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ABOUT CONVERTING ACTIVE ANALOG FILTERS TO DIGITAL FILTERS

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Abstract: The goal of this paper is to present the state-space approach used to obtain the digital filter directly from the netlist of the active analog filter. To reach this goal, the state-space approach has been used to obtain the state-space description of the active analog filter. Then the bilinear transformation was recalled to convert from analog domain to the digital domain. The entire approach is available software and the experimental results shown in this work have been obtained using this computer application.

Keywords: *state-space, analog to digital conversion, dependent sources.*

I. INTRODUCTION

Infinite Impulse Response Filters (IIR) are often designed by using analog filters described in transform domain. The analog filters are then converted to digital filters using appropriate transformation from analog frequency domain to digital frequency domain.

Recently, two methods were proposed to obtain the digital filter directly from the netlist of the analog filter. The first one is the Analog Filter Netlist to Digital Filter Statements (AFN-DFS) approach, where every component is replaced by its corresponding companion model and a software code for the digital filter may be available rapidly [1-4]. The second one is the state-space (SS) based approach, which allows automated conversion of an analog filter described by a diagram or a netlist to a digital filter described in system function form [5, 6]. However, previous works have introduced the equations only for passive analog filters.

In this work, we shall present in details (all the equations) the SS approach for converting an analog filter described by its netlist to a digital filter for an active analog filter. The paper is organized as follows. First the SS approach is reviewed (Section II) and the specific implementation for circuits having dependent sources is presented in a step-by-step procedure. Then as an example, an active analog circuit (notch filter based double T-shaped bridge) is converted into a digital filter. Note that the SS approach is available software as MATLAB code [7] and the experimental results shown in this work have been obtained using the computer application.

II. THE STATE-SPACE APROACH

The state-space based approach for converting an active analog filter described by a circuit diagram or a netlist to a digital filter described in system function form consists of three main parts, as in the case of passive filters [5]:

1. First the state-space of the active analog filter is computed based on diagram or netlist of the active analog circuit;
2. Then a conversion from analog domain to digital domain is used;
3. Finally, the digital filter is delivered in system function form.

In the following we shall present in details the first part for dependent sources, then we shall briefly discuss the second step.

II.1 State-space formulation for a network containing dependent sources

The state-equation formulation for active filters is actually the same procedure as for networks containing independent sources [5]. The development is similar, with few exceptions [8]. In the case of a network containing dependent sources, the cut-set matrix **Q** is given by:

$$\begin{array}{c} \\ C_t \\ R_t \\ E \\ E_d \end{array} \begin{array}{c} \begin{array}{cccccccc} C_t & R_t & E & E_d & L_l & R_l & J & J_d \end{array} \\ \left[\begin{array}{cccc} & : \mathbf{Q}_{lCL} & \mathbf{Q}_{lCR} & \mathbf{Q}_{lCJ} & \mathbf{Q}_{lCD} \\ \mathbf{I} & : \mathbf{Q}_{lRL} & \mathbf{Q}_{lRR} & \mathbf{Q}_{lRJ} & \mathbf{Q}_{lRD} \\ & : \mathbf{Q}_{lEL} & \mathbf{Q}_{lER} & \mathbf{Q}_{lEJ} & \mathbf{Q}_{lED} \\ & : \mathbf{Q}_{lDL} & \mathbf{Q}_{lDR} & \mathbf{Q}_{lDJ} & \mathbf{Q}_{lDD} \end{array}\right] \end{array} \quad (1)$$

and the tree current vector can be partitioned into four vectors corresponding to tree capacitance, tree resistance, voltage source, and dependent voltage source. Also, the link current vector can be partitioned into inductance, link resistance, current source, and dependent current source. The state-space procedure may be implemented as follows:

1. The link branch voltages are expressed in terms of tree-branch voltages;
2. Capacitance currents and inductance voltages are expressed in terms of tree-branch voltages and link currents;
3. The resistive tree-branch currents and resistive link voltages are expressed in terms of tree-branch voltages

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and link currents;

4. The state equations are obtained after the vectors relating the resistive quantities and dependent sources are eliminated.

