HUMAN AUDIOGRAM FIR MODELING WITH MATLAB TOOLKITS

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Abstract: The paper presents the synthesis of FIR filters with the magnitude characteristic that matches an arbitrary human audiogram, by using two of the toolkits provided by Matlab: the *levinson* function, that implements the Levinson-Durbin algorithm, and the standard routine *fir2*, based on the frequency sampling method. The performances of the filters synthesized with the two Matlab toolkits are evaluated by comparing their magnitude characteristics against the given audiogram, for the same length of the filters. Moreover, for each of the two implemented filters having the optimal length, for the same maximum approximation error, the impulse response and phase characteristic are also discussed.

Keywords: *FIR filter, Levinson-Durbin algorithm, human audiogram.*

I. INTRODUCTION

The sensitivity of the human ear varies with frequency, being less sensitive at very low and very high frequencies, and most sensitive to sounds in the range of frequencies used by human spoken language (500 Hz - 4 kHz). When hearing loss of a person is suspected, an audiometry is performed. The audiologists measure the threshold intensity of the human hearing in the range of 125–8000 Hz, generally in octave steps and often including the half-octave frequencies; hence standardized frequencies include: 125, 250, 500, 750, 1000, 1500, 2000, 3000, 4000, 6000 and 8000 Hz.

The *audiogram* is the graphical representation of the sound level required for a person to just detect specific sounds across the frequencies that it can hear. The audiogram charts the hearing level relative to a standardized curve representing normal hearing. Therefore, the units of the y-axis are *decibels hearing level* (dB HL). Normal hearing is classified as being between -10 dB HL and 15 dB HL [1].

Hearing loss is usually accompanied by a reduction in the loudness range of sound that a person can process. This range is different across the frequency spectrum of hearing. A deteriorated hearing of a person can be improved by fitting an electro-acoustic device, called *hearing aid*, which selectively amplify the sounds based on these two factors: the frequency of the sound and the loudness of the sound source.

The field of research of this paper is the synthesis of a digital FIR filter that models the human hearing response, based on the data recorded in an audiogram. Compared to IIR filters, digital FIR filters proved to be easier to design in order to meet the design specifications and are better suited to multi-rate applications (decimation and interpolation);also they are easier to implement using fixed-point arithmetic; usually they are linear phase [2], [3]. Due to lack of feedback from previous filter output, FIR filters are less sensitive to quantization errors in the coefficients. Their disadvantage is that they require more memory or calculation to achieve a given filter response characteristic than a similar IIR filter.

Our previously work in the designing of FIR filters that approximates a given audiogram includes the implementation in Labview of a frequency sampling design procedure in combination with various interpolation methods [4] and also the use of a non-uniform frequency sampling algorithm [5].

In the present paper, the digital FIR filter that models the audiogram is synthesized in Matlab using the *levinson* function that implements the Levinson-Durbin algorithm [6] and *fir2* standard routine provided by Matlab [7].

The paper is organized into five sections. Section II presents the theoretical background of the Levinson-Durbin algorithm and the *fir2* routine. In Section III a few details of the Matlab implementation are discussed. Section IV deals with the results obtained from the simulation in Matlab by comparing the magnitude characteristics, the phase characteristics and the impulse responses of the filters. The last Section summarizes the paper and presents the main conclusions.

II. THE THEORETICAL BACKGROUND

The Matlab program is very flexible in modeling and simulating various signal-processing techniques. It provides extensive libraries with toolboxes, routines and functions including the area of filter design, analysis and implementation.

Two of Matlab functions are used in the present paper for the synthesis of the filter that models the audiogram:

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levinson – implementing the Levinson-Durbin algorithm – and *fir2* – implementing the frequency sampling procedure.

A. The Levinson-Durbin algorithm

The Levinson-Durbin algorithm has applications in filter design, coding, and spectral estimation. In filter design area, it can be used for finding an all-pole IIR filter whose impulse response matches a given autocorrelation sequence [6].

