. Analyzed time begins with the first available listing day of each index (1997 – BET; 1987 – CAC40; 1987 – DAX; 1978 – FTSE; 1993 – PX; 2000 – SOFX; 1988 – SWISS; 1991 – WIG) and ends on June 27, 2012. All closing values of the indices are collected from Datastream database, respectively are denominated in local currency.

We analyzed the stochastic dominance before and after the appearance of financial crisis. Thus, we divided the analyzed period in two subsamples. The breaking point was considered the first day of decreasing index after the registered high value of the index: August 25, 2007 (BET); June 4, 2007 (CAC40); July 16, 2007 (DAX); June 18, 2007 (FTSE); October 15, 2007 (PX); October 22, 2007 (SOFX); June 4, 2007 (SWISS); June 7, 2007 (WIG).

The main descriptive statistics of daily return series corresponding to the twelve analyzed indices for the period before the current crisis are presented in Table 1. We can observe that the mean return series are positive in all examined markets, to the extremes being placed Bulgaria (18.83%) and France (3.72%). A first argument that returns do not follow a normal distribution law is given by the Kurtosis coefficient (has higher values of 3), which means that the distribution is leptokurtic, which is much less sharp than the normal distribution, and by the asymmetry coefficient (Skeweness) which is different from zero indicating a left asymmetry (except Romania, Czech Republic, Bulgaria and Poland), i.e. – the left tail is larger.

Table 1: Descriptive statistics of return series before financial crisis

Ticker	*Mean*	*Median*	*Std. Dev.*	*Skewness*	*Kurtosis*
BET	0.10833	0.05759	1.7551	0.321717	10.5536
CAC40	0.03721	0.04388	1.3187	-0.16043	7.37975
DAX	0.04361	0.08352	1.3921	-0.25349	8.36711
FTSE	0.05099	0.06803	1.1476	-0.29291	23.4088
PX	0.06083	0.04962	1.3712	1.388451	19.7358
SOFIX	0.18833	0.10449	1.8356	0.559474	39.1065
SWISS	0.04496	0.07343	1.1250	-0.33995	9.83766
WIG	0.13830	0.08194	2.1241	0.201971	8.74738

Return series became negative after the appearance of financial crisis for all analyzed stock markets (Table 2). Kurtosis coefficients remain higher than the value of three, therefore the distributions are leptokurtic, and these do not follow the normal law (according to Jarque-Bera test). A remark useful in the experimental part, one can state that only distributions of BET and PX return indices have a right asymmetry, and for the other indices the distribution remains have a left elongated tail. As regards the asymmetry coefficients, they present positive values for seven return series (CAC40, DAX, FTSE, and SWISS). Thus, these indices have right asymmetry.

Table 2: Descriptive statistics of return series after financial crisis

Ticker	*Mean*	*Median*	*Std. Dev.*	*Skewness*	*Kurtosis*
BET	-0.0451	0.014495	2.076829	-0.27369	8.21568
CAC40	-0.0371	-0.01277	1.800336	0.315593	8.10097
DAX	-0.0202	0.040623	1.72665	0.323315	8.61703
FTSE	-0.0031	0.007804	1.557533	0.100992	8.45624
PX	-0.0458	-0.06456	1.945186	-0.04032	12.9783
SOFIX	-0.1506	-0.056481	1.614512	-0.70771	9.87889
SWISS	-0.0261	0.014663	1.411577	0.292486	9.62672
WIG	-0.0145	0.000599	1.537889	-0.18153	5.75293

V. EXPERIMENTAL RESULTS

There are many important aspects in regards with the obtained results, which we want to point out in order to emphasis the relevance of the present study. Before going into o detailed analysis regarding how the financial crisis has affected the investor preferences for a certain country represented by its main stock index, some remarks are necessary.

