

# Supratensiuni la transformatoare

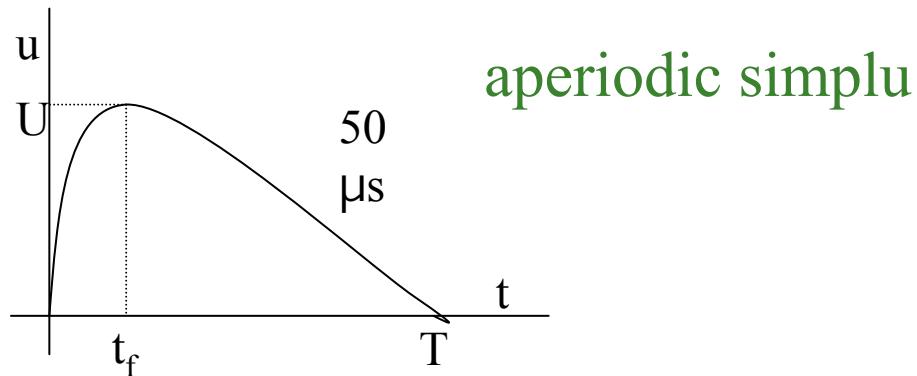
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Repartizarea inițială și finală  
Măsuri de protecție

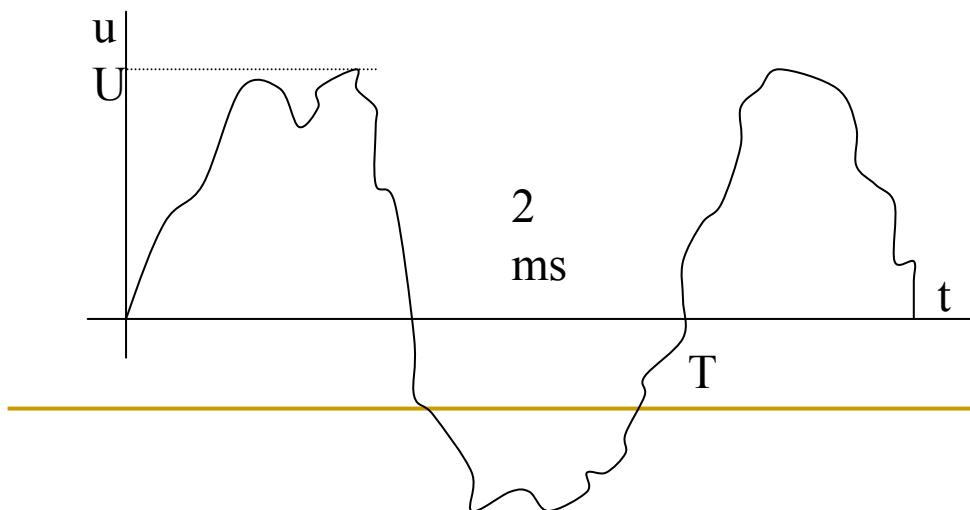
# Formele de unde de supratensiuni

Cauzele și formele supratensiunilor:

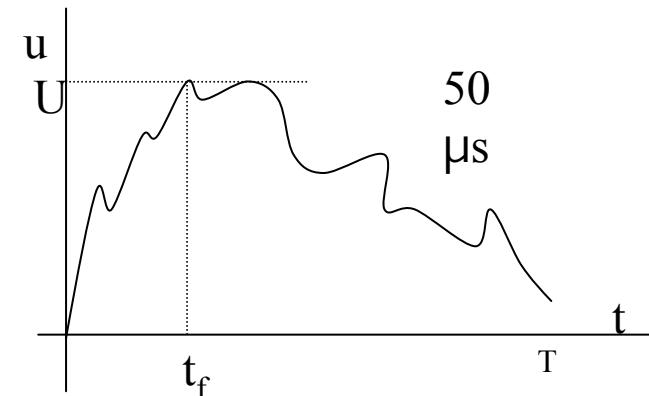
- atmosferice - 8....12 Un



- datorită avariilor - 7.....8 Un



aperiodic complex



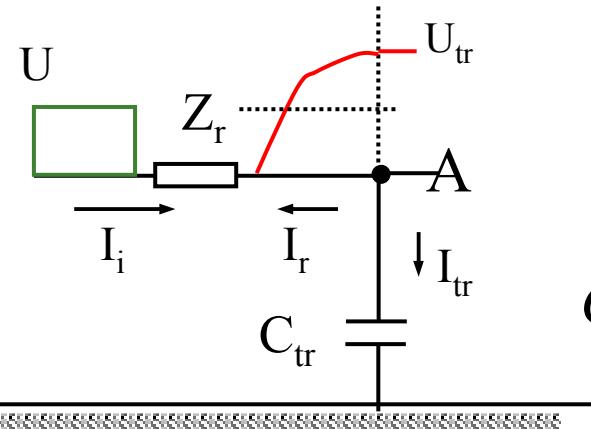
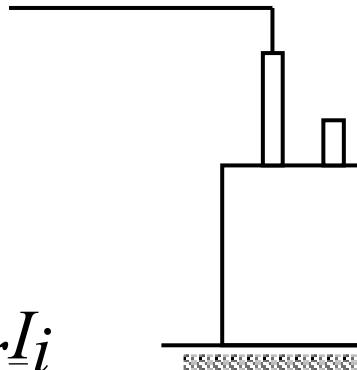
Periodic – simplu  
- complex

