
Regimurile nesimetrice ale mașinii de inducție

Alimentare statorică nesimetrică

Metoda componentelor simetrice la mașina de inducție

Ipoteze :

- circuit magnetic liniar,
- parametrii constanți,
- regimul de motor,
- puterea rețelei foarte mare,
- regimurile sunt staționare.

Regimuri nesimetrice:

- rețeaua este nesimetrică, nesimetria nu depinde de sarcina motorului,
 - rețeaua este simetrică, nesimetria se crează la bornele mașinii fiind dependentă de sarcină.
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Metoda componentelor simetrice la mașina de inducție

1. Tensiunile sunt nesimetrice

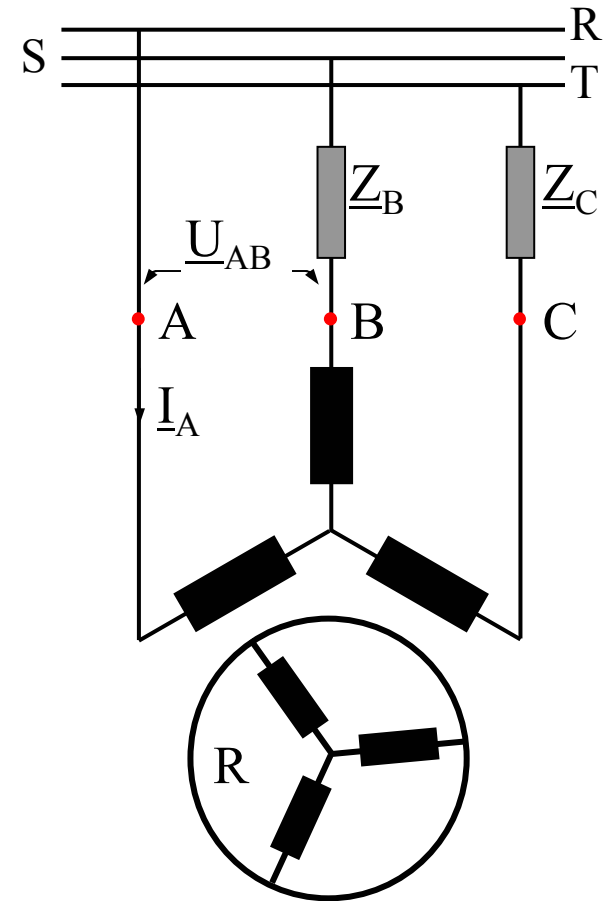
$$\begin{bmatrix} \underline{U}_d \\ \underline{U}_i \\ \underline{U}_h \end{bmatrix} = \frac{1}{3} \times \begin{vmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{vmatrix} \times \begin{bmatrix} \underline{U}_A \\ \underline{U}_B \\ \underline{U}_C \end{bmatrix}$$

2. Se crează nesimetria cu ajutorul unor impedanțe

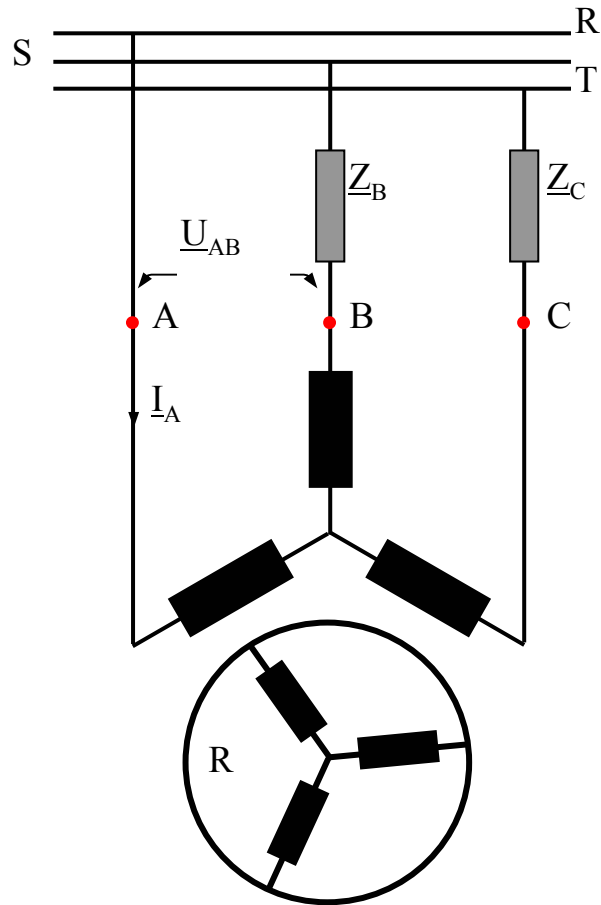
$$\underline{U}_A = \underline{U}_R$$

$$\underline{U}_B = \underline{U}_S - \underline{Z}_B \cdot \underline{I}_B$$

$$\underline{U}_C = \underline{U}_T - \underline{Z}_C \cdot \underline{I}_C$$



Metoda componentelor simetrice la mașina de inducție



$$\underline{U}_A = \underline{U}_R$$

$$\underline{U}_B = \underline{U}_S - \underline{Z}_B \cdot \underline{I}_B$$

$$\underline{U}_C = \underline{U}_T - \underline{Z}_C \cdot \underline{I}_C$$

$$\begin{bmatrix} \underline{Z}_d \\ \underline{Z}_i \\ \underline{Z}_h \end{bmatrix} = \frac{1}{3} \times \begin{vmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{vmatrix} \times \begin{vmatrix} \underline{Z}_A \\ \underline{Z}_B \\ \underline{Z}_C \end{vmatrix}$$

Metoda componentelor simetrice la mașina de inducție

Componentele simetrice ale tensiunii de alimentare:

$$\begin{bmatrix} \underline{U}_d \\ \underline{U}_i \\ \underline{U}_h \end{bmatrix}_A = \begin{bmatrix} \underline{U}_R \\ 0 \\ 0 \end{bmatrix} - \begin{vmatrix} \underline{Z}_h & \underline{Z}_i & \underline{Z}_d \\ \underline{Z}_d & \underline{Z}_h & \underline{Z}_i \\ \underline{Z}_i & \underline{Z}_d & \underline{Z}_h \end{vmatrix} \times \begin{vmatrix} \underline{I}_d \\ \underline{I}_i \\ \underline{I}_h \end{vmatrix}_A$$