Basically the Levinson-Durbin algorithm solves the linear equation:

$$\begin{bmatrix} r_1 & r_2^* & \cdots & r_n^* \\ r_2 & r_1 & \cdots & r_{n-1}^* \\ \vdots & \vdots & & \vdots \\ r_{n+1} & \cdots & r_2 & r_1 \end{bmatrix} \cdot \begin{bmatrix} a_2 \\ a_3 \\ \vdots \\ a_{n+1} \end{bmatrix} = \begin{bmatrix} -r_2 \\ -r_3 \\ \vdots \\ -r_{n+1} \end{bmatrix} \quad (1)$$

where $\boldsymbol{r} = [r_1 \quad r_2 \quad \dots \quad r_{n+1}]$ represents the autocorrelation sequence of the filter's impulse response, with r_1 being the zero-lag value, r_i^* denotes the complex conjugate of r_i and the vector $\boldsymbol{a} = [a_2 \quad a_3 \quad \dots \quad a_{n+1}]$ contains the coefficients of the polynomial:

$$A(z) = 1 + a_2 z^{-1} + \dots + a_{n+1} z^{-n} \quad (2)$$

Usually, the algorithm is used to model an all-pole IIR filter with the transfer function given by:

$$H(z) = \frac{1}{A(z)} \quad (3)$$

but it can be used as well to synthesis a FIR filter having the transfer function given by:

$$G(z) = A(z) \quad (4)$$

For the implementation of the algorithm, the Matlab program provides the function *levinson* having the syntax:

$$\boldsymbol{a} = levinson(\boldsymbol{r}, \boldsymbol{n}) \quad (5)$$

The *levinson* function receives as inputs the autocorrelation sequence (***r***) of the filter's impulse response and the number of polynomial coefficients (n) and returns a vector containing the filter coefficients (***a***), ordered in descending powers of z^{-1}.

B. The Matlab's routine fir2.m

The synthesis routine *fir2* provided by Matlab can be used to design FIR filters with arbitrarily shaped frequency response [7]. The routine is based on the frequency sampling method and presents the syntax:

$$\boldsymbol{b} = fir2(N, \boldsymbol{f}, \boldsymbol{m}) \quad (6)$$

where N is the filter's order, ***f*** is the vector of frequencies and ***m*** is the vector of the desired magnitude response at the specified frequencies. The frequencies are normalized in the range [0,1], where 1 represents the Nyquist frequency. The output of *fir2* routine represents the vector ***b*** of length $N+1$ containing the filter coefficients, ordered in descending powers of z^{-1}:

$$B(z) = b_1 + b_2 z^{-1} + \dots + b_{n+1} z^{-n} \quad (7)$$

First, the desired frequency response is uniformly sampled. The inverse DFT is then applied to the resulted N equally spaced samples, providing the impulse response coefficients of the FIR filter. Finally, a window function (Hamming window by default) is used in order to reduce the oscillations produced in the magnitude response.

III. IMPLEMENTATION

An audiogram consists of a set of hearing thresholds recorded at specific frequencies. The frequencies of the audiogram are not equally spaced. This unevenly distribution of the samples has a few drawbacks. First, the coefficients of the synthesized filter result widely spread, which can lead to difficulties in implementation [7]. Secondly, with the standard method for designing FIR filters based on the frequency-sampling algorithm [8], the magnitude of the synthesized filter can be controlled only at the frequencies specified in the audiogram. Consequently, in the large intervals between the samples the magnitude can have large deviations from the wanted characteristic.

The first step was therefore to redistribute the thresholds using an interpolation method. In this paper the cubic spline interpolation was chosen, due to its stability and calculation simplicity. The maximum optimal spacing between the samples was calculated as the greatest common divisor of the initial frequencies from the audiogram, in order for the initial samples to be among the interpolated samples. The maximum spacing resulted of 125 Hz, consequently 64 samples resulted after the interpolation.

For the implementation using the *fir2* routine, the vector of interpolated frequencies was then normalized in the range [0,1].

Regarding the implementation using *levinson* function, we have mentioned above that it uses the autocorrelation vector of the impulse response of an *IIR filter*. For this reason, the magnitude characteristic had to be vertically flipped with respect to its mean value prior to calculate the autocorrelation sequence. The next step was to calculate the Fourier transform of the autocorrelation coefficients, as the magnitude of the flipped characteristic:

$$R(\omega) = |H(\omega)|^2 \quad (8)$$

The autocorrelation vector r was then calculated as the inverse Fourier transform of $R(\omega)$, then applied to the *levinson* function which returns the polynomial coefficients from (2). The last step was to obtain the transfer function of the FIR filter having the coefficients of the nominator (4).

IV. SIMULATION RESULTS

In order to illustrate the experimental results, we have chosen a real-life audiogram containing 11 hearing thresholds, corresponding to the 11 standard frequencies, as presented in *Table I.*

f [Hz]	125	250	500	750	1k	1.5k	2k	3k	4k	6k	8k
A [dB]	22	24	28	30	35	38	41	44	48	46	43

Table I. The 11 values for the frequencies and the thresholds of the experimental audiogram.