A complete list of computations is available below. We start from the parts of the cut-set matrix given by (1). First, we compute the capacitance currents and the inductance voltages:

$$\begin{bmatrix}\mathbf{i}_{tC}\\ \mathbf{v}_{lL}\end{bmatrix}=\begin{bmatrix}0 & -\mathbf{Q}_{lCL}\\ \mathbf{Q}_{lCL}^t & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_{tC}\\ \mathbf{i}_{lL}\end{bmatrix}+\begin{bmatrix}0 & -\mathbf{Q}_{lCR}\\ \mathbf{Q}_{lRL}^t & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_{tR}\\ \mathbf{i}_{lR}\end{bmatrix}+\begin{bmatrix}0 & -\mathbf{Q}_{lCJ}\\ \mathbf{Q}_{lEL}^t & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_{tE}\\ \mathbf{i}_{lJ}\end{bmatrix}+\begin{bmatrix}0 & -\mathbf{Q}_{lCD}\\ \mathbf{Q}_{lDL}^t & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_{tE_d}\\ \mathbf{i}_{lJ_d}\end{bmatrix} \quad (2)$$

Then the resistive tree-branch currents and resistive link voltages can be written as

$$\begin{bmatrix}\mathbf{v}_{lR}\\ \mathbf{i}_{tR}\end{bmatrix}=\begin{bmatrix}\mathbf{Q}_{lCR}^t & 0\\ 0 & -\mathbf{Q}_{lRL}\end{bmatrix}\begin{bmatrix}\mathbf{v}_{tC}\\ \mathbf{i}_{lL}\end{bmatrix}+\begin{bmatrix}\mathbf{Q}_{lRR}^t & 0\\ 0 & -\mathbf{Q}_{lRR}\end{bmatrix}\begin{bmatrix}\mathbf{v}_{tR}\\ \mathbf{i}_{lR}\end{bmatrix}+\begin{bmatrix}\mathbf{Q}_{lER}^t & 0\\ 0 & -\mathbf{Q}_{lRJ}\end{bmatrix}\begin{bmatrix}\mathbf{v}_{tE}\\ \mathbf{i}_{lJ}\end{bmatrix}+\begin{bmatrix}\mathbf{Q}_{lDR}^t & 0\\ 0 & -\mathbf{Q}_{lRD}\end{bmatrix}\begin{bmatrix}\mathbf{v}_{tE_d}\\ \mathbf{i}_{lJ_d}\end{bmatrix} \quad (3)$$

Since the dependent voltage and current sources are functions of the other network variables, it is possible to represent the dependence quantities as

$$\begin{bmatrix}\mathbf{v}_{tED}\\ \mathbf{i}_{lJ_d}\end{bmatrix}=K\begin{bmatrix}\mathbf{v}_{tC}\\ \mathbf{i}_{lL}\end{bmatrix}+\hat{K}\begin{bmatrix}\mathbf{v}_{tR}\\ \mathbf{i}_{lR}\end{bmatrix}+\tilde{K}\begin{bmatrix}\mathbf{v}_{tE}\\ \mathbf{i}_{lJ}\end{bmatrix} \quad (4)$$

Now (4) is substituted into (2) and (3) to eliminate the dependent source quantities. We have

$$\begin{bmatrix}\mathbf{i}_{tC}\\ \mathbf{v}_{lL}\end{bmatrix}=J\begin{bmatrix}\mathbf{v}_{tC}\\ \mathbf{i}_{lL}\end{bmatrix}+\hat{J}\begin{bmatrix}\mathbf{v}_{tR}\\ \mathbf{i}_{lR}\end{bmatrix}+\tilde{J}\begin{bmatrix}\mathbf{v}_{tE}\\ \mathbf{i}_{lJ}\end{bmatrix} \quad (5)$$

where:

$$J=\begin{bmatrix}0 & -\mathbf{Q}_{lCL}\\ \mathbf{Q}_{lCL}^t & 0\end{bmatrix}+\begin{bmatrix}0 & -\mathbf{Q}_{lCD}\\ \mathbf{Q}_{lDL}^t & 0\end{bmatrix}K \quad (6)$$

$$\hat{J}=\begin{bmatrix}0 & -\mathbf{Q}_{lCR}\\ \mathbf{Q}_{lRL}^t & 0\end{bmatrix}+\begin{bmatrix}0 & -\mathbf{Q}_{lCD}\\ \mathbf{Q}_{lDL}^t & 0\end{bmatrix}\hat{K} \quad (7)$$

$$\tilde{J}=\begin{bmatrix}0 & -\mathbf{Q}_{lCJ}\\ \mathbf{Q}_{lEL}^t & 0\end{bmatrix}+\begin{bmatrix}0 & -\mathbf{Q}_{lCD}\\ \mathbf{Q}_{lDL}^t & 0\end{bmatrix}\tilde{K} \quad (8)$$

and

$$\begin{bmatrix}\mathbf{v}_{lR}\\ \mathbf{i}_{tR}\end{bmatrix}=I\begin{bmatrix}\mathbf{v}_{tC}\\ \mathbf{i}_{lL}\end{bmatrix}+\hat{I}\begin{bmatrix}\mathbf{v}_{tR}\\ \mathbf{i}_{lR}\end{bmatrix}+\tilde{I}\begin{bmatrix}\mathbf{v}_{tE}\\ \mathbf{i}_{lJ}\end{bmatrix} \quad (9)$$

where:

$$I=\begin{bmatrix}\mathbf{Q}_{lCR}^t & 0\\ 0 & -\mathbf{Q}_{lRL}\end{bmatrix}+\begin{bmatrix}\mathbf{Q}_{lDR}^t & 0\\ 0 & -\mathbf{Q}_{lRD}\end{bmatrix}K \quad (10)$$

$$\hat{I}=\begin{bmatrix}\mathbf{Q}_{lRR}^t & 0\\ 0 & -\mathbf{Q}_{lRR}\end{bmatrix}+\begin{bmatrix}\mathbf{Q}_{lDR}^t & 0\\ 0 & -\mathbf{Q}_{lRD}\end{bmatrix}\hat{K} \quad (11)$$

$$\tilde{I}=\begin{bmatrix}\mathbf{Q}_{lER}^t & 0\\ 0 & -\mathbf{Q}_{lRJ}\end{bmatrix}+\begin{bmatrix}\mathbf{Q}_{lDR}^t & 0\\ 0 & -\mathbf{Q}_{lRD}\end{bmatrix}\tilde{K} \quad (12)$$

The resistive voltages and currents are related by

$$\begin{bmatrix}\mathbf{v}_{lR}\\ \mathbf{i}_{tR}\end{bmatrix}=\begin{bmatrix}0 & \mathbf{R}_l\\ \mathbf{G}_t & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_{tR}\\ \mathbf{i}_{lR}\end{bmatrix} \quad (13)$$

where $\mathbf{R}_l$ is a matrix whose elements are the link branch resistances and $\mathbf{G}_t$ is a matrix whose elements are the tree-branch conductance.

The next step is to eliminate the resistive quantities. We get:

$$\begin{bmatrix}\mathbf{v}_{tR}\\ \mathbf{i}_{lR}\end{bmatrix}=M\begin{bmatrix}\mathbf{v}_{tC}\\ \mathbf{i}_{lL}\end{bmatrix}+\hat{M}\begin{bmatrix}\mathbf{v}_{tE}\\ \mathbf{i}_{lJ}\end{bmatrix} \quad (14)$$

where

$$M=\left\{\begin{bmatrix}0 & \mathbf{R}_l\\ \mathbf{G}_t & 0\end{bmatrix}-\hat{I}\right\}^{-1} I \quad (15)$$

