Numerical Methods

Prof. Bogdan Gavrea

IETTI 2017

Lecture 1

Syllabus

- Transforms
  - Z–transform: discrete signals.
  - The Discrete Fourier Transform (DFT): discrete signals.
  - The Laplace Transform: continuous signals.
  - The Fourier Transform: continuous signals.

- Probability theory
  - Basic probability.
  - Conditional probability.
  - Random variables. Random vectors. Functions of random vectors.
  - Numerical characteristics of random variables: mean(expectation), variance, covariance, correlation coefficient.
  - Basic statistics.

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- l_1 and l_2 signals:

$$l_1 = \left\{ x \in S \;\middle|\; \sum_{n=-\infty}^{\infty} |x(n)| < \infty \right\},\ l_2 = \left\{ x \in S \;\middle|\; \sum_{n=-\infty}^{\infty} |x(n)|^2 < \infty \right\}$$

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6) **(Scaling)** Let $x \in S_+$ and $a \in C^*$. Then,

$$Z\left\{a^n x(n)\right\}(z) = X\left(\frac{z}{a}\right).$$