First we compared the magnitude characteristics of the filters designed with the two functions against the interpolated samples of the audiogram. The two synthesized filters had the same number of coefficients, *16*. *Figure 1* presents in the upper subplot the interpolated samples and the magnitude characteristic of the *levinson* filter and, in the lower subplot, the magnitude of the *fir2* filter. It can be noticed that the magnitude characteristic stays very close to most points of the audiogram in the case of *levinson* function. The maximum deviation from the wanted characteristic is *0.992 dB*, measured at the frequency *1133* Hz, for *levinson* filter, while *fir2* characteristic has a maximum deviation of *7.05 dB*, at the frequency of *125 Hz*.

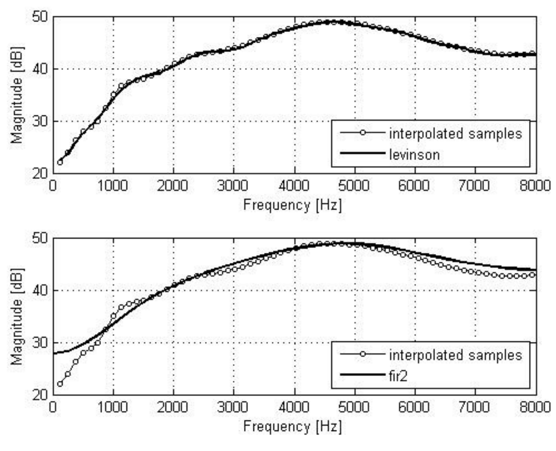

Figure 1. The interpolated samples and the magnitude characteristics given by levinson *and* fir2 *respectively (16 coefficients).*

The small deviations noticed on the characteristic obtained with *levinson* are confirmed by the small values of the error registered, as illustrated in *Figure 2.*

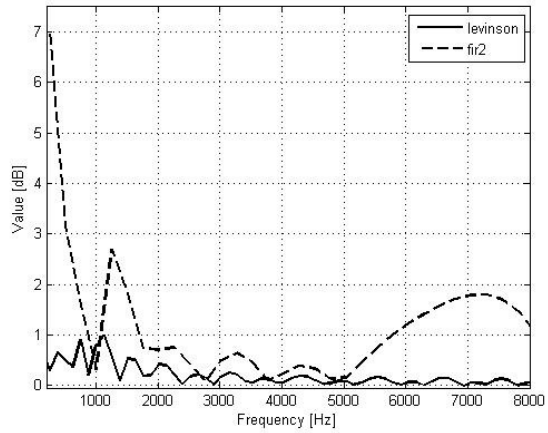

Figure 2. The error in the approximation of the interpolated samples for levinson *and* fir2.m *respectively (16 coefficients).*

The error was calculated as the absolute values of the difference between the hearing thresholds and the values of the magnitude characteristics in the two cases of toolkits of Matlab.

The filter length is important to be as small as possible in order to increase the efficiency of the implementation. In this context, the first experiment consisted in finding the optimal number of polynomial coefficients for each of the transfer function of the filters obtained with *levinson* and *fir2* respectively.

The amplitude levels in the audiogram are usually determined by subjective measurements, with a resolution of *1 dB*, hence we set the maximum allowed deviation from the audiogram to *1 dB*. For the filter obtained with *levinson*, the optimal number of coefficients resulted *16*, while for the *fir2* routine, this number resulted *54*.

Further, we compared the phase characteristics and impulse responses for the two filters having the optimal number of coefficients; that is, *16* for the *levinson* filter and *54* for the *fir2* filter respectively.

The phase characteristics are illustrated in *Figure 3* (the values are in degrees and the characteristics were plotted at different *y*-scales). The filter obtained with *levinson* is minimum phase and nonlinear, while the filter modeled with *fir2* is not minimum phase but linear (excluding phase wraps at +/- 180 degrees). A minimum-phase filter has less delay than the linear-phase filter with the same amplitude response, at the cost of a non-linear phase characteristic.

Figure 4 presents the impulse responses of the two filters under comparison. The linearity of the phase of the *fir2* filter is reflected into the symmetry of the impulse response, as seen in the lower subplot.