$$\hat{M}=\left\{\begin{bmatrix}0 & \mathbf{R}_l\\ \mathbf{G}_t & 0\end{bmatrix}-\hat{I}\right\}^{-1} \tilde{I} \quad (16)$$

and

$$\begin{bmatrix}\mathbf{i}_{tC}\\ \mathbf{v}_{lL}\end{bmatrix}=N\begin{bmatrix}\mathbf{v}_{tC}\\ \mathbf{i}_{lL}\end{bmatrix}+\hat{N}\begin{bmatrix}\mathbf{v}_{tE}\\ \mathbf{i}_{lJ}\end{bmatrix} \quad (17)$$

where

$$N=J+\hat{J}M;\quad \hat{N}=\tilde{J}+\hat{J}\hat{M} \quad (18)$$

Since

$$\begin{bmatrix}\mathbf{i}_{tC}\\ \mathbf{v}_{lL}\end{bmatrix}=\begin{bmatrix}\mathbf{C}_t & 0\\ 0 & \mathbf{L}_l\end{bmatrix}\frac{d}{dt}\begin{bmatrix}\mathbf{v}_{tC}\\ \mathbf{i}_{lL}\end{bmatrix} \quad (19)$$

we finally get the SS formulation for a network containing dependent sources:

$$\frac{d}{dt}\begin{bmatrix}\mathbf{v}_{tC}\\ \mathbf{i}_{lL}\end{bmatrix}=\underbrace{\begin{bmatrix}\mathbf{C}_t & 0\\ 0 & \mathbf{L}_l\end{bmatrix}^{-1}N}_{\mathbf{A}}\begin{bmatrix}\mathbf{v}_{tC}\\ \mathbf{i}_{lL}\end{bmatrix}+\underbrace{\begin{bmatrix}\mathbf{C}_t & 0\\ 0 & \mathbf{L}_l\end{bmatrix}^{-1}\hat{N}}_{\mathbf{B}}\begin{bmatrix}\mathbf{v}_{tE}\\ \mathbf{i}_{lJ}\end{bmatrix} \quad (20)$$

II.2 Filter conversion from analog domain to digital domain

Given the SS canonical representation [9] of an analog filter

$$\begin{cases}\dot{\mathbf{x}}=\mathbf{Ax}+\mathbf{Bu}\\ \mathbf{y}=\mathbf{Cx}+\mathbf{Du}\end{cases}$$

we can use it to evaluate the transfer function of the analog filter:

$$H(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \qquad (21)$$

To obtain the digital filter by use of the bilinear transformation, we can either

- Convert the continuous-time state-space system in matrices **A**, **B**, **C**, **D** to the discrete-time system:

$$\begin{cases} \mathbf{x}(n+1) = \mathbf{A}_d\mathbf{x}(n) + \mathbf{B}_d\mathbf{u}(n) \\ \mathbf{y}(n) = \mathbf{C}_d\mathbf{x}(n) + \mathbf{D}_d\mathbf{u}(n) \end{cases}$$

  then compute the transfer function of the digital filter from state-space representation;
- Or we can use the numerator and the denominator coefficients of the analog transfer function obtained previously and then apply the bilinear transform.

Similar procedure can be used for other transformations from analog domain to digital domain.

III. CASE STUDY

Now we are presenting the way an analog active filter [10] is converted to a digital filter. The SPICE netlist, corresponding to circuit in Fig. 1 is presented in Table 1.

To obtain the state-space equation we build the network graph (Fig. 2) and the proper tree (Fig. 3). The tree branches have been selected as follows: the branches which contain the capacitors, the input voltage, the branches containing the voltages which controlled the voltage source and the output of voltage controlled source. The link branches have been asserted to link resistances. Thus, the state vector contains mainly the voltages across the capacitors.

