INFLUENCE OF CONSTITUTIVE EQUATIONS ON THE ACCURACY OF PREDICTION IN SHEET METAL FORMING SIMULATION

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ABSTRACT: During the last three decades, numerical simulation has gradually extended its applicability in the field of sheet metal forming. Constitutive modelling is one of the domains closely related to the development of numerical simulation tools. The paper is focused on the development of new phenomenological yield criteria developed in the CERTETA research centre able to describe the anisotropic response of sheet metals. The numerical tests presented in the paper prove the capability of the equivalent stresses to model the inelastic response of a large variety of materials (steel and aluminium alloys). The last section of the paper is devoted to a comprehensive testing of the new yield criterion as implemented in the finite-element code AUTOFORM 4.1. With this aim in view, the authors have chosen the bulging and cross deep drawing benchmarks. The test proves the capability of the yield criterion to describe the effects of the plastic anisotropy of the sheet metals subjected to industrial forming processes.

KEYWORDS: Anisotropic yield criteria, BBC2005 model, AUTOFORM 4.1, sheet forming simulation

1 INTRODUCTION

The accuracy of the simulation results is given mainly by the accuracy of the material model. In the last years, the scientific research is oriented in developing of new material models able to describe the material behaviour (mainly the anisotropic one) as accurate as possible [1], [2]. The computer simulation of the sheet metal forming processes needs a quantitative description of the plastic anisotropy by the yield locus [3]. An evaluation of some recent yield criteria for industrial simulations of sheet forming processes has been presented by Mattiasson and Sigvant [3]. During the last years, new yield functions were introduced in order to improve the fitting of the experimental results, especially for aluminium and magnesium alloys. In order to remove the disadvantages of the Barlat 1994 and Barlat 1997 yield criteria, but aiming to preserve their flexibility, Barlat proposed in 2000 [4] a new model particularized for plane stress (2D). The linear transformation method is used to introduce the anisotropy. This model has been implemented in the LS Dyna commercial program. Vegter [5] proposed the representation of the yield function with the help of Bezier's interpolation using directly the test results. The Vegter model has been implemented in the PAMSTAMP commercial program developed by ESI. The CERTETA team has developed several anisotropic yield criteria [6], [7]. A description of the model BBC2005 implemented in the AUTOFORM 4.1 program is presented in the next section.

2. PRESENTATION OF THE BBC2005 YIELD CRITERION

2.1 BBC 2005 YIELD FUNCTION

In 2000 the members of the CERTETA team started a research programme having as principal objective the development of a model able to

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provide an accurate description of the yield surfaces [6]. BBC2000 formulation was developed on the basis of the isotropic formulation proposed by Hershey [8]. By adding weight coefficients to that model, the researchers succeeded to develop a flexible yield criterion, named BBC 2005 [7]. The last version incorporates a number of 8 coefficients and, consequently, its identification procedure uses 8 mechanical parameters (3 uniaxial yield stresses, 3 uniaxial coefficients of anisotropy, the biaxial yield stress and the biaxial coefficient of plastic anisotropy).

The new formulation implemented in the AUTOFORM 4.1 version is different from the version published in [7]. The equivalent stress is defined by the following formula:

$$\bar{\sigma}=\left[a\left(\Lambda+\Gamma\right)^{2k}+a\left(\Lambda-\Gamma\right)^{2k}+b\left(\Lambda+\Psi\right)^{2k}+b\left(\Lambda-\Psi\right)^{2k}\right]^{\frac{1}{2k}} \quad (1)$$

where $k \in \Im^{\geq 1}$ and a,b>0 are material parameters, while Γ, Λ and Ψ are functions depending on the planar components of the stress tensor:

$$\begin{aligned}\Gamma&=L\sigma_{11}+M\sigma_{22}\\ \Lambda&=\sqrt{\left(N\sigma_{11}-P\sigma_{22}\right)^2+\sigma_{12}\sigma_{21}}\\ \Psi&=\sqrt{\left(Q\sigma_{11}-R\sigma_{22}\right)^2+\sigma_{12}\sigma_{21}}\end{aligned} \quad (2)$$

Nine material parameters are involved in the expression of the BBC equivalent stress: k, a, b, L, M, N, P, Q and R (see Eqns (1) and (2)). The integer exponent k has a special status, due to the fact that its value is fixed from the very beginning in accordance with the crystallographic structure of the material: k=3 for BCC materials; k=4 for FCC materials.