- de comutatie  
- 2...5 Un

periodic

## Propagarea undei de supratensiuni

$$U_i = Z_r I_i$$



$$Z_r \cong 500\Omega$$

$$C_{tr} \approx 0.1\text{ nF} \div 0.1\text{ }\mu\text{F}$$

$$U_r = Z_r I_r$$

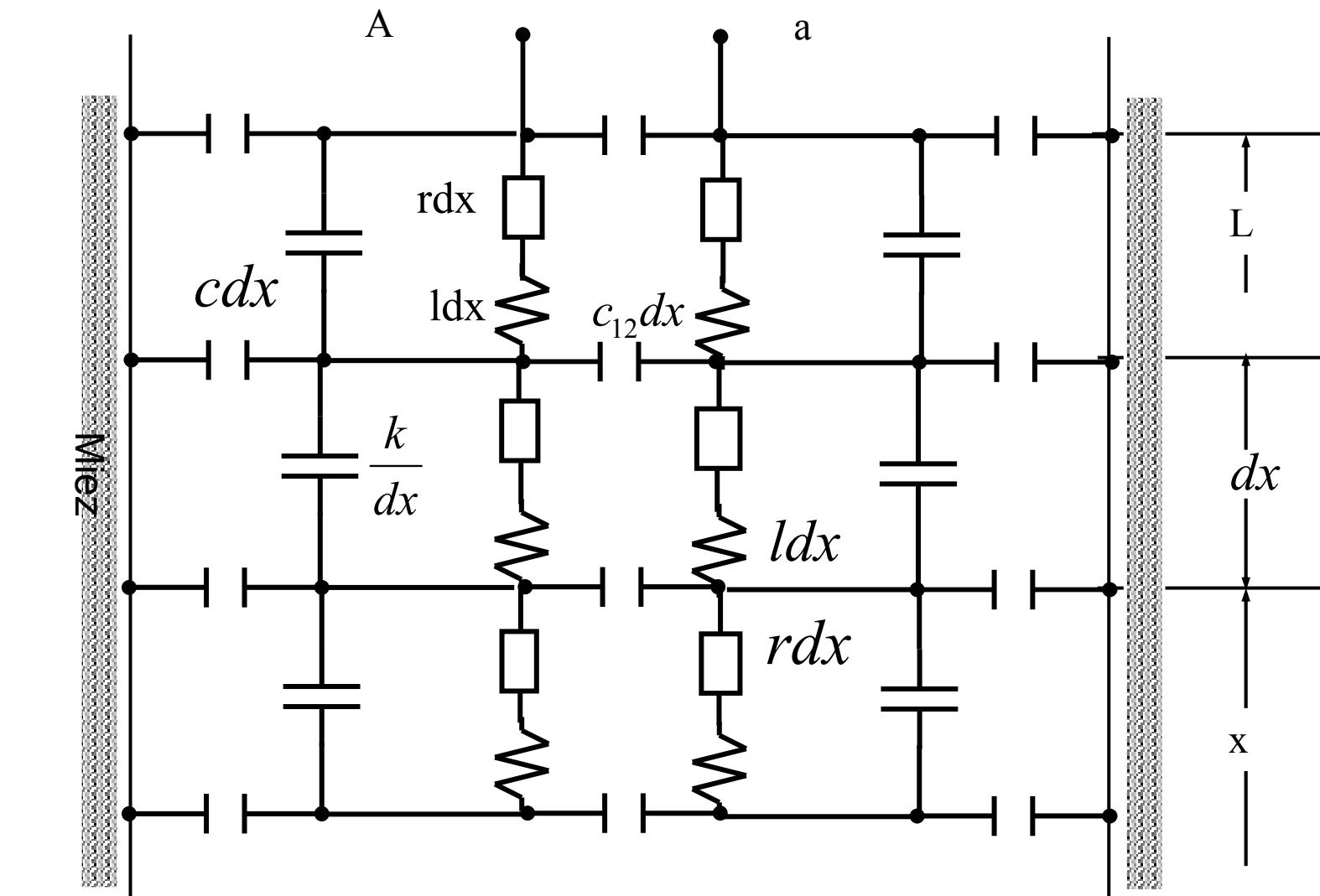
$$u_{tr} = \frac{1}{C_{tr}} \int i_{tr} dt$$

$$u_{tr} = 2U \left( 1 - e^{-\frac{t}{C_{tr} Z_r}} \right)$$

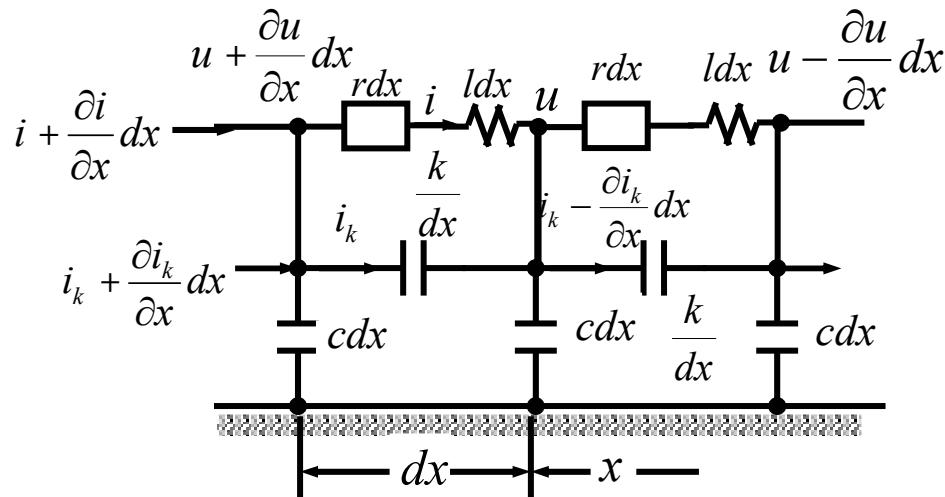
$$i_t = i_i - i$$

$$C_{tr} \cdot Z_r \cong 0.05 \div 50 \text{ }\mu\text{s}$$

## Schema echivalentă a transformatorului



## Ecuăriile schemei echivalente



$$\left( u + \frac{\partial u}{\partial x} dx \right) - u = rdx \cdot i + ldx \cdot \frac{\partial i}{\partial t}$$

$$\frac{\partial u}{\partial x} = r \cdot i + l \frac{\partial i}{\partial t}$$

$$\frac{1}{k} \int_{dx} i_k dt = \left( u + \frac{\partial u}{\partial x} dx \right) - u$$

$dx$

$$\frac{dx}{k} \int i_k dt = \frac{\partial u}{\partial x} dx$$

$$\frac{i_k}{k} = \frac{\partial u}{\partial x \partial t}$$

$$q = u \cdot c$$

$$\sum i = \frac{\partial u}{\partial t} c$$

$$i_k - \left( i_k - \frac{\partial i_k}{\partial x} dx \right) + i - \left( i - \frac{\partial i}{\partial x} dx \right) = \frac{\partial u}{\partial t} c dx$$

$$\frac{\partial i_k}{\partial x} + \frac{\partial i}{\partial x} = \frac{\partial u}{\partial t} c$$

## Deducerea ecuației generale

$$\frac{1}{k} \frac{\partial i}{\partial x} = \frac{\partial^3 u}{\partial x^2 \partial t} \Rightarrow k \frac{\partial^3 u}{\partial x^2 \partial t} + \frac{\partial i}{\partial x} = c \frac{\partial u}{\partial t} \Rightarrow \frac{\partial i}{\partial x} = c \frac{\partial u}{\partial t} - k \frac{\partial^3 u}{\partial x^2 \partial t}$$

$$\frac{\partial^2 u}{\partial x^2} = r \frac{\partial i}{\partial x} + l \frac{\partial^2 i}{\partial x \partial t} = r \left( c \frac{\partial u}{\partial t} - k \frac{\partial^3 u}{\partial x^2 \partial t} \right) + l \left( c \frac{\partial^2 u}{\partial t^2} - k \frac{\partial^4 u}{\partial x^2 \partial t^2} \right)$$

$$l \cdot k \frac{\partial^4 u}{\partial x^2 \partial t^2} + r \cdot k \frac{\partial^3 u}{\partial x^2 \partial t} - r \cdot c \frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} = l \cdot c \frac{\partial^2 u}{\partial t^2}$$

$$k \frac{\partial^4 u}{\partial x^2 \partial t^2} + \frac{r}{l} k \frac{\partial^3 u}{\partial x^2 \partial t} - \frac{r}{l} c \frac{\partial u}{\partial t} + \frac{1}{l} \frac{\partial^2 u}{\partial x^2} = c \frac{\partial^2 u}{\partial t^2}$$

## Aplicarea transformatei Laplace

$$\frac{\partial}{\partial t} \rightarrow p \quad u \rightarrow U$$

$$\left( k \cdot p^2 + \frac{1}{l} \right) \frac{d^2 U}{dx^2} + \frac{r}{l} \left( k \cdot p \frac{d^2 U}{dx^2} - c \cdot p \cdot U \right) = c \cdot p^2 \cdot U$$

$$\left( k \cdot p^2 + k \frac{r}{l} p + \frac{1}{l} \right) \frac{d^2 U}{dx^2} = c \left( p^2 + \frac{r}{l} p \right) U$$

$$\frac{d^2 U}{dx^2} = \alpha^2 \cdot U \quad \alpha^2 = \frac{c \cdot p \cdot (l \cdot p + )}{k \cdot l \cdot p^2 + k \cdot r \cdot p + 1}$$