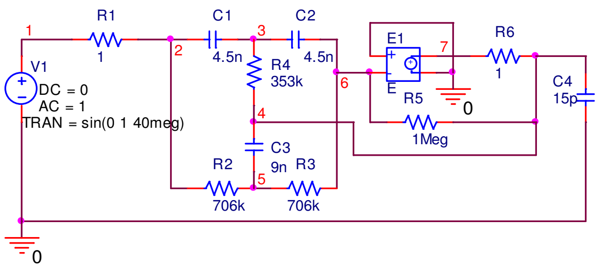

Figure 1. Notch filter based double T-shaped bridge circuit.

```
* source NOTCH_FILTER_T
R_R3   5   6   706k
R_R4   4   3   353k
V_V1   1   0   DC 0 AC 1 sin(0 1 40meg)
R_R6   7   4   1
E_E1   7   0   0  6  1e6
R_R5   6   4   1Meg
C_C1   2   3   4.5n
C_C2   3   6   4.5n
R_R1   1   2   1
C_C3   5   4   9p
C_C4   0   4   15p
R_R2   4   5   706k
```

Table 1. The SPICE netlist of the active notch filter shown in Fig. 1.

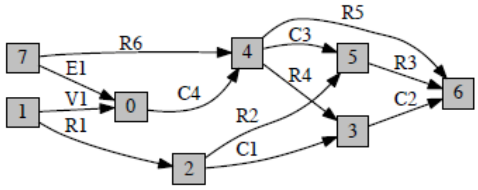

Figure 2. The network graph of the filter shown in Fig. 1.

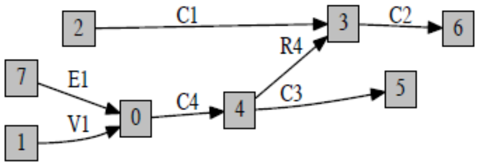

Figure 3. The proper tree of the filter shown in Fig. 1.

The partitioned matrix $\mathbf{Q}_l$ is the following:

$$\mathbf{Q}_l = \begin{array}{c} \\ C1 \\ C2 \\ C3 \\ C4 \\ R4 \\ V1 \\ E1 \end{array} \begin{array}{c} \begin{array}{ccccc} R1 & R2 & R3 & R5 & R6 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 \\ -1 & 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array} \qquad (22)$$

The matrixes $\mathbf{R}_l$, $\mathbf{G}_t$, and $\mathbf{C}_t$ are given by:

$$\begin{aligned} \mathbf{R}_l &= \mathrm{diag}(R1, R2, R3, R5, R6) \\ &= \mathrm{diag}(1, 706\cdot 10^3, 706\cdot 10^3, 10^6, 1) \end{aligned} \qquad (23)$$

$$\mathbf{G}_t = \frac{1}{R4} = 2.83\cdot 10^{-6} \qquad (24)$$

$$\begin{aligned} \mathbf{C}_t &= \mathrm{diag}(C1, C2, C3, C4) \\ &= \mathrm{diag}(4.5\cdot 10^{-9}, 4.5\cdot 10^{-9}, 9\cdot 10^{-9}, 15\cdot 10^{-12}) \end{aligned} \qquad (25)$$

and $\mathbf{L}_l$ is empty.

We apply the sequence of computations presented in Section II.1 and we get the state-space equation of the analog filter:

$$\frac{d}{dt}[\mathbf{v}_{tC}] = \mathbf{A}[\mathbf{v}_{tC}] + \mathbf{B}[\mathbf{v}_{tE}]$$

where

$$\mathbf{A} = \begin{bmatrix} -1.17\cdot 10^3 & 5.37\cdot 10^2 & 3.15\cdot 10^2 & 1.17\cdot 10^3 \\ -5.37\cdot 10^2 & -5.37\cdot 10^2 & 3.15\cdot 10^2 & 5.37\cdot 10^2 \\ 1.57\cdot 10^2 & 1.57\cdot 10^2 & -3.15\cdot 10^2 & -3.15\cdot 10^2 \\ 6.67\cdot 10^{16} & 6.67\cdot 10^{16} & 1.89\cdot 10^{11} & 3.78\cdot 10^{11} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1.17\cdot 10^{3} \\ 5.37\cdot 10^{2} \\ -3.15\cdot 10^{2} \\ -6.67\cdot 10^{16} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}; \quad \mathbf{D} = 0$$

