MATHEMATICAL MODELS IN SEISMOLOGY

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Advanced Nonlinear Inelastic Analysis Methods for Seismic Performance Evaluation of Frame Structures

Prof. C. G. Chiorean

(www.cosminchiorean.com)

Technical University of Cluj-Napoca, Romania

Overview of research significance

- In recent years, non-linear inelastic analysis methods of frame structures has become the focus of intense research efforts **<u>because of rapid development of computer technology</u>** and the need of implementation in the new design codes, the more rational advanced analysis techniques and performance-based seismic design procedures.

- **<u>In spite of the availability of some FEM algorithms and powerful computer programs</u>**, the non-linear inelastic analysis of **<u>real large-scale frame structures</u>** still posses huge demands on the most powerful of available computers and still represents unpractical tasks to most designers.

- **<u>Structural response to strong earthquake ground motions cannot be accurately predicted due to large uncertainities *and the randomness of structural properties and ground motion parameters*</u>**. Excessive sophistication in structural analysis is not waranted.

- The need for accurate yet computational efficient tools for the non-linear analysis of 3D frame structures; developing integrated systems for advanced structural analysis and seismic performance evaluation of 3D steel and composite steel-concrete building frameworks with rigid or flexible connections.

Outline

Part I.

- Applications of the advanced nonlinear analysis for structures
- Sources of nonlinearities;
- Seismic performance evaluation methods: nonlinear dyamic analysis vs nonlinear
- static analysis;
- Formulation of time history analysis; Formulation of static nonlinear analysis (concentrated plasticity and distributed plasticity);
- Advanced inelastic analysis of cross-sections;
- Computational examples and discussions.

Part II.

- Advanced inelastic analysis of 3D composite steel-concrete frame structures; computational examples and discussions.
- Computer programs:

  NEFCAD : *Computer program for large deflection elasto-plastic analysis of spatial frame structures*

  ASEP: A computer program for ultimate strength analysis of composite steel- concrete cross-section

Applications of advanced nonlinear static analysis

Accurate yet computational efficient tools for nonlinear inelastic analysis of 3D steel and composite steel-concrete frame structures

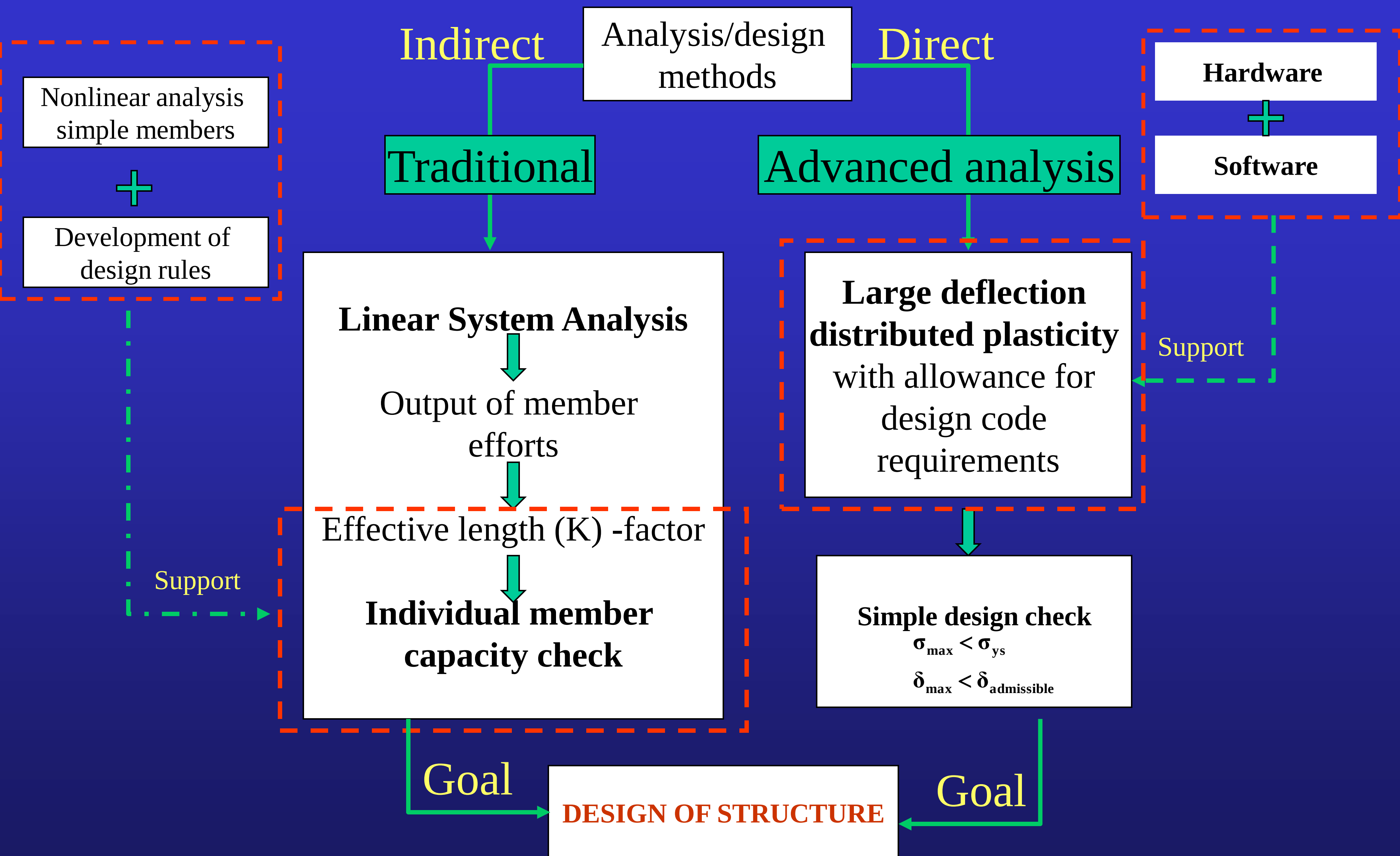
Advanced analysis procedures

Performance-based earthquake engineering

Developing advanced inelastic analysis methods which can sufficiently represent the behavioral effects associated with member primary limit states such that the separated specification member capacity checks are not required. In other words, there is a potential for direct handling of limit states within analysis models. The reason for this is that, since *advanced analysis* can directly asses the strength and stability of the overall structural system as well as interdependence of member and system strength and stability, separate member capacity checks are not required.

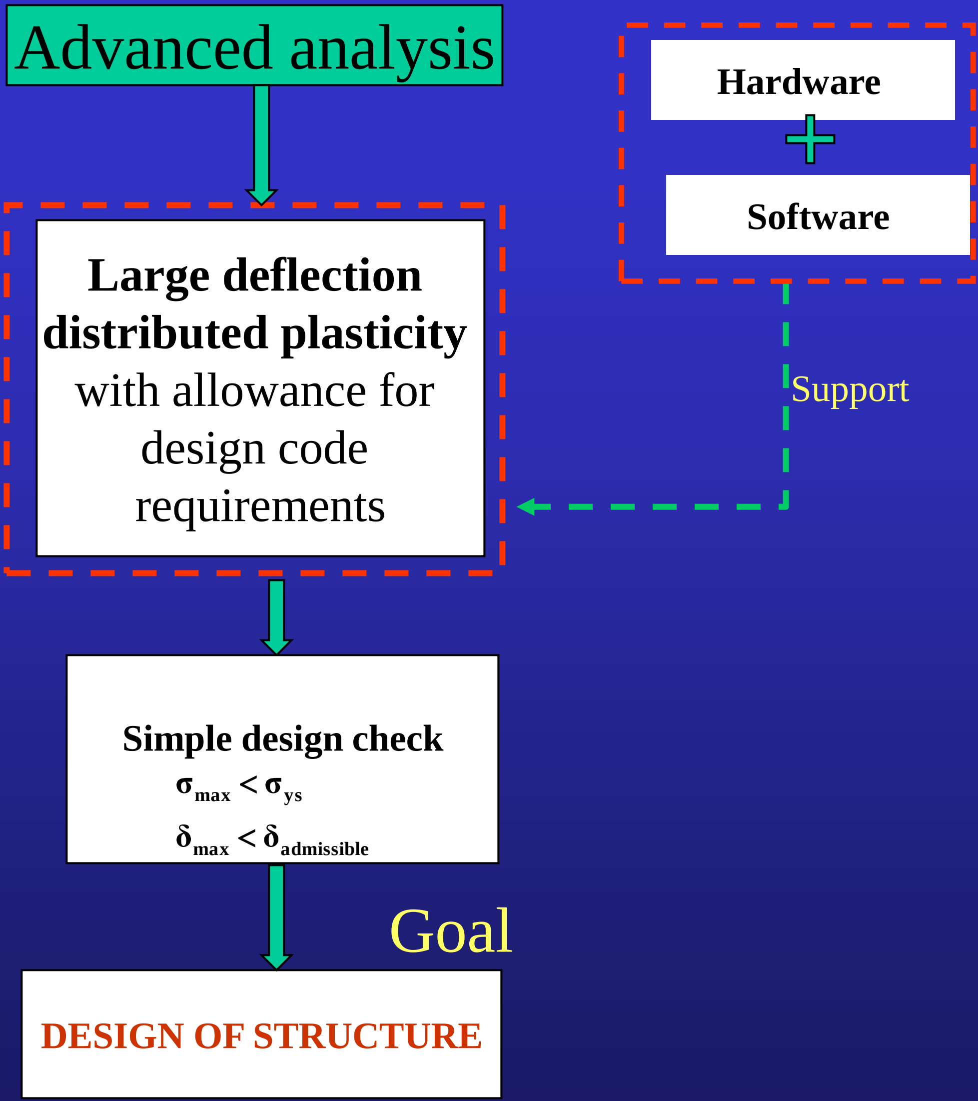
Accurate and computationally efficient numerical models that represent the nonlinear behavior in beam-columns elements are required to simulate the seismic response and evaluate the performance of structural system. On the other hand, structural response to strong earthquake ground motions cannot be accurately predicted due to large uncertainties and the randomness of structural properties and ground motion parameters. Therefore, for the time being, the most rational analysis and performance evaluation methods for practical applications seem to be simplified inelastic procedures, which combine the non-linear static (pushover) analysis of a relatively simple mathematical model and the response spectrum approach.

Advanced analysis procedure



Advanced analysis procedure

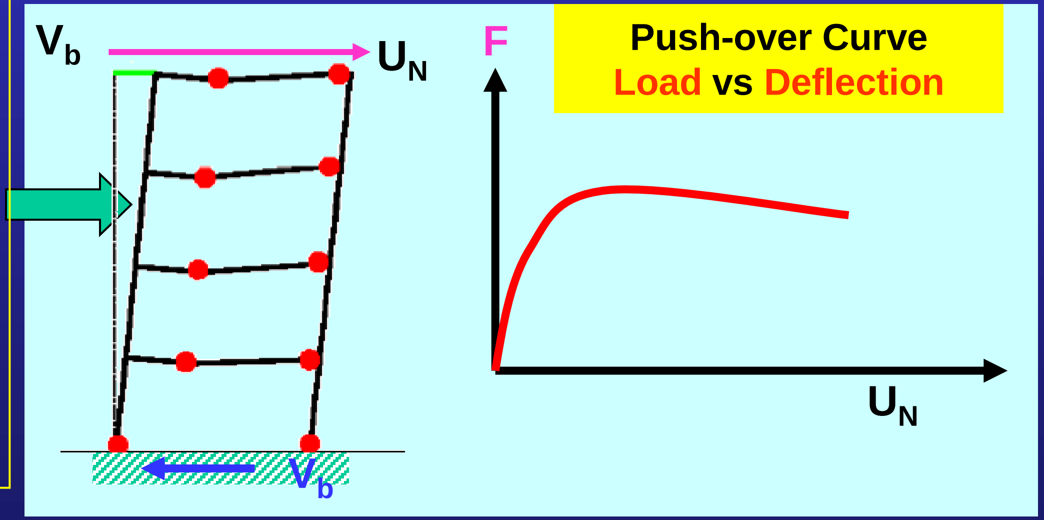
The conventional approach for design of frame structures is to treat theory of structural stability and plastic analysis/design as two separate topics. Little time has been spent to formulate solution methods and solve problems combining the theories of plasticity and stability. This philosophy has lead to an analysis and design approach in which forces and deformations demands are estimated from an elastic analysis and acceptability of a structure is assessed by comparing the demands with the component capacities defined in traditional limit states checks. In other words, the design specifications provide guidance to perform ultimate strength checks of structural components based on elastic forces obtained from global analysis. They are not adequate in addressing behavioral issues related to system limit-states. Advanced inelastic analysis methods represents a potential for direct handling of limit states within analysis models. The reason for this is that, since *advanced analysis* can directly asses the strength and stability of the overall structural system as well as interdependence of member and system strength and stability, separate member capacity checks are not required.



Seismic performance – Inelastic Types of analysis

The structural analysis in earthquake engineering is a complex task because (a) the problem is dynamic and usually non-linear, (b) the structural system is usually complex, and (c) input data (structural properties and ground motions) are random and uncertain. In principle, the non-linear time-history analysis is the correct approach. However, such an approach, for the time being, is not practical for everyday design use. It requires **additional input data (time-histories of ground motions and detailed hysteretic behavior of structural members) which cannot be reliably predicted**. Non-linear dynamic analysis is, at present, appropriate for research and for design of important structures. It represents a long-term trend. On the other hand, the methods applied in the great majority of existing building codes are based on the assumption of linear elastic structural behavior and do not provide information about real strength, ductility and energy dissipation. They also fail to predict expected damage in quantitative terms. For the time being, the most rational analysis and performance evaluation methods for practical applications seem to be simplified inelastic procedures, which combine the non-linear static (pushover) analysis of a relatively simple mathematical model and the response spectrum approach.

- **Nonlinear dynamic analysis- time history :**
  - **"exact solution"**
  - **large uncertainities of ground motion parameters.**
- **Push-over Analysis:**
  - **Static Nonlinear analysis**
  - **efficient method for the seismic performance evaluation of high-rise buildings**



Advanced analysis requirements

A plastic zone analysis can be considered as advanced analysis procedure if includes:

- the spread of plasticity (gradual yielding of cross sectional fibbers and gradual developing along the member length)
- three dimensional plastic interaction curves
- local and global geometrical nonliniarities
- lateral-torsional buckling effects
- local buckling effects
- bowing effect
- nonlinear behavior of semi-rigid connections
- local and global geometrical imperfections
- material imperfections, for instance the residual stresses in the case of hot-rolled steel members
- elastic unloading
- shear deformations effects
- and any other second-order behavioral effects.

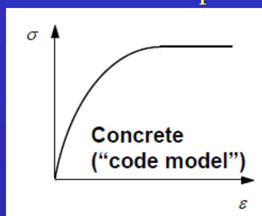
Although the plastic-zone solution my be considered “exact”, it is not yet conducive to daily use in engineering design, because it is too computationally intensive and too costly in the case of 3D real-large scale frame structures.

Sources of nonlinearity

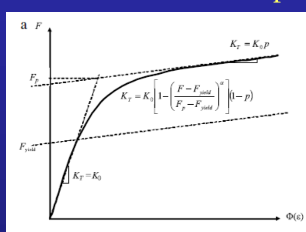
Three types of nonlinearities may arise for structures

Material (or physical)

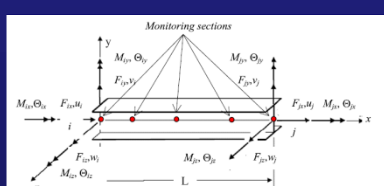
- Fiber level: nonlinear stress-strain relationships



- Cross-section level: nonlinear force strain relationships

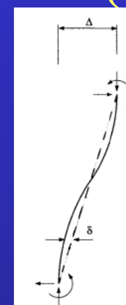


- Element level: nonlinear force displacement relationships

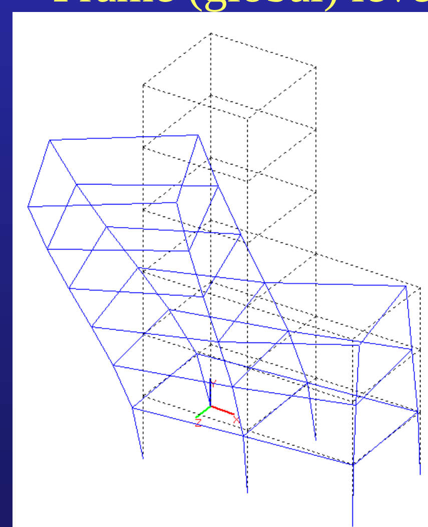


Geometrical

- Element (local) level

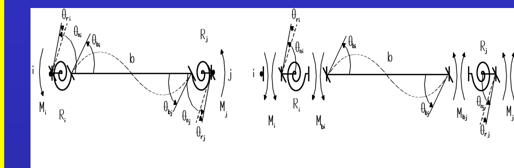


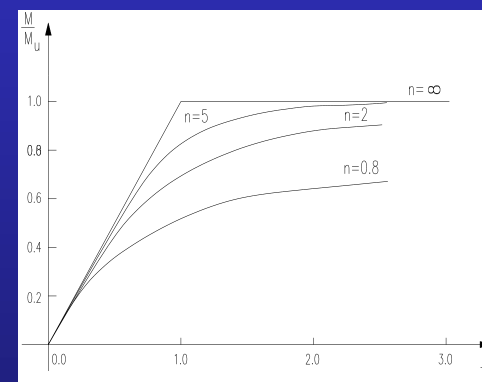
- Frame (global) level

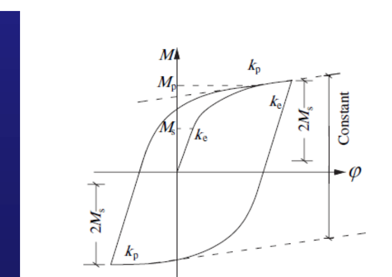


Connections

Joint level

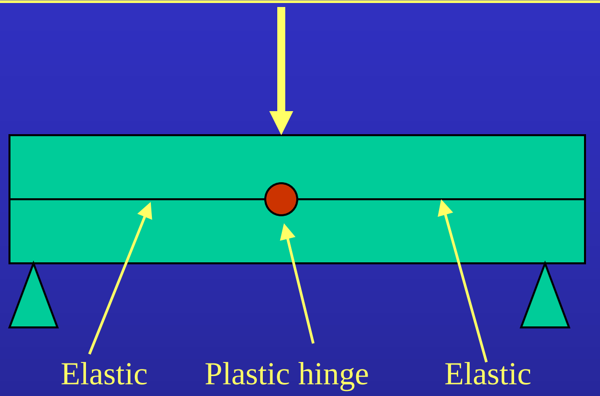






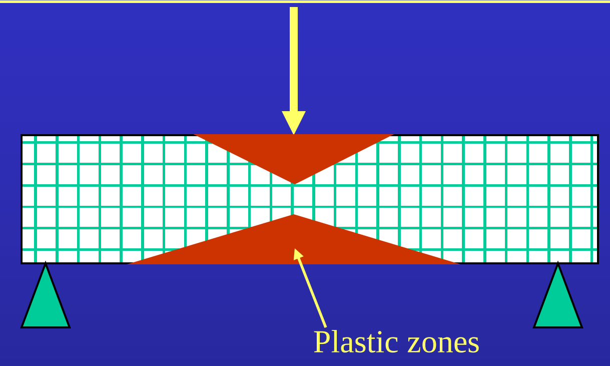
Inelastic analysis models

Concentrated plasticity



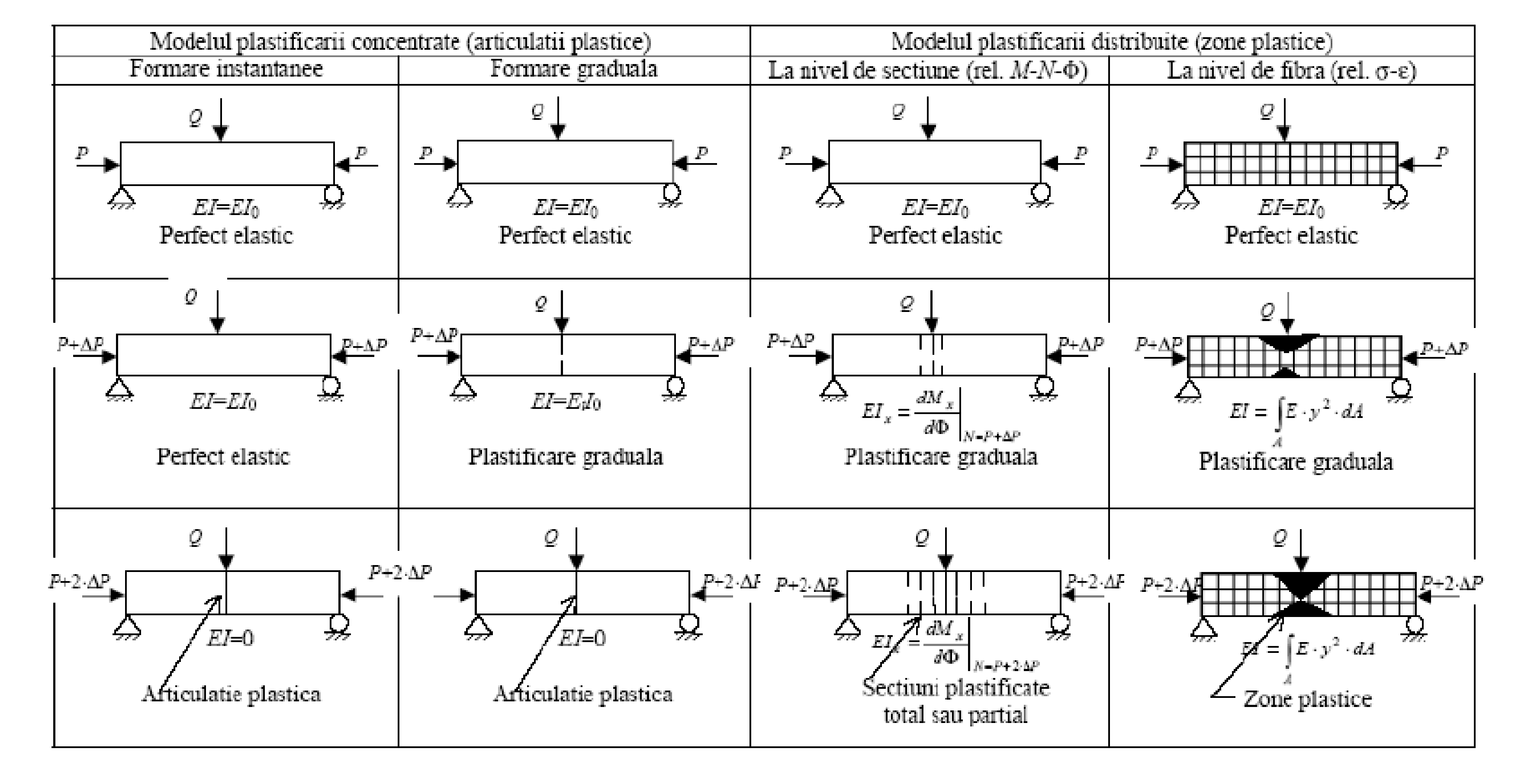
- Dimensionless plastic hinge
- Interaction surface
- Return mapping plasticity algorithms
- Computationally efficient but limited accuracy

Distributed plasticity(proposed)



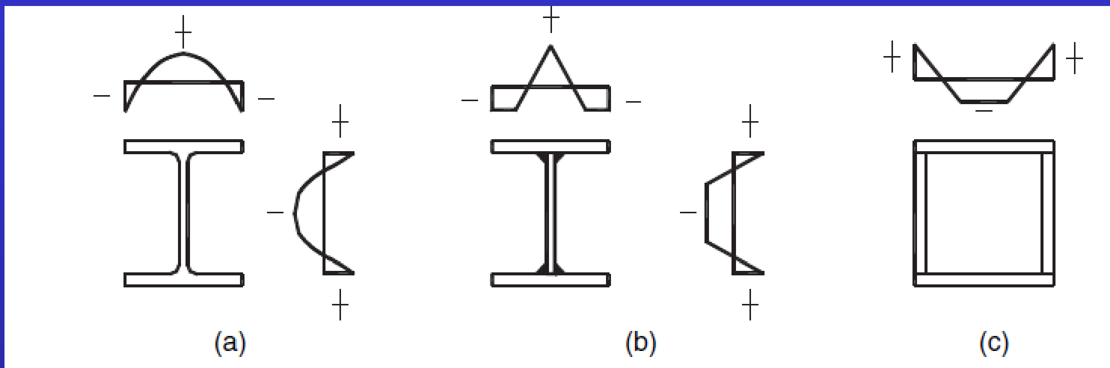
- Plastic zones
- Force-strain curves: quasi plastic hinge approach
- Stress-strain curves: fiber element approach
- Plastic flow rules
- High accuracy but computational expensive