## Analiza procesului

La începutul procesului

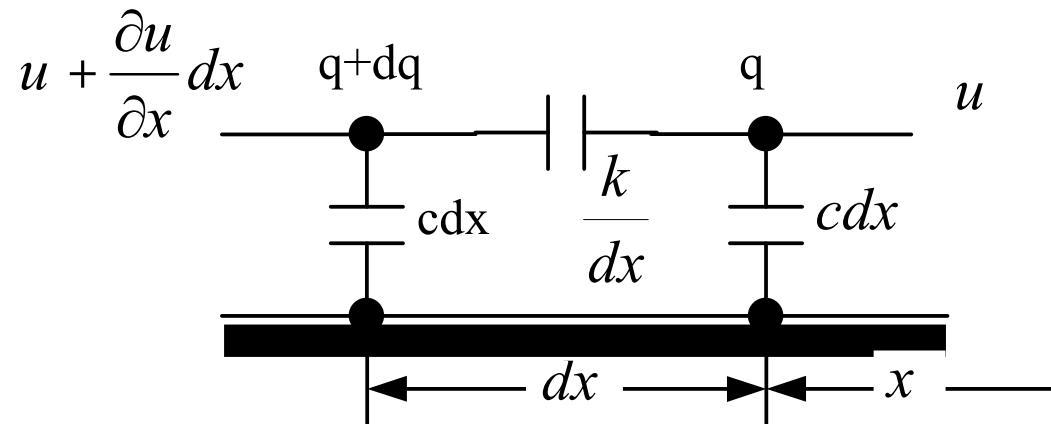
$$t = 0^+$$

$$p \rightarrow \infty$$

$$\alpha^2 = \frac{c}{k}$$

Inductivitatea nu are influență

se consideră  $l \rightarrow \infty$



$$k \frac{\partial^4 u}{\partial x^2 \cdot \partial t^2} = c \frac{\partial^2 u}{\partial t^2}$$

$$\alpha = \sqrt{\frac{c}{k}}$$

## Repartizarea inițială a supratensiunii

$$\frac{\partial^2 u}{\partial t^2} \rightarrow p^2 \cdot U$$

$$U = A \cdot \sinh(\alpha \cdot x) + B \cdot \cosh(\alpha \cdot x)$$

$$\frac{\partial^2 U}{\partial x^2} = \alpha^2 \cdot U$$

nulul transformatorului este legat la pămănt

la  $x=0$  (nulul) potențialul este al pământului  $U = 0$

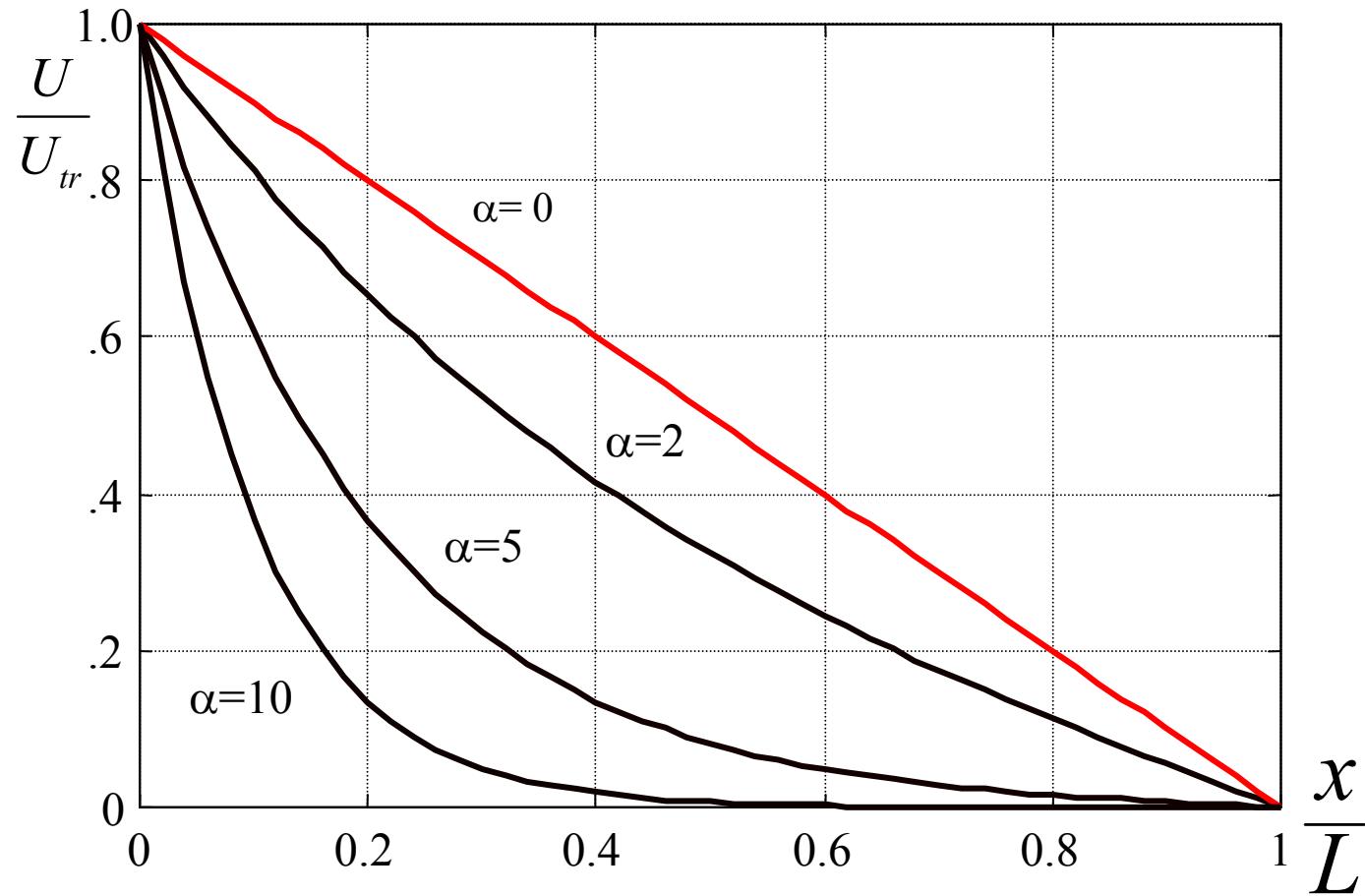
$$0 = A \cdot \sinh(\alpha \cdot 0) + B \cdot \cosh(\alpha \cdot 0) \implies B = 0$$

la  $x = L$  (borna de intrare) este tensiunea  $U = U_{tr}$

## Repartizarea inițială a supratensiunii

$$U_{tr} = A \cdot \operatorname{sh}(\alpha \cdot L)$$

$$U = U_{tr} \frac{\operatorname{sh}(\alpha \cdot x)}{\operatorname{sh}(\alpha \cdot L)}$$



## Repartizarea inițială a supratensiunii

Nulul transformatorului este izolat față de pămănt

$$q = \left[ u - \left( u - \frac{\partial u}{\partial x} dx \right) \right] \frac{k}{dx} = k \frac{\partial u}{\partial x}$$

$$q = [ A \cdot \alpha \cdot \operatorname{sh} \alpha \cdot x - B \cdot \alpha \cdot \operatorname{ch} \alpha \cdot x ] \quad k = 0$$