Ecuatiile de tensiuni ale motorului de inducție

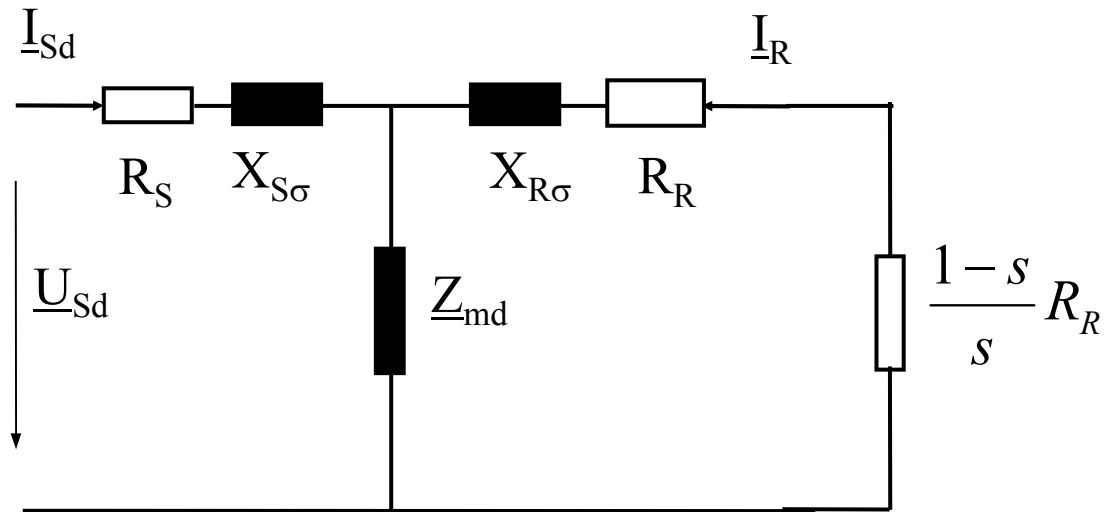
$$\underline{U}_d = \underline{Z}_{ed} \cdot \underline{I}_d$$

$$\underline{U}_i = \underline{Z}_{ei} \cdot \underline{I}_i$$

$$\underline{U}_h = \underline{Z}_{eh} \cdot \underline{I}_h$$

Parametrii mașinii de inducție

1. Sistem simetric de succesiune directă.



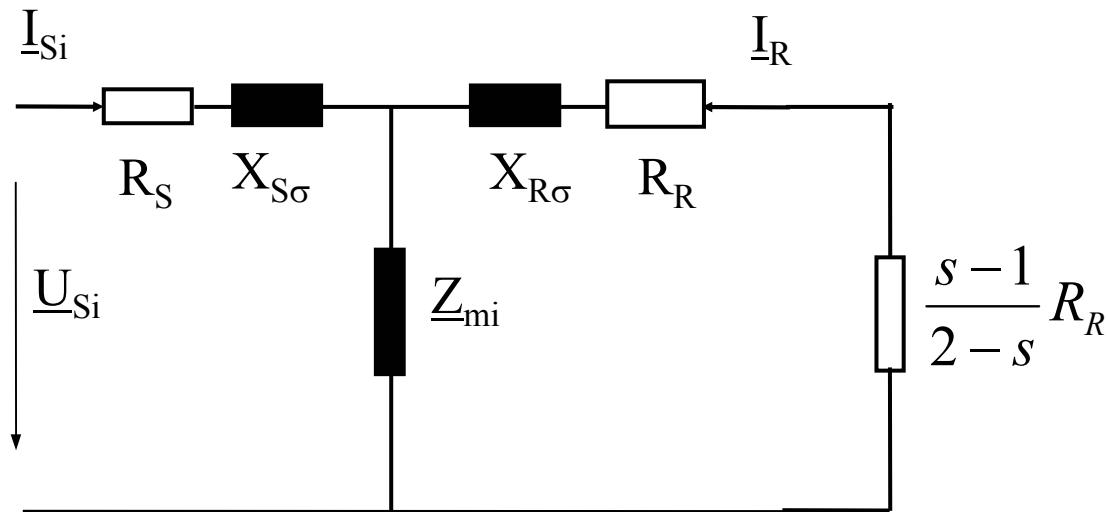
$$\underline{Z}_{ed} = \underline{Z}_S + \frac{\underline{Z}_{md} \cdot \underline{Z}_{Rd}}{\underline{Z}_{md} + \underline{Z}_{Rd}}$$

$$\underline{Z}_{Rd} = \frac{R_R}{s} + jX_{R\sigma}$$

Parametrii mașinii de inducție

2. Sistem simetric de succesiune inversă

Alunecarea $s_i = 2-s$



$$\underline{Z}_{ei} = \underline{Z}_S + \frac{\underline{Z}_{mi} \cdot \underline{Z}_{Ri}}{\underline{Z}_{mi} + \underline{Z}_{Ri}}$$

$$\underline{Z}_{Ri} = \frac{R_R}{2-s} + jX_{R\sigma}$$

Parametrii mașinii de inducție

3. Sistemul homopolar

In statorul mașinii pot exista curenți homopolari numai la conexiunea înfășurărilor în stea cu nul și triunghi întrerupt.

