In: Proc. of 7th **COLD METAL FORMING** Conference (Ed. D. Banabic), May 11-12, 2000, Cluj Napoca, Romania, pag. 217-224. (http://www.utcluj.ro/conf/tpr2000)

A NEW YIELD CRITERION FOR ORTHOTROPIC SHEET METALS UNDER PLANE-STRESS CONDITIONS

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Abstract

The paper presents a new yield criterion for orthotropic sheet metals under plane-stress conditions. The criterion is derived from the one proposed by Barlat and Lian in 1989. Two additional coefficients have been introduced in order to allow a better representation of the plastic behaviour of the orthotropic sheet metals. The predictions of the new yield criterion are compared with the experimental data for two materials.

Keywords: sheet metals, anisotropy, yield criterion

1 Contents

The computer simulation of sheet metal forming processes needs a quantitative description of plastic anisotropy by the yield locus of the material. For taking into account the anisotropy, the von Mises classical yield criterion must be modified. A presentation of the historical development of the anisotropic yield criteria may be found in [1, 2].

In this work, the precise description of complex yielding behaviour exhibited by sheet metals is approached from the theoretical viewpoint. A new yield criterion combining the advantages of the Barlat and Karafillis-Boyce criteria is developed. The ability of the new criterion to represent the plastic behaviour of orthotropic sheet metals is investigated.

2 Equation of the yield surface

A yield surface is generally described by an implicit equation of the form

$$\Phi\left(\bar{s}, Y\right) := \bar{s} - Y = 0 \qquad (1)$$

where $\bar{s}$ is the equivalent stress and Y is a yield parameter. In practice, Y may be chosen as one of the following parameters of the sheet metal: s_0^{exp} (uniaxial yield stress along the rolling direction), s_{90}^{exp} (uniaxial yield stress along the transverse direction), s_{45}^{exp} (uniaxial yield stress at 45° from the rolling direction), an average of s_0^{exp}, s_{90}^{exp} and s_{45}^{exp}, or s_b^{exp} (equi-biaxial yield stress). The equivalent stress is defined by the following relationship:

$$\bar{s} = \left[a(b\Gamma + c\Psi)^{2k} + a(b\Gamma - c\Psi)^{2k} + (1-a)(2c\Psi)^{2k}\right]^{\frac{1}{2k}} \quad (2)$$

where a, b, c, and k are material parameters, while G and ? are functions of the second and third invariants of a fictitious deviatoric stress tensor **s'** which will be described later on. One may notice that the above expression of the equivalent stress is derived from the one proposed by Barlat and Lian for orthotropic materials under plane-stress state [3]. Two additional parameters, namely b and c, have been introduced in order to allow a better representation of the plastic behaviour of the sheet metal. The convexity of the yield surface described by Eqns (1) and (2) is ensured if a ∈ [0, 1] and k is a strictly positive integer number.

As we have already mentioned, G and ? are functions of the second and third invariants of a fictitious deviatoric stress tensor **s'**. This tensor is related to the actual stress tensor **s** by the Karafillis-Boyce linear transformation [4]:

$$\begin{aligned} &s'_{11} = d s_{11} + e s_{22}, \quad s'_{22} = e s_{11} + f s_{22}, \quad s'_{33} = -(d+e) s_{11} - (e+f) s_{22}, \\ &s'_{12} = g s_{12}, \quad s'_{21} = g s_{21}, \quad s'_{23} = s'_{32} \equiv 0, \quad s'_{31} = s'_{32} \equiv 0 \end{aligned} \quad (3)$$

where d, e, f, and g are also material parameters. The components of the stress tensors in Eqns (3) are expressed in the system of orthotropic axes (1 is the rolling direction - RD, 2 is the transverse direction - TD, and 3 is the normal direction - ND).