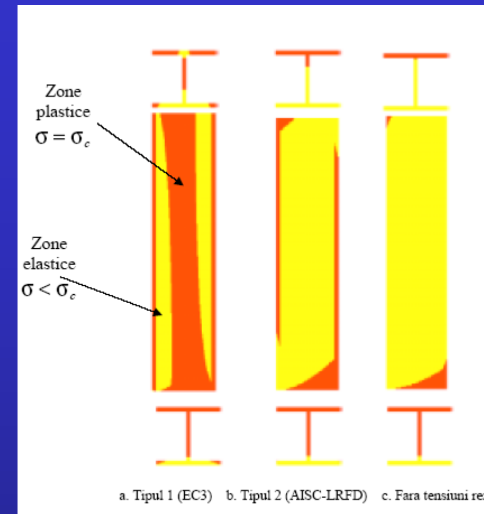
Inelastic analysis models

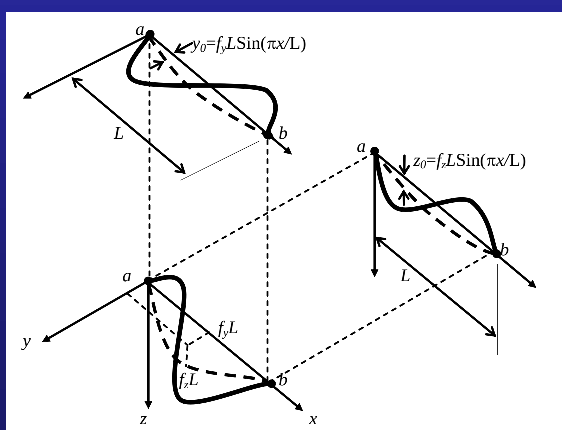


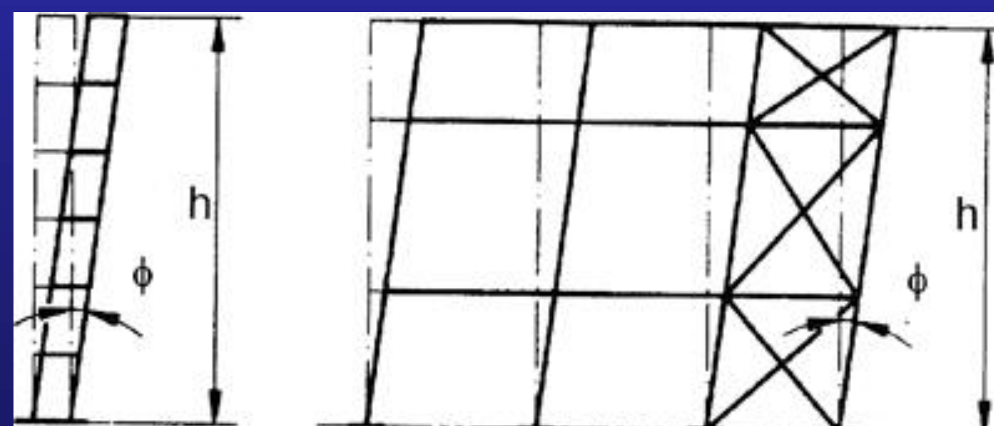
Sources of nonlinearity

Imperfections



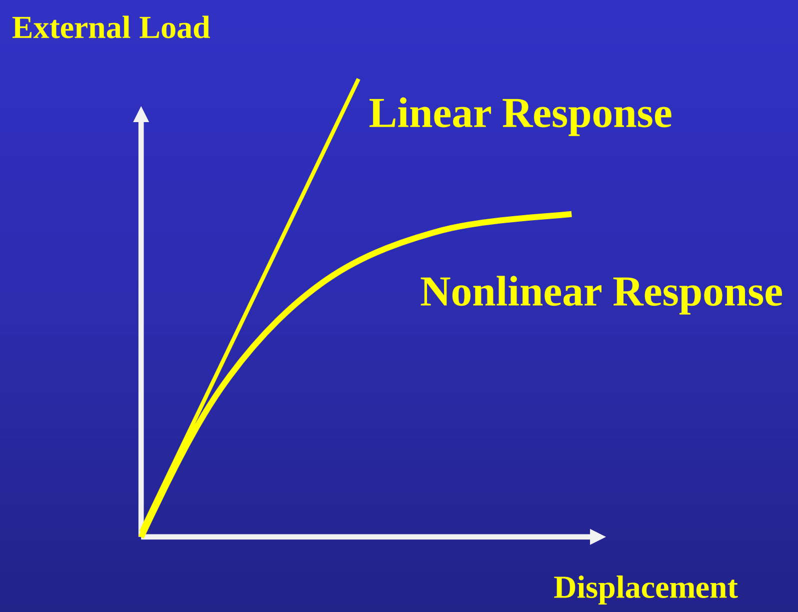






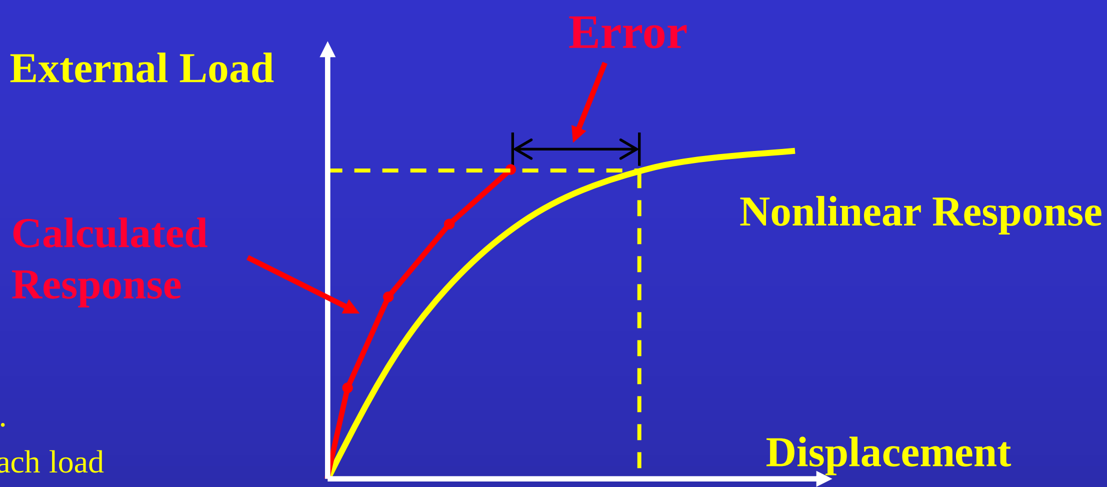
Linear vs Nonlinear

- Linearity
  - The response is directly proportional to excitation
  - (Deflection doubles if load is doubled)
- Non-Linearity
  - The response is not directly proportional to excitation
  - (deflection may become 4 times if load is doubled)
- Non-linear response may be produced by:
  - Geometric Effects (Geometric non-linearity)
  - Material Effects (Material non-linearity)
  - Both



Static Nonlinear Analysis-Basics

- One approach is to apply the load gradually by dividing it into a series of increments and adjusting the stiffness matrix at the end of each increment.
- The problem with this approach is that errors accumulate with each load increment, causing the final results to be out of equilibrium.



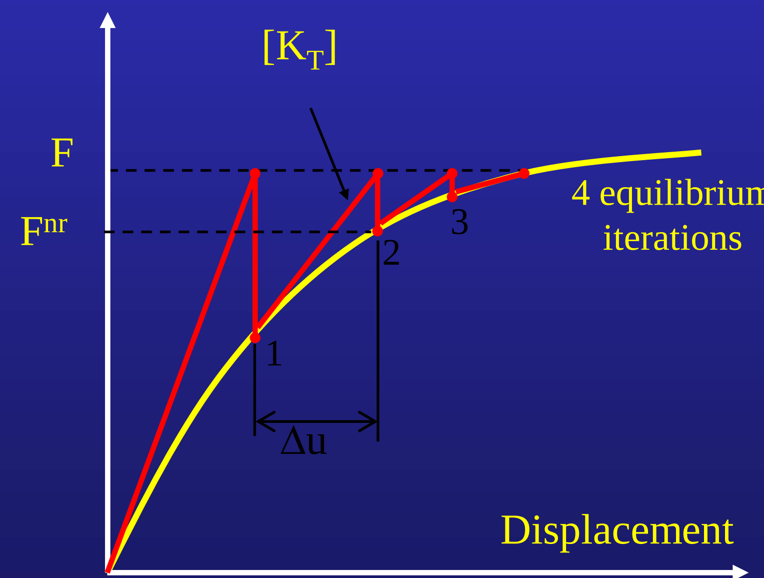
- General algorithm:
  - Applies the load gradually, in increments.
  - Also performs *equilibrium iterations* at each load increment to drive the incremental solution to equilibrium.
  - Solves the equation $[K_T]\{Du\} = \{F\} - \{F^{nr}\}$

    $[K_T]$ = tangent stiffness matrix

    $\{\Delta u\}$ = displacement increment

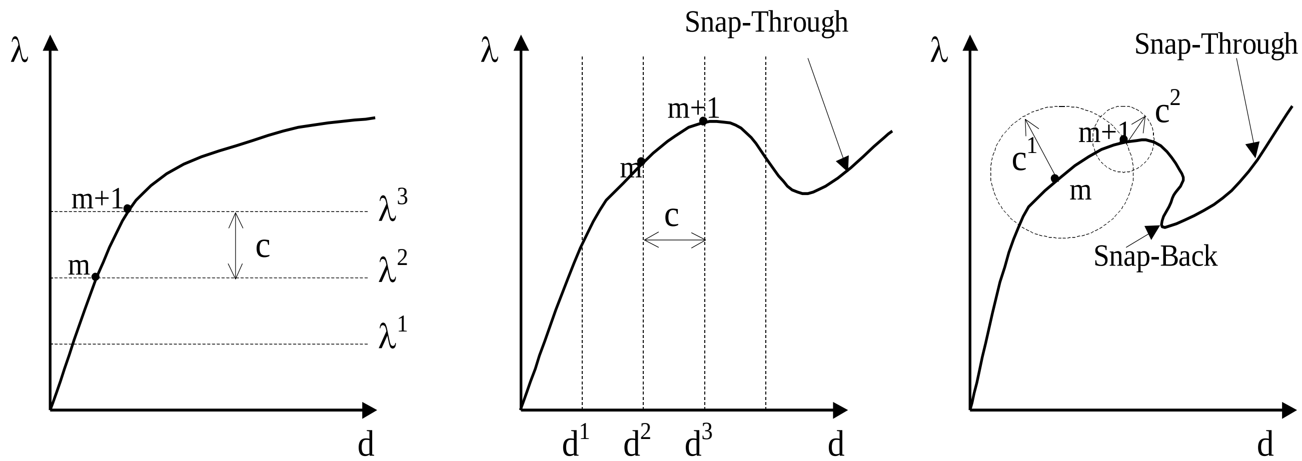
    $\{F\}$ = external load vector

    $\{F^{nr}\}$ = internal force vector
  - Iterations continue until $\{F\} - \{F^{nr}\}$ (difference between external and internal loads) is within a tolerance.
  - Some nonlinear analyses have trouble converging. Advanced analysis techniques are available in such cases.



Advanced incremetal-iterative strategies

Solution strategy

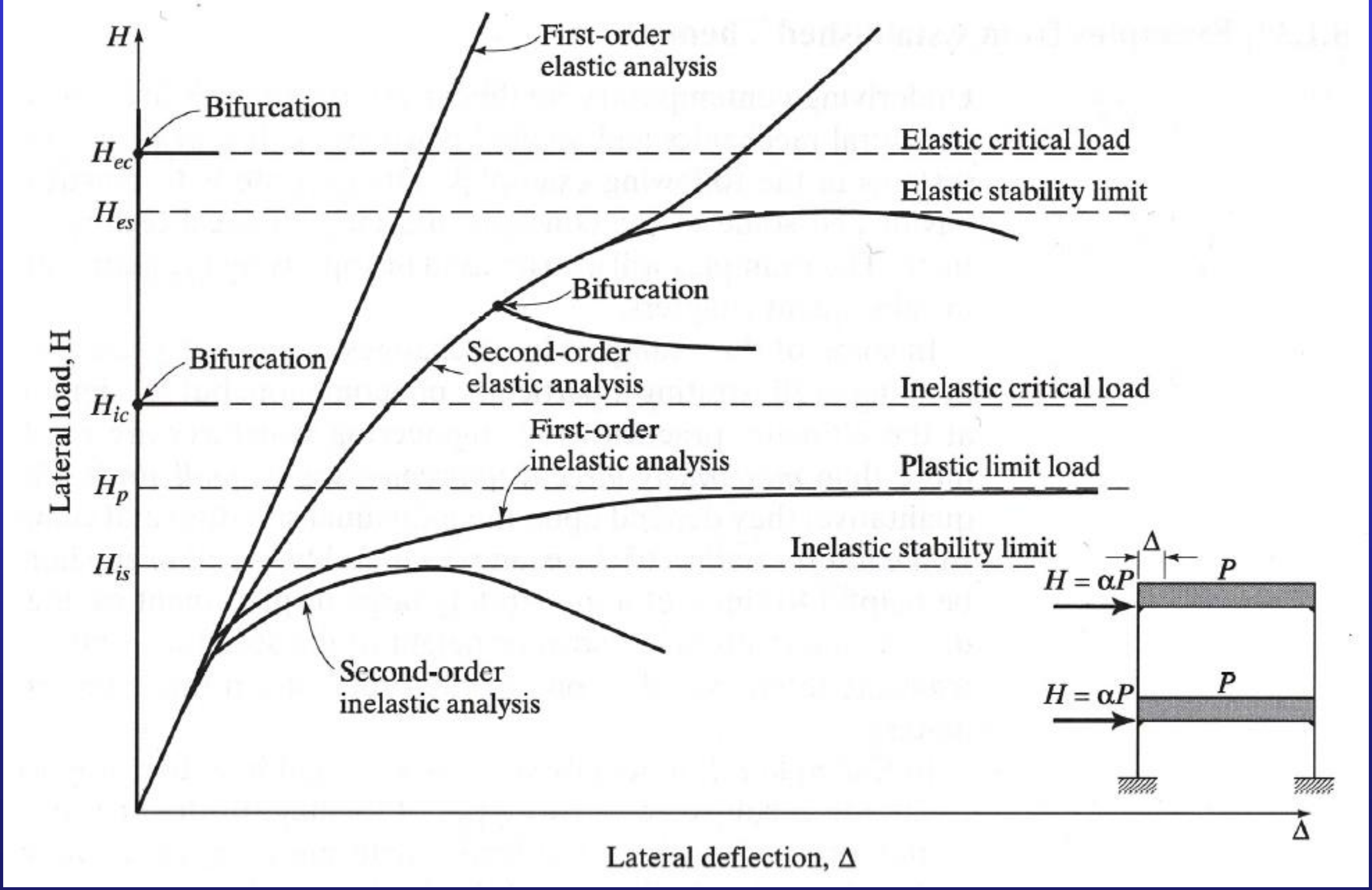


Control in incarcari	Control in deplasari	Control in lungimea de arc
$\lambda^{i+1} - \lambda^{i} = c$	$d^{i+1} - d^{i} = c$	$\left(\lambda^{i+1} - \lambda^{i}\right)^2 + \left(d^{i+1} - d^{i}\right)^2 = c^2$
(a)	(b)	(c)

$$\mathbf{P}(\mathbf{d}) - \lambda \cdot \mathbf{F} = 0$$

$$\sum_{k=1}^{n} \beta_k \cdot \left(d_k^m - d_k^{m-1}\right)^2 + \beta_{n+1} \cdot \alpha^2 \cdot \left(\lambda^m - \lambda^{m-1}\right)^2 = c^2$$

Nonlinear analysis models-STATIC



Types of analyses

1. **Nonlinear-Dynamic (Elastic OR Inelastic)**

$$M\ddot{u}(t) + C\dot{u}(t) + F(t)_{NL} = F(t)$$

2. **Nonlinear – Static (Elastic OR Inelastic)**

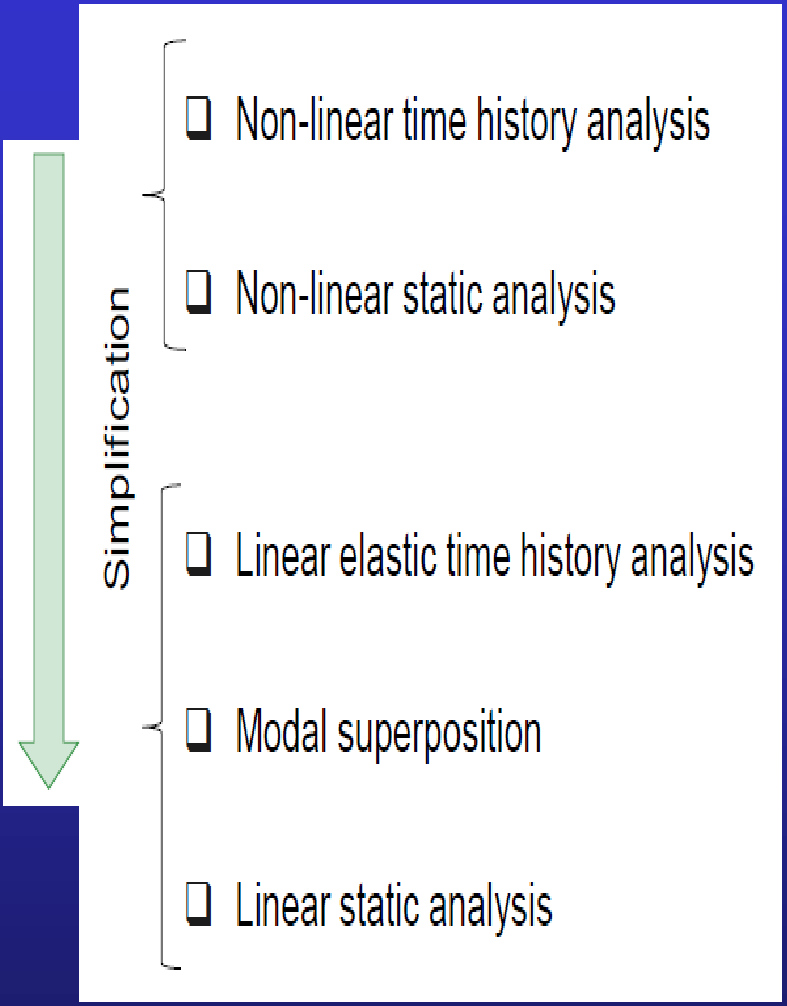
$$Ku + F_{NL} = F$$

3. **Linear-Dynamic (Elastic)**

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = F(t)$$

4. **Linear-Static (Elastic OR Inelastic)**

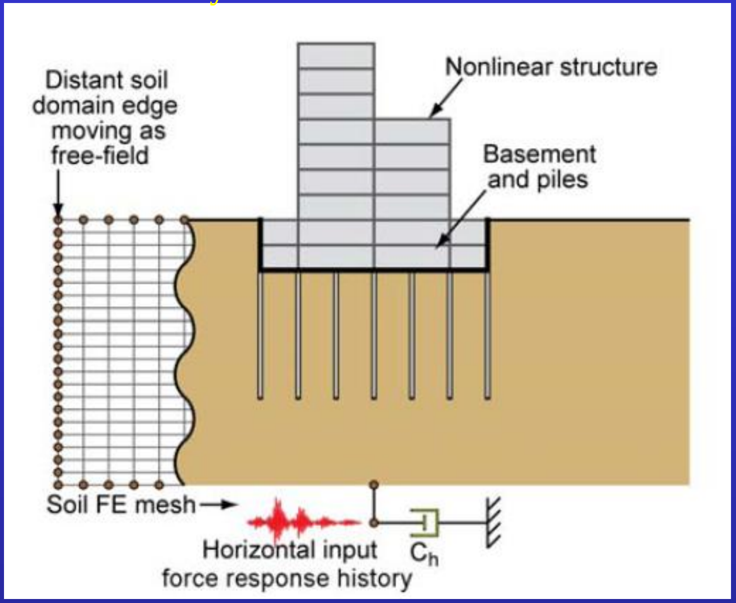
$$Ku = F$$



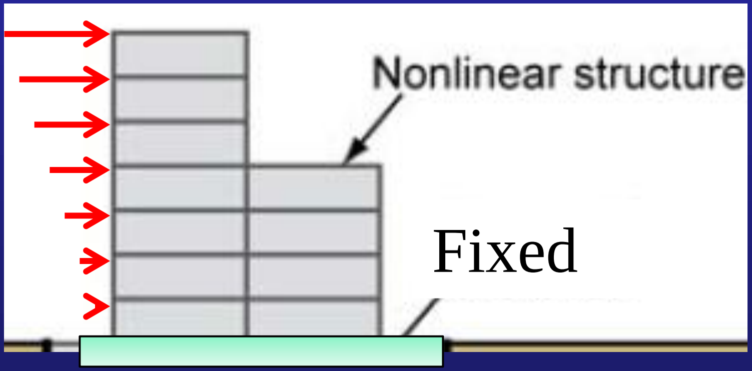
Static vs Dynamic

- **Static Excitation**
  - When the Excitation (Load) does not vary rapidly with Time
  - When the Load can be assumed to be applied "Slowly"
- **Dynamic Excitation**
  - When the Excitation varies rapidly with Time
  - When the "Inertial Force" becomes significant
- **Most Real Excitation are Dynamic but are considered "Quasi Static"**
- **<u>Most Dynamic Excitation can be converted to "Equivalent Static Loads"</u>**

- **Dynamic Excitation**



Equivalent Static Loads



Nonlinear dynamic analysis-”time history”

Linear differential equation of dynamic equilibrium

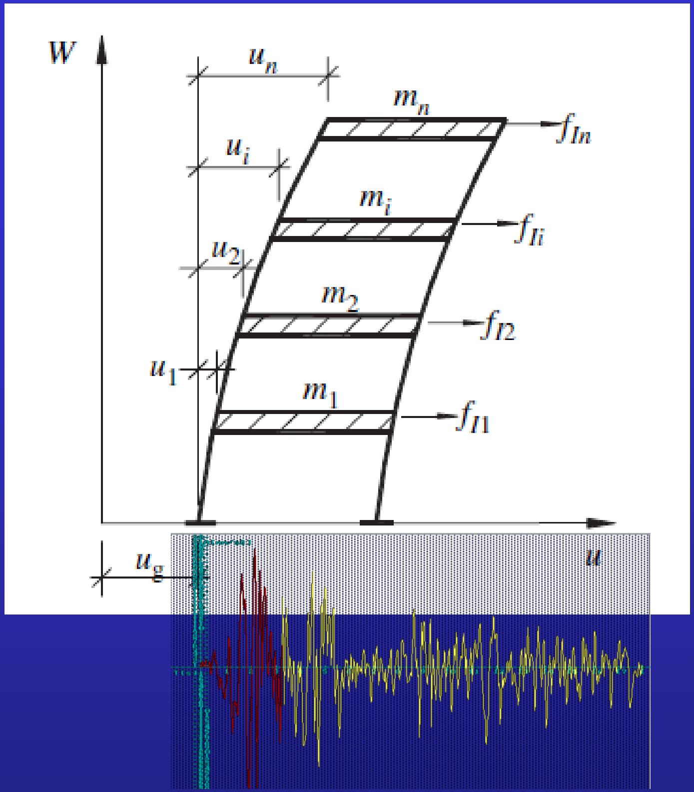
$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K_e]\{u\} = -[M]\{1\}\ddot{u}_g.$$

NonLinear differential equation of dynamic equilibrium at the time “t”

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + \{f(u(t))\} = -[M]\{1\}\ddot{u}_g(t),$$