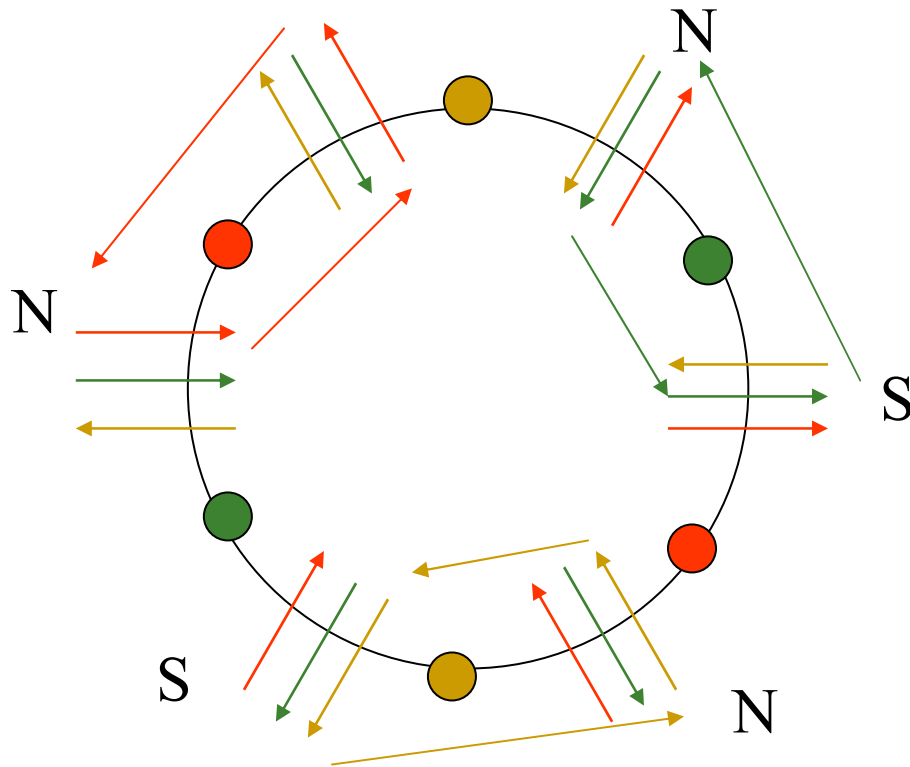
In rotorul mașinii pot exista curenți homopolari numai la înfășurarea în colivie. In acest caz înfășurarea fiind diametrală reactanța homopolară a rotorului este:

$$X_{Rh} = X_{R\sigma} + X_{arm}$$

Dacă există simultan în stator și în rotor curenți homopolari, atunci produc un **cuplu homopolar**, care are caracterul unui cuplu asincron.

Parametrii mașinii de inducție

S



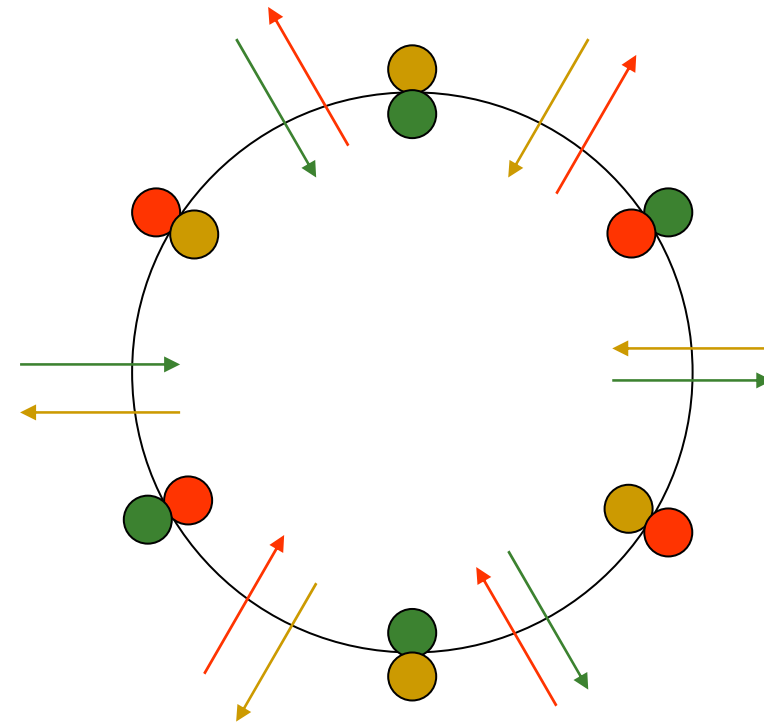
Câmpul homopolar la înfășurare într-un strat, câmp pulsator cu

$$p_h = 3 \cdot p$$

$$X_h = X_{S\sigma} + X_{\sigma arm}$$

$$R_h \approx R_S$$

$$X_h = X_{\sigma arm}$$



Câmpul homopolar la înfășurarea în dublu strat cu scurtare $y = 2/3 \tau$

$$p_h = 3 \cdot p$$

$$X_h = X_{S\sigma} + X_{\sigma arm}$$

$$R_h \approx R_S$$

$$X_h = X_{\sigma arm}$$

Regim nesimetric dependent de sarcină

Se consideră o mașina de inducție cu înfășurarea conectată în stea.
în stator nu există curent homopolar

$$\begin{aligned} \underline{U}_d &= \underline{Z}_{ed} \cdot \underline{I}_d \\ \underline{U}_i &= \underline{Z}_{ei} \cdot \underline{I}_i \\ \underline{U}_h &= \underline{Z}_{eh} \cdot \underline{I}_h \end{aligned} \quad \begin{bmatrix} \underline{U}_d \\ \underline{U}_i \\ \underline{U}_h \end{bmatrix}_A = \begin{bmatrix} \underline{U}_R \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \underline{Z}_h & \underline{Z}_i & \underline{Z}_d \\ \underline{Z}_d & \underline{Z}_h & \underline{Z}_i \\ \underline{Z}_i & \underline{Z}_d & \underline{Z}_h \end{bmatrix} \times \begin{bmatrix} \underline{I}_d \\ \underline{I}_i \\ \underline{I}_h \end{bmatrix}_A$$

$$\underline{Z}_{ed} \cdot \underline{I}_d = \underline{U}_R - \underline{Z}_h \cdot \underline{I}_d - \underline{Z}_i \cdot \underline{I}_i$$

$$\underline{Z}_{ei} \cdot \underline{I}_i = -\underline{Z}_d \cdot \underline{I}_d - \underline{Z}_h \cdot \underline{I}_i$$

$$\underline{U}_R = (\underline{Z}_{ed} + \underline{Z}_h) \cdot \underline{I}_d + \underline{Z}_i \cdot \underline{I}_i$$

$$0 = \underline{Z}_d \cdot \underline{I}_d + (\underline{Z}_{ei} + \underline{Z}_h) \cdot \underline{I}_i$$