The second and third invariants of the deviatoric tensor **s'** have the following expressions:

$$J'_2 = (s'_{gg})^2 - \det s'_{ab}, \quad J'_3 = -(\det s'_{ab}) s'_{gg} \quad (4)$$

where the Greek indices take the values 1 and 2. The quantities

$$I'_2 = s'_{gg}, \quad I'_3 = \det s'_{ab} \quad (5)$$

are not affected by the rotations that leave unchanged the third axis (ND). Thus, in the case of the plane-stress of sheet metals, we can use I'_2 and I'_3 instead of J'_2 and J'_3 in order to define the functions G and ? . We have adopted the following expressions for these functions:

$$\Gamma = I'_2, \quad \Psi = \left[\left(\frac{I'_2}{2}\right)^2 - I'_3\right]^{\frac{1}{2}} \quad (6)$$

By using Eqns (6), (5) and (3), we can express G and ? as explicit dependencies of the actual stress components:

$$\Gamma = M s_{11} + N s_{22}, \quad \Psi = \sqrt{(P s_{11} + Q s_{22})^2 + R s_{12} s_{21}} \quad (7)$$

where

$$M = d + e, \quad N = e + f, \quad P = \frac{d-e}{2}, \quad Q = \frac{e-f}{2}, \quad R = g^2 \quad (8)$$

The above equations show that the shape of the yield surface is defined by the material parameters a, b, c, d, e, f, g, and k. From these parameters, k has a distinct status. More precisely, its value is set in accordance with the crystallographic structure of the material [5]: k = 3 for BCC alloys, and k = 4 for FCC alloys. The other parameters are established in such a way that the constitutive equation associated to the yield surface reproduce as well as possible the plastic behaviour of the actual material. The procedure used for identifying the parameters a, b, c, d, e, f, and g is described in §4.

3 Flow rule

The flow rule associated to the yield surface presented in §1 is

$$\dot{e}^{p}_{ab} = l \frac{\partial \Phi}{\partial s_{ab}}, \quad a, b = 1, 2 \tag{9}$$

where $\dot{e}^{p}_{ab}$ are in-plane components of the plastic strain-rate tensor, and ? = 0 is a scalar multiplier. The values of the non-planar components of the plastic strain-rate tensor are restricted by the plane-stress condition and the isochoric character of the plastic deformation:

$$\dot{e}^{p}_{23} = \dot{e}^{p}_{32} \equiv 0, \quad \dot{e}^{p}_{31} = \dot{e}^{p}_{13} \equiv 0, \quad \dot{e}^{p}_{33} = -\left(\dot{e}^{p}_{11} + \dot{e}^{p}_{22}\right) \tag{10}$$

Assuming a purely isotropic hardening of the material, only one scalar state parameter is needed in order to describe the evolution of the yield surface. This parameter is the so-called equivalent plastic strain computed as a time-integral of the equivalent plastic strain-rate:

$$\bar{e}^{p} = \int_{0}^{t} \dot{\bar{e}}^{p} dt \tag{11}$$

The equivalent plastic strain-rate is defined by equating the power developed by to the stress tensor and the power associated to the equivalent stress:

$$\bar{s}\,\dot{\bar{e}}^{p} = s_{ab}\dot{e}^{p}_{ab} \tag{12}$$

Using the homogeneity of the equivalent stress (see Eqn (2)), one can prove that the scalar multiplier ? is in fact the equivalent plastic strain-rate. Thus, the flow rule (9) takes the following form:

$$\dot{e}^{p}_{ab} = \dot{\bar{e}}^{p} \frac{\partial \Phi}{\partial s_{ab}}, \quad a, b = 1, 2 \tag{13}$$

4 Identification procedure

The parameters a, b, c, d, e, f, and g in the expression of the equivalent stress are established in such a way that the constitutive equation associated to the yield surface reproduce as well as possible the following characteristics of the orthotropic sheet- metal: s_0^{exp} (yield stress obtained by a uniaxial tensile test along RD), s_{90}^{exp} (yield stress obtained by a uniaxial tensile test along TD), s_{45}^{exp} (yield stress obtained by a uniaxial tensile test along a direction equally

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inclined to RD and TD), s_b^{exp} (yield stress obtained by an equi-biaxial tensile test along RD and TD), r_0^{exp} (coefficient of plastic anisotropy associated to RD), r_{90}^{exp} (coefficient of plastic anisotropy associated to TD), and r_{45}^{exp} (coefficient of plastic anisotropy asociated to a direction equally inclined to RD and TD). There are as many conditions as the material parameters in the expression of the equivalent stress. Thus it is possible to obtain their values by solving a set of seven nonlinear equations. But this is a difficult approach, because the set of equations has multiple solutions. After several trials and comparisons with experimental data, we have concluded that the best solution is to avoid the strict enforcement of the restrictions mentioned above. A more effective strategy of identification is to impose the minimization of the following error function:

$$F(a,b,c,d,e)=\left(\frac{s_0}{s_0^{exp}}-1\right)^2+\left(\frac{s_{90}}{s_{90}^{exp}}-1\right)^2+\left(\frac{s_{45}}{s_{45}^{exp}}-1\right)^2+ \\ \left(\frac{s_b}{s_b^{exp}}-1\right)^2+\left(\frac{r_0}{r_0^{exp}}-1\right)^2+\left(\frac{r_{90}}{r_{90}^{exp}}-1\right)^2+\left(\frac{r_{45}}{r_{45}^{exp}}-1\right)^2 \qquad (14)$$

where s_0, s_{90}, s_{45}, s_b, r_0, r_{90}, and r_{45} are the uniaxial yield stresses, the equi-biaxial yield stress and the coefficients of plastic anisotropy predicted by the constitutive equation. In order to use the function defined by Eqn (14) in a minimization procedure, we need some formulas for calculating these quantities.

4.1 Prediction of the uniaxial yield stress

Let $s_f > 0$ be the yield stress obtained by the uniaxial tensile test of a specimen cut at an angle $f \in [0, 90°]$ with the rolling direction. In this case, the non-zero components of the stress tensor **s** (expressed in the system of orthtropic axes) are given by the following relationships:

$$s_{11}=s_j\cos^2 j,\quad s_{22}=s_j\sin^2 j,\quad s_{12}=s_{21}=s_j\sin j\cos j \qquad (15)$$

Eqns (1), (2), (7), and (15) allow the obtention of a formula for evaluating the uniaxial yield stress at different angles with the rolling direction:

$$s_j=\frac{Y}{\left[a\left(bA_j+cB_j\right)^{2k}+a\left(bA_j-cB_j\right)^{2k}+(1-a)\left(2cB_j\right)^{2k}\right]^{\frac{1}{2k}}} \qquad (16)$$

where

$$A_j=M\cos^2 j+N\sin^2 j\quad B_j=\sqrt{\left(P\cos^2 j+N\sin^2 j\right)^2+R\sin^2 j\cos^2 j} \qquad (17)$$

4.2 Prediction of the equibiaxial yield stress

Let $s_b > 0$ be the yield stress obtained by an equi-biaxial tensile test along RD and TD. The in-plane components of the stress tensor **s** are in this case as follows:

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$$s_{11} = s_{22} = s_b, \quad s_{12} = s_{21} = 0 \tag{18}$$

Eqns (1), (2), (7), and (18) lead to a formula for evaluating the equibiaxial yield stress:

$$s_b = \frac{Y}{\left[a\left(bA_b + cB_b\right)^{2k} + a\left(bA_b - cB_b\right)^{2k} + \left(1-a\right)\left(2cB_b\right)^{2k}\right]^{\frac{1}{2k}}} \tag{19}$$

where

$$A_b = M + N, \quad B_b = \left|P + Q\right| \tag{20}$$

4.3 Prediction of the r-coefficients

The coefficient of plastic anisotropy associated to a direction inclined at an angle f ∈ [0, 90°] with the rolling direction is defined as follows:

$$r_j = \frac{\dot{e}^{p}_{j+90°}}{\dot{e}^{p}_{33}} \tag{21}$$

where $\dot{e}^{p}_{j+90°}$ is the component of the plastic strain-rate tensor associated to a direction perpendicular to the longitudinal axis of the tensile specimen, and $\dot{e}^{p}_{33}$ is the component of the same tensor associated to DN. By using the volume constancy condition, we can rewrite Eqn(21) in the form

$$r_j = \frac{\dot{e}^{p}_{j}}{\dot{e}^{p}_{11} + \dot{e}^{p}_{22}} - 1 \tag{22}$$

where $\dot{e}^{p}_{j}$ is the component of the plastic strain-rate tensor along the specimen axis. Further on, $\dot{e}^{p}_{j}$ may be written as

$$\dot{e}^{p}_{j} = \dot{e}^{p}_{11}\cos^2 j + \dot{e}^{p}_{22}\sin^2 j + 2\dot{e}^{p}_{12}\sin j \cos j \tag{23}$$