NonLinear differential equation of dynamic equilibrium at the time “t+Δt”

$$[M]\{\ddot{u}(t+\Delta t)\} + [C]\{\dot{u}(t+\Delta t)\} + \{f(u(t+\Delta t))\} = -[M]\{1\}\ddot{u}_g(t+\Delta t),$$



$$[M]\{\Delta\ddot{u}\} + [C]\{\Delta\dot{u}\} + \{\Delta f\} = -[M]\{1\}\Delta\ddot{u}_g,$$

$$\{\Delta f\} = \{f(u(t+\Delta t))\} - \{f(u(t))\},$$

$$\Delta\ddot{u}_g = \ddot{u}_g(t+\Delta t) - \ddot{u}_g(t),$$

$$\{\Delta\ddot{u}\} = \{\ddot{u}(t+\Delta t)\} - \{\ddot{u}(t)\},$$

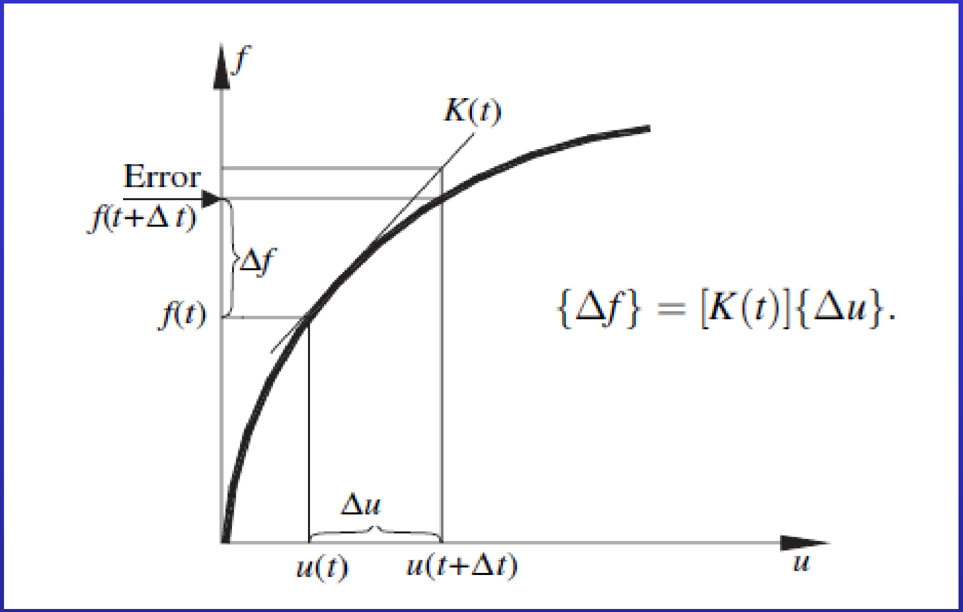
$$\{\Delta\dot{u}\} = \{\dot{u}(t+\Delta t)\} - \{\dot{u}(t)\}.$$

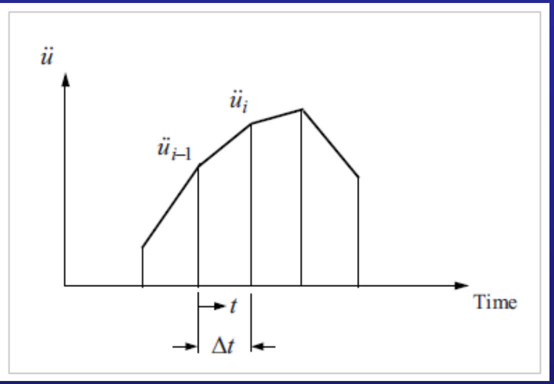
Incremental differential equation of dynamic equilibrium at the time “Δt”

$$\boxed{[M]\{\Delta\ddot{u}\} + [C]\{\Delta\dot{u}\} + [K(t)]\{\Delta u\} = -[M]\{1\}\Delta\ddot{u}_g.}$$

Nonlinear dynamic analysis-"time history"

Incremental differential equation of dynamic equilibrium at the time "Δt"





Nonlinear dynamic analysis-"time history"

Taylor seiries expansion used to reperesent the displacemet and velocity at time t+Δt

$$\{u(t+\Delta t)\} = \{u(t)\} + \{\dot{u}(t)\}\Delta t + \{\ddot{u}(t)\}\frac{\Delta t^2}{2} + \{\dddot{u}(t)\}\frac{\Delta t^3}{6} + \cdots,$$

$$\{\dot{u}(t+\Delta t)\} = \{\dot{u}(t)\} + \{\ddot{u}(t)\}\Delta t + \{\dddot{u}(t)\}\frac{\Delta t^2}{2} + \cdots.$$

Linear acceleration hypothesis

$$\{\dddot{u}(t)\} = \frac{1}{\Delta t}(\{\ddot{u}(t+\Delta t)\} - \{\ddot{u}(t)\}) = \frac{1}{\Delta t}\{\Delta\ddot{u}\},$$

$$\left\{\frac{\mathrm{d}^r u(t)}{\mathrm{d}t^r}\right\} = 0, \quad \text{for} \quad r = 4, 5, \ldots$$

$$\{\Delta u\} = \{\dot{u}(t)\}\Delta t + \{\ddot{u}(t)\}\frac{\Delta t^2}{2} + \{\dddot{u}(t)\}\frac{\Delta t^3}{6},$$

$$\{\Delta\dot{u}\} = \{\ddot{u}(t)\}\Delta t + \{\Delta\ddot{u}\}\frac{\Delta t}{2}.$$

$$\{\Delta\ddot{u}\} = \frac{6}{\Delta t^2}\{\Delta u\} - \frac{6}{\Delta t}\{\dot{u}(t)\} - 3\{\ddot{u}(t)\},$$

$$\{\Delta\dot{u}\} = \frac{3}{\Delta t}\{\Delta u\} - 3\{\dot{u}(t)\} - \frac{\Delta t}{2}\{\ddot{u}(t)\}.$$

$$\boxed{[K^*]\{\Delta u\} = \{F^*\},}$$

Quasi static equilibrium equation

$$[K^*] = [K(t)] + \frac{6}{\Delta t^2}[M] + \frac{3}{\Delta t}[C],$$

$$\{F^*\} = -[M]\{1\}\Delta\ddot{u}_g + [M]\left(\frac{6}{\Delta t}\{\dot{u}(t)\} + 3\{\ddot{u}(t)\}\right) + [C]\left(3\{\dot{u}(t)\} + \frac{\Delta t}{2}\{\ddot{u}(t)\}\right).$$

Nonlinear dynamic analysis-"time history"

Stiffness matrix- obtained by "dynamic" condensation

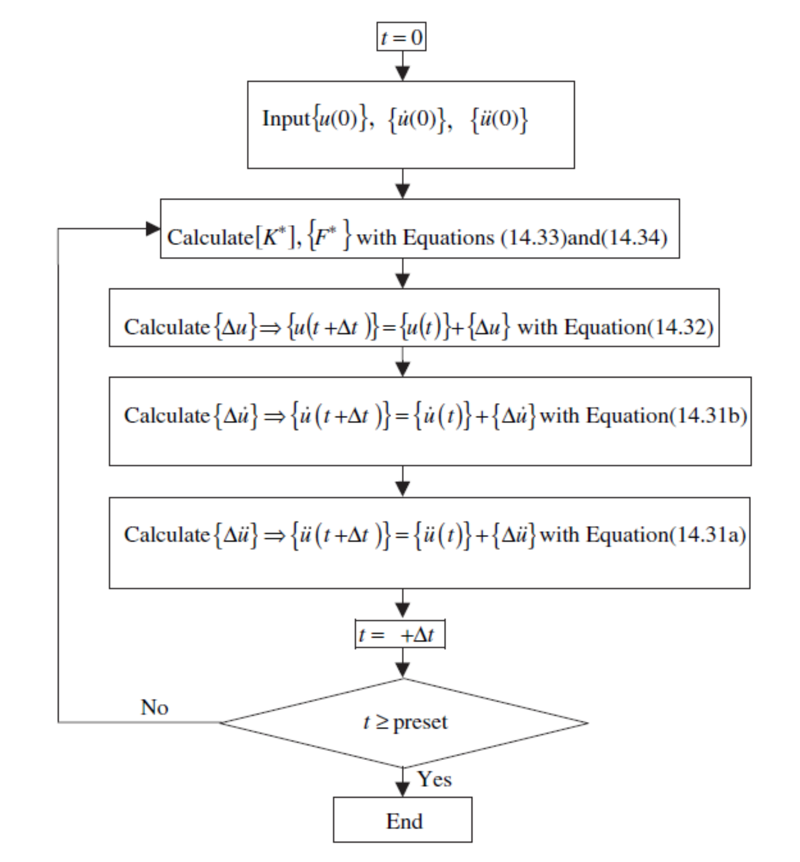
$$\begin{bmatrix} [K_{uu}(t)] & [K_{ur}(t)] \\ [K_{ru}(t)] & [K_{rr}(t)] \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta r \end{Bmatrix} = \begin{Bmatrix} \Delta F_u \\ \Delta F_r \end{Bmatrix},$$

$$[C] = a[M] + b[K_e],$$

Damping matrix

$$\{\phi_i\}^T[C]\{\phi_i\} = a\{\phi_i\}^T[M]\{\phi_i\} + b\{\phi_i\}^T[K_e]\{\phi_i\},$$
$$\{\phi_j\}^T[C]\{\phi_j\} = a\{\phi_j\}^T[M]\{\phi_j\} + b\{\phi_j\}^T[K_e]\{\phi_j\}.$$

$$a = \frac{2\omega_i\omega_j(\xi_i\omega_j - \xi_j\omega_i)}{\omega_j^2 - \omega_i^2},$$
$$b = \frac{2(\omega_j\xi_j - \omega_i\xi_i)}{\omega_j^2 - \omega_i^2}.$$



Flowchart for the analysis of structural elasto-plastic response

Nonlinear dynamic analysis-"time history"-requirements

The NDA provides more accurate calculation of the structural response to strong ground shaking.

Incorporates inelastic member behaviour under cyclic earthquake NDA explicitly simulates hysteretic energy dissipation in the nonlinear range

Due to inherent variability (due to three main sources: hazard uncertainity in the ground motion intensity, such as the spectral acceleration intensity calculated for a specified earthquake scenario; frequevency content and duration of a ground motion with a given intensity; structural behavior and modelling uncertainities such as material properties, nonlinear behavior, mathematical models) in earthquake ground motions NDA for multiple ground motions are necessary to calculate statistically robust values of the demand parameters for given excitation.

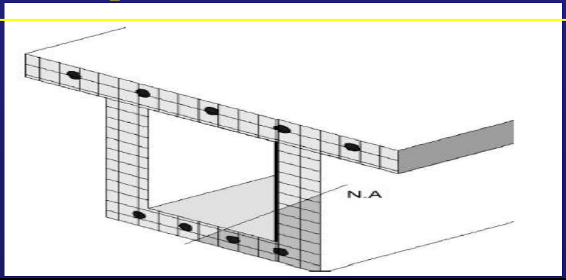
However the accuracy of results depends on the details of the analysis model and how faithfully it captures the significant behavioural effects and **<u>for the time being is considered much computational expensive.</u>**

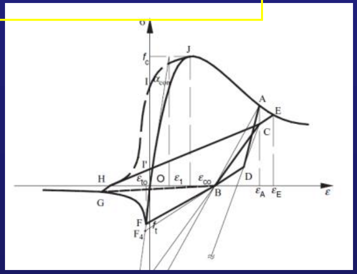
NDA generaly provide more realistic models of structural response to strong ground motions.

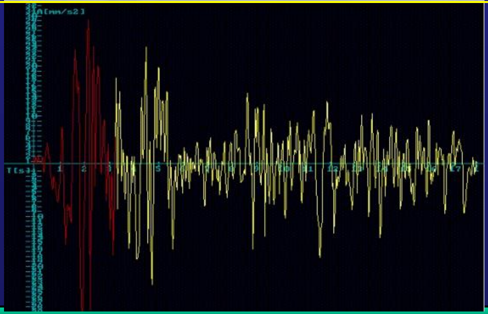
NSA provides a conveninet and fairly reliable method for structures whose dynamic response is governed by first-mode sway motions.

NSA can be an effective design tool to investigate aspects of the analysis model and the nonlinear response that are difficult to do by nonlinear dynamic analysis.

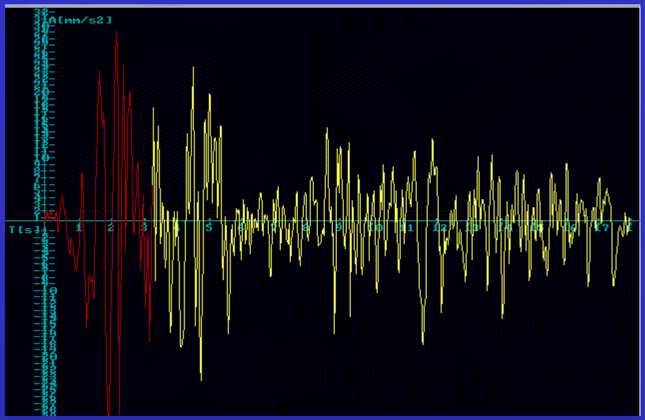
<u>NSA can be useful to: (1) check and dsebug the nonlinear analysis model; (2) augment understanding of the yielding mechanisms and deformation demands and (3) investigate alternative design parameters and how variations in the component properties may affect response.</u>

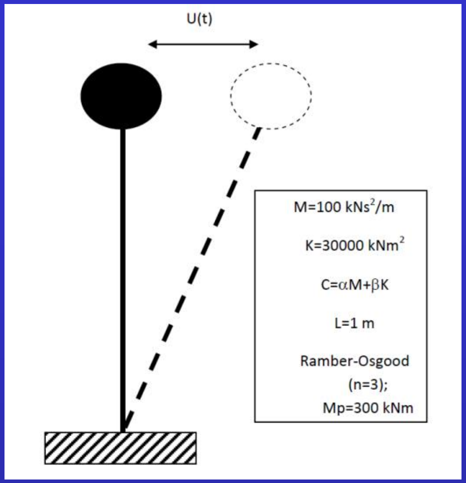


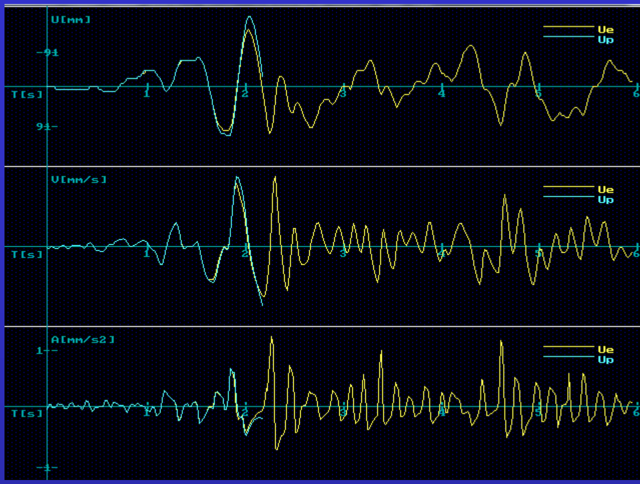


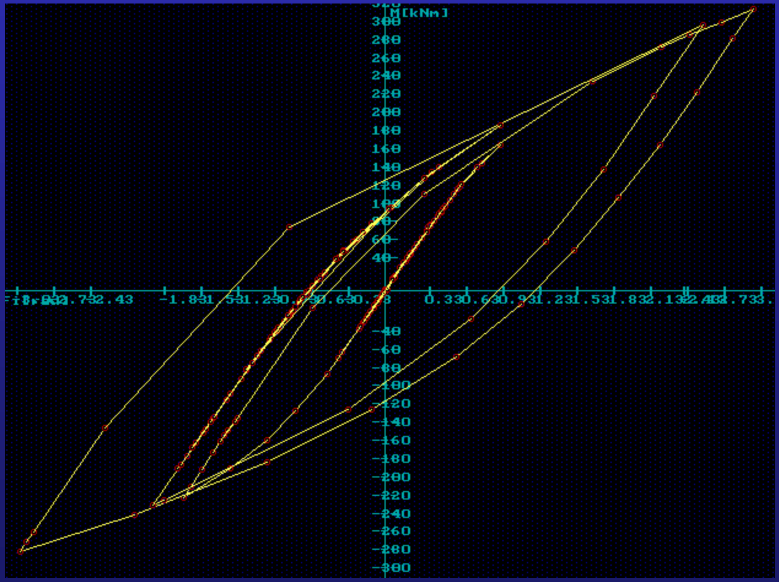


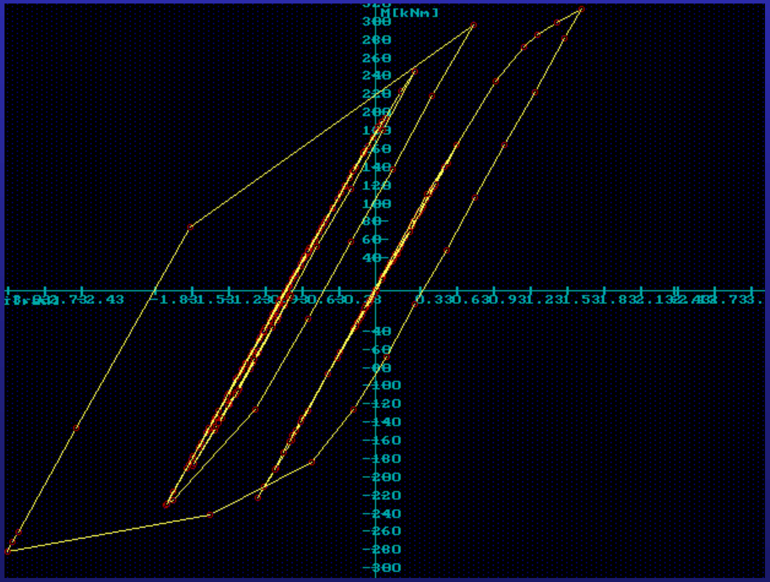
Nonlinear dynamic analysis-"time history"





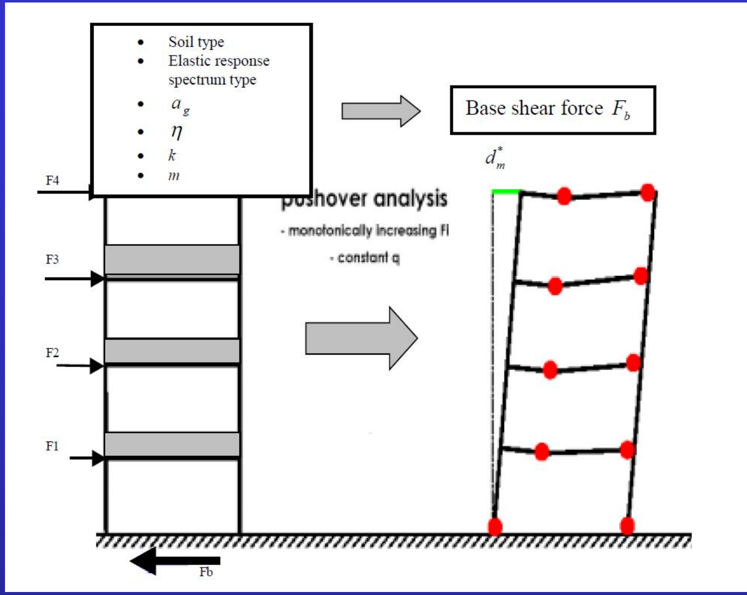


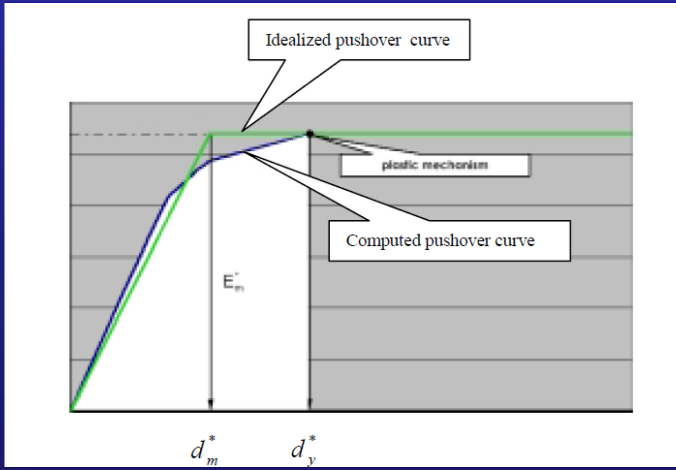




Key elements of the push-over analysis

- Nonlinear static procedure: constant gravitational loads and monotonically increasing lateral loads
- Plastic mechanisms and P-Δ effects: displacement or arc length control
- Capacity curve: Control node displacement vs base shear force
- Lateral load patterns: uniform, modal, SRSS, ELF force distribution
- Estimation of the target displacement: elastic or inelastic response spectrum for equivalent SDOF system
- Performance evaluation: global and local seismic demands with capacities of performance level.



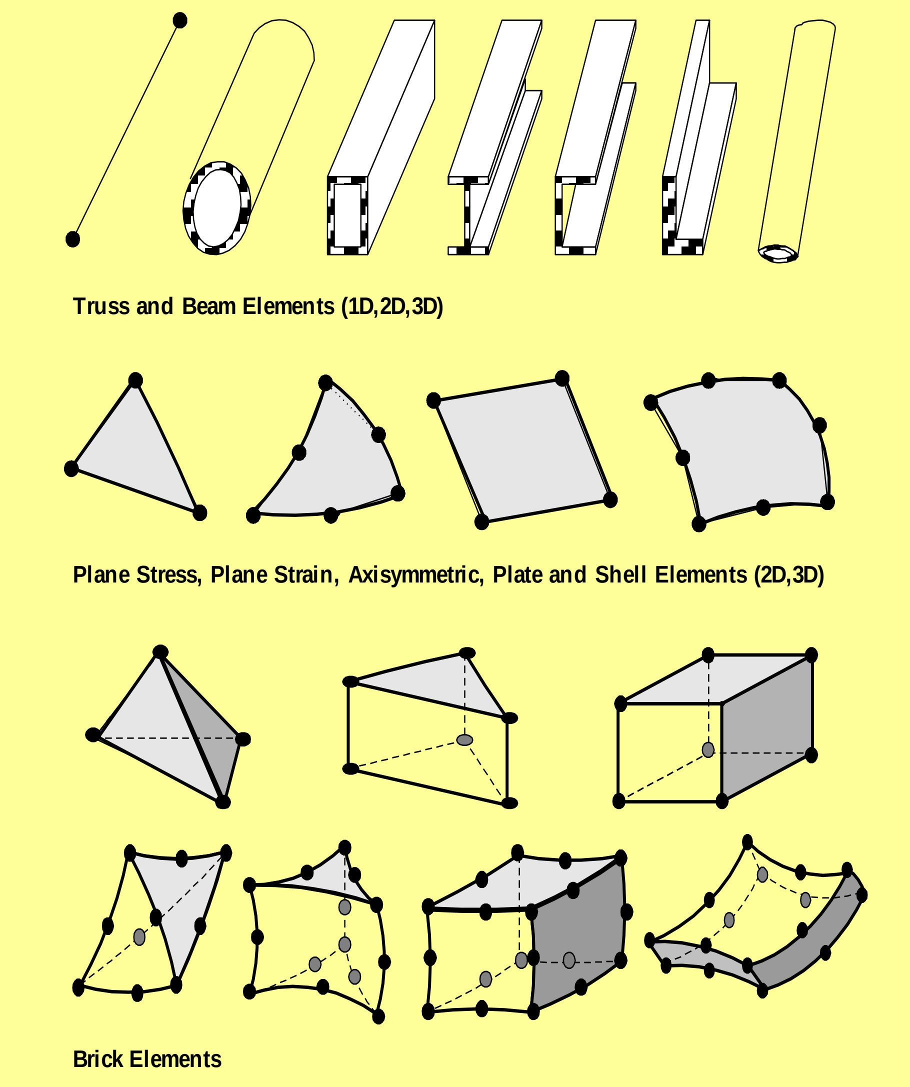


Finite elements-Types

- 1 D Elements (Beam type)
  - Can be used in 1D, 2D and 2D
  - 2-3 Nodes. A, I etc.