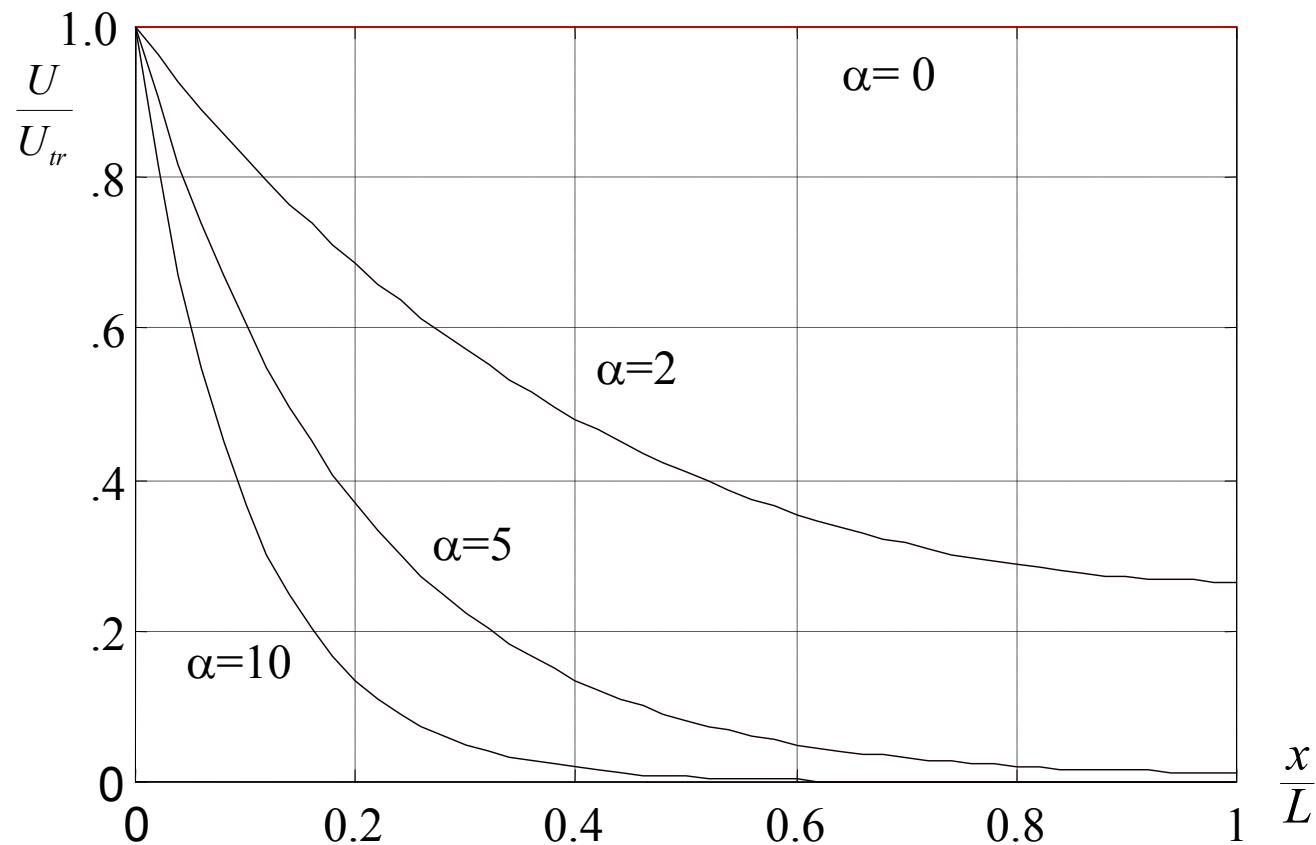
La  $x = 0$  sarcinile sunt repartizate pe condensatoare  
iar la nul  $q = 0$

$$A = 0$$

la  $x = L$  (borna de intrare) este tensiunea  $U = U_{tr}$

## Repartizarea inițială a supratensiunii

$$U_{tr} = B \cdot \text{ch } \alpha \cdot L \quad \xrightarrow{\text{red arrow}} \quad U = U_{tr} \frac{\text{ch}(\alpha \cdot x)}{\text{ch}(\alpha \cdot L)}$$



## Repartizarea finală a supratensiunii

$$t \rightarrow \infty \quad p = 0 \quad \Rightarrow \alpha = 0$$

Regimul tranzitoriu este determinat de rădăcinile ecuației caracteristice:

$$k \cdot l \cdot p^2 + k \cdot r \cdot p + 1 = 0$$

$$p_{1,2} = -\frac{r}{2 \cdot l} \pm \sqrt{\left(\frac{r}{2 \cdot l}\right)^2 - \frac{1}{k \cdot l}} = -\frac{1}{T} \pm \sqrt{\frac{1}{T^2} - \frac{1}{k \cdot l}}$$

## Repartizarea finală a supratensiunii

$$T = \frac{2 \cdot l}{r}$$

$$T = \sqrt{k \cdot l}$$

$$p_1 = p_2 = -\frac{1}{T}$$

*aperiodic*

$$T \rangle \sqrt{k \cdot l}$$

$$p_1 = -\frac{1}{T_1}$$

$$p_2 = -\frac{1}{T_2}$$

*aperiodic*

$$T \langle \sqrt{k \cdot l}$$

$$p_{1,2} = -\frac{1}{T} \pm j \cdot \rho$$

*oscilatii*

În următorul tabel sunt prezentate datele de caracteristici ale sistemelor de control.

## Măsuri de protecție

### Măsuri exterioare:

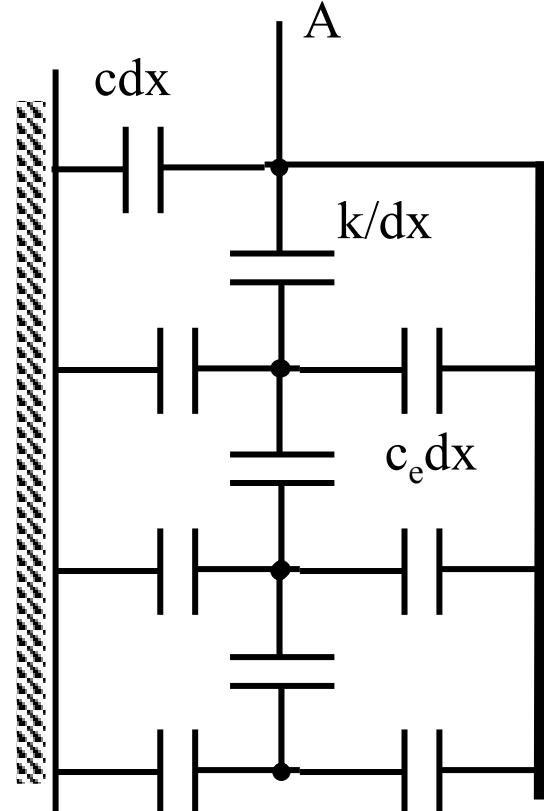
- alegerea corespunzătoare a traseului liniei electrice
- folosirea eclatoarelor și descărcătoarelor electrice
- legarea nulului la pământ prin grup RLC în paralel

### Măsuri constructive (interioare)

- izolarea suplimentară a primelor spire
- ecran electrostatic
- inel secționat pentru primele prize

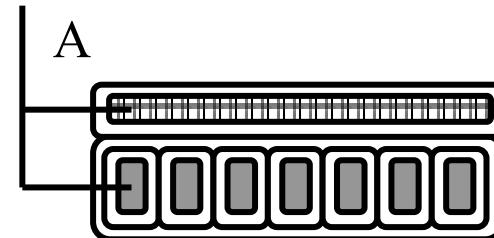
## Măsuri de protecție

ecran electrostatic



$$\alpha = \sqrt{\frac{c}{k}}$$

$$\alpha \rightarrow 0$$



inel sectionat pentru primele prize

# Regimurile tranzitorii la transformatoare

Cuplarea la rețea a transformatorului și scurtcircuitul brusc.