Regim nesimetric dependent de sarcină

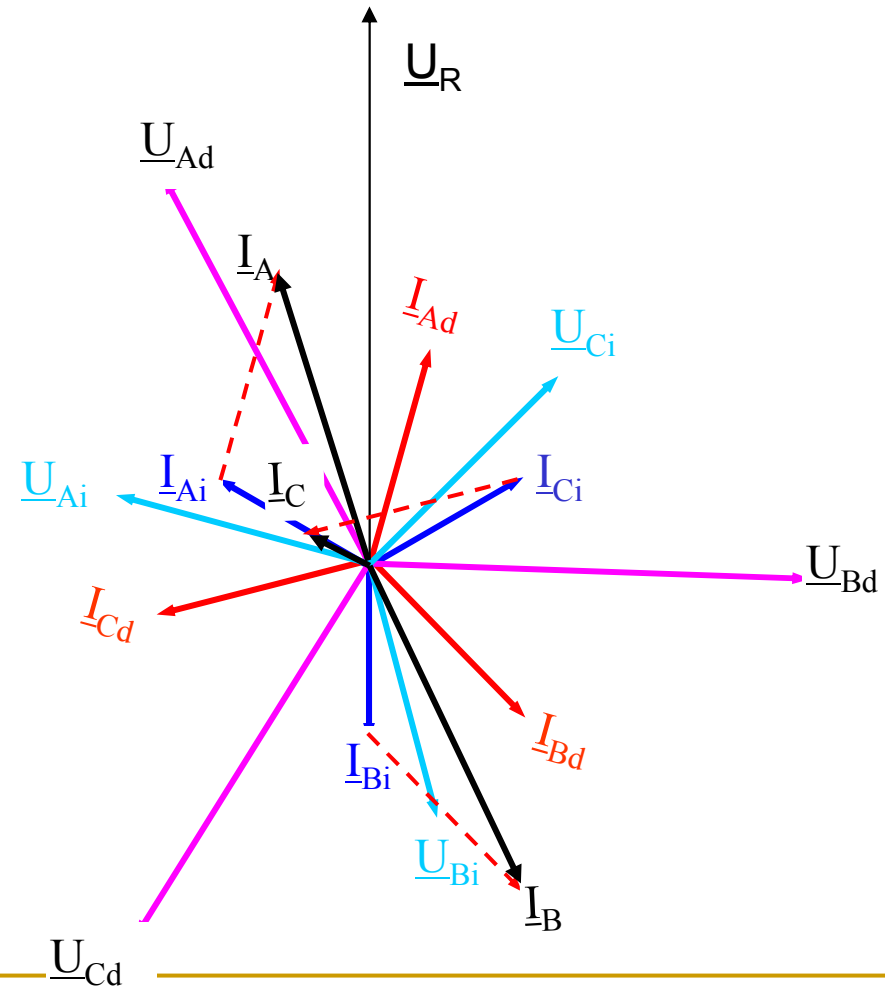
$$\underline{\Delta} = (\underline{Z}_{ed} + \underline{Z}_h) \cdot (\underline{Z}_{ei} + \underline{Z}_h) - \underline{Z}_d \cdot \underline{Z}_i$$

$$\underline{I}_{Ad} = \frac{\underline{U}_R \cdot (\underline{Z}_{ei} + \underline{Z}_h)}{\underline{\Delta}}$$

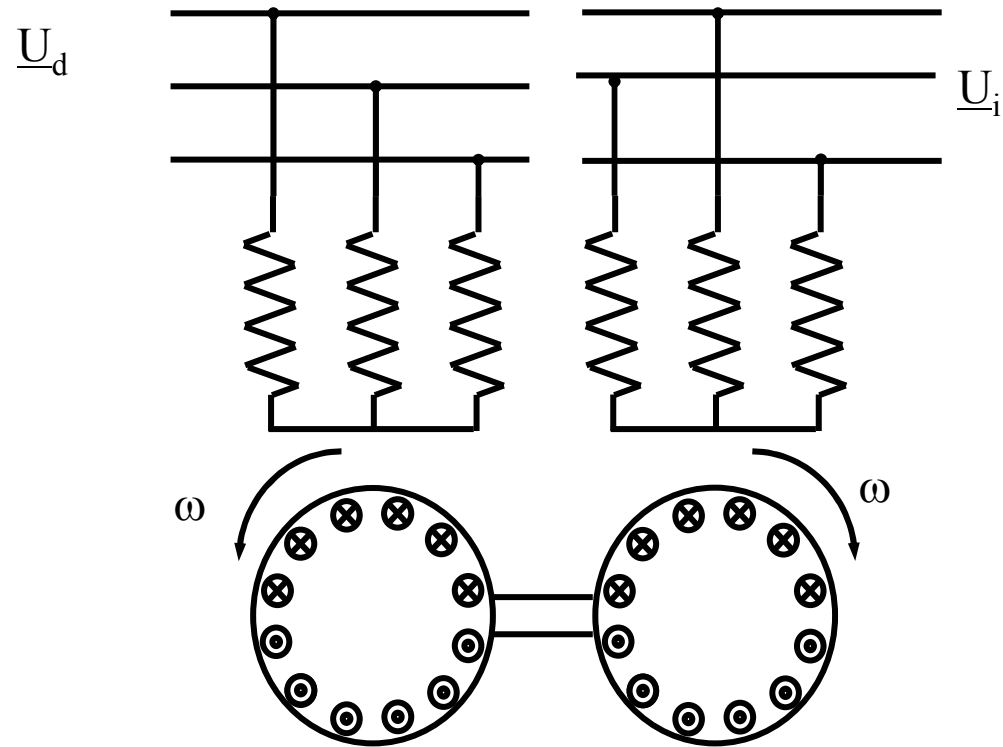
$$\underline{I}_{Ai} = -\frac{\underline{U}_R \cdot \underline{Z}_d}{\underline{\Delta}}$$