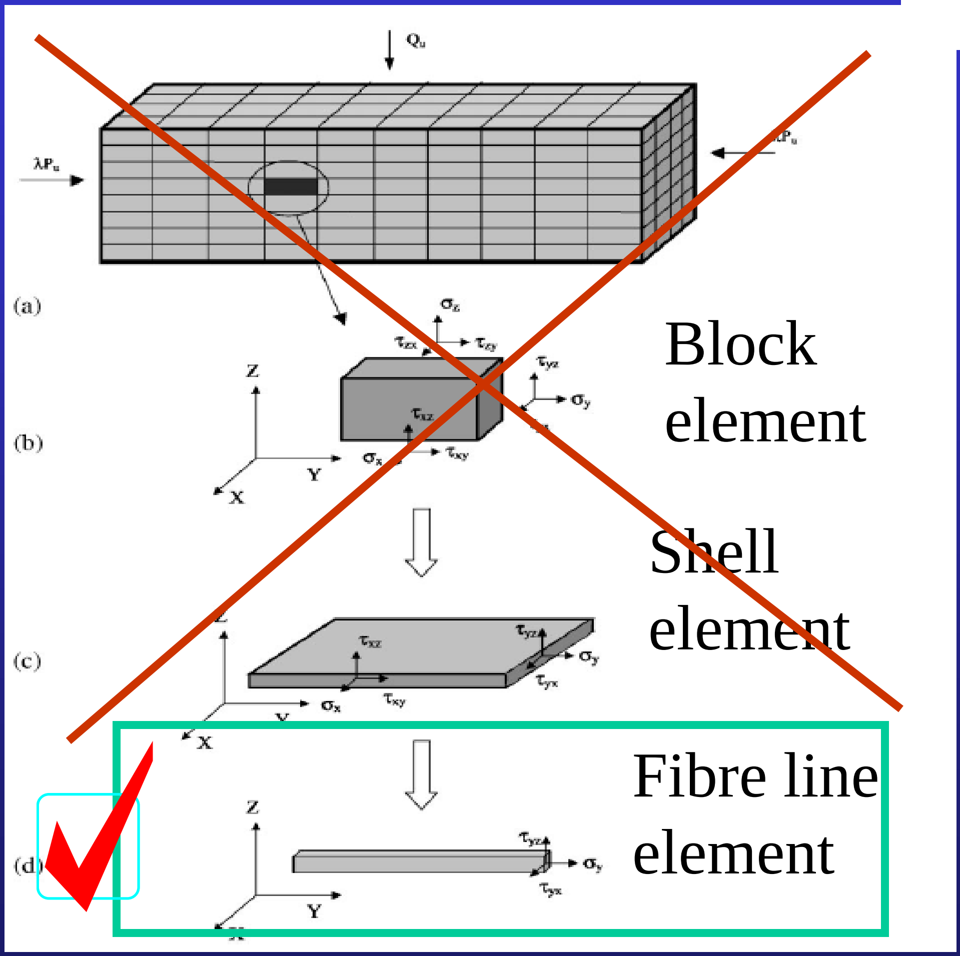
- 2 D Elements (Plate type)
  - Can be used in 2D and 3D Model
  - 3-9 nodes. Thickness

- 3 D Elements (Brick type)
  - Can be used in 3D Model
  - 6-20 Nodes.

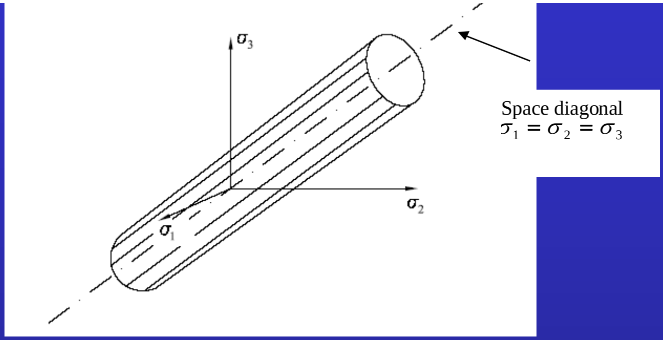


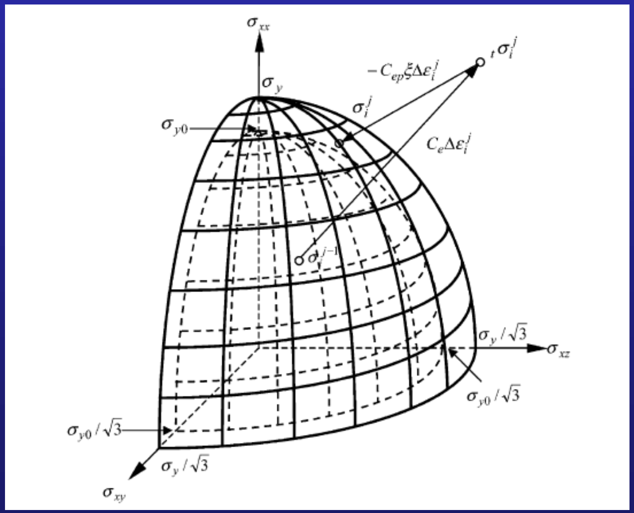
Inelastic analysis models- Distributed plasticity

Finite element discretization



Advanced nonlinear constitutive models





Inelastic analysis models: Concentrated vs Distributed

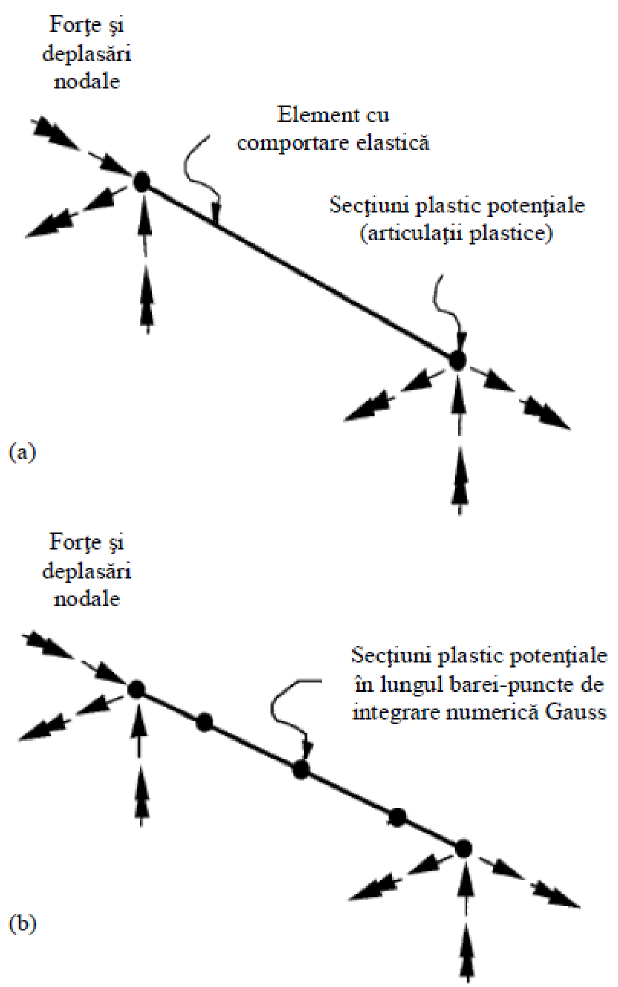
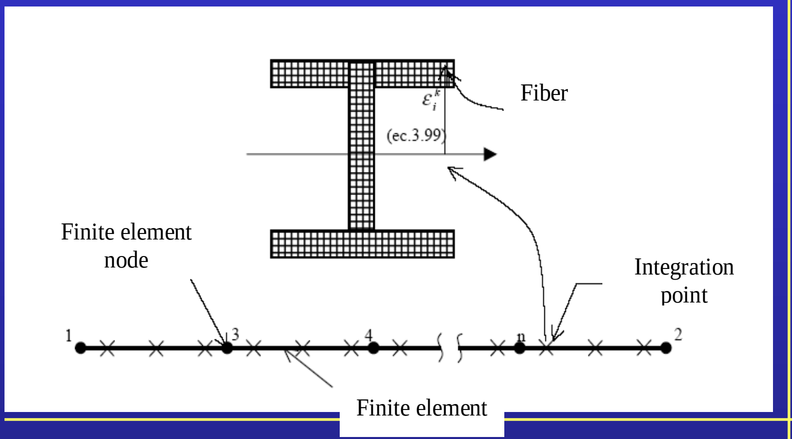
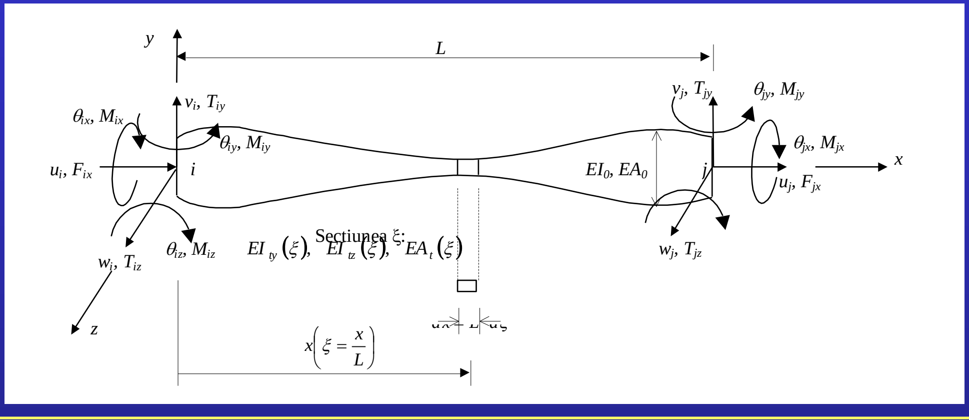


Fig. 2.1 Elemente liniare unidimensionale utilizate în analiza elasto-plastică. (a) Modelul plastificării concentrate; (b) Modelul plastificării distribuite.

Inelastic analysis models- Line elements

Displacement -based (finite element)	Flexibility-based (proposed approach)
	
$\boxed{\mathbf{u}(x)=\mathbf{N}(x)\cdot\mathbf{d}}$	$\boxed{\mathbf{S}(x)=\mathbf{B}(x)\cdot\mathbf{E}}$
•Displacement shape functions •Influenced by the inelastic effects •Do not satisfy equilibrium •Many finite elements -**EXPENSIVE**	• Assumed force distribution •Satisfy equilibrium-distribution function is exact •Is not senzitive to the inelastic effects •One element per member -**EFFICIENT**

Finite fiber element approach

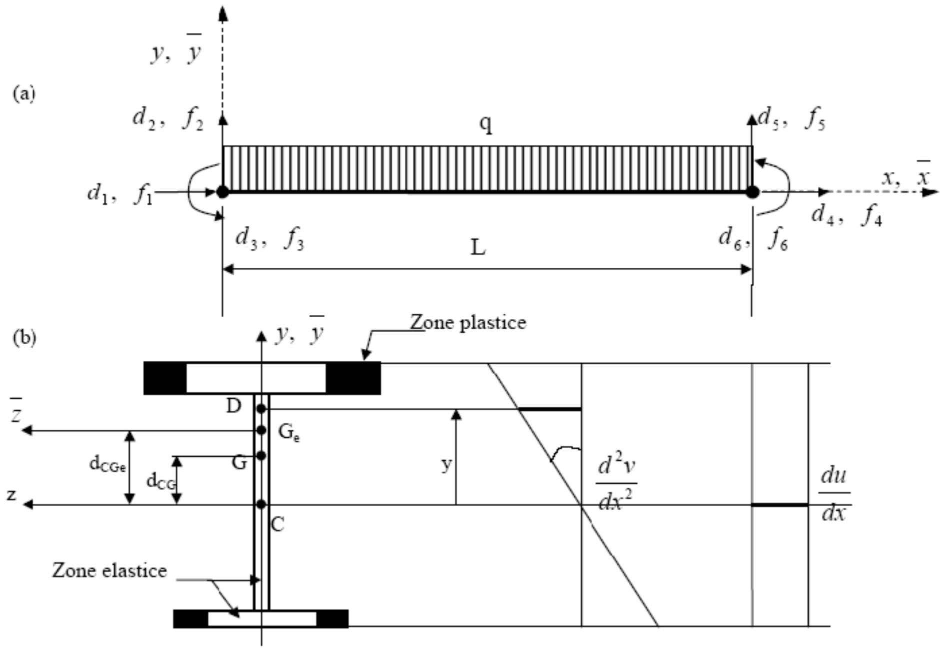
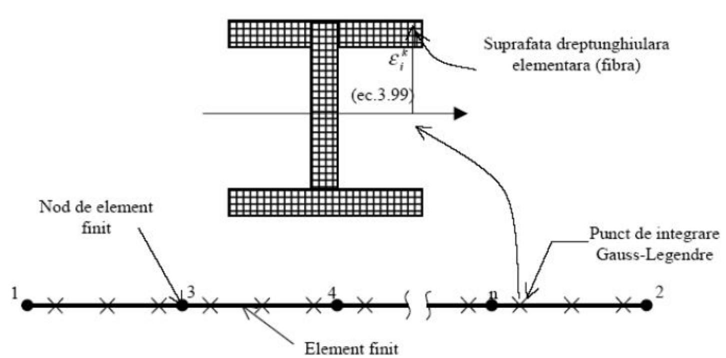
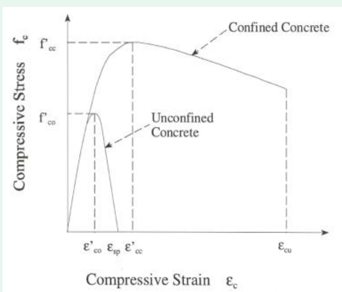


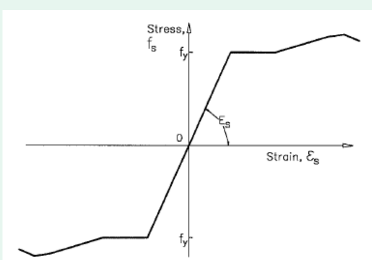
Fig. 2.29. Elementul finit de fibră.

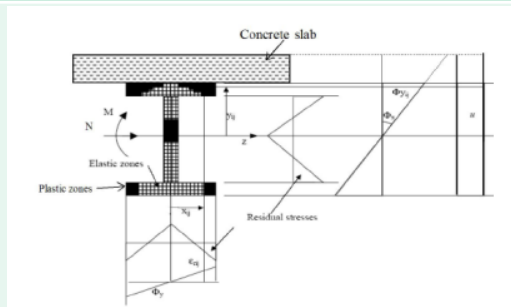


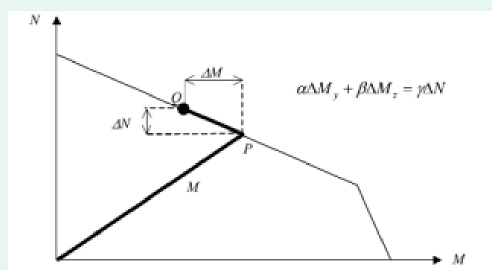
Distributed plasticity approach

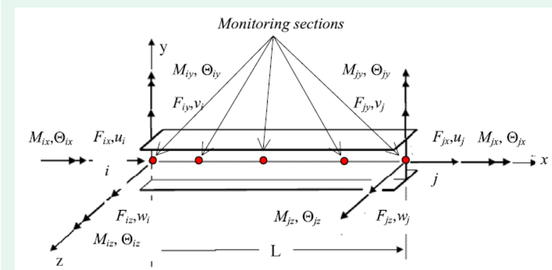
Fiber level	Cross-section level	Element level
Nonlinear stress-strain relationships and failure criteria $\varepsilon_0 + \phi_x y_c(\phi_x,\phi_y) + \phi_y x_c(\phi_x,\phi_y) - \varepsilon_u = 0$	$f_N(u,\Phi_y,\Phi_z) \equiv \sum_i \int_{\Omega_i} \sigma(\varepsilon)\cdot d\Omega_i - N = 0$ $f_{My}(u,\Phi_y,\Phi_z) \equiv \sum_i \int_{\Omega_i} \sigma(\varepsilon)\cdot y \cdot d\Omega_i - M_y = 0$ $f_{Mz}(u,\Phi_y,\Phi_z) \equiv \sum_i \int_{\Omega_i} \sigma(\varepsilon)\cdot z \cdot d\Omega_i - M_z = 0$	$\Delta W = \frac{1}{2}\cdot\int_0^L \frac{N^2}{EA_t(x)}dx + \frac{1}{2}\cdot\int_0^L \frac{\left(\frac{M_{jz}-M_{iz}}{L}x + M_{iz}\right)^2}{EI_{tz}(x)}dx + \frac{1}{2}\cdot\int_0^L \frac{\left(\frac{M_{jy}-M_{iy}}{L}x + M_{iy}\right)^2}{EI_{ty}(x)}dx + \frac{1}{2}\cdot\int_0^L \frac{\left(\frac{M_{jz}-M_{iz}}{L}\right)^2}{GA_z(x)}dx + \frac{1}{2}\cdot\int_0^L \frac{\left(\frac{M_{jy}-M_{iy}}{L}\right)^2}{GA_y(x)}dx + \frac{1}{2}\cdot\int_0^L \frac{M_x^2}{GI_t(x)}dx$

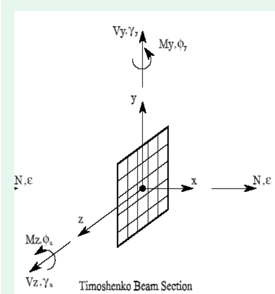


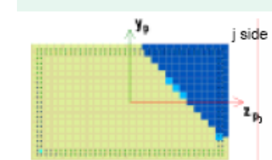










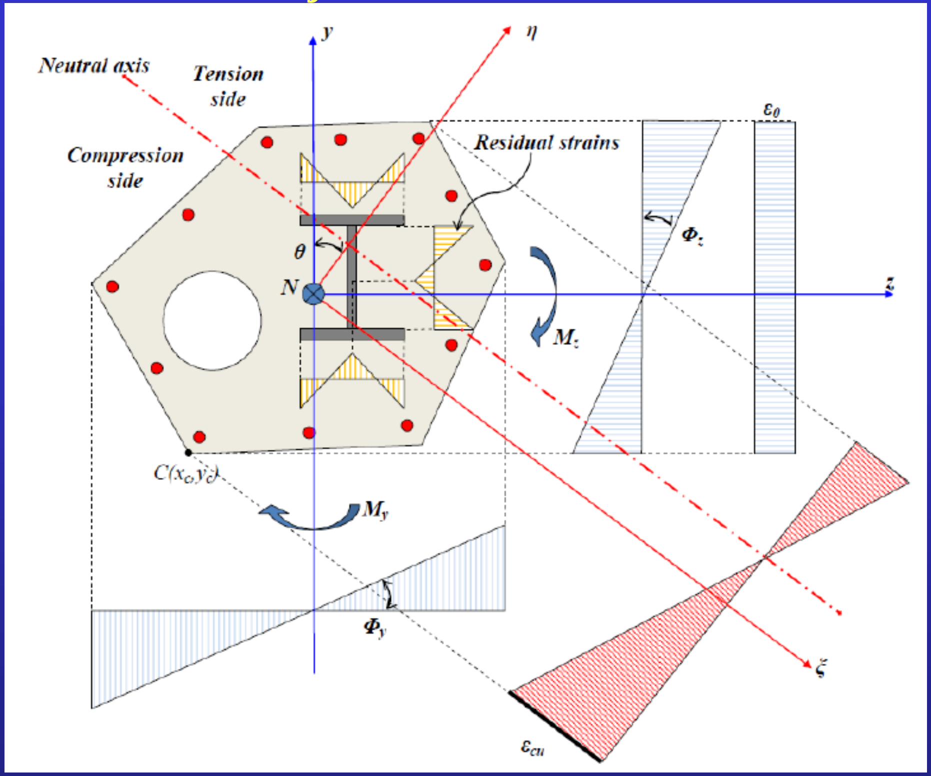


Advanced inelastic analysis of cross-sections

- Moment curvature analysis
- Computerized interaction diagrams
- Automatic design of composite steel-concrete cross-sections

Mathematical formulation: General

Arbitrary cross-section subjected to axial force and biaxial bending moments



Equilibrium equations

$$\begin{cases} \int_{A_{cs}} \sigma\left(\varepsilon\left(\varepsilon_0, \phi_y, \phi_z\right)\right) dA_{cs} - N = 0 \\ \int_{A_{cs}} \sigma\left(\varepsilon\left(\varepsilon_0, \phi_y, \phi_z\right)\right) z\, dA_{cs} - M_y = 0 \\ \int_{A_{cs}} \sigma\left(\varepsilon\left(\varepsilon_0, \phi_y, \phi_z\right)\right) y\, dA_{cs} - M_z = 0 \end{cases}$$

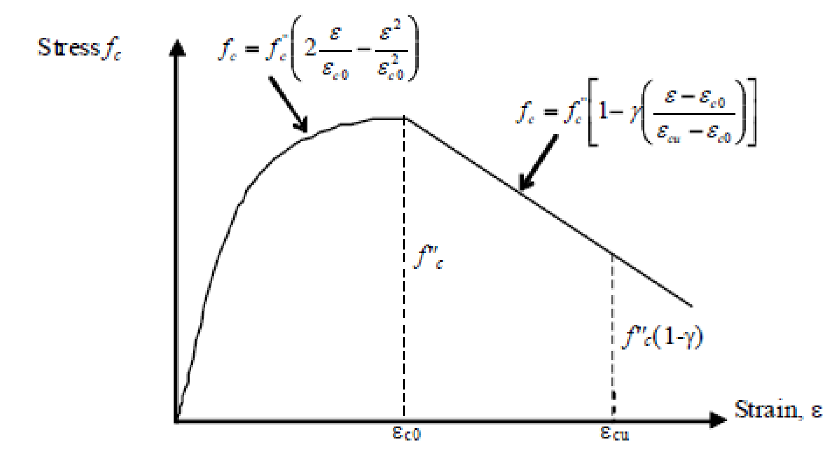
\+

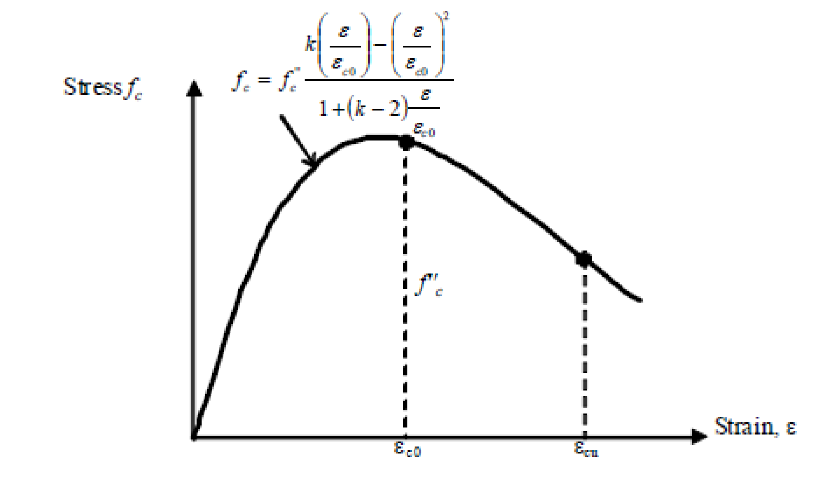
Plane section hypothesis (Bernouli)

$$\varepsilon = \varepsilon_0 + \Phi_y z + \Phi_z y + \varepsilon_r = \varepsilon_0 + \mathbf{\Phi r}^T + \varepsilon_r$$

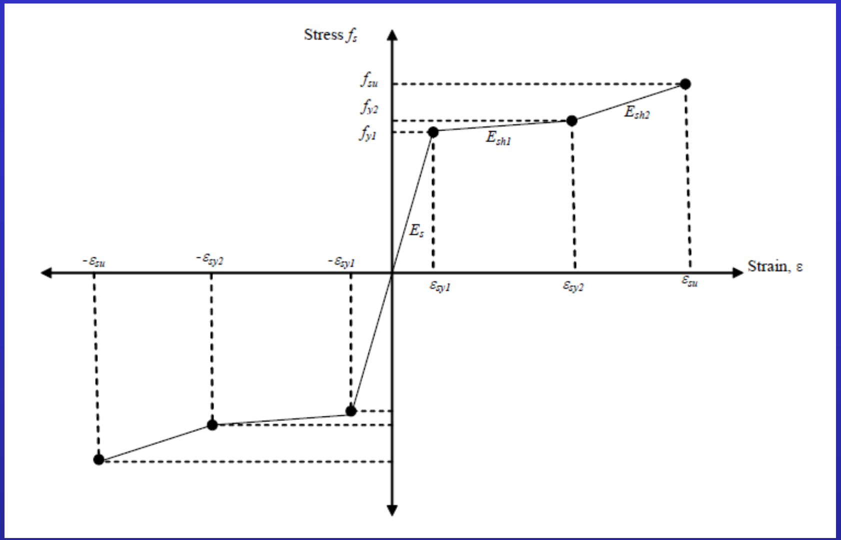
Mathematical formulation: General

Stress strain relationships for concrete in compression

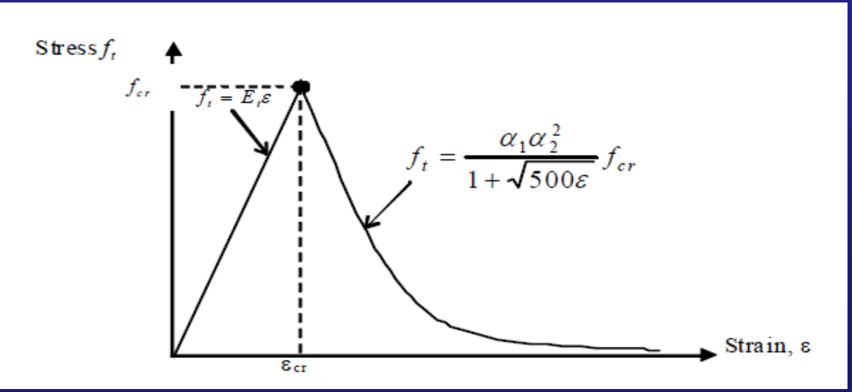




Stress strain relationships for steel



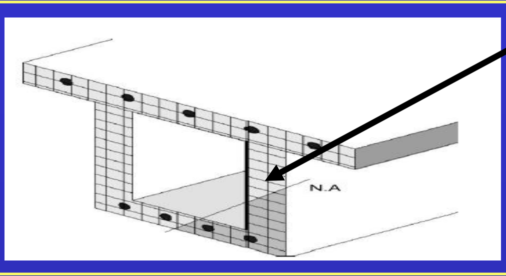
Stress strain relationships for concrete in tension