The identification procedure calculates the other parameters (a, b, L, M, N, P, Q and R) by forcing the constitutive equations associated to the BBC yield criterion to reproduce the following experimental data: the uniaxial yield stresses associated to the directions defined by 0°, 45° and 90° angles measured from RD (denoted as Y_0, Y_{45} and Y_{90}); the coefficients of uniaxial plastic anisotropy associated to the directions defined by 0°, 45° and 90° angles measured from RD (denoted as r_0, r_{45} and r_{90}); the biaxial yield stress associated to RD and TD (denoted as Y_b); the coefficient of biaxial plastic anisotropy associated to RD and TD (denoted as r_b).

There are 8 constraints acting on 8 material parameters. The identification procedure has enough data to generate a set of equations having a, b, L, M, N, P, Q and R as unknowns.

2.2 IDENTIFICATION PROCEDURE

As mentioned in §2.1, the parameters a, b, L, M, N, P, Q and R are obtained by constraining the constitutive equations associated to the BBC yield criterion to reproduce the following experimental data: Y_0, Y_{45}, Y_{90}, r_0, r_{45}, r_{90} Y_b and r_b. In fact, the identification procedure will solve the following set of 8 equations considering a, b, L, M, N, P, Q and R as unknowns:

$$\begin{aligned}&\tilde{Y}_0=Y_0,\tilde{Y}_{45}=Y_{45},\tilde{Y}_{90}=Y_{90}\\ &\tilde{r}_0=r_0,\tilde{r}_{45}=r_{45},\tilde{r}_{90}=r_{90}\\ &\tilde{Y}_b=Y_b,\tilde{r}_b=r_b\end{aligned} \quad (3)$$

where: $\tilde{Y}_0$, $\tilde{Y}_{45}$ and $\tilde{Y}_{90}$ are the theoretical yield stresses corresponding to pure tension along the directions defined by 0°, 45° and 90° angles measured from RD; $\tilde{r}_0$, $\tilde{r}_{45}$ and $\tilde{r}_{90}$ are the theoretical coefficients of uniaxial plastic anisotropy associated to the directions mentioned above; $\tilde{Y}_b$ is the theoretical yield stress corresponding to biaxial tension along RD and TD; $\tilde{r}_b$ is the theoretical coefficient of biaxial plastic anisotropy associated to RD and TD. It is obvious that the identification procedure needs formulas for evaluating $\tilde{Y}_0$, $\tilde{Y}_{45}$, $\tilde{Y}_{90}$, $\tilde{r}_0$, $\tilde{r}_{45}$, $\tilde{r}_{90}$, $\tilde{Y}_b$, and $\tilde{r}_b$. These formulas will be presented below.

The formula for evaluating the uniaxial yield stress at the angle θ with the rolling direction is:

$$\tilde{Y}_\theta=\frac{Y}{F(\theta)} \quad (4)$$

where:

$$F(\theta)=\left[a\left(\Lambda_\theta+\Gamma_\theta\right)^{2k}+a\left(\Lambda_\theta-\Gamma_\theta\right)^{2k}+b\left(\Lambda_\theta+\Psi_\theta\right)^{2k}+b\left(\Lambda_\theta-\Psi_\theta\right)^{2k}\right]^{\frac{1}{2k}} \quad (5)$$

and

$$\begin{aligned}\Gamma_\theta&=L\cos^2\theta+M\sin^2\theta\\ \Lambda_\theta&=\sqrt{\left(N\cos^2\theta-P\sin^2\theta\right)^2+\sin^2\theta\cos^2\theta}\\ \Psi_\theta&=\sqrt{\left(Q\cos^2\theta-R\sin^2\theta\right)^2+\sin^2\theta\cos^2\theta}\end{aligned} \quad (6)$$

The formula for evaluating the coefficient of uniaxial plastic anisotropy is:

$$\tilde{r}_\theta=\frac{\left[F(\theta)\right]^{2k}}{G(\theta)}-1 \quad (7)$$

where:

$$\begin{aligned}G(\theta)=\,&a\left[\frac{(N-P)\left(N\cos^2\theta-P\sin^2\theta\right)}{\Lambda_\theta}+L+M\right]\left(\Lambda_\theta+\Gamma_\theta\right)^{2k-1}+\\ &a\left[\frac{(N-P)\left(N\cos^2\theta-P\sin^2\theta\right)}{\Lambda_\theta}-L-M\right]\left(\Lambda_\theta-\Gamma_\theta\right)^{2k-1}+\\ &b\left[\frac{(N-P)\left(N\cos^2\theta-P\sin^2\theta\right)}{\Lambda_\theta}+\frac{(Q-R)\left(Q\cos^2\theta-R\sin^2\theta\right)}{\Psi_\theta}\right]\left(\Lambda_\theta+\Psi_\theta\right)^{2k-1}+\\ &b\left[\frac{(N-P)\left(N\cos^2\theta-P\sin^2\theta\right)}{\Lambda_\theta}-\frac{(Q-R)\left(Q\cos^2\theta-R\sin^2\theta\right)}{\Psi_\theta}\right]\left(\Lambda_\theta-\Psi_\theta\right)^{2k-1}\end{aligned} \quad (8)$$