$$\underline{U}_d = \underline{Z}_{ed} \cdot \underline{I}_d$$

$$\underline{U}_i = \underline{Z}_{ei} \cdot \underline{I}_i$$

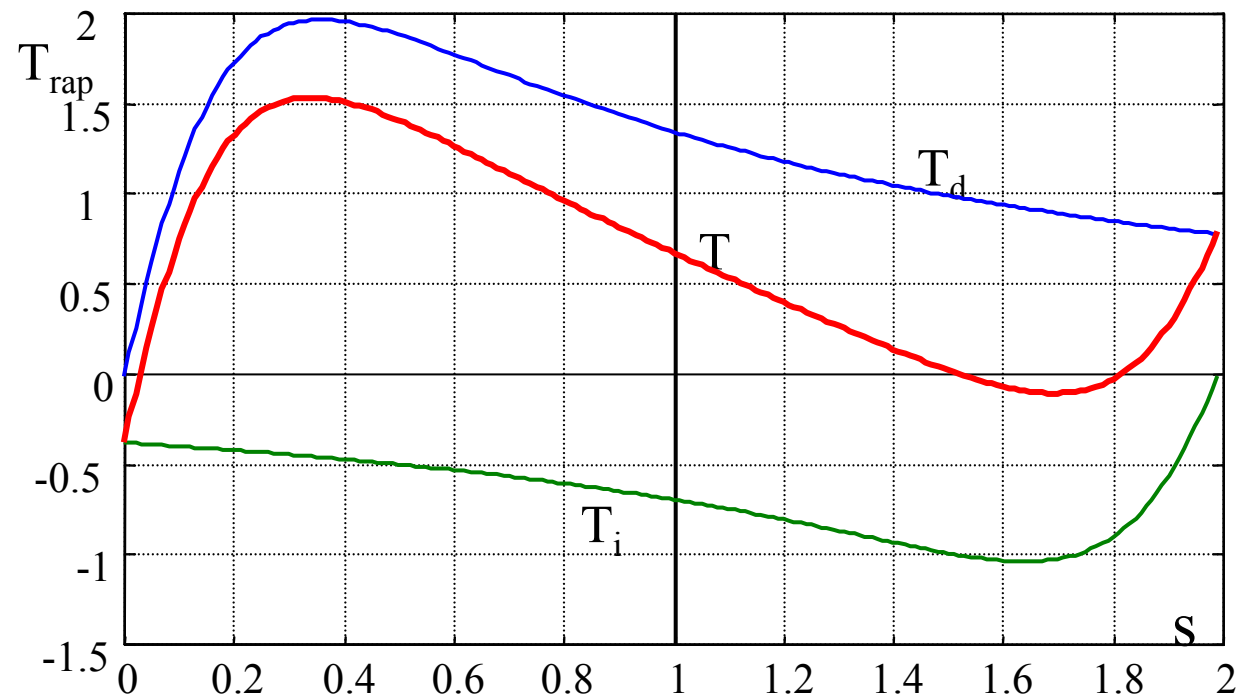


Regim nesimetric dependent de sarcină

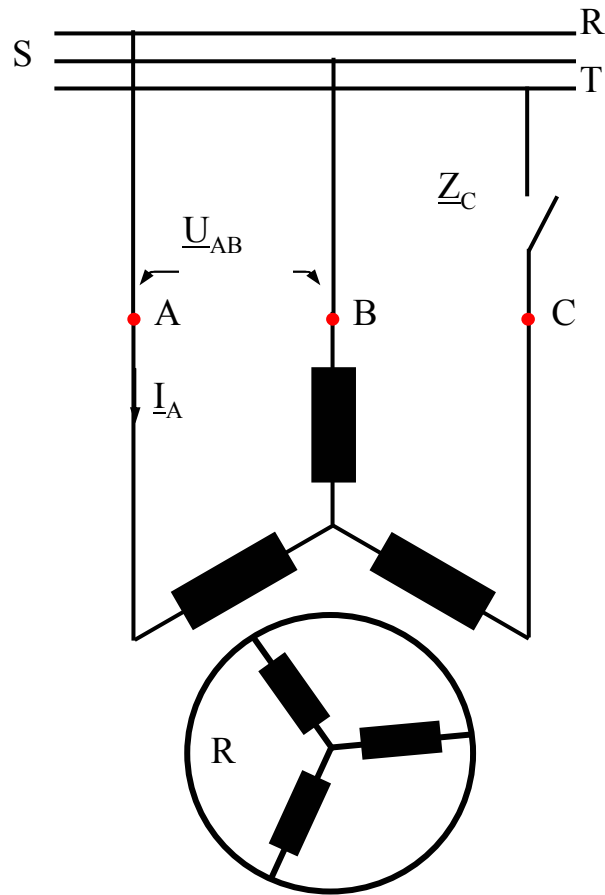


Cuplul electromagnetic

$$T_d = \frac{p \cdot m}{\omega} \frac{\frac{R_R}{s} U_d^2}{\left(R_S + \frac{R_R}{s}\right)^2 + (X_{S\sigma} + X_{R\sigma})^2} \quad T_i = \frac{p \cdot m}{\omega} \frac{\frac{R_R}{2-s} U_i^2}{\left(R_S + \frac{R_R}{2-s}\right)^2 + (X_{S\sigma} + X_{R\sigma})^2}$$



Regimul nesimetric cu o fază întreruptă



$$\underline{Z}_B = 0 \quad \underline{Z}_C \rightarrow \infty$$

$$\begin{bmatrix} \underline{Z}_d \\ \underline{Z}_i \\ \underline{Z}_h \end{bmatrix} = \frac{1}{3} \times \begin{vmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{vmatrix} \times \begin{vmatrix} \underline{Z}_A \\ \underline{Z}_B \\ \underline{Z}_C \end{vmatrix}$$