# Regimurile tranzitorii la transformatoare

Cauzele regimurilor tranzitorii:

- modificarea bruscă a condițiilor de exploatare
  - condiții de mediu
  - condiții de funcționare
  - valorile parametrilor

Manifestarea regimului tranzitorii:

- regimuri de supracurenți
  - cuplarea la rețea
  - scurtcircuit brusc
- regimuri de supratensiuni

## Cuplarea la rețea a transformatorului

### Ipoteze:

- transformatorul este fără sarcină,
- fluxul remanent al miezului  $\Psi_r$  este redus,
- tensiunea de alimentare este sinusoidală

$$u_1 = \sqrt{2} \cdot U_1 \cdot \sin(\omega \cdot t + \alpha)$$

- conectarea se face în momentul când tensiunea are valoarea:

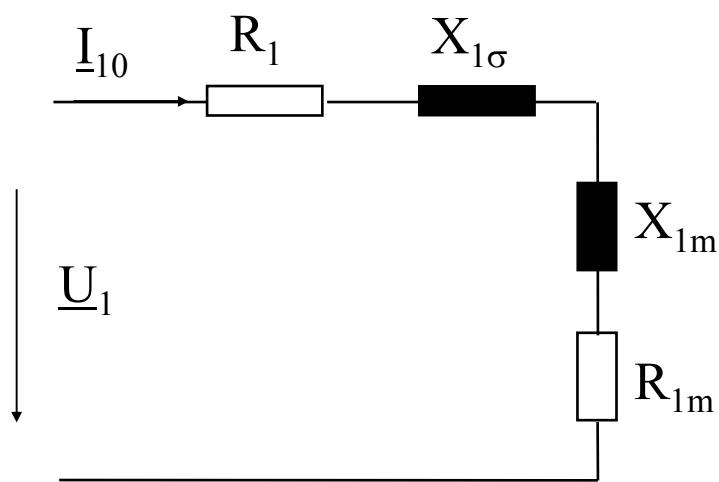
$$u_{10} = \sqrt{2} \cdot U_1 \cdot \sin(\alpha)$$

Ecuatia de tensiune:

$$u_1 = R_1 \cdot i_{10} + \frac{d\Psi_1}{dt}$$

## Cuplarea la rețea a transformatorului

Fluxul total la mersul în gol



$$\Psi_1 = L_1 \cdot i_{10} = (L_{1\sigma} + L_{1m}) \cdot i_{10}$$

Rezultă:

$$i_{10} = \frac{\Psi_1}{L_1}$$

$$\sqrt{2} \cdot U_1 \cdot \sin(\omega \cdot t + \alpha) = \frac{R_1}{L_1} \Psi_1 + \frac{d\Psi_1}{dt}$$

## Cuplarea la rețea a transformatorului

Soluția:

$$\Psi_1 = C \cdot e^{-\frac{t}{T_1}} + \Psi_{1m} \cdot \sin(\omega \cdot t + \gamma)$$

Unde:

$$T_1 = \frac{L_1}{R_1} \quad \text{Constanta de timp}$$

$$\Psi_{1m} = \frac{\sqrt{2} \cdot U_1}{\sqrt{\omega^2 + \frac{1}{T_1^2}}} \quad \text{Amplitudinea fluxului alternativ}$$

$$\tan(\gamma - \alpha) = -\omega \cdot T_1 = -\frac{X_1}{R_1}$$

## Cuplarea la rețea a transformatorului

Condiții initiale: la  $t = 0$   $\Psi_1 = \Psi_r$

Rezultă:  $C = \Psi_r - \Psi_{1m} \cdot \sin \gamma$

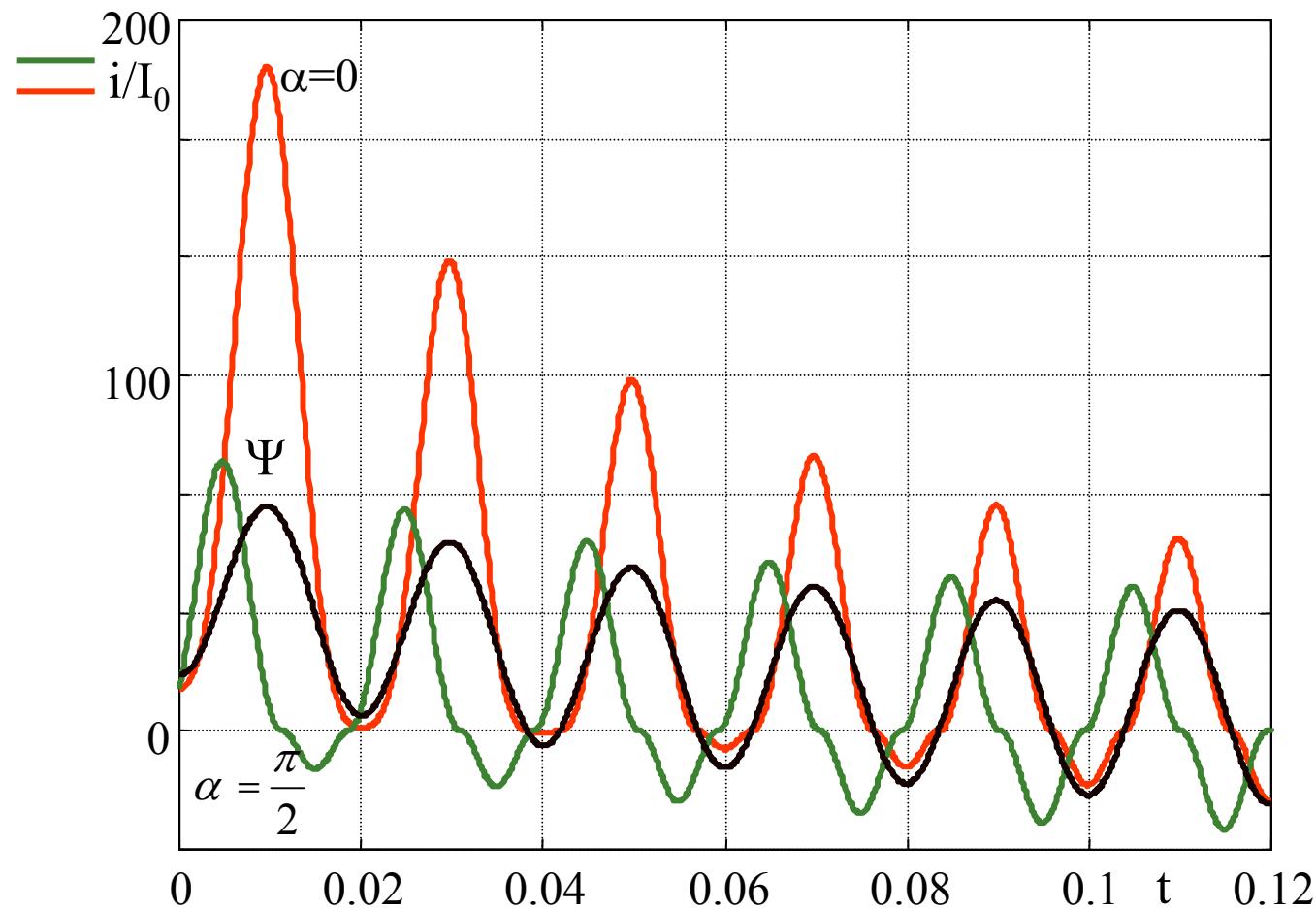
$$\Psi_1 = \Psi_{1m} \cdot \sin(\omega \cdot t + \gamma) + (\Psi_r - \Psi_{1m} \cdot \sin \gamma) \cdot e^{-\frac{t}{T_1}}$$

Valoarea maximă pentru:  $\gamma = -\frac{\pi}{2}$        $\omega \cdot t + \gamma = \frac{\pi}{2}$