The expression of $\tilde{Y}_b$ is the theoretical yield stress corresponding to biaxial tension is:

$$\tilde{Y}_b = \frac{Y}{F_b} \tag{9}$$

where:

$$F_b = \left[a\left(\Lambda_b + \Gamma_b\right)^{2k} + a\left(\Lambda_b - \Gamma_b\right)^{2k} + b\left(\Lambda_b + \Psi_b\right)^{2k} + b\left(\Lambda_b - \Psi_b\right)^{2k} \right]^{\frac{1}{2k}} \tag{10}$$

The theoretical coefficient of biaxial plastic anisotropy is calculated as follow:

$$\tilde{r}_b = \frac{F_b^{2k}}{G_b} - 1 \tag{11}$$

where:

$$G_b = a\left[\frac{N(N-P)}{\Lambda_b} + L\right]\left(\Lambda_b + \Gamma_b\right)^{2k-1} + a\left[\frac{N(N-P)}{\Lambda_b} - L\right]\left(\Lambda_b - \Gamma_b\right)^{2k-1} + b\left[\frac{N(N-P)}{\Lambda_b} + \frac{Q(Q-R)}{\Psi_b}\right]\left(\Lambda_b + \Psi_b\right)^{2k-1} + b\left[\frac{N(N-P)}{\Lambda_b} - \frac{Q(Q-R)}{\Psi_b}\right]\left(\Lambda_b - \Psi_b\right)^{2k-1} \tag{12}$$

Now all the parameters needed to construct the identification conditions (see Eqns (3)) have been determined. The identification procedure uses Newton's method to obtain its numerical solution.

3 IMPLEMENTATION OF THE BBC 2005 MODEL IN THE AUTOFORM 4.1 PROGRAM

The identification procedure described in the previous section is implemented in the "Material Generator" module of the sheet metal forming software AUTOFORM 4.1. The user has to enter the six material parameters Y_0, Y_{45}, Y_{90}, r_0, r_{45}, r_{90}, that are routinely measured in conventional tensile tests. The input of the biaxial material parameters Y_b and r_b is optional. However, as will be demonstrated in the next section, at least Y_b should be accurately measured in order to take full advantage of the BBC2005 model.

The finite element implementation of the 2D plane stress elastic plastic constitutive equations follows [9]. The constitutive equations are integrated over a finite time step with help of the General Closest Point Projection Method (GCPPM) of Simo. The first and second order derivatives of Eqn. (1) that are needed for the GCPPM are computed in closed form; no numerical differentiation is employed. The increase in total computation time compared to a simulation with the Hill48 model is between 5 and 15 % for a typical AUTOFORM simulation.

4 RESULTS OF THE SIMULATIONS

4.1 BULGE TEST

In an EGKS European research project [10], an experimental program was conducted consisting of tensile tests and cruciform tests for material characterization, complemented by bulge tests for validation. In the following, results for DC04-IF (0.81 mm sheet thickness) are presented.

4.1.1 Material data

Hardening parameters for a combined Swift/HS law, Eqn. (13), are given in Table 1. Anisotropy parameters are listed in Table 2. All stress units are in MPa.

$$\sigma = (1-\alpha)\{C(\varepsilon_{pl} + \varepsilon_0)^m\} + \alpha\{\sigma_{Sat} - (\sigma_{Sat} - \sigma_i)e^{a\varepsilon_{pl}^p}\} \tag{13}$$

***Table 1:** Hardening parameters of DC04-IF*

α	ε_0	m	C	σ_i	σ_{Sat}	a	p
0.50	0.0044	0.27	580	140	432	5.47	0.46

***Table 2:** Anisotropy parameters of DC04-IF*

Y_0	Y_{45}	Y_{90}	Y_b	r_0	r_{45}	r_{90}	r_b
137	142	142	154	1.9	1.8	2.5	0.76

In Figure 1 yield surfaces of the Hill48 and BBC2005 models are displayed in the principal stress plane. Within the Hill48 model, the biaxial yield stress is fully determined by the r-values, $Y_{b|Hill48}$ = 176 MPa. On the other hand, the BBC2005 model has the flexibility of exactly taking the measured biaxial yield stress Y_b = 154 MPa into account. Note that in Figure 1 not the initial yield surface but the yield surface corresponding to an equivalent plastic strain of 0.05 is plotted.