$$\underline{Z}_d = \frac{1}{3} a^2 \cdot \underline{Z}_C$$

$$\underline{Z}_i = \frac{1}{3} a \cdot \underline{Z}_C$$

$$\underline{Z}_h = \frac{1}{3} \cdot \underline{Z}_C$$

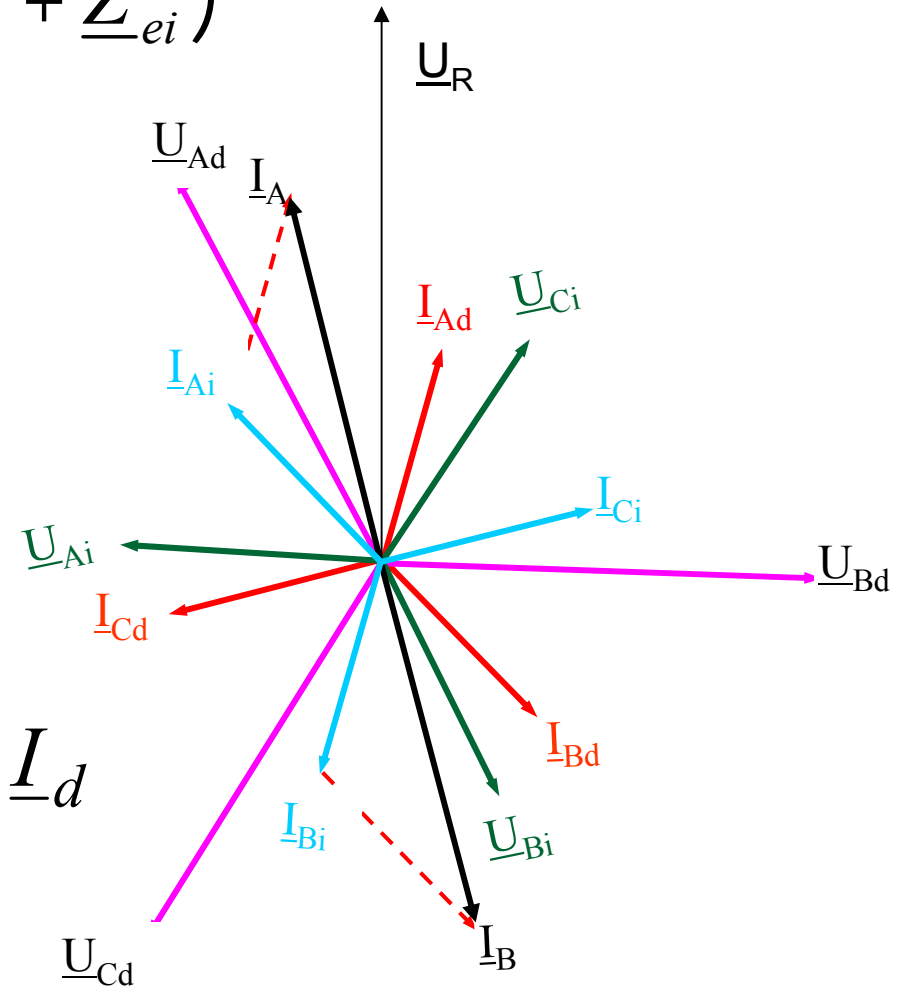
Regimul nesimetric cu o fază întreruptă

$$\underline{\Delta} = \underline{Z}_{ed} \cdot \underline{Z}_{ei} + \underline{Z}_h \cdot (\underline{Z}_{ed} + \underline{Z}_{ei})$$

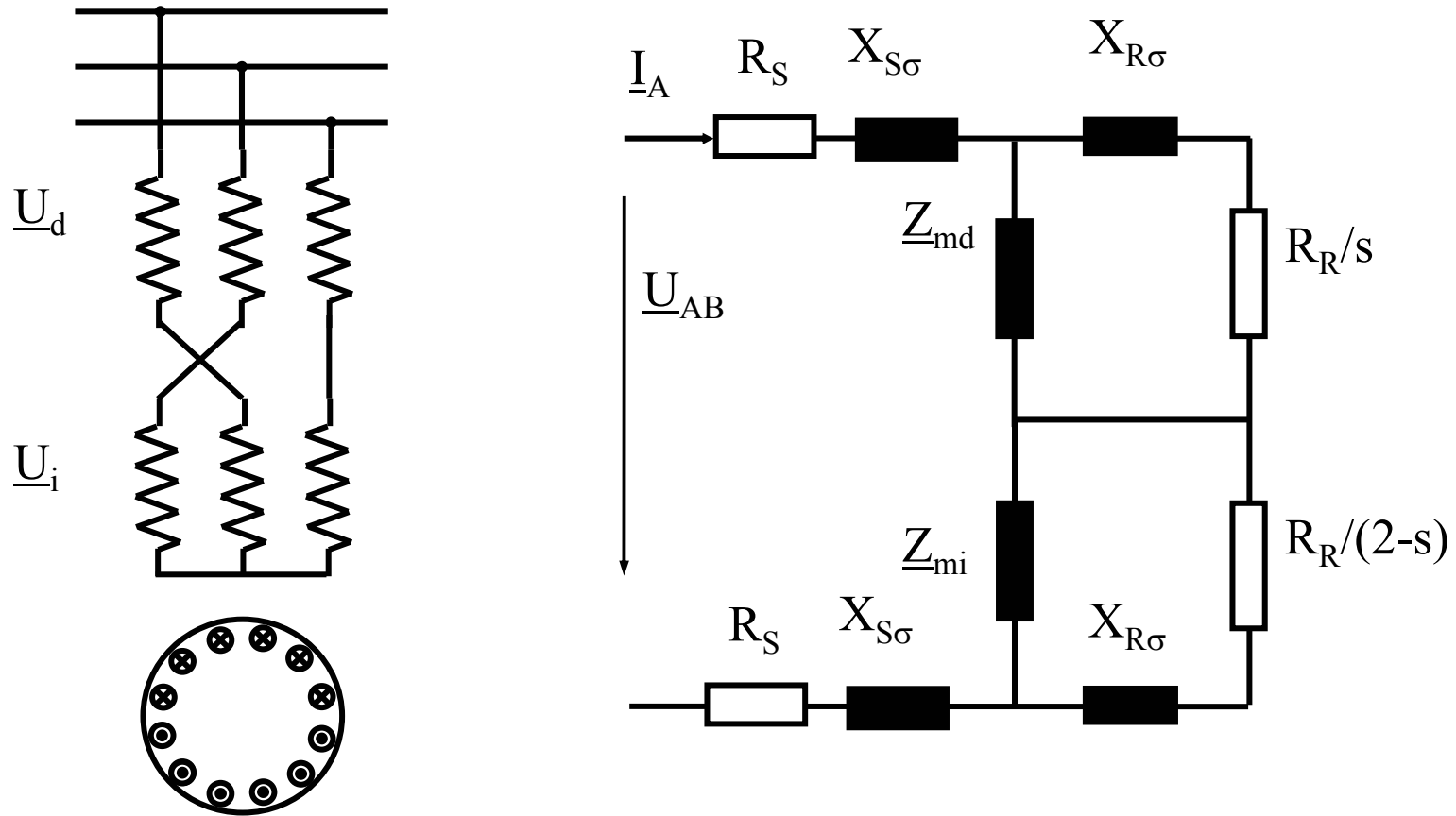
$$\underline{Z}_d \cdot \underline{Z}_i = \underline{Z}_h^2$$

$$\underline{I}_d = \frac{\underline{U}_R}{\underline{Z}_{ed} + \underline{Z}_{ei}}$$

$$\underline{I}_i = -\frac{a^2 \cdot \underline{U}_R}{\underline{Z}_{ed} + \underline{Z}_{ei}} = -a^2 \cdot \underline{I}_d$$