Mathematical formulation: Numerical integration

Fiber decompostion method:

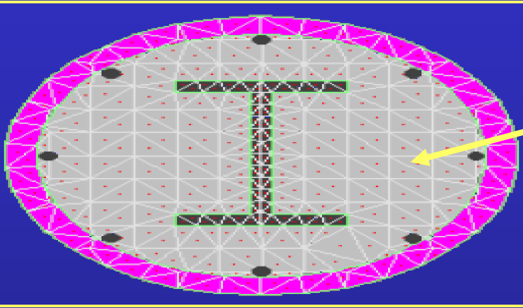
the cross-section is decomposed in filaments or layers



Filaments or cells

Gauss cuadrature integration:

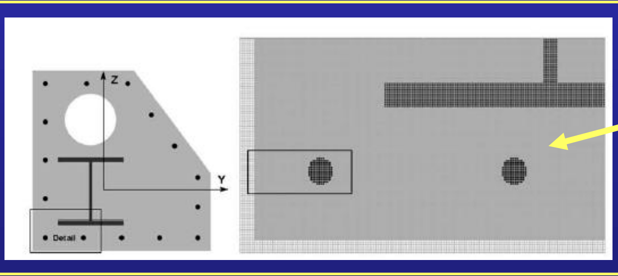
the cross-section is decomposed in a triangular mesh



Triangular mesh

Computer graphics approach:

a grid mesh =pictures elements (pixels) of the computer monitor



Pixels grid

Path integral approach:

Double integral ➔ Path integral

$$\iint \rightarrow \oint$$

Evaluation of tangent stiffness and stress resultant

Green theorem:

$\iint \rightarrow \oint$

$$\iint_A (\partial Q/\partial x - \partial P/\partial y)dxdy = \oint_L Pdx + \oint_L Qdy$$

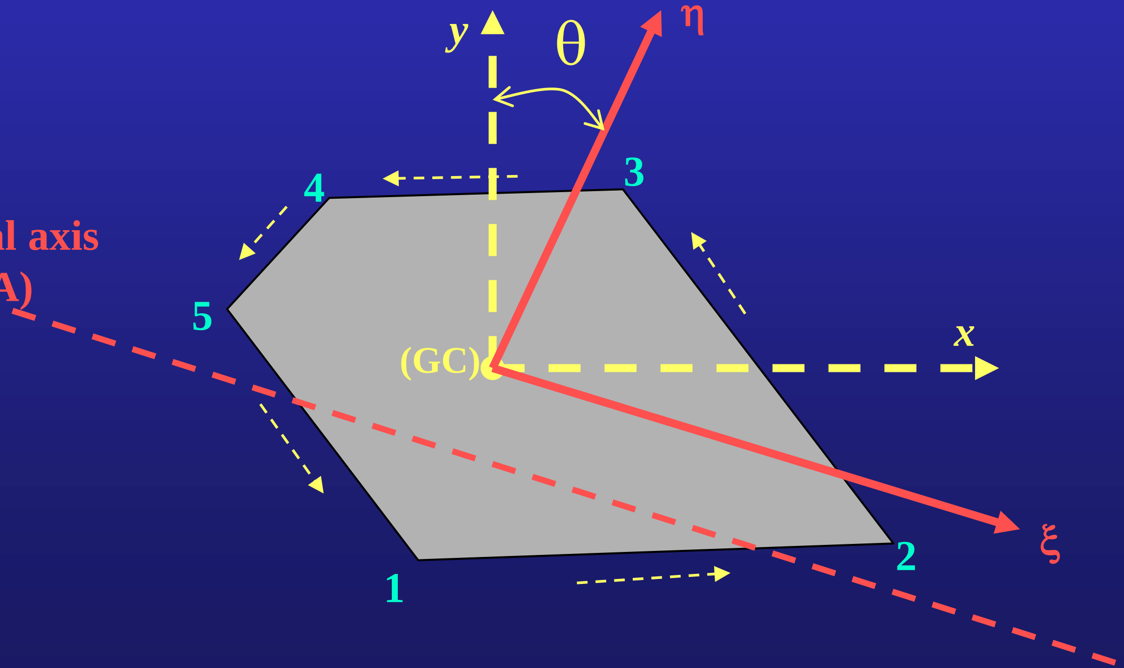
Coordinate transformation

$$\begin{cases} x = \xi\cos\theta + \eta\sin\theta \\ y = -\xi\sin\theta + \eta\cos\theta \end{cases}$$

Stress field uniform in direction parallel with neutral axis

$$P = 0$$

$$Q = \frac{1}{r+1}\int x^{r+1} y^s g(y)dx$$

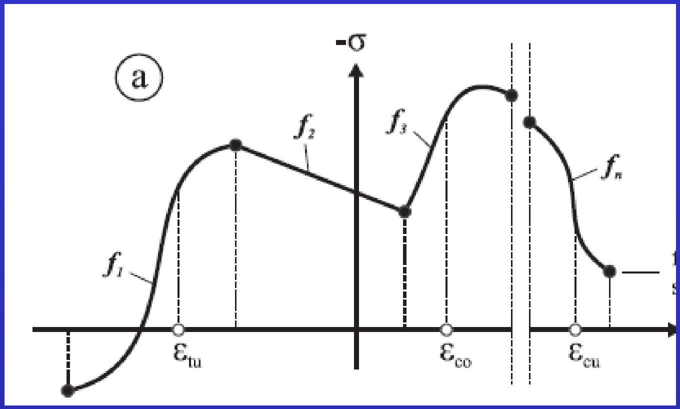


$\tan\theta = \Phi_y/\Phi_x$

$$\sigma(\xi,\eta) = \sigma(\eta)$$

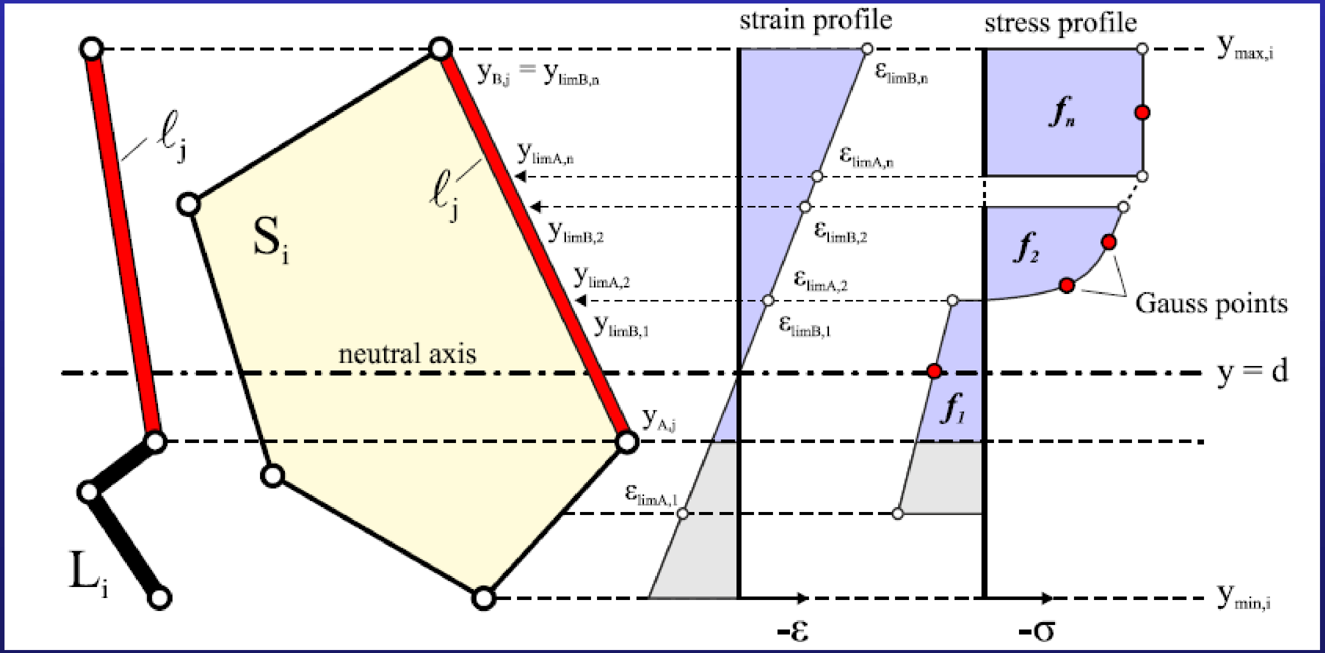
$$E_T(\xi,\eta) = E_T(\eta)$$

Evaluation of tangent stiffness and stress resultant

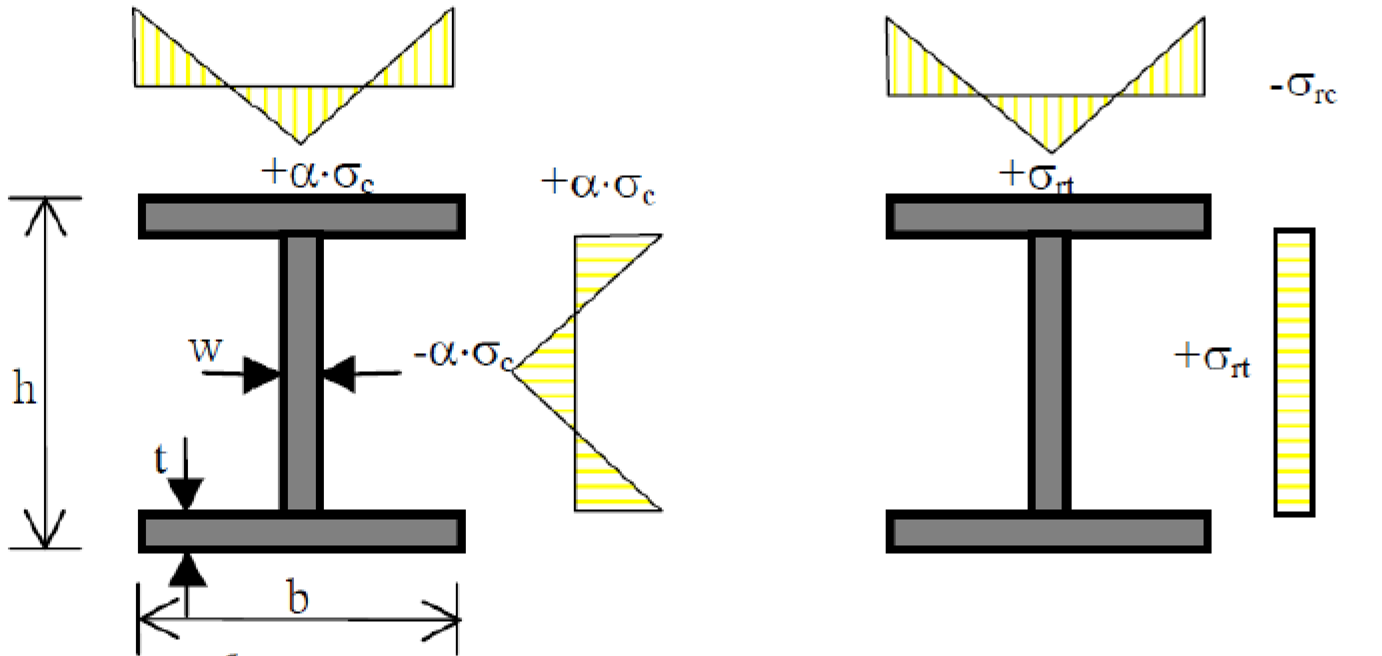


Discontinuity in strees-strain relationships

Adaptive Gauss-Lobatto integration rule



Mathematical formulation: Residual stresses



For $\frac{h}{b} \leq 1.2$ α=0.5

For $\frac{h}{b} > 1.2$ α=0.3

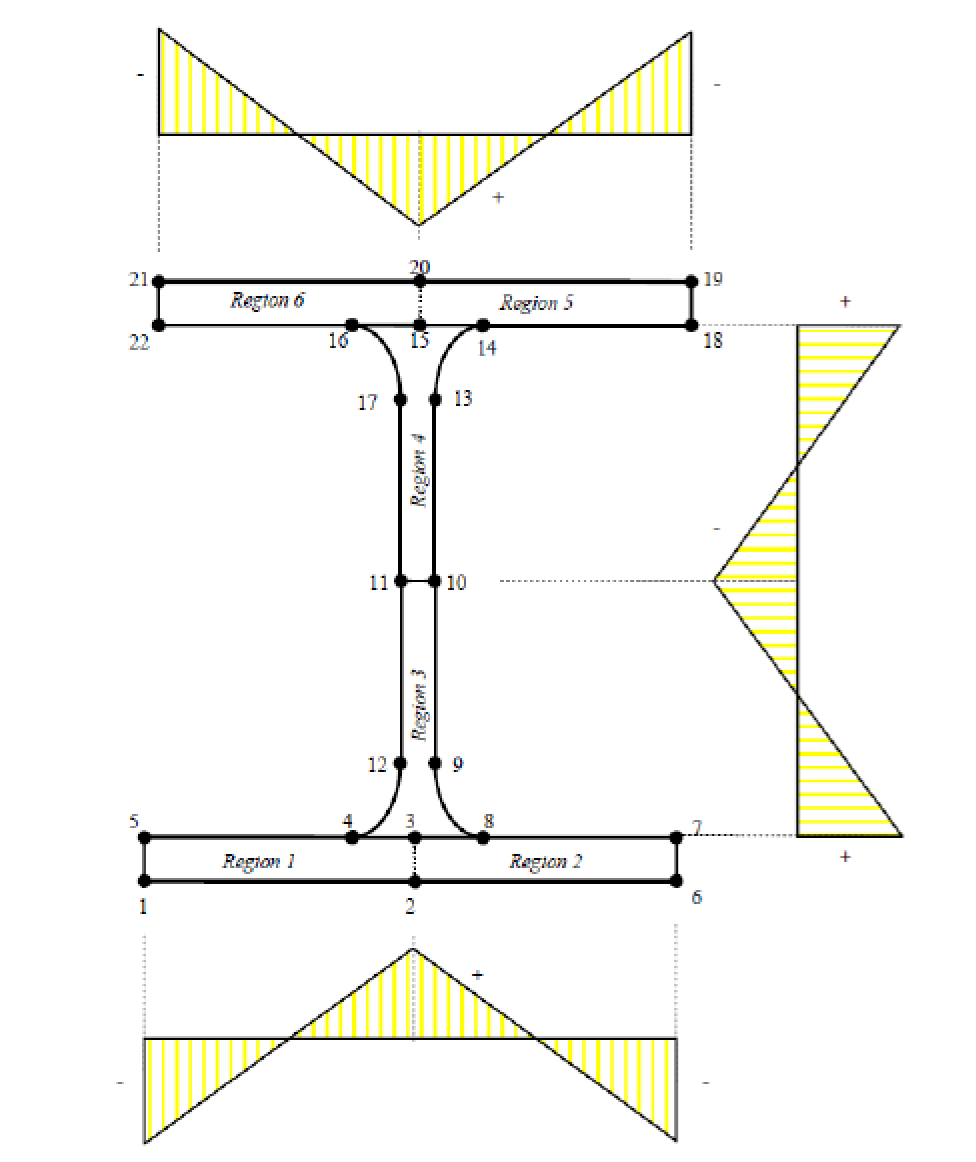
a) EC3

$$\sigma_{rc} = 0{,}3 \cdot \sigma_c$$

$$\sigma_{rt} = \frac{b \cdot t\sigma_c}{b \cdot t + w \cdot (h - 2 \cdot t)}$$

b) AISC-LRFD

Mathematical formulation: Residual stresses



Mathematical formulation: Moment-curvature analysis

Nonlinear equilibrium equations

$$\mathbf{F}(\mathbf{X}) = \mathbf{f}^{\text{int}} - \mathbf{f}^{ext} = 0$$

Jacobian of the system of equations

$$\mathbf{F}' = \left(\frac{\partial \mathbf{F}}{\partial \mathbf{X}}\right) = \begin{bmatrix} \dfrac{\partial N^{\text{int}}}{\partial \varepsilon_0} & \dfrac{\partial N^{\text{int}}}{\partial \phi_y} & \dfrac{\partial N^{\text{int}}}{\partial \phi_z} \\ \dfrac{\partial M_y^{\text{int}}}{\partial \varepsilon_0} & \dfrac{\partial M_y^{\text{int}}}{\partial \phi_y} & \dfrac{\partial M_y^{\text{int}}}{\partial \phi_z} \\ \dfrac{\partial M_z^{\text{int}}}{\partial \varepsilon_0} & \dfrac{\partial M_z^{\text{int}}}{\partial \phi_y} & \dfrac{\partial M_z^{\text{int}}}{\partial \phi_z} \end{bmatrix}$$

Tangent stiffness matrix coefficients

$$k_{11} = \frac{\partial N^{int}}{\partial \varepsilon_0} = \frac{\partial}{\partial \varepsilon_0}\left[\int_{A_{cs}} \sigma(\varepsilon(\varepsilon_0, \phi_y, \phi_z))dA_{cs}\right] = \int_{A_{cs}} \frac{\partial \sigma}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \varepsilon_0} dA_s = \int_{A_{cs}} E_T dA_{cs}$$

$$k_{12} = \frac{\partial N^{int}}{\partial \phi_y} = \frac{\partial}{\partial \phi_y}\left[\int_{A_{cs}} \sigma(\varepsilon(\varepsilon_0, \phi_y, \phi_z))dA_{cs}\right] = \int_{A_{cs}} \frac{\partial \sigma}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \phi_y} dA_{cs} = \int_{A_{cs}} E_T z dA_{cs}$$

$$k_{13} = \frac{\partial N^{int}}{\partial \phi_z} = \frac{\partial}{\partial \phi_z}\left[\int_{A_{cs}} \sigma(\varepsilon(\varepsilon_0, \phi_y, \phi_z))dA_{cs}\right] = \int_{A_{cs}} \frac{\partial \sigma}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \phi_z} dA_s = \int_{A_{cs}} E_T y dA_{cs}$$

$$k_{21} = \frac{\partial M_y^{int}}{\partial \varepsilon_0} = \frac{\partial}{\partial \varepsilon_0}\left[\int_{A_{cs}} \sigma(\varepsilon(\varepsilon_0, \phi_y, \phi_z))z dA_{cs}\right] = \int_{A_{cs}} \frac{\partial \sigma}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \varepsilon_0} z dA_s = \int_{A_{cs}} E_T z dA_{cs}$$

$$k_{22} = \frac{\partial M_y^{int}}{\partial \phi_y} = \frac{\partial}{\partial \phi_y}\left[\int_{A_{cs}} \sigma(\varepsilon(\varepsilon_0, \phi_y, \phi_z))z dA_{cs}\right] = \int_{A_{cs}} \frac{\partial \sigma}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \phi_y} z dA_s = \int_{A_{cs}} E_T z^2 dA_{cs}$$

....

Newton method: rapid and local convergent

+

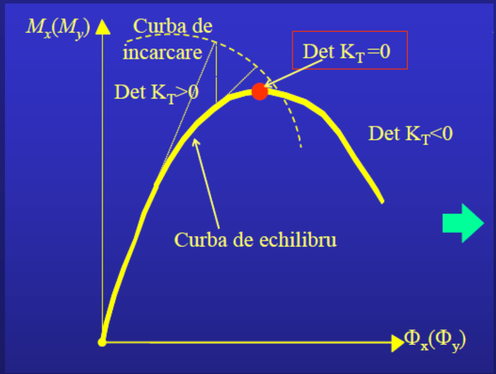
Enhanced with line search=> Global Convergence

$$\mathbf{X}^{k+1} = \mathbf{X}^k - s \cdot \mathbf{F}'\left(\mathbf{X}^k\right)^{-1}\mathbf{F}\left(\mathbf{X}^k\right), k \geq 0$$

$$\boxed{s = \frac{1}{2}}$$

$$\boxed{\mathbf{X} = \left[\varepsilon_0 \quad \phi_y \quad \phi_z\right]^T}$$

Arc length incremental-iterative algorithm



Moment curvature relationships parametrized with axial force

Mathematical formulation: Moment-curvature analysis

Tangent flexural rigidity coefficients

$$\begin{bmatrix} \delta N \\ \delta M_x \\ \delta M_y \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} \delta u \\ \delta \Phi_x \\ \delta \Phi_y \end{bmatrix}$$

$$k_{11} = \iint E_T \, dA$$

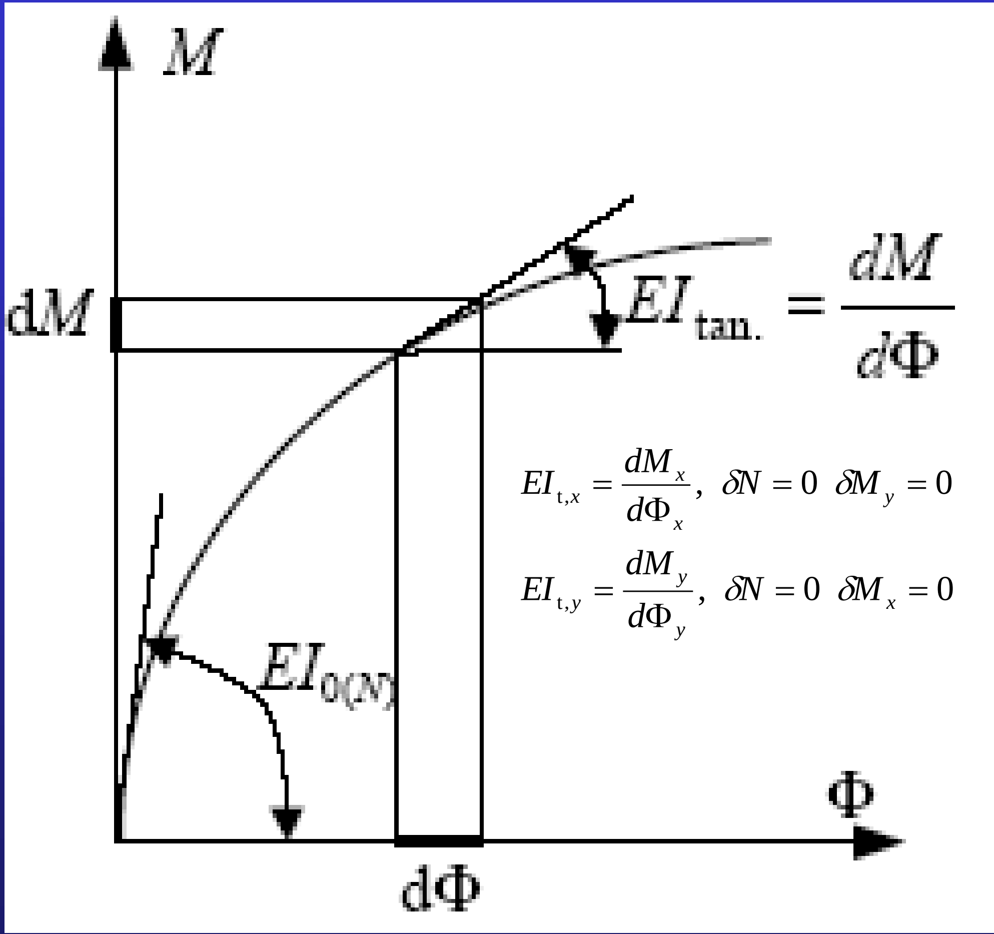
$$k_{12} = k_{21} = \iint E_T \, y \, dA$$