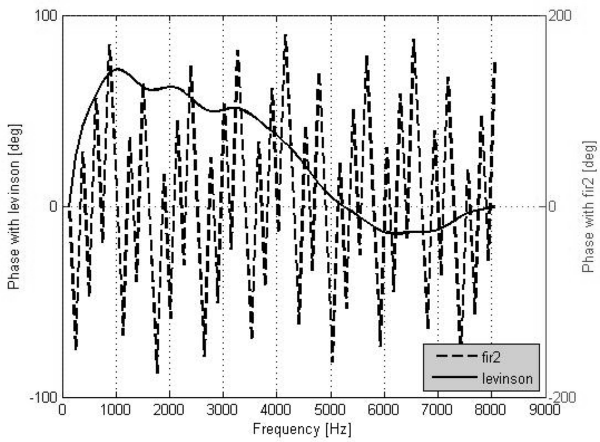

Figure 3. The phase characteristics of the filters synthesized with levinson *(16 coefficients) and* fir2 *respectively (54 coefficients).*

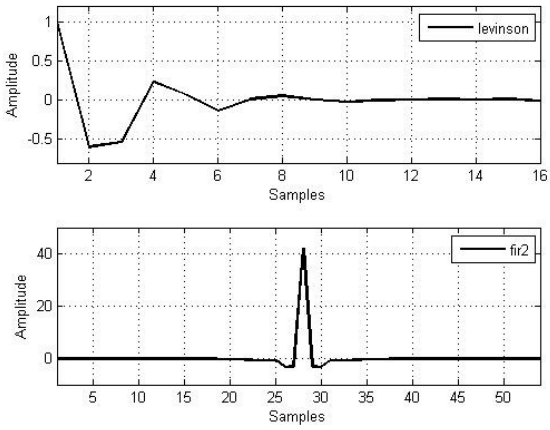

Figure 4. The impulse response in the case of levinson *(16 coefficients) and* fir2 *respectively (54 coefficients).*

At every frequency, the phase and also the group delay of the *fir2* filter are equal to (N-1)/2 samples of plain delay, which represents half the total filter delay. The response in the case of the *levinson* function has a typical shape for a non-symmetrical FIR filter (the upper subplot).

Note that even if for the *fir2* filter only half of the coefficients are used (due to symmetry), the resulting optimal length of the filter (27) is still greater than optimal length obtained in the *levinson* case (16).

V. CONCLUSIONS

This paper deals with the synthesis of the FIR filters that model the nonlinear human hearing mechanism response. The filter was synthesized using two of the Matlab's functions, *levinson* and *fir2* respectively.

The wanted magnitude characteristic (the audiogram) is defined through a set of frequency-magnitude samples. Due to the disadvantages of the unevenly distribution of the initial samples, they were first redistributed using the cubic spline interpolation.

For both filters having the same length (*16*), their magnitudes characteristics were compared against the interpolated samples of the audiogram. The synthesis using Levinson-Durbin algorithm yielded significantly better results than the filter obtained with the *fir2* routine, in respect to matching the wanted magnitude characteristic. *Fir2* proved to be more rigid in the magnitude characteristic, unable to touch all the points of the audiogram.

In order to obtain a deviation less than *1 db*, the optimal number of polynomial coefficients for the filter resulted *16* for the *levinson* case and *54* for the *fir2* filter. The impulse responses and phase characteristics were illustrated for these two filters with optimal length.

Though Levinson-Durbin provides models with fewer coefficients than *fir2* at the same specified error, *fir2* has the advantage of having linear phase. Consequently, the choice of the appropriate toolbox for modeling the audiogram depends on the envisaged application: *levinson* – if a low number of FIR taps is more important – or *fir2* – if linear phase is required together with the approximation of the magnitude characteristics.

Future work includes the implementation in Matlab of the modeling procedures previously developed in Labview in [4], [5], for the purpose of comparing them against the resulted filters discussed in this paper.

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REFERENCES

[1] Thomas D. Rossing [Ed.], "*Springer Handbook of Acoustics*", Springer Science & Business Media, 2nd ed., 2014.
[2] T. W. Parks, C. S. Burrus, "*Digital Filter Design*", John Wiley & Sons, 1987.
[3] J.G. Proakis and D.G. Manolakis, "*Digital Signal Processing: Principles, Algorithms, and Applications*", 3*rd* ed., Prentice Hall, 1996.
[4] E. Szopos, M. Topa, L. Festila, H. Hedesiu, "*FIR Synthesis of The Human Hearing Mechanism Response*", Acta Technica Napocensis – Electronics and Telecommunications, vol. 51, no. 4, pp. 41- 44, 2010.
[5] E. Szopos, M. Neag, I. Sărăcuţ, H. Hedesiu, L. Festila, "*A method for designing FIR filters with arbitrary magnitude characteristic used for modeling human audiogram*", Advances in Electrical and Computer Engineering, vol. 12, no. 2, pp. 51-56, 2012.
[6] A. Zacknich,"*Principles of Adaptive Filters and Self-learning Systems*", Springer, 2005.
[7] The MathWorks Inc, *"Filter Design Toolbox for use with Matlab",* User's Guide, 2000.
[8] D. Schlichtharle, "*Digital filters. Basics and design*", Springer, 2011.