The stochastic dominance analysis is a concept that strongly relays on the distribution of the analyzed assets (prospects). The way this distribution is constructed has an important influence on the experimental results. It is possible to build the distribution of the prices (value of indexes, in our case) or the distribution of the returns. Since the returns are presenting a higher interest in the stock market world and also due to the fact that the distribution of returns is quite similar to a normal distribution (which could lead to a better modeling), we chose to use this representation as a basis for constructing the cumulative distribution functions for each of the analyzed index.

The empirical cumulative distributions for stock market returns data have similar shapes (forms), so that the situations presented in Figure 1 and 2 are rarely encountered in real applications. Therefore, in the current experiment a clear stochastic dominance of order 1 or 2 is not present.

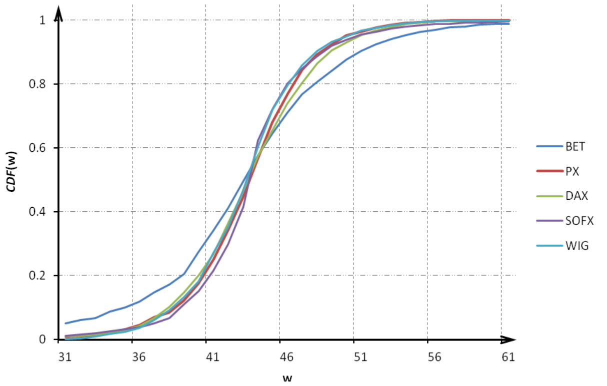

Figure 3. Cumulative distribution functions of indexes' returns from some European stock markets (Source: author's calculations in the own implementation software).

One can see the previous figure that it is rather difficult to estimate what order of dominance exist between the returns variables. Thus, the Meyer generalized stochastic dominance algorithm is a good alternative for having proper measurements in this direction.

Figure 3 presents the cumulative distribution functions (CDF) for the return of stock indexes from six countries. The figure is part of the representation of the entire set of repartition functions. The represented indexes are from the Eastern Europe countries and they are put together since they have some similar characteristics, which are reflecting in their associated returns distributions too. Empirically it has been observed that the size of the bins for constructing the histograms does not affect the form of cumulative distribution functions unless there are used extreme values. Thus, the numbers of bins used in the experimental part was 100 since the length of the series is in the range 1000 – 5000 observations. A higher number of bins does not increase significantly the performance and does not modify the result.

Another interesting part of the analysis concerns the risk coefficient values. More details about risk aversion coefficient could be found also in [16]. We used only constant value for the risk aversion function since we considered that the lower and the upper limits bound the risk aversion in a proper way. For a proper analysis, we chose as a range for risk aversion coefficient the interval [-2;+2].

Hence, we present two tables, the first one is focusing on the results before the start of the crisis and the second one on the preferences of stock market players in regards with certain indexes after the beginning of the crisis. The tables are quite big since we grouped the results for all analyzed indexes. The value from each cell is representing the value for J^* measure described by (17).

Table 3: Generalized stochastic dominance for principal stock indexes from Europe before the start of financial crisis

	BET	CAC	DAX	FT	PX	SOFX	SWISS	WIG
BET	0							
CAC	3.849	0						
DAX	3.764	0.792	0					
FT100	4.623	1.436	1.564	0				
PX	1.639	-0.024	-0.004	-0.003	0			
SOFX	-0.147	-0.187	-0.148	-0.115	-0.12	0		
SWISS	3.617	-0.002	0.657	0.016	2.494	5.349	0	
WIG	2.075	-0.023	-0.023	0.028	0.541	4.443	-0.368	0

The results presented in the previous table are reflecting the stochastic dominance in the preferences of risk adverse investors during the period before the financial crisis. In this case the time frame for each index is very different since there are countries for one can get data for very long periods since for others the period is relatively short. Independently to length of the period, the distributions have the same size and therefore the results are referring the so-called "period before the 2007/2008 crisis". There are several aspects, which can be commented, since the information from the table can cover several topic topics. We just want to point out that before the crisis the investors' preferences from the stochastic dominance point of view are in favor of stocks indexes from emerged countries. On the other side, it is possible to make a top of dominance, but one has to take into account that if the value of J_N^* for a certain asset is different compared with that obtained in case of other asset, the only which is taken into account is the sign. Unfortunately, this study is not covering also the topic related to size of J_N^*, which could lead to interesting conclusions to a refined result.