$$k_{13} = k_{31} = \iint E_T \, x \, dA$$

$$k_{22} = \iint E_T \, y^2 \, dA$$

$$k_{23} = k_{32} = \iint E_T \, xy \, dA$$

$$k_{33} = \iint E_T \, x^2 \, dA$$

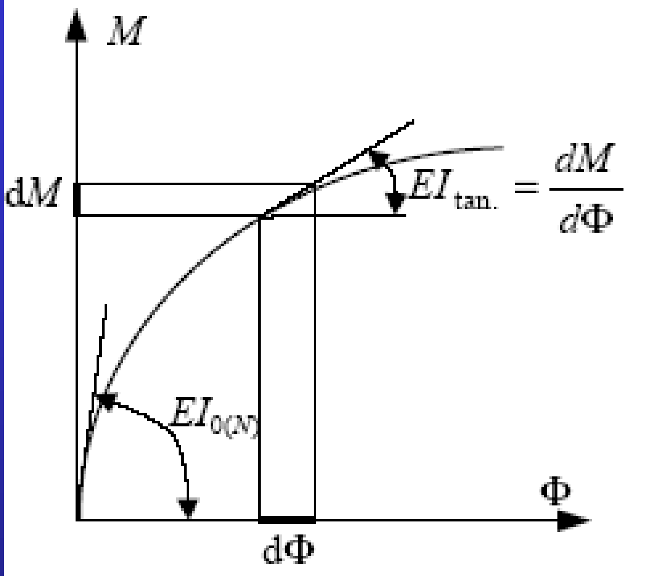


$$EI_{t,x} = \frac{dM_x}{d\Phi_x}, \quad \delta N = 0 \quad \delta M_y = 0$$

$$EI_{t,y} = \frac{dM_y}{d\Phi_y}, \quad \delta N = 0 \quad \delta M_x = 0$$

Mathematical formulation: Moment-curvature analysis

Tangent flexural rigidity coefficients



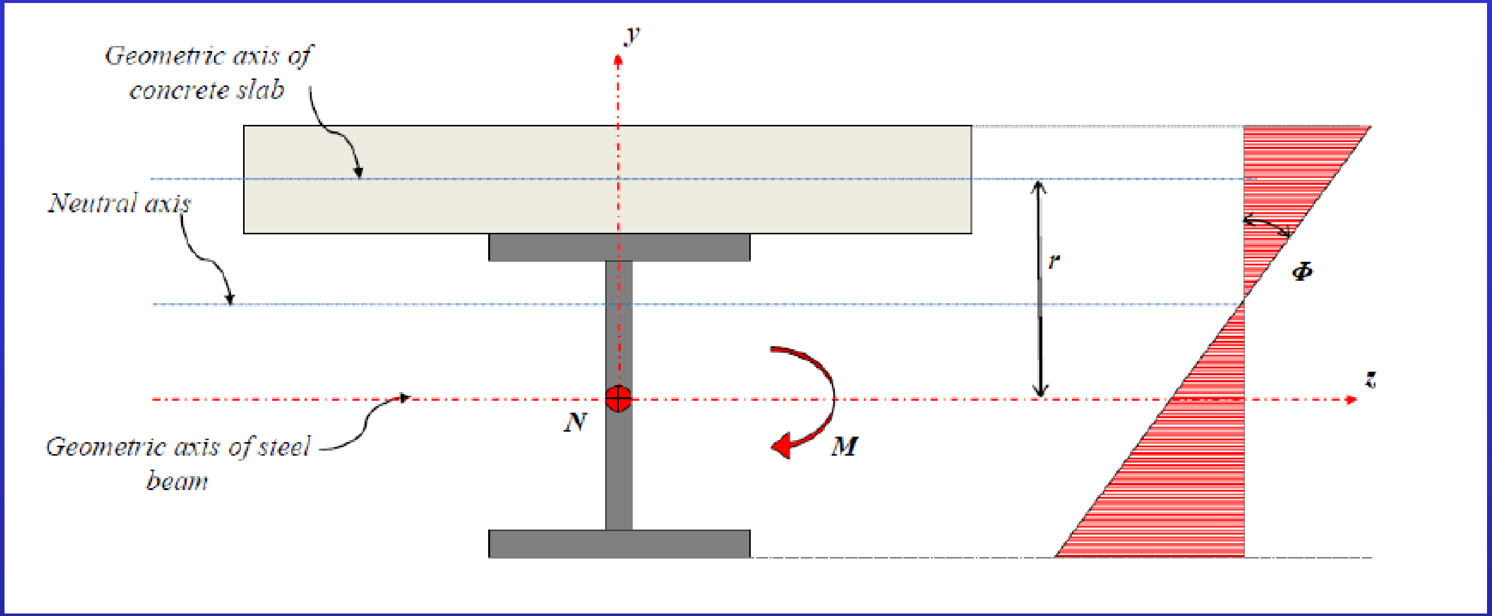
$$EI_{tx} = \frac{\Delta M_x}{\Delta \phi_x}, \quad \Delta N = 0; \ \ \Delta M_y = 0$$

$$EI_{ty} = \frac{\Delta M_y}{\Delta \phi_y}, \quad \Delta N = 0; \ \ \Delta M_x = 0$$

$$EI_{t,x} = \frac{dM_x}{d\Phi_x} = \frac{k_{11}k_{22}k_{33} - k_{11}k_{23}^2 - k_{12}^2 k_{33} + 2k_{12}k_{13}k_{23} - k_{13}^2 k_{22}}{k_{11}k_{33} - k_{13}^2}$$

$$EI_{t,y} = \frac{dM_y}{d\Phi_y} = \frac{k_{11}k_{22}k_{33} - k_{11}k_{23}^2 - k_{12}^2 k_{33} + 2k_{12}k_{13}k_{23} - k_{13}^2 k_{22}}{k_{11}k_{22} - k_{12}^2}$$

Mathematical formulation: Moment-curvature analysis



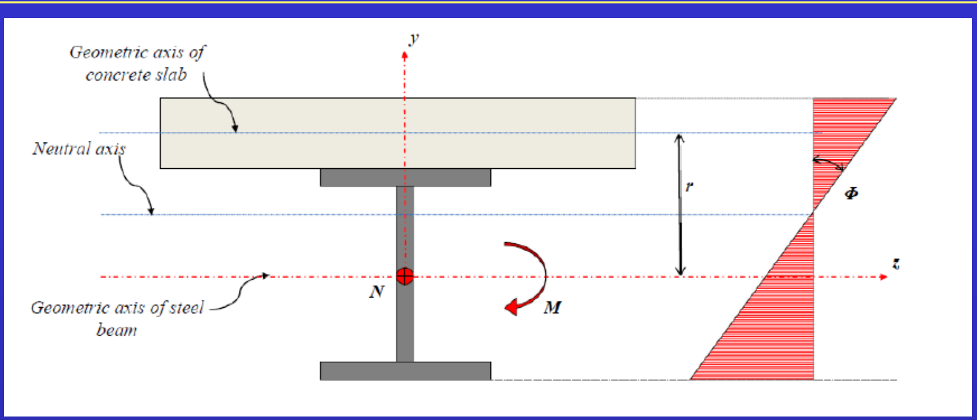
$$EI_{tz} = \int_{A_{cs}} E_t y^2 dA_{cs} - \frac{\left(\int_{A_{cs}} E_t y dA_{cs}\right)^2}{\int_{A_{cs}} E_t dA_{cs}}$$

$$EI_{tz} = \int_{A_{cs}} E_t (y - y_p)^2 dA_{cs}$$

$$y_p = \frac{\int_{A_{cs}} E_t y dA_{cs}}{\int_{A_{cs}} E_t dA_{cs}}$$

$$\boxed{(EI)_0 = E_c I_c + E_s I_s + \frac{E_c A_c \cdot E_s A_s}{E_c A_c + E_s A_s} r^2}$$

Mathematical formulation: Moment-curvature analysis



$$EI = (1-k)EI_{steel} + kEI_{comp}$$

$$M^{\text{int}} = (1-k)M^{\text{int}}{}_{steel} + kM^{\text{int}}{}_{comp}$$

$$k = \sqrt{\frac{N}{N_f}}$$

Partial interaction coefficient

$$\begin{cases} \int\limits_{A_{cs}} \sigma(\varepsilon(\varepsilon_0, \phi_z))dA_{cs} - N = 0 \\ (1-k)\int\limits_{A_s} \sigma(\varepsilon(\varepsilon_0, \phi_z))y dA_s + k\int\limits_{A_{cs}} \sigma(\varepsilon(\varepsilon_0, \phi_z))y dA_{cs} - M_z = 0 \end{cases}$$

Equilibrium equations

$$k_{11} = \frac{\partial N^{int}}{\partial \varepsilon_0} = \frac{\partial}{\partial \varepsilon_0}\left[\int\limits_{A_{cs}} \sigma(\varepsilon(\varepsilon_0, \phi_z))dA_{cs}\right] = \int\limits_{A_{cs}} \frac{\partial \sigma}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \varepsilon_0} dA_s = \int\limits_{A_{cs}} E_T dA_{cs}$$

$$k_{12} = \frac{\partial N^{int}}{\partial \phi_z} = \frac{\partial}{\partial \phi_z}\left[\int\limits_{A_{cs}} \sigma(\varepsilon(\varepsilon_0, \phi_z))dA_{cs}\right] = \int\limits_{A_{cs}} \frac{\partial \sigma}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \phi_z} dA_s = \int\limits_{A_{cs}} E_T y dA_{cs}$$

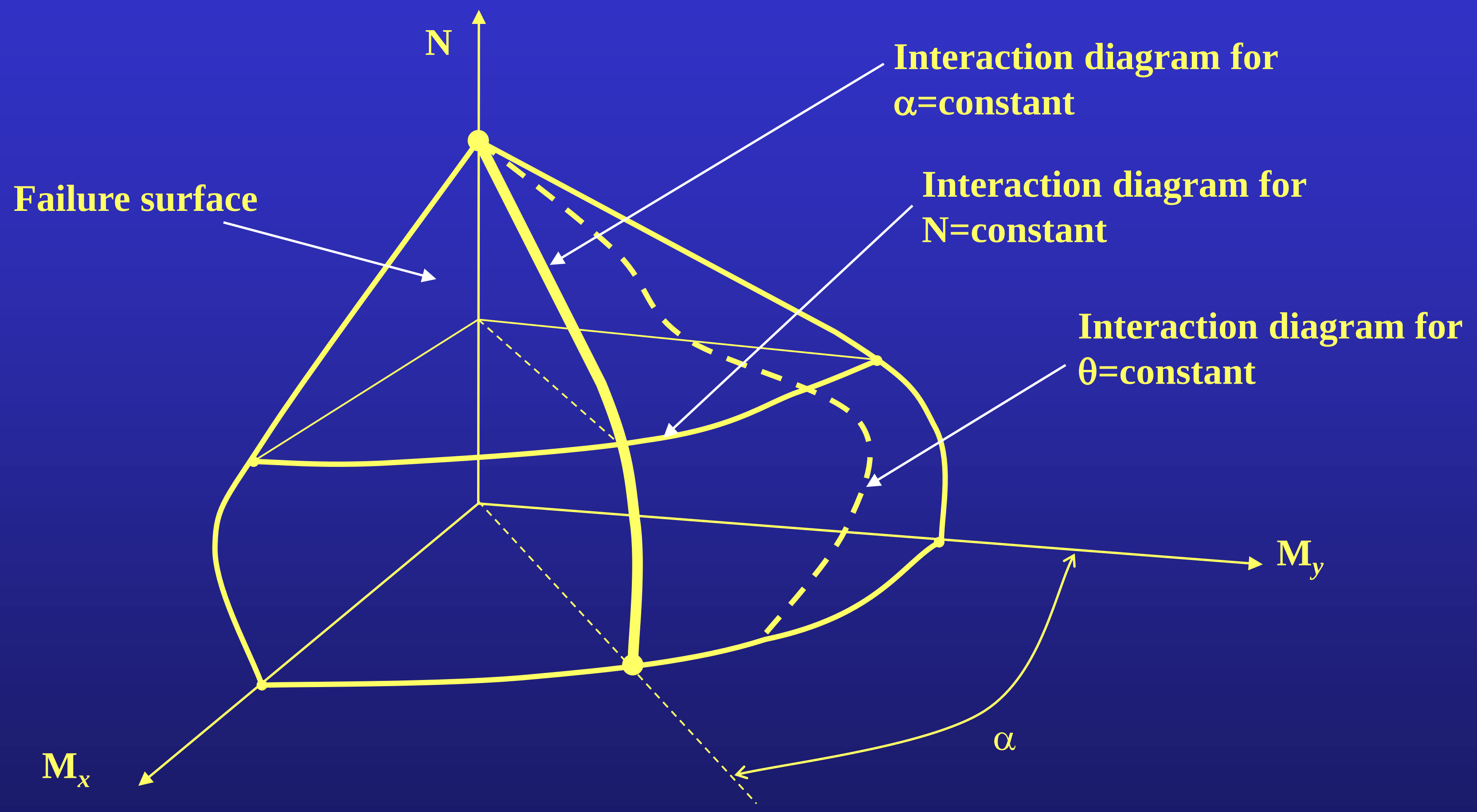
$$k_{21} = \frac{\partial M_z^{int}}{\partial \varepsilon_0} = (1-k)\int\limits_{A_s} E_T y dA_s + k\int\limits_{A_{cs}} E_T y dA_{cs}$$

$$k_{22} = \frac{\partial M_z^{int}}{\partial \phi_z} = (1-k)\int\limits_{A_s} E_T y^2 dA_s + k\int\limits_{A_{cs}} E_T y^2 dA_{cs}$$

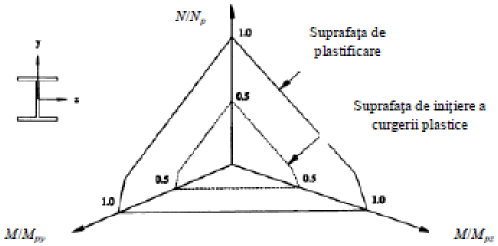
Tangent stiffness matrix coefficients

Mathematical formulation: Interaction diagrams

Ultimate limit state condition (plastic surface requirements)



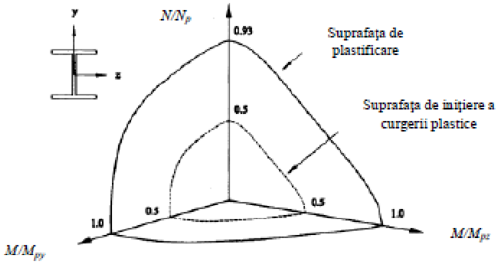
Mathematical formulation: Interaction diagrams



AISC-LRFD (Kanchanaly 1977)-sectiune metalică (profil I)

$$\frac{N}{N_p}+\frac{8}{9}\frac{M_y}{M_{py}}+\frac{8}{9}\frac{M_z}{M_{pz}}-1=0,\quad \frac{N}{N_p}\geq\frac{2}{9}\frac{M_y}{M_{py}}+\frac{2}{9}\frac{M_z}{M_{pz}}$$

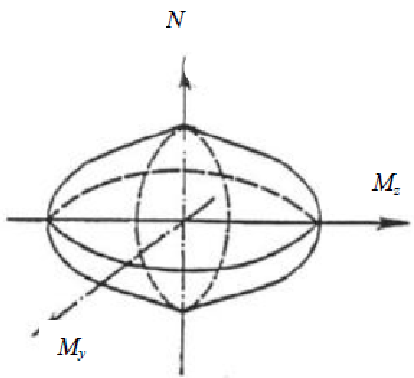
$$\frac{N}{2N_p}+\frac{M_y}{M_{py}}+\frac{M_z}{M_{pz}}-1=0,\quad \frac{N}{N_p}<\frac{2}{9}\frac{M_y}{M_{py}}+\frac{2}{9}\frac{M_z}{M_{pz}}$$



Orinison, 1982- sectiune metalică (profil I)

$$1.15n^2+m_z^2+m_y^4+3.67n^2m_z^2+3.0n^6m_y^2+4.65m_z^2m_y^2-1=0$$

$$n=\frac{N}{N_p},\quad m_z=\frac{M_z}{M_{pz}},\quad m_y=\frac{M_y}{M_{py}}$$



Powel-Chen, 1986

$$\sqrt{\left(\frac{M_y}{M_{py}}\right)^2+\left(\frac{M_z}{M_{pz}}\right)^2}+\left(\frac{N}{N_p}\right)^\alpha-1=0$$

Fig. 2.5 Suprafeţe de plastificare utilizate în calculul secţiunilor metalice şi din beton armat.

Mathematical formulation: Interaction diagrams

$$\left(\Delta \mathbf{U}^{p}\right)^{T}{}_{(1\times12)} \cdot \Delta \mathbf{F}_{(12\times1)} = 0$$

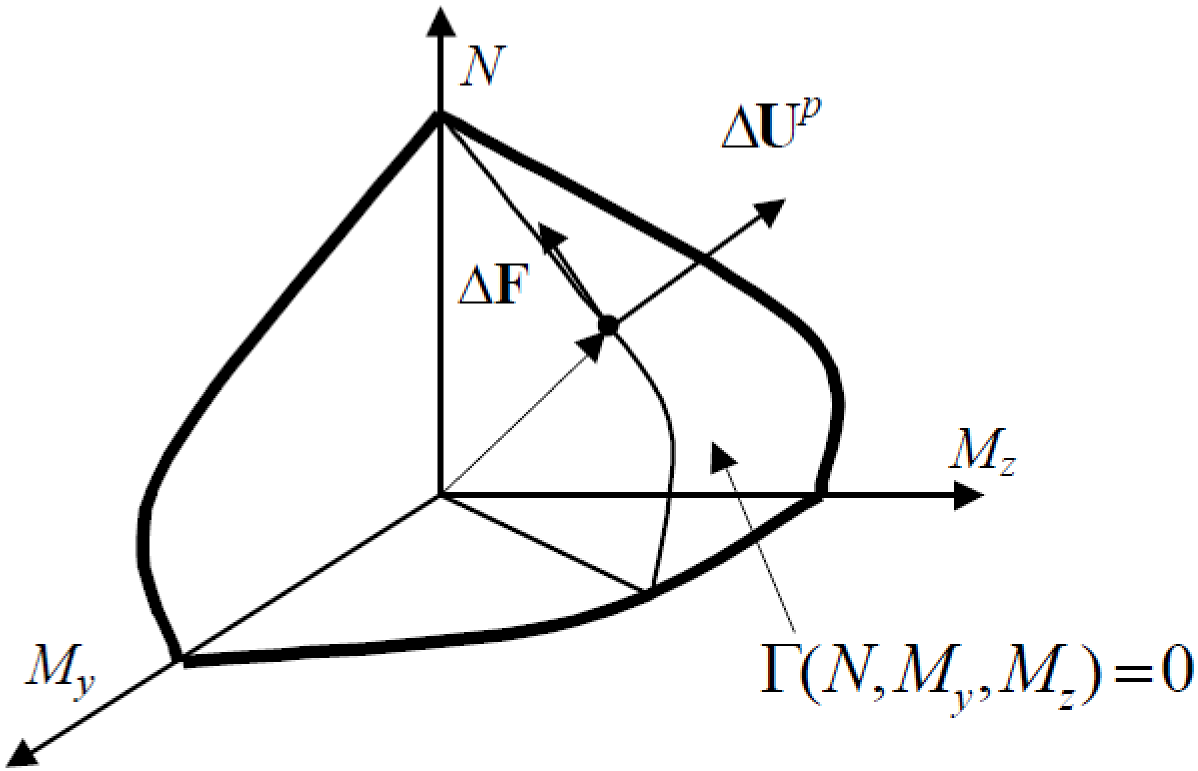
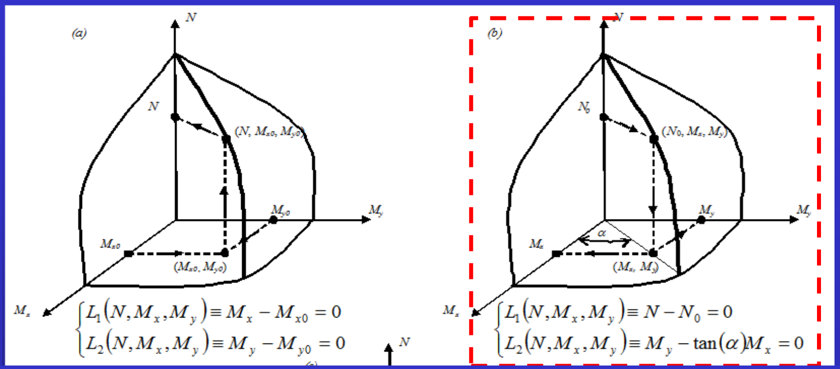


Fig. 2.8 Legea de normalitate.

Mathematical formulation: Interaction diagrams



Failure condition

$$\boxed{\varepsilon_0} + \boxed{\phi_x} y_c(\phi_x, \phi_y) + \boxed{\phi_y} x_c(\phi_x, \phi_y) - \varepsilon_u = 0$$

Equilibrium equations

$$\begin{cases} \int_A \sigma(\varepsilon(\varepsilon_0, \phi_x, \phi_y)) dA - \boxed{N} = 0 \\ \int_A \sigma(\varepsilon(\varepsilon_0, \phi_x, \phi_y)) y dA - \boxed{M_x} = 0 \\ \int_A \sigma(\varepsilon(\varepsilon_0, \phi_x, \phi_y)) x dA - \boxed{M_y} = 0 \end{cases}$$

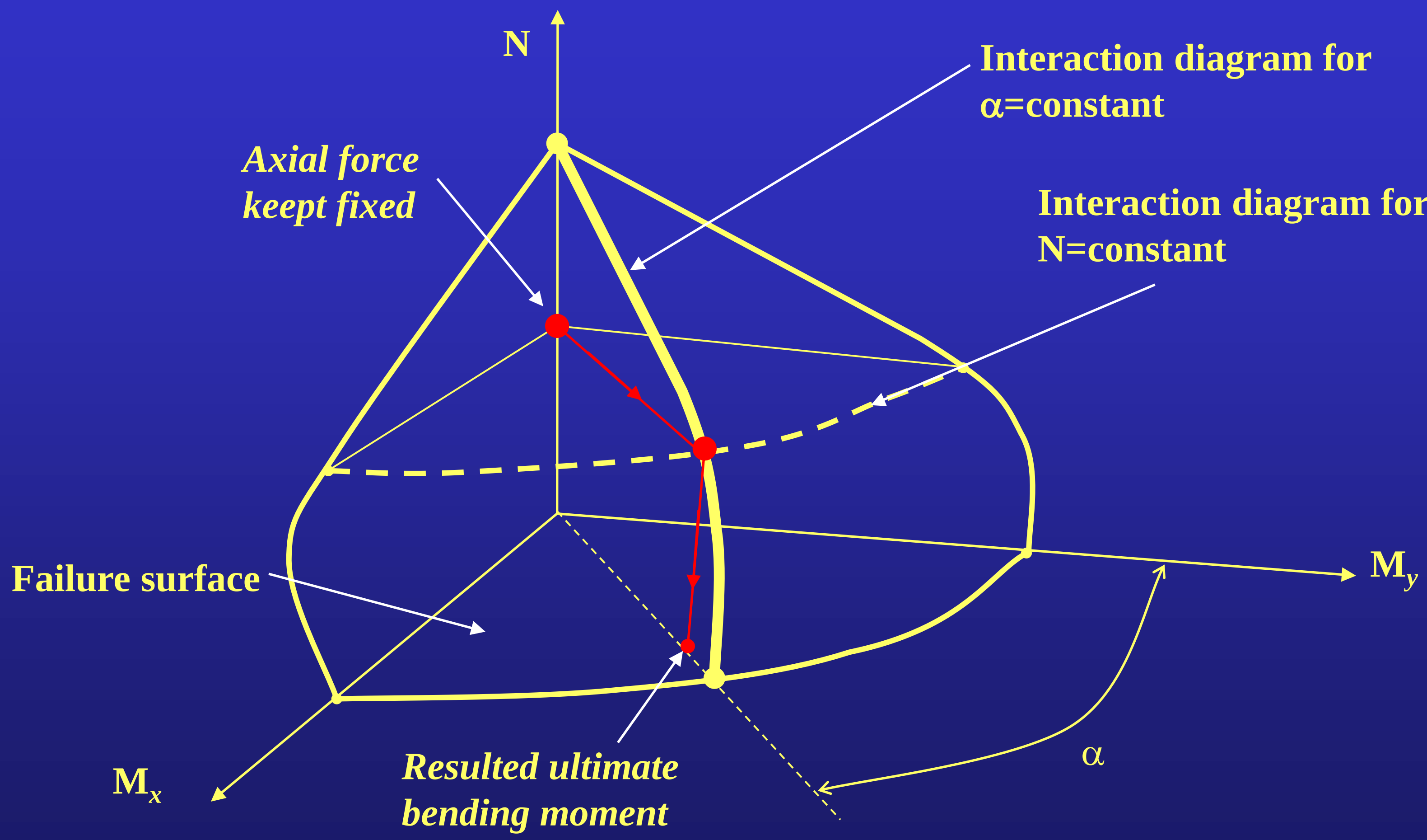
Linear constraints

$$\begin{cases} L_1(N, M_x, M_y) \equiv M_x - M_{x0} = 0 \\ L_2(N, M_x, M_y) \equiv M_y - M_{y0} = 0 \end{cases}$$

With strain distribution corresponding to a failure condition, find the ultimate resistances N, M_x, M_y so as to fulfil the basic equations of equilibrium and one of the imposed linear constraints.