By using Eqns (22), (23), (13), (1) and the homogeneity of the equivalent stress, we arrive at the following expression of the plastic anisotropy coefficient

$$r_j = \frac{Y}{s_j\left[\frac{\partial\Phi}{\partial\Gamma}\left(\frac{\partial\Gamma}{\partial s_{11}} + \frac{\partial\Gamma}{\partial s_{22}}\right) + \frac{\partial\Phi}{\partial\Psi}\left(\frac{\partial\Psi}{\partial s_{11}} + \frac{\partial\Psi}{\partial s_{22}}\right)\right]} - 1 \tag{24}$$

where s_f is given by Eqn (19) and

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$$\frac{\partial\Phi}{\partial\Gamma}=b\left(\frac{s_j}{Y}\right)^{2k-1}\left[a\left(bA_j+cB_j\right)^{2k-1}+a\left(bA_j-cB_j\right)^{2k-1}\right]$$

$$\frac{\partial\Phi}{\partial\Psi}=c\left(\frac{s_j}{Y}\right)^{2k-1}\left[a\left(bA_j+cB_j\right)^{2k-1}-a\left(bA_j-cB_j\right)^{2k-1}+2(1-a)\left(2cB_j\right)^{2k-1}\right] \qquad (25)$$

$$\frac{\partial\Gamma}{\partial s_{11}}=M, \quad \frac{\partial\Gamma}{\partial s_{22}}=N,$$

$$\frac{\partial\Psi}{\partial s_{11}}=\frac{P}{B_j}\left(P\cos^2 j+Q\sin^2 j\right) \quad \frac{\partial\Psi}{\partial s_{22}}=\frac{Q}{B_j}\left(P\cos^2 j+Q\sin^2 j\right)$$

Eqns (16), (19) and (24) are used in order to evaluate the quantities involved in the error function F. We have adopted the downhill simplex method proposed by Nelder and Mead [6] for the numerical minimization, because it does not need the evaluation of the gradients. The minimization procedure has been implemented into a computer programme written in the C language. The numerical results presented in the next section have been obtained using this programme.

5 Comparison with experiments

The predictions of the new yield criterion have been tested for two sorts of sheet metals: A6XXX-T4 and SPCE. The theoretical results have been compared with the experimental data published in [7, 8].

Table 1 shows the experimental values needed as input data by the computer programme used for the numerical identification of the material parameters involved in the expression of the yield criterion.

Table 1. Experimental values used as input data for the numerical identification of the material parameters for A6XXX-T4 and SPCE sheet metals

		A6XXX-T4	SPCE
s_0^{exp}	[MPa]	125	180
s_{90}^{exp}	[MPa]	122	184
s_{45}^{exp}	[MPa]	123	188
s_b^{exp}	[MPa]	121	184
r_0^{exp}		0.78	2.01
r_{90}^{exp}		0.53	2.42
r_{45}^{exp}		0.47	1.52
Y	[MPa]	125	180
k		4	3

Table2. Material parameters for A6XXX-T4 and SPCE sheet metals obtained by numerical identification

		A6XXX-T4	SPCE
a		0.6512	0.2115
b		0.9521	0.9941
c		0.0987	0.8390
M		0.4881	0.5811
N		0.5659	0.5571
P		5.2209	0.5923
Q		-5.2598	-0.5803
R		98.6719	1.1548
k		4	3
Y	[MPa]	125	180

Table 2 shows the values of the material parameters a, b, c, M, N, P, Q, and R obtained by numerical identification for the two sorts of sheet metals. The corresponding values of k and Y are also presented for completeness.

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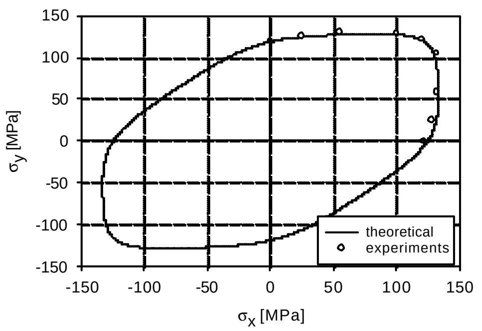