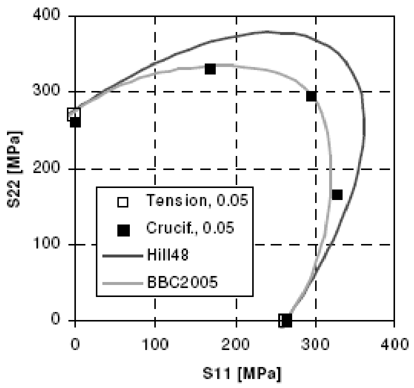

***Figure 1:** Yield surfaces for DC04-IF*

4.1.2 Simulation of bulge tests

In the project [10], the bulge equipment had an internal diameter of 100 mm and a tool radius of 2 mm. The sheets were gridded to allow optical strain measurements. For the DC04-IF sheets, the bulge height of 20 mm was reached at an internal pressure of 7.75 MPa.

The simulations are run with the Hydromech-module of AUTOFORM 4.1, both with the Hill48 and the BBC2005 models. Three node shell elements with 5 integration points through the thickness are used. The initial element size is set to 8 mm in all simulations, and adaptive refinement with "accuracy fine" is used.

It is to be expected for the bulge test that the simulation results are very sensitive to the choice of the biaxial stress point in the yield surface model. In fact, the simulation with the BBC2005 matches the measured strain values much better than the Hill48 simulation, see Figure 2.

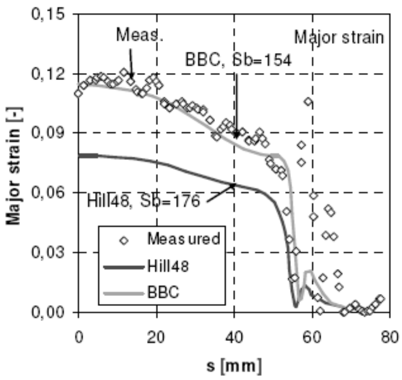

Figure 2: *Measured and computed major strain values for bulge test with DC04-IF*

4.2 CROSS DIE TEST

A material characterization programme with tensile and bulge tests was carried out in the European project "FOMM" [11], accompanied by experiments with cross dies. In this section, results will be presented for the mild steel DC04 (sheet thickness 0.79 mm) and the aluminium alloy Ac121-T4 (sheet thickness 1.01 mm).

4.2.1 Material data

Material parameters are given in Table 3 to Table 6. Yield surfaces are presented in Figure 3 to Figure 4.

Table 3: *Hardening parameters of DC04*

α	ε_0	m	C	σ_i	σ_{Sat}	a	p
0.5	0.0061	0.26	561	153	415	6.13	0.8

Table 4: *Anisotropy parameters of DC04*

Y_0	Y_{45}	Y_{90}	Y_b	r_0	r_{45}	r_{90}	r_b
151	166	163	192	1.83	1.39	2.11	0.87

Table 5: *Hardening parameters of Ac-121-T4*

α	ε_0	m	C	σ_i	σ_{Sat}	a	p
0.75	0.0070	0.29	482	130	330	9.08	0.96

Table 6: *Anisotropy parameters of Ac121-T4*

Y_0	Y_{45}	Y_{90}	Y_b	r_0	r_{45}	r_{90}	r_b
126	122	121	137	0.65	0.40	0.77	0.67

For the aluminium alloy Ac121-T4 the Barlat89 yield surface model is used as a reference. The BC2005 model has the flexibility to describe both the Y_0 and Y_{90} values from tension tests and the Y_b value from the bulge test. On the other hand, the yield surface of the Hill48/Barlat89 models is fully determined by the r-values and the Y_{90} and Y_b values are not matched by the model.