Mathematical formulation: Interaction diagrams

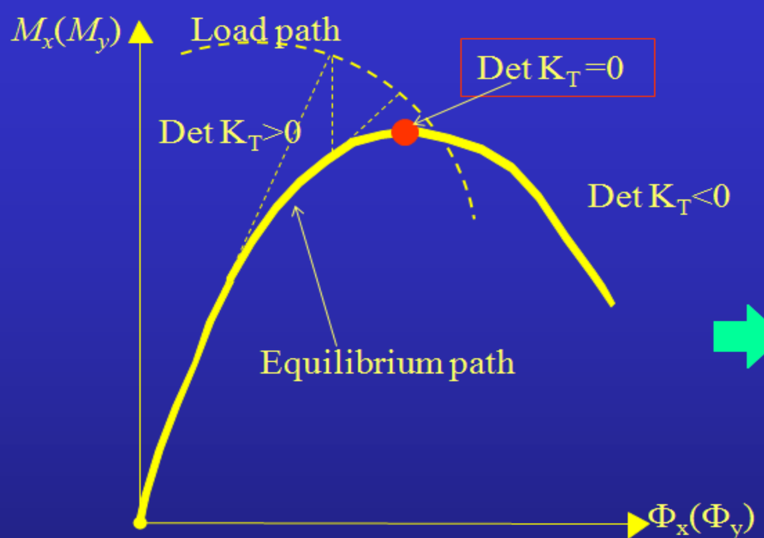
Ultimate limit state condition (plastic surface requirements)



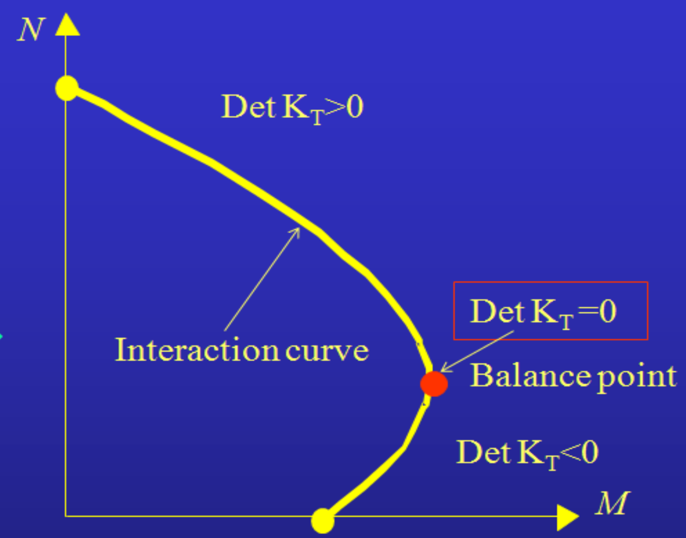
Mathematical formulation: Interaction diagrams

Geometrical representation of the proposed method

(a) Moment-curvature space



(b) Axial force-bending moment space



Initial data: Bending moments $\boldsymbol{M_x}$ **and** $\boldsymbol{M_y}$

Iterative procedure:
the curvatures Φ_x **and** Φ_y and loading factor λ

$$\begin{cases} N^* : \text{numerical integration of stress resultant} \\ M_x^* = M_x \times \lambda \\ M_y^* = M_y \times \lambda \end{cases}$$

Mathematical formulation: Interaction diagrams

Key elements of the proposed method

Initial equilibrium equations

$$\int_A \sigma(\varepsilon_0, \Phi_x, \Phi_y)\,dA - N_{ext} = 0$$
$$\int_A \sigma(\varepsilon_0, \Phi_x, \Phi_y)\,y\,dA - \lambda M_{x,ref} = 0$$
$$\int_A \sigma(\varepsilon_0, \Phi_x, \Phi_y)\,x\,dA - \lambda M_{y,ref} = 0$$

Resultant strain distribution

$$\varepsilon(\Phi_x, \Phi_y) = \varepsilon_u + \Phi_x (y - y_c) + \Phi_y (x - x_c)$$

Reduced system of equations

$$\int_A \sigma(\varepsilon(\Phi_x, \Phi_y))\,y\,dA - \lambda M_{x,ext} = 0$$
$$\int_A \sigma(\varepsilon(\Phi_x, \Phi_y))\,x\,dA - \lambda M_{y,ext} = 0$$

$$\mathbf{F}(\lambda, \Phi) = \mathbf{f}^{\text{int}} - \lambda \mathbf{f}^{ext} = 0$$

+

$$g(\lambda, \Phi) = 0$$

Arc-length constrained equation

Newton-Raphson procedure

$$\begin{Bmatrix} {}^{\tau+\Delta\tau}\Phi \\ \Delta\lambda \end{Bmatrix}^{k+1} = \begin{Bmatrix} {}^{\tau+\Delta\tau}\Phi \\ \Delta\lambda \end{Bmatrix}^{k} - \begin{bmatrix} \dfrac{\partial \mathbf{F}}{\partial \Phi} & \dfrac{\partial \mathbf{F}}{\partial \lambda} \\ \dfrac{\partial g}{\partial \Phi} & \dfrac{\partial g}{\partial \lambda} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{F}(\lambda, \Phi)^k \\ g(\lambda, \Phi)^k \end{bmatrix}$$

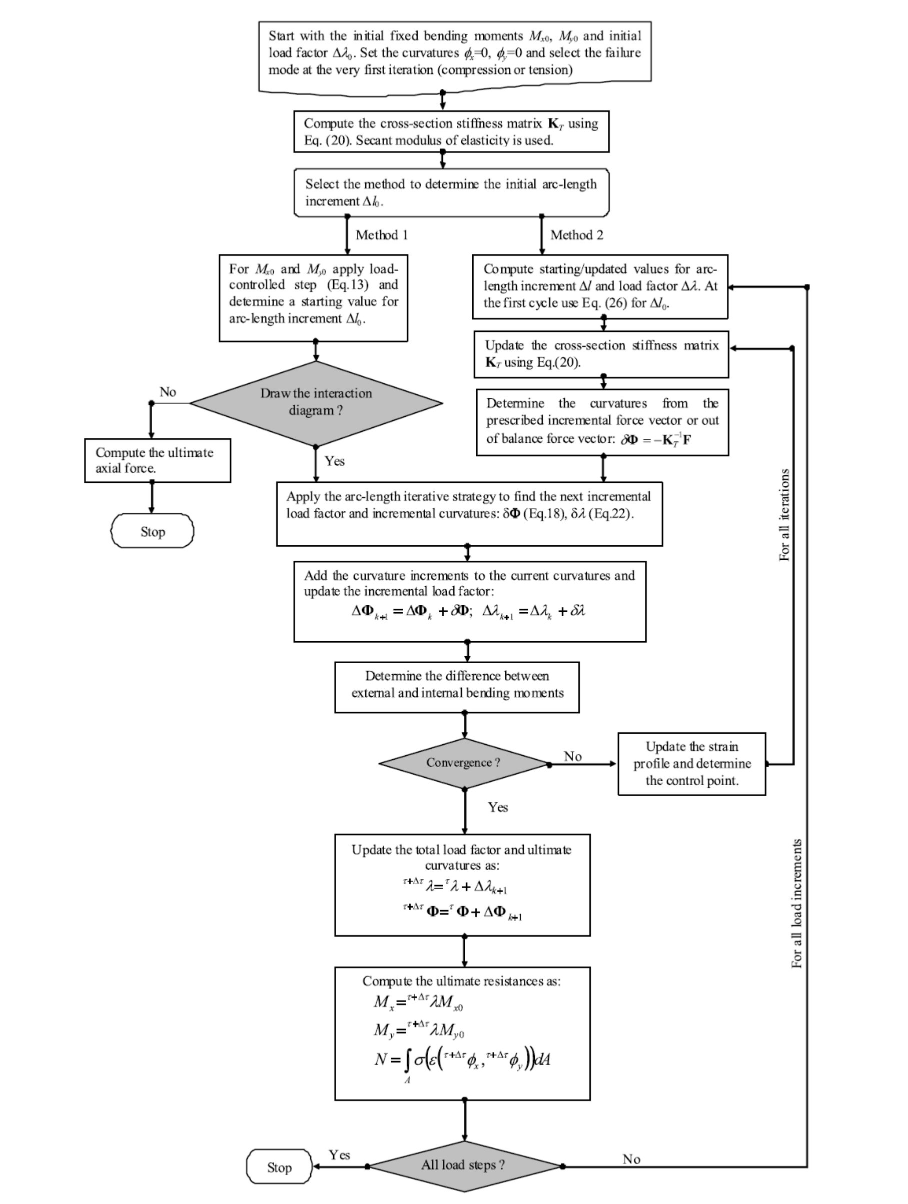
$$\mathbf{J} = \begin{bmatrix} \mathbf{K}_T & -\mathbf{f}_{ext} \\ \dfrac{\partial g}{\partial \Phi} & \dfrac{\partial g}{\partial \lambda} \end{bmatrix}$$

Jacobian of the nonlinear system

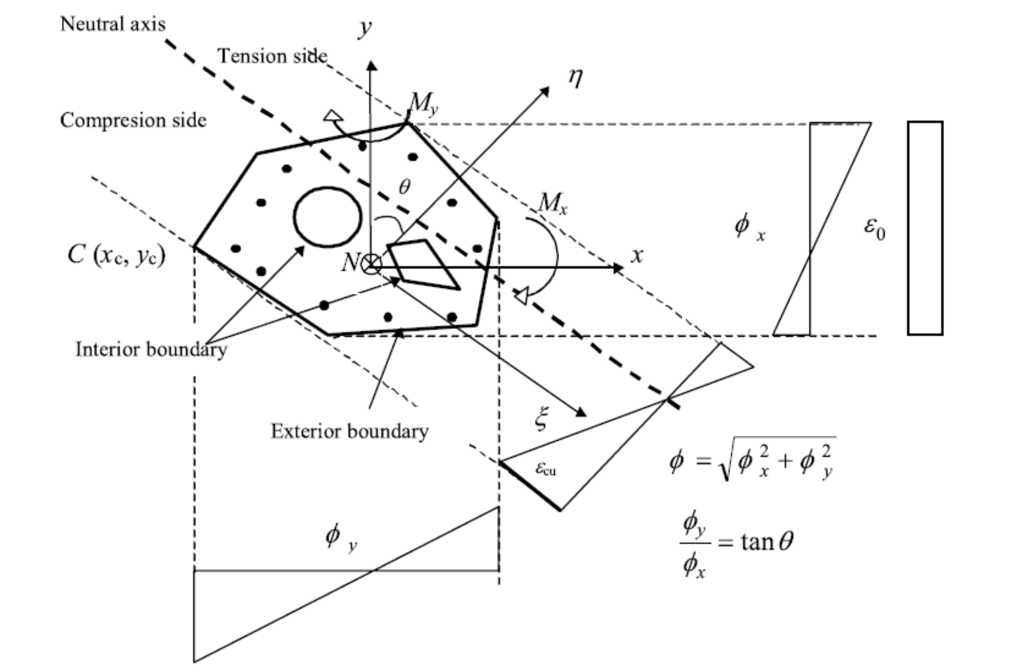
$$\mathbf{K}_T = \left(\frac{\partial F}{\partial \Phi}\right) = \begin{bmatrix} \dfrac{\partial M_{x,\text{int}}}{\partial \Phi_x} & \dfrac{\partial M_{x,\text{int}}}{\partial \Phi_y} \\ \dfrac{\partial M_{y,\text{int}}}{\partial \Phi_x} & \dfrac{\partial M_{y,\text{int}}}{\partial \Phi_x} \end{bmatrix}$$

Tangent stiffnes matrix of cross-section

Mathematical formulation: Interaction diagrams



Mathematical formulation: Design procedure



$$\begin{cases} \int_A \sigma\left(\varepsilon\left(\varepsilon_0,\phi_x,\phi_y\right)\right)dA + A_{tot}\sum_{i=1}^{n}\sigma\left(\varepsilon_i\left(\varepsilon_0,\phi_x,\phi_y\right)\right)\alpha_i - N = 0; \int_A \sigma\left(\varepsilon\left(\varepsilon_0,\phi_x,\phi_y\right)\right)y\,dA + A_{tot}\sum_{i=1}^{n}\sigma\left(\varepsilon_i\left(\varepsilon_0,\phi_x,\phi_y\right)\right)y_i\alpha_i - M_x = 0 \\ \int_A \sigma\left(\varepsilon\left(\varepsilon_0,\phi_x,\phi_y\right)\right)x\,dA + A_{tot}\sum_{i=1}^{n}\sigma\left(\varepsilon_i\left(\varepsilon_0,\phi_x,\phi_y\right)\right)x_i\alpha_i - M_y = 0; \varepsilon_0 + \phi_x y_c\left(\phi_x,\phi_y\right) + \phi_y x_c\left(\phi_x,\phi_y\right) - \varepsilon_u = 0 \end{cases}$$

$$\mathbf{X}^{k+1} = \mathbf{X}^k - \mathbf{F}'\left(\mathbf{X}^k\right)^{-1}\mathbf{F}\left(\mathbf{X}^k\right), k \geq 0$$

Mathematical formulation: Design procedure

$$\mathbf{F}' = \left(\frac{\partial \mathbf{F}}{\partial \mathbf{X}}\right) = \begin{bmatrix} \frac{\partial N^{\text{int}}}{\partial \phi_x} & \frac{\partial N^{\text{int}}}{\partial \phi_y} & \frac{\partial N^{\text{int}}}{\partial A_{tot}} \\ \frac{\partial M_x^{\text{int}}}{\partial \phi_x} & \frac{\partial M_x^{\text{int}}}{\partial \phi_y} & \frac{\partial M_x^{\text{int}}}{\partial A_{tot}} \\ \frac{\partial M_y^{\text{int}}}{\partial \phi_x} & \frac{\partial M_y^{\text{int}}}{\partial \phi_y} & \frac{\partial M_y^{\text{int}}}{\partial A_{tot}} \end{bmatrix}$$

$$\frac{\partial N^{\text{int}}}{\partial \phi_x} = \int_{A_{cs}} E_t (y - y_c) dA_{cs} + A_{tot} \sum_{i=1}^{N_{rs}} E_{ti} (y_i - y_c) \alpha_i + \sum_{j=1}^{N_{ps}} E_{tj} (y_i - y_c) A_{psj}$$

$$\frac{\partial N^{\text{int}}}{\partial \phi_y} = \int_{A_{cs}} E_t (x - x_c) dA_{cs} + A_{tot} \sum_{i=1}^{N_{rs}} E_{ti} (x_i - x_c) \alpha_i + \sum_{j=1}^{N_{ps}} E_{tj} (x_j - y_c) A_{psj}$$

$$\frac{\partial N^{\text{int}}}{\partial A_{tot}} = \sum_{i=1}^{N_{rs}} \sigma(\varepsilon_i) \alpha_i$$

$$\frac{\partial M_x^{\text{int}}}{\partial \phi_x} = \int_{A_{cs}} E_t y (y - y_c) dA_{cs} + A_{tot} \sum_{i=1}^{N_{rs}} E_{ti} y_i (y_i - y_c) \alpha_i + \sum_{j=1}^{N_{ps}} E_{tj} y_j (y_j - y_c) A_{psj}$$

$$\frac{\partial M_x^{\text{int}}}{\partial \phi_y} = \int_{A_{cs}} E_t y (x - x_c) dA_{cs} + A_{tot} \sum_{i=1}^{N_{rs}} E_{ti} y_i (x_i - x_c) \alpha_i + \sum_{j=1}^{N_{ps}} E_{tj} y_j (x_j - x_c) A_{psj}$$

$$\frac{\partial M_x^{\text{int}}}{\partial A_{tot}} = \sum_{i=1}^{N_{rs}} \sigma(\varepsilon_i) y_i \alpha_i$$

$$\frac{\partial M_y^{\text{int}}}{\partial \phi_x} = \int_{A_{cs}} E_t x (y - y_c) dA_{cs} + A_{tot} \sum_{i=1}^{N_{rs}} E_{ti} x_i (y_i - y_c) \alpha_i + \sum_{j=1}^{N_{ps}} E_{tj} x_j (y_j - y_c) A_{psj}$$

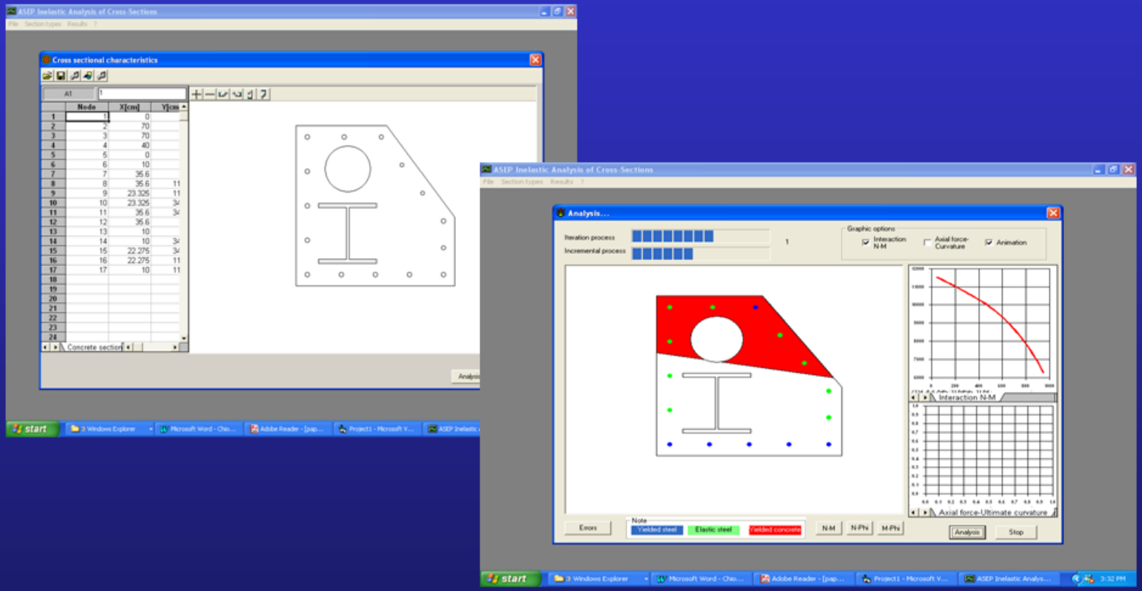
$$\frac{\partial M_y^{\text{int}}}{\partial \phi_y} = \int_{A_{cs}} E_t x (x - x_c) dA_{cs} + A_{tot} \sum_{i=1}^{N_{rs}} E_{ti} x_i (x_i - x_c) \alpha_i + \sum_{j=1}^{N_{ps}} E_{tj} x_j (x_j - x_c) A_{psj}$$

$$\frac{\partial M_y^{\text{int}}}{\partial A_{tot}} = \sum_{i=1}^{N_{rs}} \sigma(\varepsilon_i) x_i \alpha_i$$

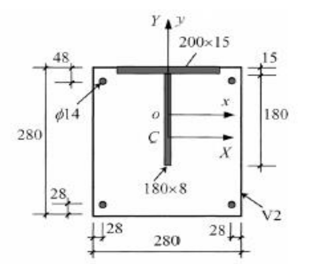
COMPUTER PROGRAM

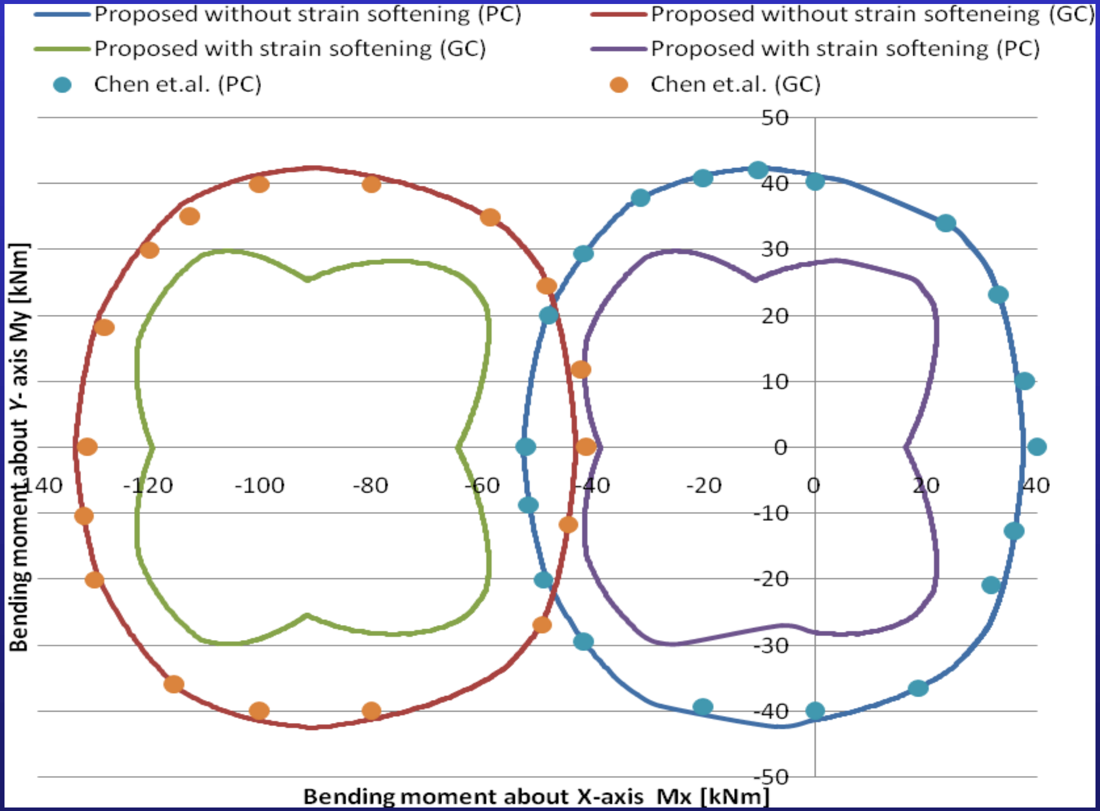
Computer program - ASEP

- ***ASEP** Computer program for inelastic analysis of arbitrary reinforced and composite concrete sections*