Regimul nesimetric cu o fază întreruptă

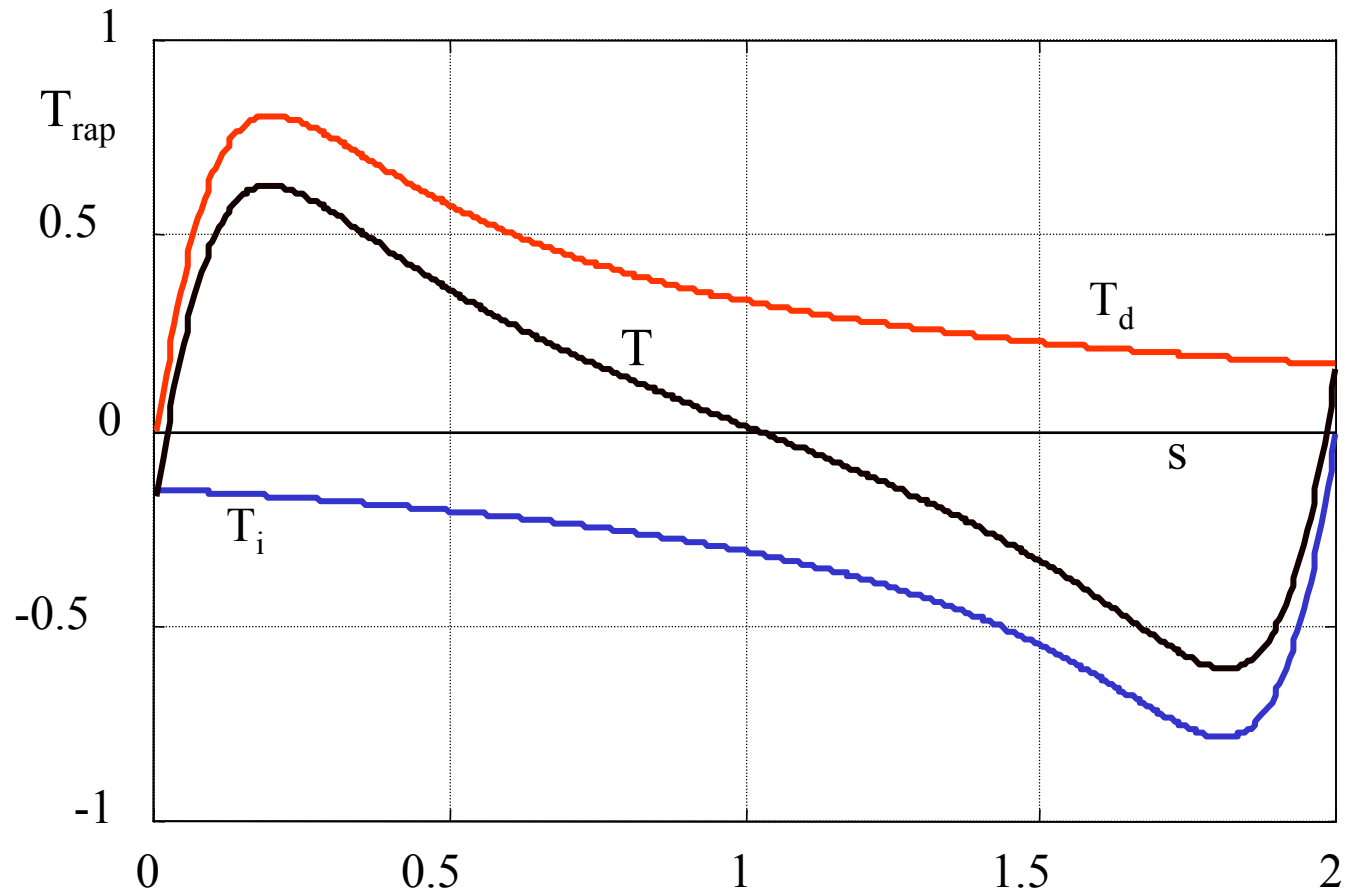


Regimul nesimetric cu o fază întreruptă

$$T_d = \frac{p \cdot m}{\omega} \frac{\frac{R_R}{s} U_d^2}{\left(R_S + \frac{R_R}{s} \right)^2 + \left(X_{S\sigma} + X_{R\sigma} \right)^2}$$

$$T_i = \frac{p \cdot m}{\omega} \frac{\frac{R_R}{2-s} U_i^2}{\left(R_S + \frac{R_R}{2-s} \right)^2 + \left(X_{S\sigma} + X_{R\sigma} \right)^2}$$

Regimul nesimetric cu o fază întreruptă



Exemplu

Datele motorului: $p = 3$

$$R_s := 1.1 \quad \Omega \quad R_r := 1.4 \quad \Omega \quad R_m := 2.8 \quad \Omega$$

$$X_{s\sigma} := 2.4 \quad \Omega \quad X_{r\sigma} := 3 \quad \Omega \quad X_m := 39.8 \quad \Omega$$

Parametrii de funcționare

$$U_s := 380 \quad V \quad s_d := 0.045$$

Impedanțe înseriate

$$Z_a := 0 \quad Z_b := 0 \quad Z_c := 1 + 14 \cdot i \quad \Omega$$

Impedanțe de succesiune simetrice:

$$Z_d := \frac{a^2 \cdot Z_c}{3} \quad Z_i := a \cdot \frac{Z_c}{3} \quad Z_h := \frac{Z_c}{3}$$

$$Z_d = 3.875 - 2.622i \quad \Omega \quad Z_i = -4.208 - 2.045i \quad \Omega \quad Z_h = 0.333 + 4.667i \quad \Omega$$