After the crisis, the situation changed in the sense that there are other distributions which became dominant, compared with those before the crisis. A similar table with Table 3 is presented bellow in order to emphasis the effect of crises and the imbalance in stock indexes' preferences changes in the emerged and developed European countries.

Table 4: Generalized stochastic dominance for principal stock indexes from Europe after the beginning of crisis

	BET	CAC	DAX	FT	PX	SOFX	SWISS	WIG
BET	0							
CAC	2.171	0						
DAX	1.856	0.007	0					
FT100	-1.922	0.009	0.789	0				
PX	1.536	-0.071	-0.001	-0.071	0			
SOFX	-1.044	0.001	0.079	0.119	0.605	0		
SWISS	2.044	0.233	0.079	0.189	0.465	-0.004	0	
WIG	-0.319	0.009	0.397	-0.007	0.080	-0.002	0.395	0

It is interesting that there are situations when we cannot state exactly if there exists completely dominance between two distributions of the indexes for both periods. There are situation when the change in sign indicate also a change in preferences of investors. There are also very interesting situations when the dominance is expressed a very small value, from numerical point of view which means that there is a weak dominance based on Meyer' algorithm.

The presented results from both tables are based on the same values for risk aversion coefficient. The coefficient values, which were suited to be used for a more precise analysis, were close to zero as indicated also the work of [McCarl1990]. We tried to use a uniform approach so that for both analyzed periods the same values for risk coefficients have been used. The upper bound was set to be equal to 0.004, the lower bound was equal to -0.035, and their values have been chose based on an empirical approach. These values are not considered the optimal ones, but still they are good measure for risk aversion.

The descriptive statistics for each time series have changed after the analyzed event (i.e. – the beginning of the crisis), whose influence is revealed by the empirical distributions.

It could be seen that in countries from Eastern Europe the changes in dominance are influenced also by the higher volatility, which characterizes these markets. The volatility, considered to be the a measure for sudden changes in return for a price series it is an important indicator used in stock market field. In this cases the structure of volatility that has a strong randomly character and the influence of the crisis had a higher impact on the preferences of investor with high aversion at risks. The risk aversion is defined by the risk aversion coefficient described in a previous paragraph.

VI. CONCLUSIONS AND FURTHER RESEARCH

There are many applications of stochastic dominance concepts. Some of them could be frequently encountered in finance and economics. Although, the stochastic dominance was applied in the early phase of this concept only in economics and agricultural economy for various (random) variables, the recent studies are covering topics like portfolio optimization and stocks dominance for different levels of risk. Therefore, accordingly with the obtained results, this concept is recommended as a good risk measurement approach.

The findings of our study reveal the fact that the financial crisis, which started in 2007, had a different impact on stock markets across Europe. The changes in preferences for certain stock index are reflecting by the change in sign of stochastic dominance measure proposed by Meyer and implemented also in our approach. The results of our approach show that constant relative risk aversion plays an important role in explaining the attitude for return distribution (e.g. – stock market index) selection.

Stochastic dominance is measure of uncertainty, which apparently involves simple methods, but for a more complex analysis more advanced mathematical and statistical tools could be required. The approach used in this paper, the Meyer algorithm, is a good tool, which offers the possibility to have an overview of the possible preferences of individuals with aversion to risk.

The latest researches that are using stochastic dominance as decision tool can be applied in other areas and not only in the financial markets field. Therefore, the presented approach could be enhanced by implementing some methods, which use other data from signal processing field where a decision tool for a proper selection between distributions might be useful.

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