Rezultă:  $t = \frac{\pi}{\omega}$        $\alpha = \lambda + \arctg \left( \frac{X_1}{R_1} \right) \approx 0$

$$\Psi_{1\max} = \Psi_{1m} + (\Psi_r + \Psi_{1m}) \cdot e^{-\frac{\pi}{\omega \cdot T_1}}$$

## Cuplarea la rețea a transformatorului

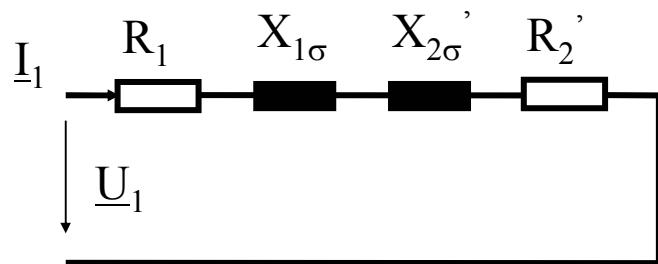


## Scurtcircuitul brusc

Ipoteze:

- funcționează în sarcină având curentul  $I_0$
- la apariția scurtcircuitului tensiunea are valoarea

$$u_1 = \sqrt{2} \cdot U_1 \cdot \sin(\alpha)$$



$$Z_{1sc} = \sqrt{R_{1sc}^2 + X_{1sc}^2}$$

$$T_{1sc} = \frac{L_{1sc}}{R_{1sc}}$$

## Scurtcircuitul brusc

$$\sqrt{2} \cdot U_1 \cdot \sin(\omega \cdot t + \alpha) = R_{1sc} \cdot i_{1sc} + L_{1sc} \frac{di_{1sc}}{dt}$$

Soluția:

$$i_{1sc} = I_{1msc} \cdot \sin(\omega \cdot t + \gamma_{sc}) + C \cdot e^{-\frac{t}{T_{1sc}}}$$

Curentul initial  $I_0$

$$I_0 = I_{1msc} \cdot \sin(\gamma_{sc}) + C$$

$$I_{1msc} = \frac{\sqrt{2} \cdot U_1}{Z_{1sc}} \quad \quad \quad \operatorname{tg}(\gamma_{sc} - \alpha) = -\omega \cdot T_{1sc}$$

$$i_{1sc} = \frac{\sqrt{2} \cdot U_1}{Z_{1sc}} \sin(\omega \cdot t + \gamma_{sc}) + \left( I_0 - \frac{\sqrt{2} \cdot U_1}{Z_{1sc}} \sin \gamma_{sc} \right) \cdot e^{-\frac{t}{T_{1sc}}}$$

## Scurtcircuitul brusc

Valoarea maximă la

$$t = \frac{\pi}{\omega}$$

$$\omega \cdot t + \gamma_{sc} = \frac{\pi}{2} \quad \gamma_{sc} = -\frac{\pi}{2}$$

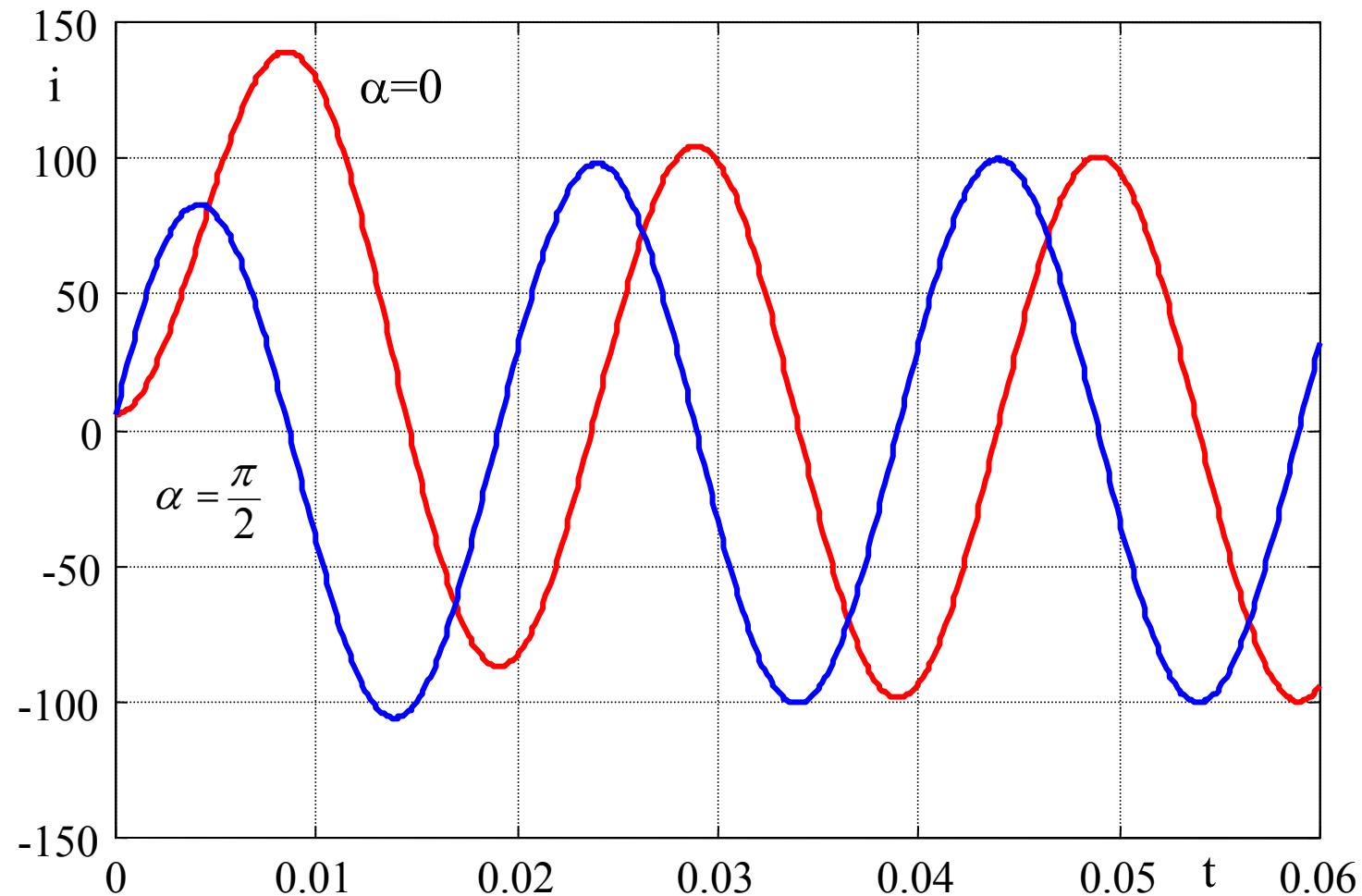
$$i_{1sc\max} = \frac{\sqrt{2} \cdot U_1}{Z_{1sc}} \left( 1 + e^{-\frac{\pi}{\omega \cdot T_{1sc}}} \right) + I_0 \cdot e^{-\frac{\pi}{\omega \cdot T_{1sc}}}$$