Exemplu

Impedanțele motorului

$$Z_s := R_s + X_{s\sigma} \cdot 1i$$

$$Z_{rd} := X_{r\sigma} \cdot 1i + \frac{R_r}{s_d}$$

$$Z_m := R_m + X_m \cdot 1i$$

$$Z_{ri} := X_{r\sigma} \cdot 1i + \frac{R_r}{2 - s_d}$$

$$Z_{ed} := Z_s + Z_m \cdot \frac{Z_{rd}}{(Z_m + Z_{rd})}$$

$$Z_{ei} := Z_s + Z_m \cdot \frac{Z_{ri}}{Z_m + Z_{ri}}$$

$$Z_{ed} = 18.626 + 17.041i \quad \Omega$$

$$Z_{ei} = 1.733 + 5.195i \quad \Omega$$

Exemplu

Componentele simetrice ale curenților;

$$I_d := (Z_{ei} + Z_h) \cdot \frac{U_s}{\sqrt{3} \cdot \Delta}$$

$$I_i := -Z_d \cdot \frac{U_s}{\sqrt{3} \cdot \Delta}$$

$$\Delta := (Z_{ed} + Z_h) \cdot (Z_{ei} + Z_h) - Z_d \cdot Z_i$$

$$\Delta = -153.233 + 228.707i \quad \Omega$$

$$I_d = 5.613 - 5.742i \quad A$$

$$I_i = 3.455 + 1.402i \quad A$$

Exemplu

Componentele simetrice ale tensiunii

$$U_d := Z_{ed} \cdot I_d$$

$$U_i := Z_{ei} \cdot I_i$$

$$U_d = 202.396 - 11.313i \quad \text{V} \quad |U_d| = 202.712 \quad \text{V}$$

$$U_i = -1.299 + 20.377i \quad \text{V} \quad |U_i| = 20.418 \quad \text{V}$$

Curentul pe faza A

$$I_a := I_d + I_i$$

$$I_a = 9.067 - 4.34i \quad \text{A}$$

$$|I_a| = 10.052 \quad \text{A}$$

Exemplu

Cuplul:

$$T_d := 3 \cdot \frac{p}{100 \cdot \pi} \cdot (|U_d|)^2 \cdot \frac{\left(\frac{R_r}{s_d}\right)}{\left(R_s + \frac{R_r}{s_d}\right)^2 + (X_{s\sigma} + X_{r\sigma})^2}$$

$$T_d = 34.334 \quad \text{Nm}$$

$$T_i := 3 \cdot \frac{p}{100 \cdot \pi} \cdot (|U_i|)^2 \cdot \frac{\left(\frac{R_r}{2 - s_d}\right)}{\left(R_s + \frac{R_r}{2 - s_d}\right)^2 + (X_{s\sigma} + X_{r\sigma})^2}$$

$$T_i = 0.263 \quad \text{Nm}$$

$$T := T_d - T_i \quad T = 34.07 \quad \text{Nm}$$

Exemplu

Componentele simetrice la întreruperea unei faze

$$\text{Cu } Z_d * Z_i = Z_h^2 \quad \Delta := (Z_{ed} + Z_{ei}) \cdot Z_h$$

$$I_{d1} := \frac{U_s}{\sqrt{3} \cdot (Z_{ed} + Z_{ei})} \quad I_{i1} := -a \cdot I_{d1}$$

$$I_{d1} = 4.914 - 5.367i \quad \text{A}$$

$$I_{i1} = -2.191 - 6.939i \quad \text{A}$$

$$U_{d1} := Z_{ed} \cdot I_{d1}$$

$$U_{d1} = 182.997 - 16.229i \quad \text{V}$$

$$|U_{d1}| = 183.715 \quad \text{V}$$

$$U_{i1} := Z_{ei} \cdot I_{i1}$$

$$U_{i1} = 32.253 - 23.405i \quad \text{V}$$

$$|U_{i1}| = 39.851 \quad \text{V}$$

Exemplu

Curentul de faza A

$$I_1 := I_{d1} + I_{i1}$$

$$I_1 = 2.723 - 12.307i \quad \text{A}$$

$$|I_1| = 12.604 \quad \text{A}$$

Cuplul direct

$$T_{d1} := 3 \cdot \frac{p}{100 \cdot \pi} \cdot (|U_{d1}|)^2 \cdot \frac{\left(\frac{R_r}{s_d}\right)}{\left(R_s + \frac{R_r}{s_d}\right)^2 + (X_{s\sigma} + X_{r\sigma})^2}$$

$$T_{d1} = 28.2 \quad \text{Nm}$$

Exemplu

Cuplul invers

$$T_{i1} := 3 \cdot \frac{p}{100 \cdot \pi} \cdot (|U_{i1}|)^2 \cdot \frac{\left(\frac{R_r}{2 - s_d} \right)}{\left(R_s + \frac{R_r}{2 - s_d} \right)^2 + (X_{s\sigma} + X_{r\sigma})^2}$$

$$T_{i1} = 1.004 \quad \text{Nm}$$

Cuplul rezultat

$$T_1 := T_{d1} - T_{i1} \quad T_1 = 27.196 \quad \text{Nm}$$

Exemplu

La pornire: impedantele

$$Z_r := R_r + X_{r\sigma} \cdot 1i$$

$$Z_{ep} := Z_s + Z_m \cdot \frac{Z_r}{(Z_m + Z_r)} \quad Z_{ep} = 2.322 + 5.218i \quad \Omega$$

Componente de curenți

$$I_{dp} := \frac{U_s}{\sqrt{3} \cdot 2 \cdot Z_{ep}} \quad I_{dp} = 7.808 - 17.549i \quad A$$

$$I_{ip} := -a \cdot I_{dp} \quad I_{ip} = -11.294 - 15.536i \quad A$$

Exemplu

Curentul din faza A

$$I_{ap} := I_{dp} + I_{ip} \quad I_{ap} = -3.486 - 33.085i \quad |I_{ap}| = 33.268 \quad A$$

Componente de tensiune

$$U_{dp} := Z_{ep} \cdot I_{dp} \quad U_{dp} = 109.697 \quad V$$

$$U_{ip} := Z_{ep} \cdot I_{ip} \quad U_{ip} = 54.848 - 95i \quad V$$

$|U_{ip}| = 109.697 \quad V$

Exemplu

Cuplul de pornire

$$T_{dp} := 3 \cdot \frac{p}{100 \cdot \pi} \cdot (|U_{dp}|)^2 \cdot \frac{(R_r)}{(R_s + R_r)^2 + (X_{s\sigma} + X_{r\sigma})^2}$$

$T_{dp} = 13.63 \quad \text{Nm}$

$$T_{ip} := T_{dp}$$

$$T_p := T_{dp} - T_{ip} \quad T_p = 0 \quad \text{Nm}$$