Fig. 1. Yield surface for A6XXX-T4

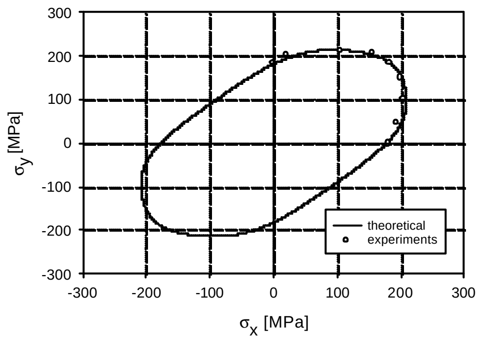

Fig. 2. Yield surface for SPCE

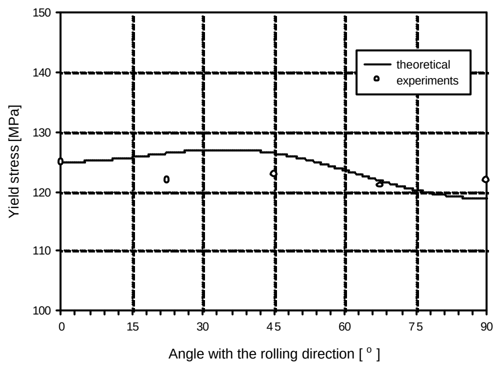

Fig. 3. Distribution of the uniaxial yield stress for A6XXX-T4

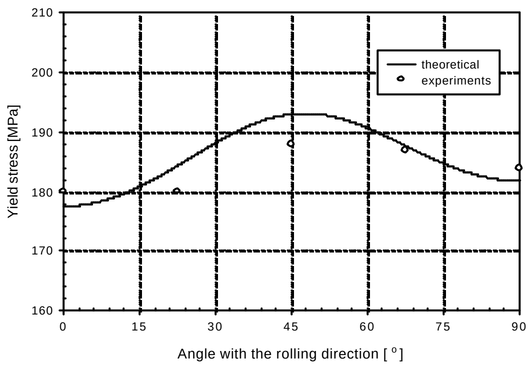

Fig. 4. Distribution of the uniaxial yield stress for SPCE

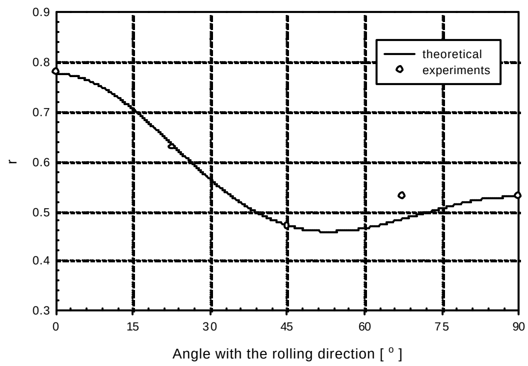

Fig. 5. Distribution of the r-coefficient for A6XXX-T4

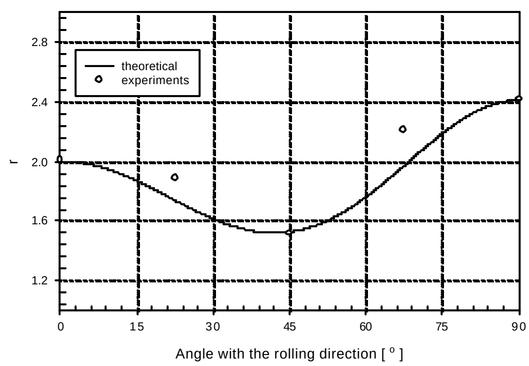

Fig. 6. Distribution of the r-coefficient for SPCE

The yield surfaces predicted by the new yield criterion for the A6XXX-T4 and SPCE sheet metals are presented in Figures 1 and 2, respectively. The experimental data are also plotted on the diagrams. The predicted distribution of the uniaxial yield stress with respect to the angle with the rolling direction is shown in Figures 3 and 4 for A6XXX-T4 and SPCE sheet metals, respectively. The predicted distribution of the r-coefficient with respect to the angle with the rolling direction is shown in Figures 5 and 6 for A6XXX-T4 and SPCE sheet metals, respectively.

6 Conclusions

A new yield criterion derived from the one introduced by Barlat and Lian [3] has been proposed. The new criterion has an increased flexibility due to the fact that it uses seven coefficients in order to describe the yield surface. The minimization of an error-function has been used for the numerical identication of the coefficients. The predicted yield surfaces for two materials (A6XXX-T4 and SPCE) are in very good agreement with the experimental data published by Kuwabara et al. [7, 8]. The associated flow rule predicts very accurately the distribution of the Lanckford coefficient and uniaxial yield stress, respectively.

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