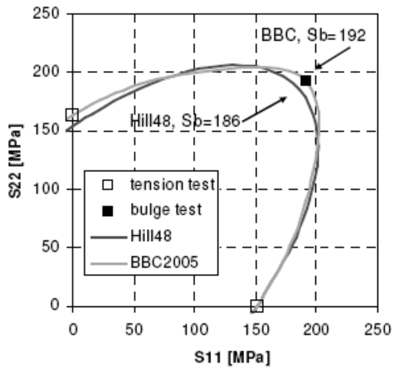

Figure 3: *Yield surfaces for DC04*

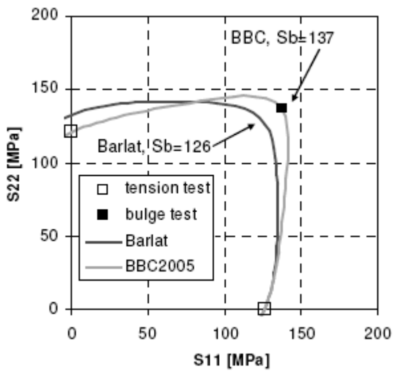

Figure 4: *Yield surfaces for Ac121-T4*

4.2.2 Simulation of cross die tests

The result of a cross die simulation is shown in Figure 5, together with two cuts where thickness measurements were done in the "FOMM" project. The tests are simulated with AUTOFORM 4.1 using all available experimental input (tool and blank geometries, lubrication, blank-holder force) and three node shell elements with 5 layers, adaptive refinement and "accuracy fine" settings. The cross die geometry is very sensitive to details in the material modelling, especially to the Y_b value. In Figure 6, the simulation results for the

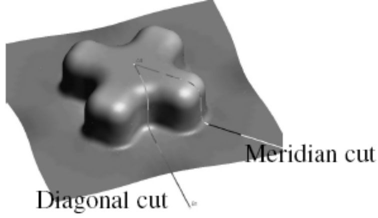

Figure 5: *Geometry of cross die test*

Hill and the BBC model deviate, although the differences in the yield surface description appear to be rather small (see Figure.3). For the Ac121-T4 material, see Figure 7, the deviations between the

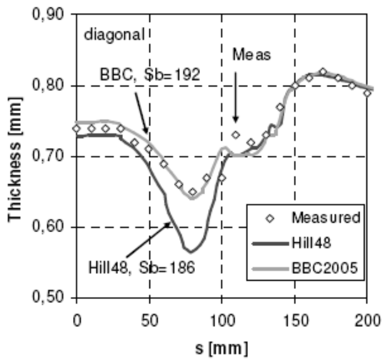

Figure 6: *Measured and computed thickness for cross die test with DC04 (diagonal cut)*

simulations with the Barlat and the BBC model are even larger. For both materials, the measured thickness is described more accurate with the BBC model than with Hill48/Barlat89.

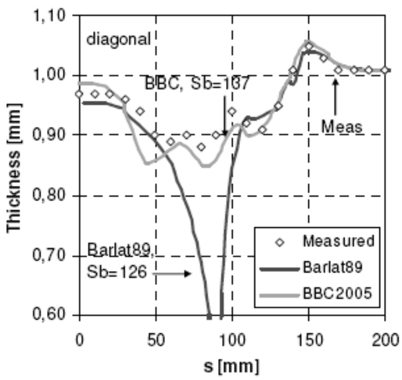

Figure 7: *Measured and computed thickness for cross die test with Ac121-T4 (diagonal cut)*

4.2.3 Sensitivity analysis

Finally, it was examined with help of the AUTOFORM 4.1 SIGMA module how sensitive the cross die simulation results react to a variation in the Y_b and r_b values. The results are presented in Figure 8 for the DC04 material.

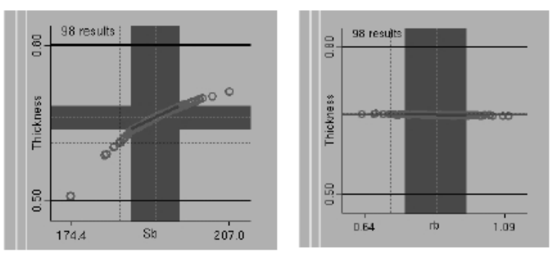

Figure 8: *Sensitivity of computed thickness in diagonal cut at s = 80 mm with respect to Y_b and r_b*

Compared with the influence of Y_b, the influence of r_b is insignificant. This conclusion also holds for automotive industry parts (not shown here). Therefore, we would suggest for practical use of the BBC2005 model in AUTOFORM 4.1 that r_b values need not to be measured.

4.3 AUTOMOTIVE INDUSTRY CASE

In this case, a H180BD was the material sample. The anisotropy parameters used in the simulations are presented in Table 6. The value of the 2k has been choose 5. These parameters have been determined at Volvo Cars with experimental data from tensile test, viscous bulge test [12], LDH test and stretch forming test with a spherical punch.