The input-output relationship of the corresponding digital notch filter is:

$$\begin{aligned} y(n) = & -2.65\cdot 10^{-3}\cdot y(n-1) + 1.99\cdot y(n-2) \\ & +2.64\cdot 10^{-3}\cdot y(n-3) - 9.95\cdot 10^{-1}\cdot y(n-4) \\ & +1.17\cdot 10^{2}\cdot x(n) - 2.34\cdot 10^{2}\cdot x(n-1) \\ & -3.68\cdot 10^{-1}\cdot x(n-2) + 2.34\cdot 10^{2}\cdot x(n-3) \\ & -1.17\cdot 10^{2}\cdot x(n-4) \end{aligned}$$

Relating the experimental results, first the frequency response of active filter has been computed using PSpice (Fig. 4). The digital filter has been obtained using a sampling frequency of 100 kHz. The frequency response characteristics of the digital filter, converted using the bilinear transformation is illustrated in Fig. 5. We can conclude that the behavior of the active filter and its digital counterpart is similar.

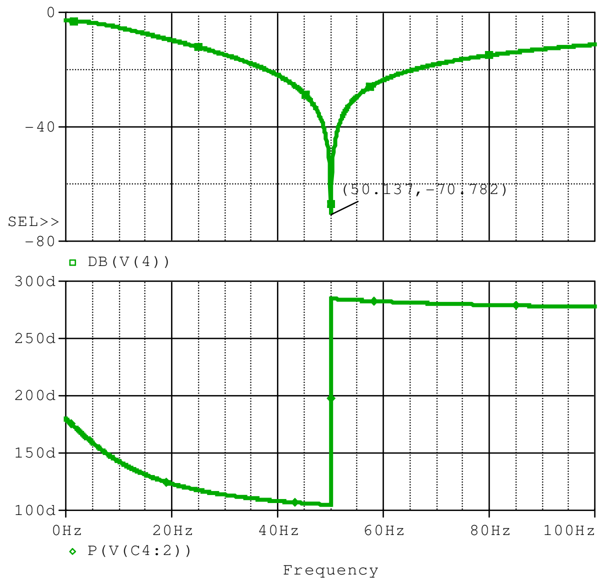

Figure 4. Frequency response characteristics for the analog notch filter based double T-shaped bridge - PSpice.

IV. CONCLUSION

In this paper, we have presented the way an active analog circuit described by its netlist can be converted to a corresponding digital filter, both having the same behavior in frequency domain. To reach this goal, the state-space approach has been used to obtain the state-space description of the active analog filter. Then the bilinear transformation was recalled to convert from analog domain to the digital domain. The entire approach is available software as MATLAB code [7].

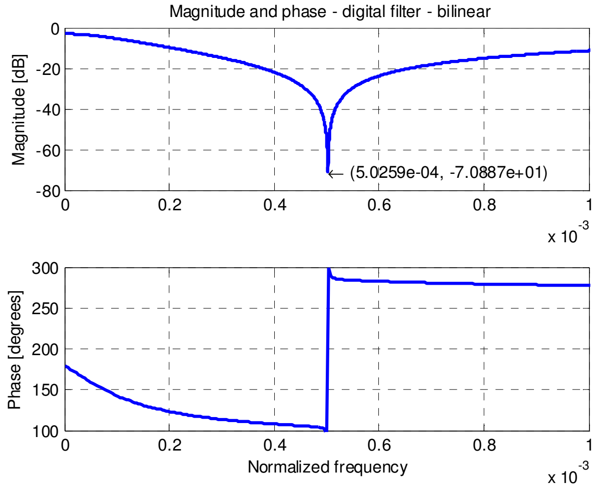

Figure 5. Frequency response characteristics for the digital notch filter based double T-shaped bridge - MATLAB.

As case study, we convert an analog active notch filter. Starting with the netlist of the analog filter, based on the SS approach, the input-output relationship of the corresponding digital filter is finally available.

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