Factorul de scurtcircuit

$$k_{sc} = 1 + e^{-\frac{\pi}{\omega \cdot T_{1sc}}} \quad 1.2 \dots 1.85$$

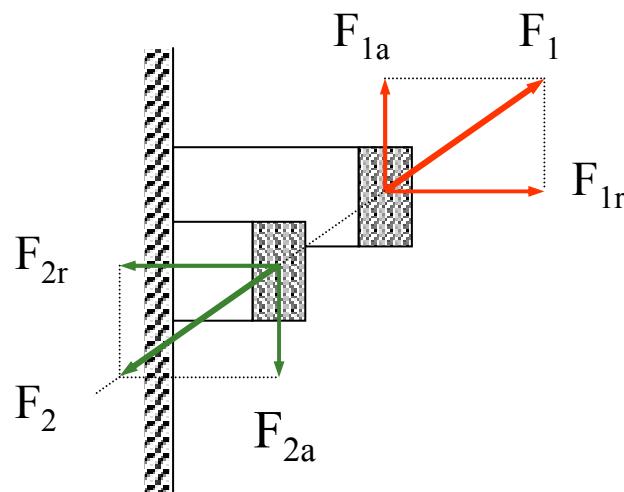
$$i_{1sc\max} = \frac{1}{u_{sc}} \cdot k_{sc} \cdot \sqrt{2} \cdot I_{1N} \quad 20 \dots 50$$

## Scurtcircuitul brusc



# Scurtcircuitul brusc

## Forțe electrodinamice



$$F_r = \left( \frac{\partial W_m \sigma}{\partial a} \right)_{i=ct.}$$

$$F_a = \left( \frac{\partial W_m \sigma}{\partial h} \right)_{i=ct.}$$

## Scurtcircuitul brusc

$$w_{m\sigma} = \frac{1}{2} \cdot L_{SC} \cdot i_{1SC}^2$$

$$L_{SC} = \frac{\mu_0}{\omega} \cdot \pi \cdot D_m \cdot K_R \cdot w^2 \cdot \frac{a}{h}$$

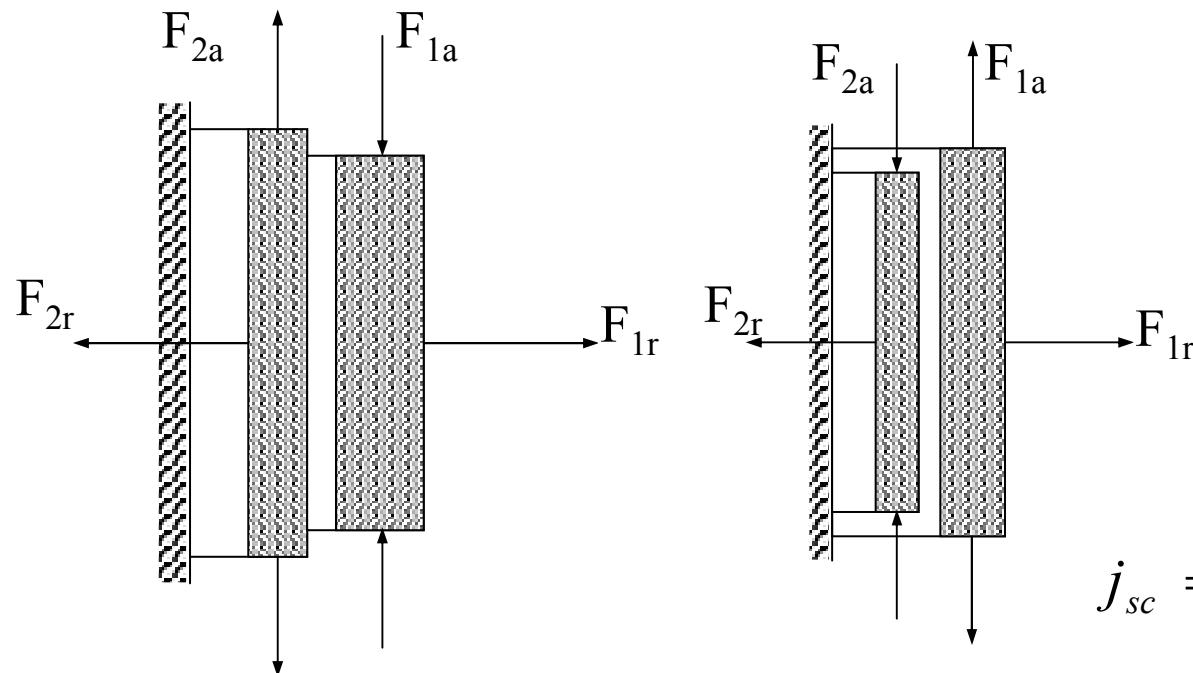
$$K_f = \frac{1}{2} \frac{\mu_0}{\omega} \cdot \pi \cdot D_m \cdot K_R \cdot w^2 \cdot i_{SC}^2$$

$$F_r = K_f \frac{1}{h}$$

$$K_a = -K_f \frac{a}{h^2} \quad \Rightarrow \frac{F_a}{F} = \frac{a}{h}$$

# Scurtcircuitul brusc

Solicitarea termică



$$\frac{d\theta}{dt} = k_\theta \cdot j_{sc}^2$$

$$j_{sc} = \frac{k_{sc}}{u_{sc}} \cdot j$$

$$k_\theta = 6.2 \cdot 10^{-15} \left[ m^4 \cdot {}^\circ C / A^2 / s \right]$$

## Exemplu

Transformator de putere în ulei

$S_n := 630 \text{ kVA}$

$p_{fe} := 1850 \text{ W}$

$U_{1n} := 15 \text{ kV}$

$p_b := 9900 \text{ W}$

$U_{2n} := 0.4 \text{ kV}$

$i_0 := 4.9 \%$

conexiune Dy-11

$usc := 6 \%$

Calculul parametrilor schemei echivalente

## Exemplu

tensiuni si curenti de fază nominați

$$U_{1f} := U_{1n}$$

$$U_{1f} = 15 \text{ kV}$$

$$U_{2f} := \frac{U_{2n}}{\sqrt{3}}$$

$$U_{2f} = 0.231 \text{ kV}$$

$$I_{1f} := \frac{S_n}{3 \cdot U_{1f}}$$

$$I_{1f} = 14 \text{ A}$$

$$I_{2f} := \frac{S_n}{3 \cdot U_{2f}}$$

$$I_{2f} = 909.327 \text{ A}$$

$$I_{10} := \frac{i_0}{100} \cdot I_{1f}$$

$$I_{10} = 0.686 \text{ A}$$

$$U_{1sc} := \frac{u_{sc}}{100} \cdot U_{1f}$$

$$U_{1sc} = 0.9 \text{ kv}$$

## Exemplu

### Impedantele

$$Z_{10} := \frac{U_{1f}}{I_{10}}$$

$$Z_{10} = 21.866 \text{ k}\Omega$$

$$R_{10} := \frac{p_{fe} \cdot 10^{-3}}{3 \cdot I_{10}^2}$$

$$R_{10} = 1.31 \text{ k}\Omega$$

$$X_{10} := \sqrt{Z_{10}^2 - R_{10}^2}$$

$$X_{10} = 21.827 \text{ k}\Omega$$

$$Z_{1sc} := \frac{U_{1sc} \cdot 10^3}{I_{1f}}$$

$$Z_{1sc} = 64.286 \text{ }\Omega$$

$$R_{1sc} := \frac{p_b}{3 \cdot I_{1f}^2}$$

$$R_{1sc} = 16.837 \text{ }\Omega$$

$$X_{1sc} := \sqrt{Z_{1sc}^2 - R_{1sc}^2}$$

$$X_{1sc} = 62.042 \text{ }\Omega$$

## Exemplu

se consideră :  $R1 := 1.1 \cdot R2$

rezulta :