Table 6: *Anisotropy parameters of H180BD*

Y_0	Y_{45}	Y_{90}	Y_b	r_0	r_{45}	r_{90}	r_b
188	205	192	227	2.09	1.03	2.59	0.97

Generally, not all data in Table 6 is always available for all materials. Therefore, it is interesting to compare three different set-ups:

1. Hill `48 material model. Only Y_0, r_0, r_{45} and r_{90} are used. In this case 2k equals 2. This is how the majority of industrial simulations are done today.
2. BBC2005 with six parameters from tensile tests: Y_0, Y_{45}, Y_{90}, r_0, r_{45} and r_{90}. Y_b, r_b and 2k are the values predicted by AUTOFORM.
3. BBC2005 with all data in Table 6

The differences between the yield loci with these three different settings are small. It is therefore easy to come to the conclusion that they should produce similar results. In order to test this conclusion, simulations of the forming of an outer trunklid with all three set-ups have been performed in AUTOFORM 4.1.

The simulation results seem very similar when compared, but there is one area where the results are quite different. The strain signatures for this area in the three different set-ups are displayed in Figure 9. The difference between the Hill `48 set-up and the BBC set-ups are huge. With Hill '48 there is clearly a failure; the strains are actually so large that AUTOFORM removes elements. With the two set-ups of BBC2005, the strains are close to the FLC and therefore the area is critical, but there is no fracture. Production tests have been made with the trunklid and the material used in simulations. In the area corresponding to Figure 4.8, the material has been subjected to very large plastic deformations but there was no fracture.

There is also a small difference in strain signature between only using tensile test data and using all data in Table 6. A comparison between measured thickness on the test part and predicted thicknesses in the simulations revealed that the results with all data in Table 6 have better agreement with the production test than only using tensile test data.

The final conclusion is that although the yield loci are similar in this case, the BBC2005 model predictions are much closer to the test results than the Hill `48 predictions. This is largely due to the fact that the BBC 2005 model gives a much better prediction of both the uniaxial yield stress and the

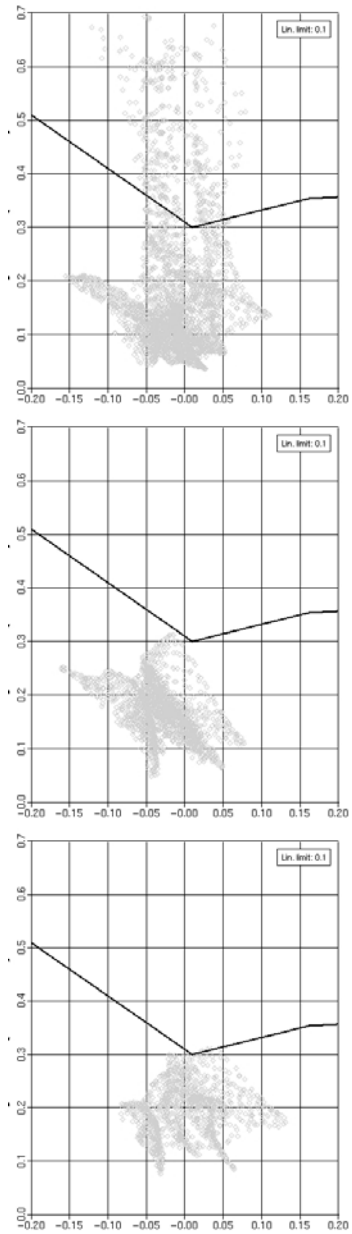

Figure 9: *Strain signature with Hill `48 (top); BBC2005 with tensile test data (middle); BCC2005 with all parameters (bellow).*

anisotropy coefficients in the sheet plane. The study also showed that using all data from the tensile test improved the accuracy of the simulation results, but in order fully take advantage of the possibilities with the BBC2005 material model, all the data in Table 6 are needed.

5 CONCLUSIONS

The results presented in the paper prove the ability of the BBC2005 yield criterion to provide an accurate description of the anisotropic behaviour both for steel and aluminium alloys. The performances of the model have been evaluated using the experimental data obtained from two benchmark tests (bulging and cross-die), as well as from an industrial forming process. In all cases the predictions of the BBC2005 model are in very good agreement with the experiments. This fact together with the flexibility of the identification procedure recommend the use of the BBC2005 yield criterion in industrial applications.

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