$$R2 := \frac{R1sc}{2.1} \quad R2 = 8.017 \quad \Omega$$

$$R1 := 1.1 \cdot R2 \quad R1 = 8.819 \quad \Omega$$

$$X2 := \frac{X1sc}{2.1} \quad X2 = 29.544 \quad \Omega$$

$$X1 := 1.1 \cdot X2 \quad X1 = 32.498 \quad \Omega$$

constantele de timp

$$T10 := \frac{X10}{100 \cdot \pi \cdot R10} \quad T10 = 0.053 \quad s$$

$$Tsc := \frac{X1sc}{100 \cdot \pi \cdot R1sc} \quad Tsc = 0.012 \quad s$$

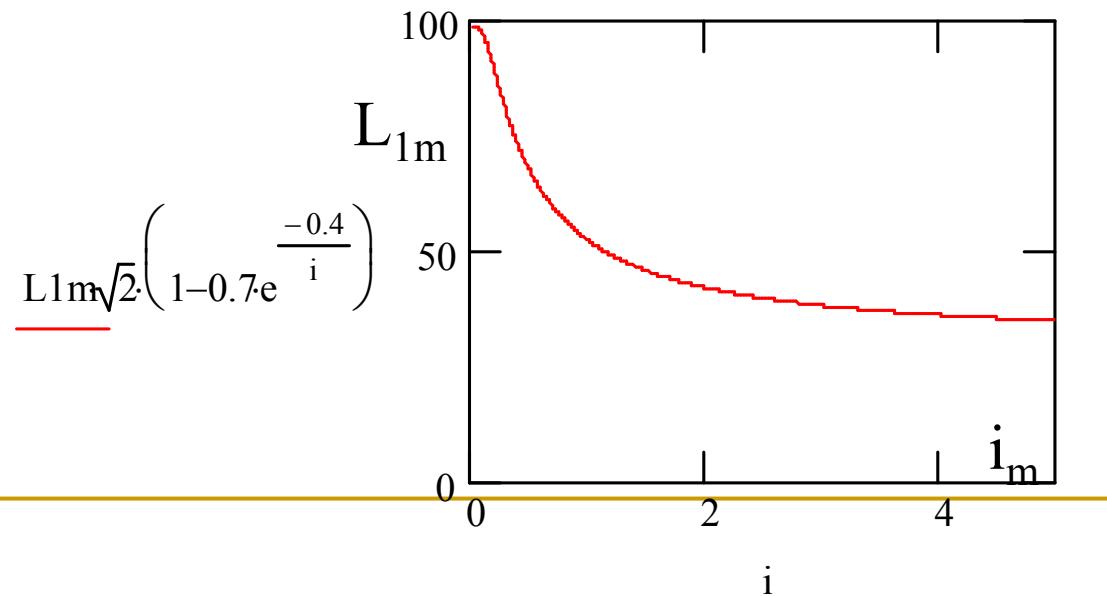
## inductivitate de magnetizare

$$L_{1m} := \frac{X10 \cdot 10^3}{100 \cdot \pi}$$

$$L_{1m} = 69.476 \text{ H}$$

considerand variatia in functie de curent de forma:

$$l_{1m} := L_{1m} \cdot \left( 1 - 0.9 \cdot e^{\frac{-0.6}{i}} \right)$$



## Exemplu

fluxul remanent

$$\Psi_r := 5 \quad \text{Wb}$$

amplitudinea fluxului

$$\alpha := 0$$

$$\Psi_{1m} := \sqrt{2} \cdot \frac{U_{1f} \cdot 10^3}{\sqrt{(100 \cdot \pi)^2 + \frac{1}{T_{10}^2}}} \quad \Psi_{1m} = 67.402 \quad \text{Wb}$$

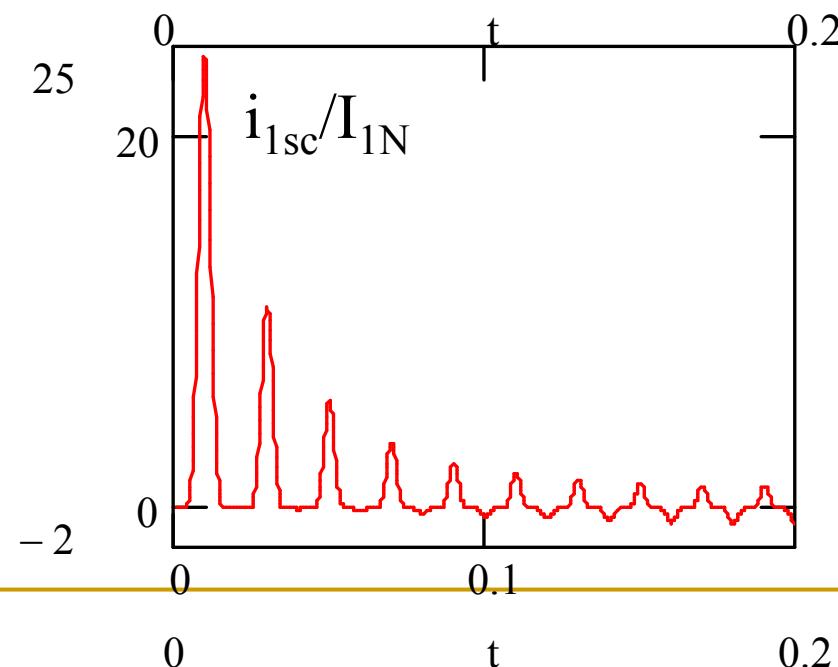
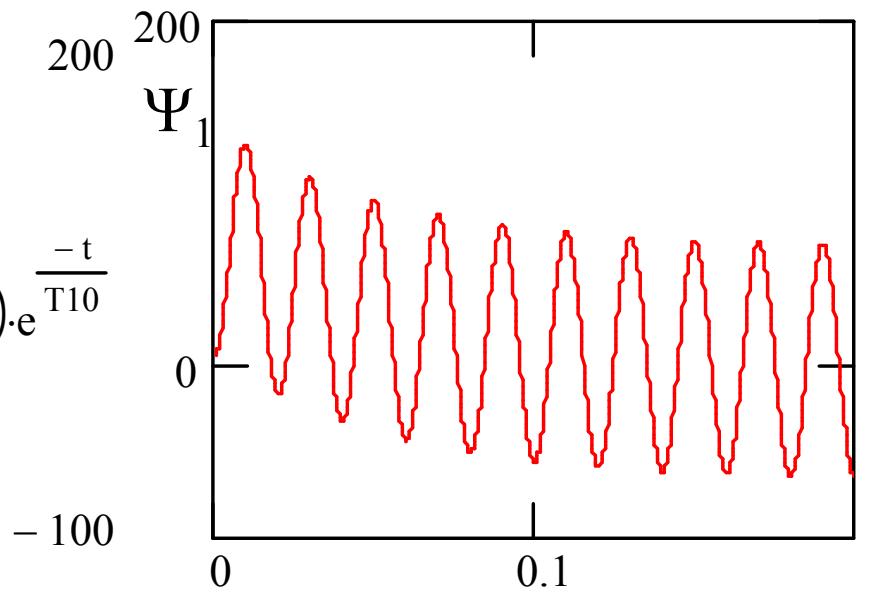
$$\gamma := \alpha - \text{atan}(100 \cdot \pi \cdot T_{10})$$

$$\gamma = -1.511 \quad \text{rad}$$

$$\pi/2 = 1.507 \text{ rad}$$

$$\underline{\Psi_1} \text{msin}(100\pi t + \gamma) + (\Psi_r - \Psi_1 \text{msin}(\gamma)) \cdot e^{\frac{-t}{T^{10}}}$$

$$\alpha = 0$$



$$\alpha = \pi/2$$

$$\underline{\Psi_{1m} \sin(100\pi t + \gamma) + (\Psi_r - \Psi_{1m} \sin(\gamma)) e^{\frac{-t}{T10}}}$$