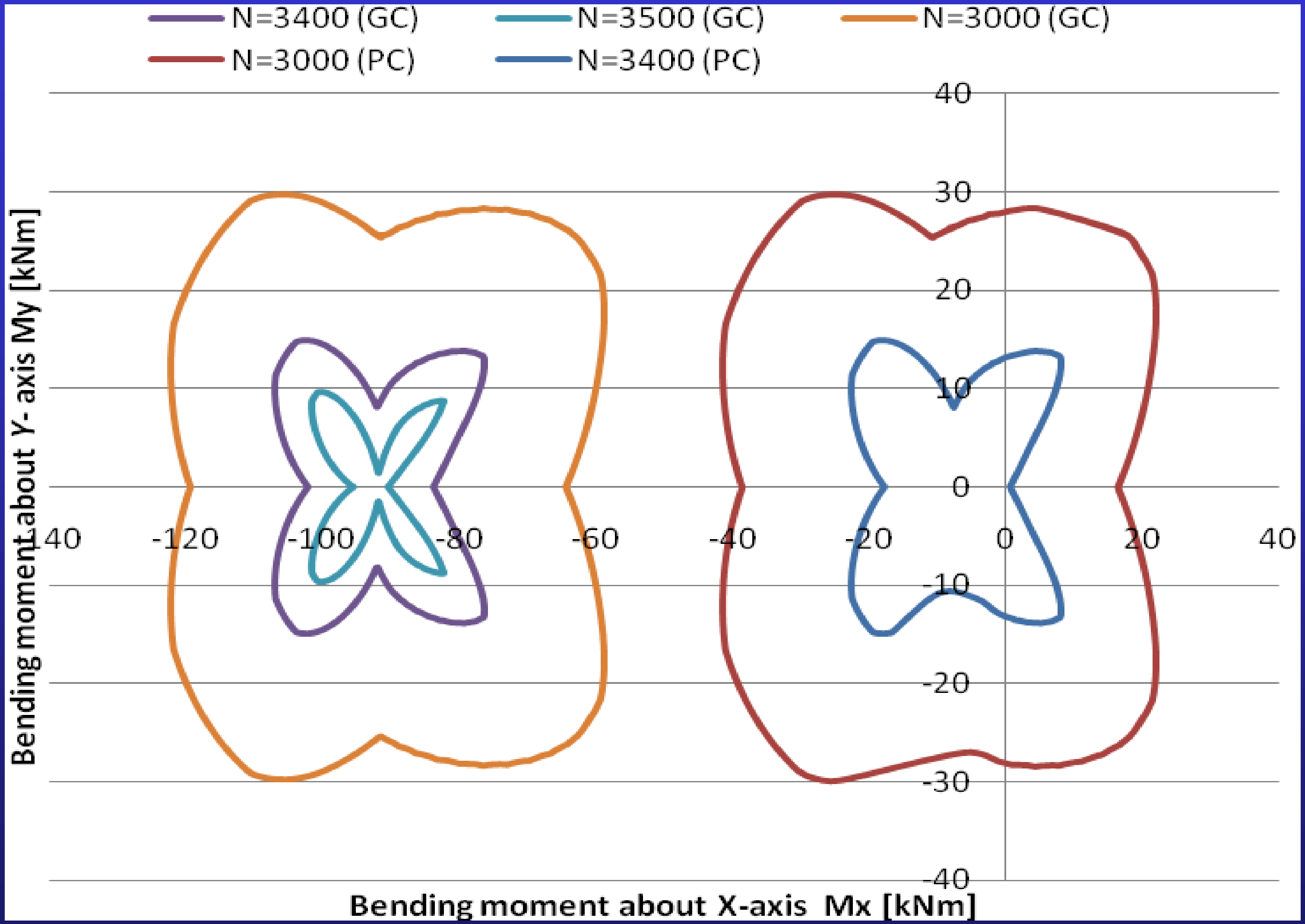


COMPUTATIONAL EXAMPLES

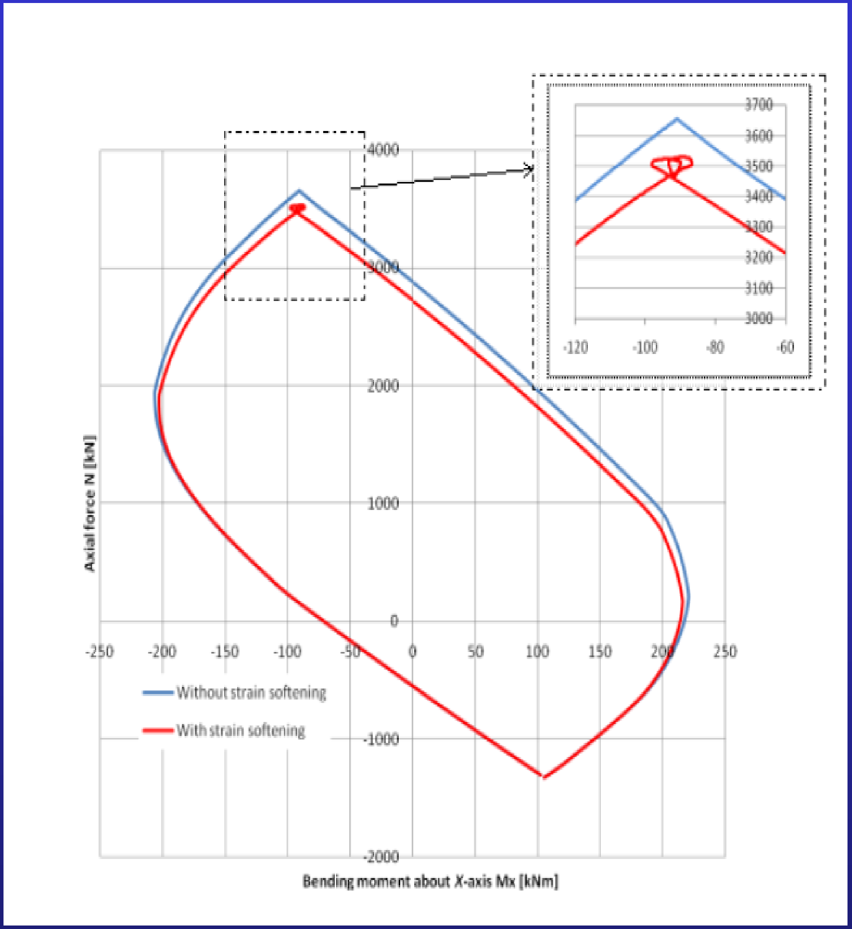


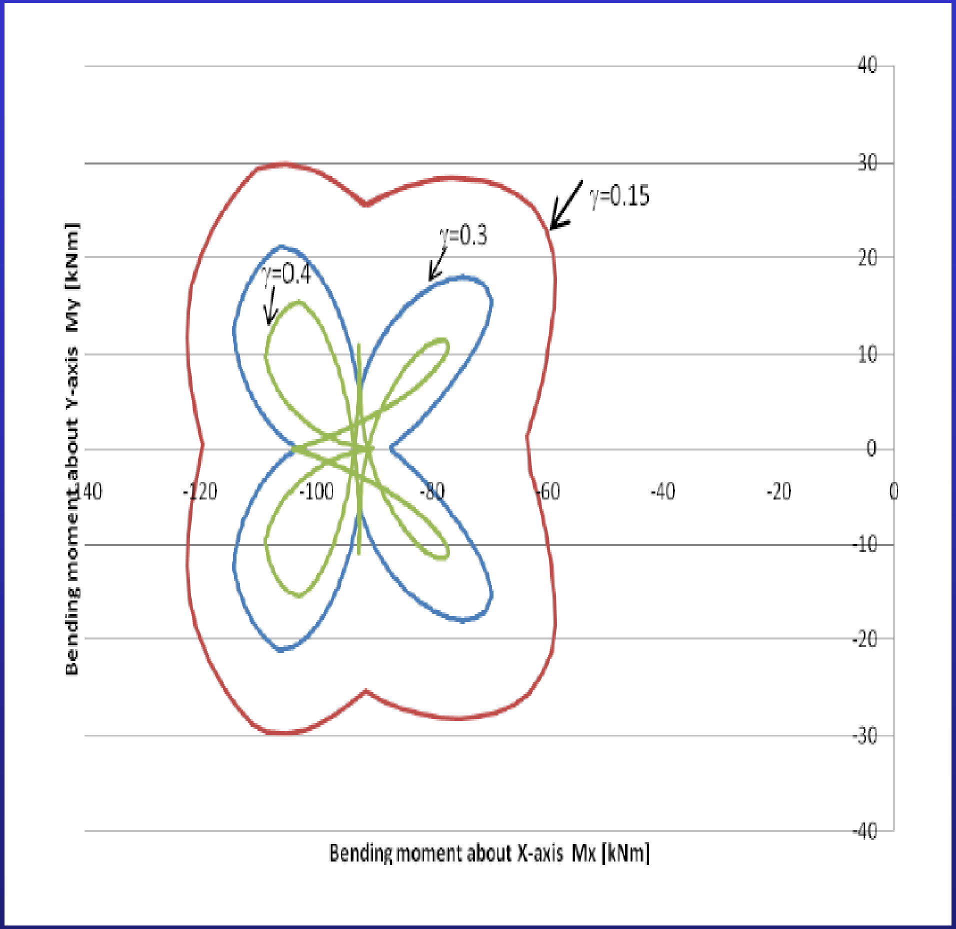


COMPUTATIONAL EXAMPLES



COMPUTATIONAL EXAMPLES





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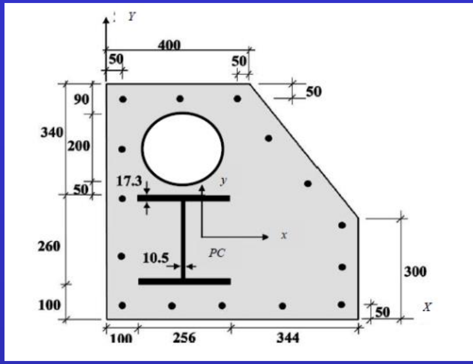
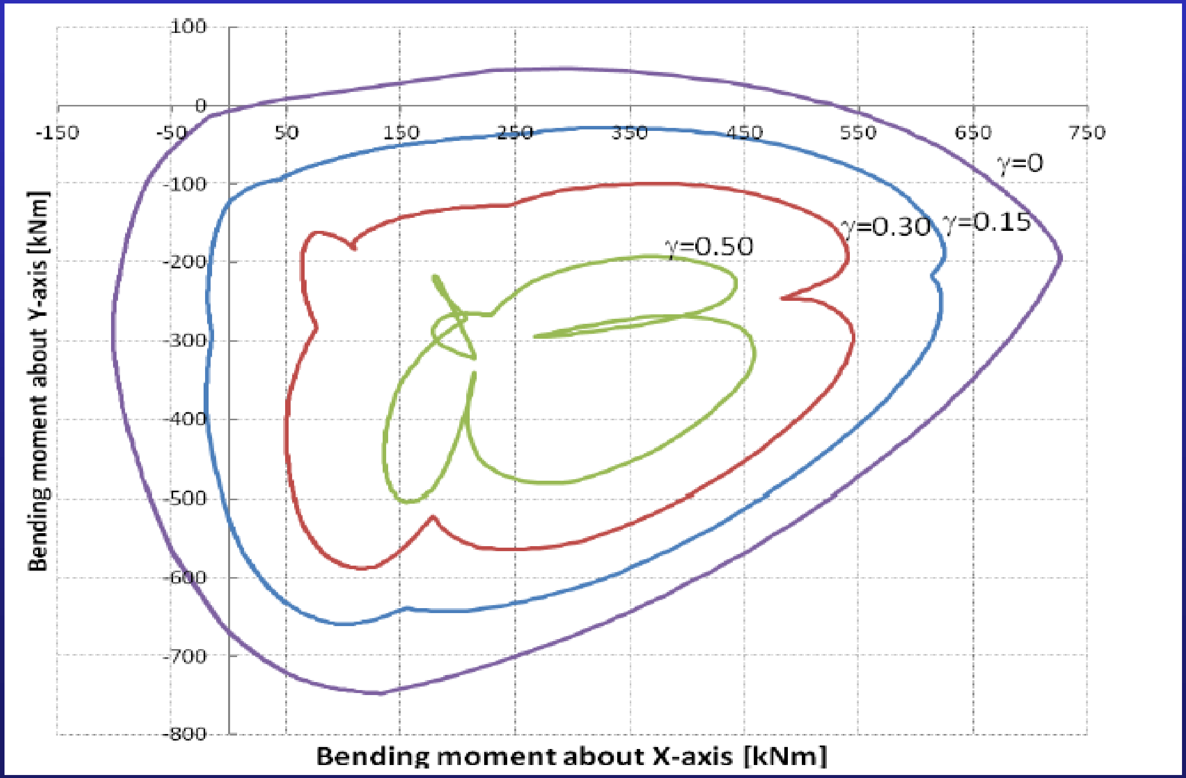
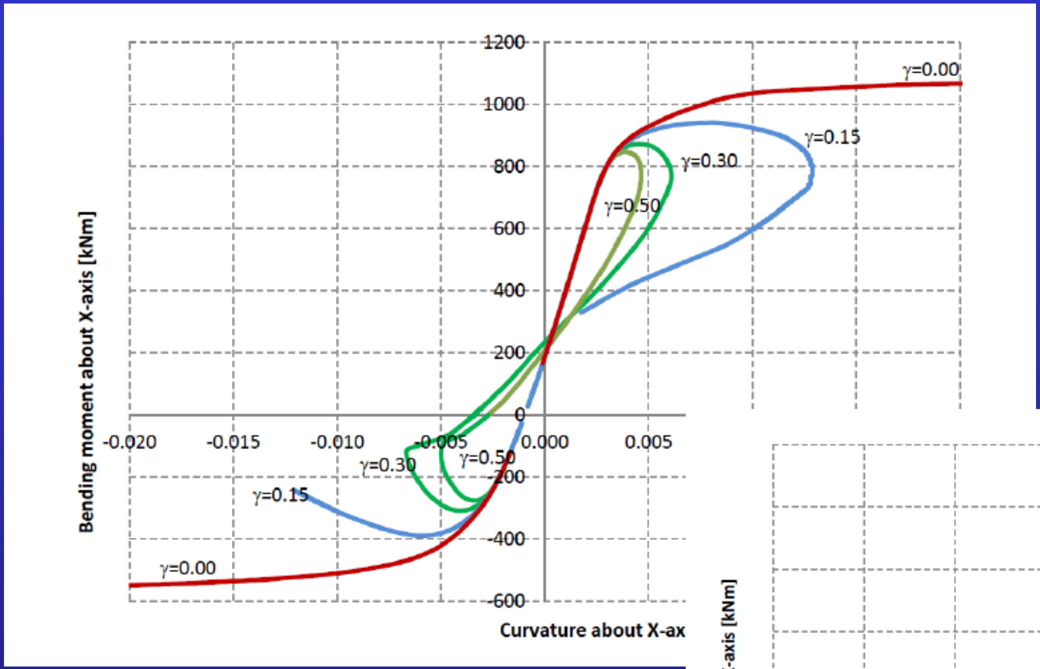
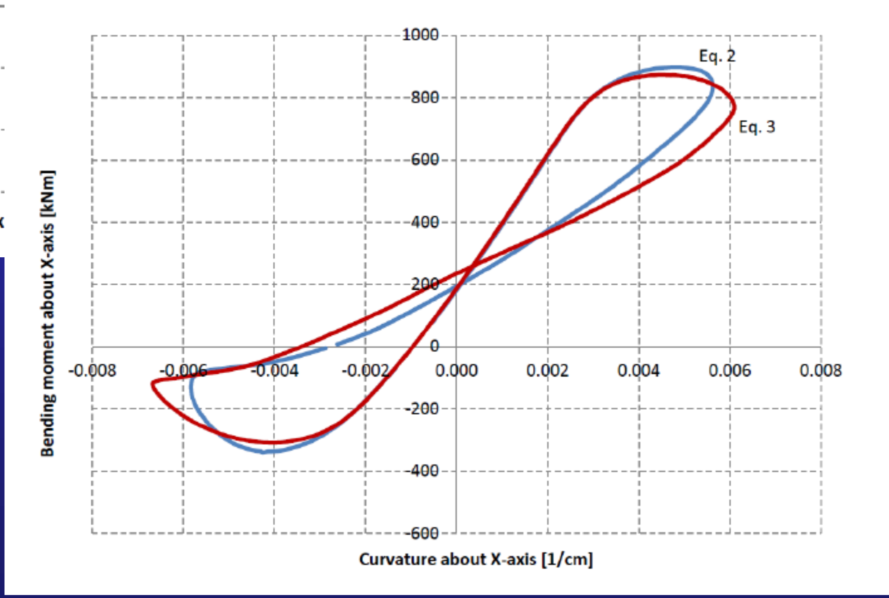


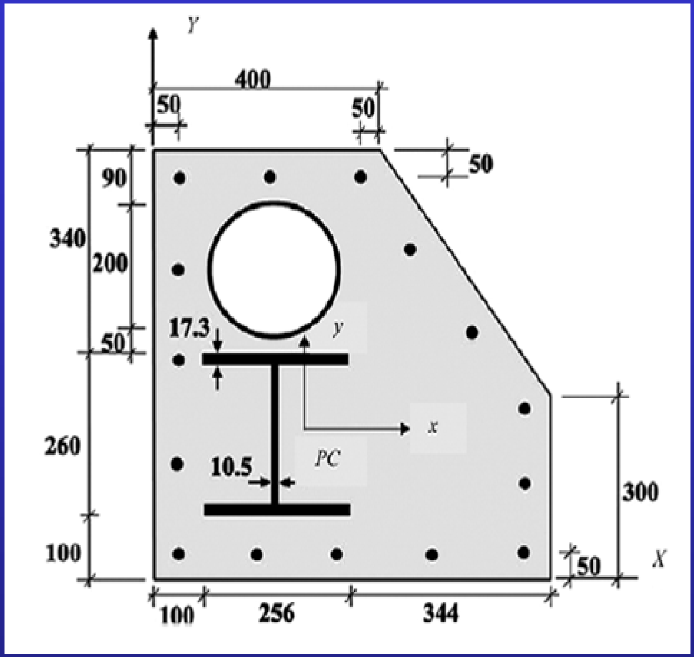


COMPUTATIONAL EXAMPLES





COMPUTATIONAL EXAMPLES



N=4120 kN,

M_x=209.70 kNm, M_y=863.70

$\phi_x=0$, $\phi_y=0$ si $A_{tot}=0$ TOL=1E-6

Proposed method: A_{tot}=35.39cm^2

Rodriugues [3]: A_{tot}=37.30cm^2

COMPUTATIONAL EXAMPLES

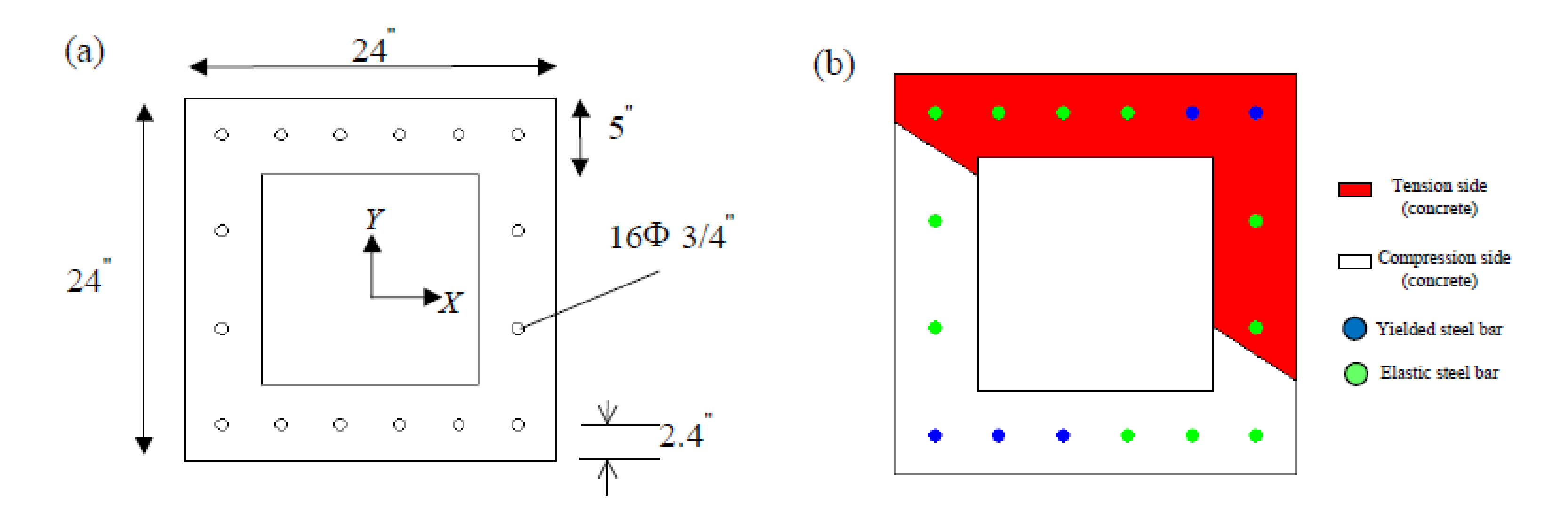
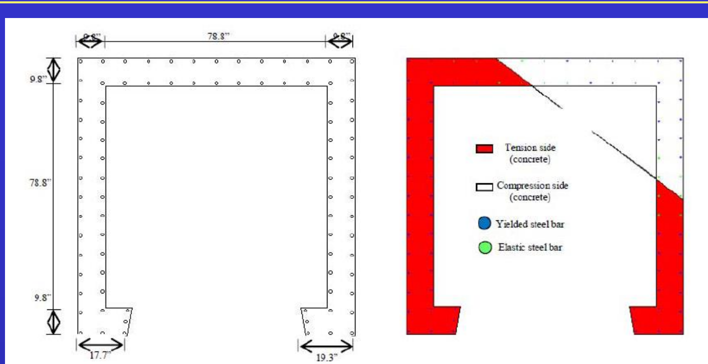


Table 3. Example 3: Main Parameters Involved in the Iterative Process

Iteration	ϕ_x	ϕ_y	A_{tot} [cm^2]	Error (Eq. 20)
Initial	0.000	0.000	$0.005A_g$=12.25	1.000
1	1.089E-4	5.576E-5	115.41	1.000146
2	6.426E-5	5.264E-5	30.816	0.599621
3	7.102E-5	4.550E-5	39.682	0.173854
4	7.084E-5	4.559E-5	40.556	0.007475
5	7.0834E-5	4.5583E-5	40.585	0.000266
6	7.08345E-5	4.5583E-5	40.586	0.00000856

COMPUTATIONAL EXAMPLES



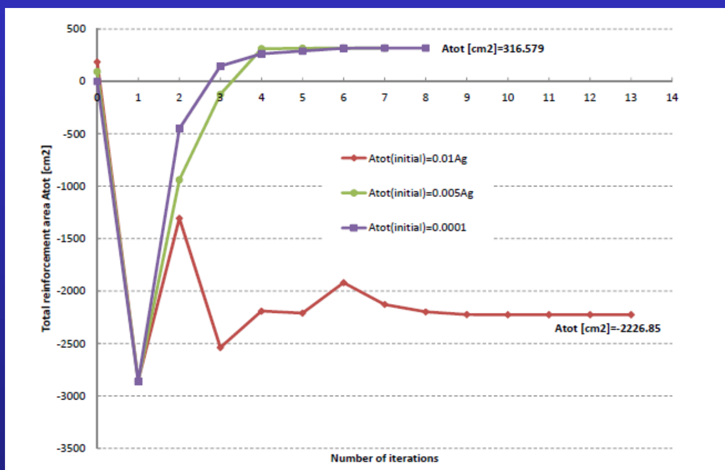
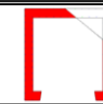
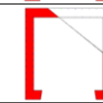
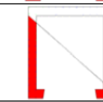
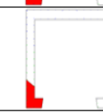
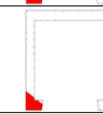


Table 9. Example 4: Variations of the Total Steel Reinforcement Area with Different Levels of the Axial Load

N [kN]	A_{tot} [cm^2]	No. of Iterations	Plastic Status
1000	559.741	6	
10000	242.105	5	
20000	12.814	4	
30000	110.903	4	
40000	505.181	6	
50000	987.723	7	
-1000	636.151	6	
-10000	994.658	7	
-20000	1409.775	7	

Inelastic analysis models: Concentrated vs Distributed

In the concentrated plasticity approach which is usually based on the plastic hinge concept, the effect of material yielding is “lumped” into a dimensionless plastic hinge. Regions in the beam-column elements other than at the plastic hinges are assumed to behave elastically. In the plastic hinge locations if the cross-section forces are less than cross-section plastic capacity, either elastic behaviour or gradual transition (*refined plastic hinge*) from elastic to plastic behaviour is assumed. The plastic hinge approach could eliminate the integration process on the cross section and permits the use of fewer elements for each member, and hence greatly reduces the computing effort. Unfortunately, as plastification in the member is assumed to be concentrated at the member ends, the plastic hinge model is usually less accurate in formulating the member stiffness, requires calibration procedures, but make possible to use **only one element** per physical member to simulate geometric and material nonlinearities in composite building frameworks.

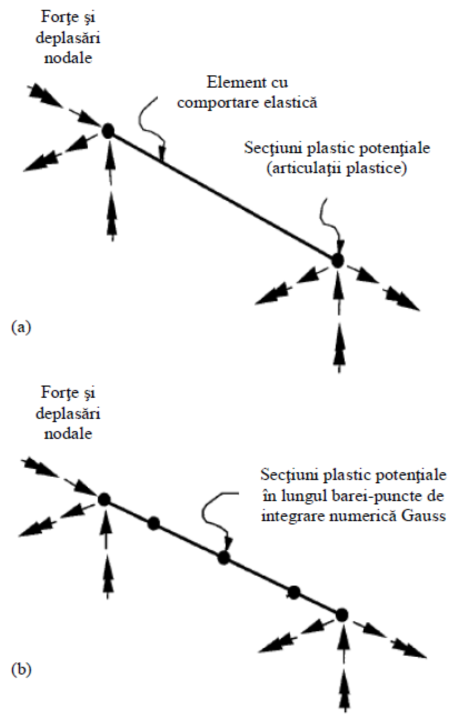
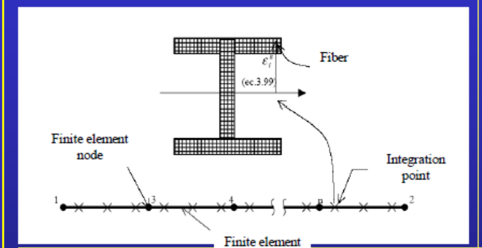
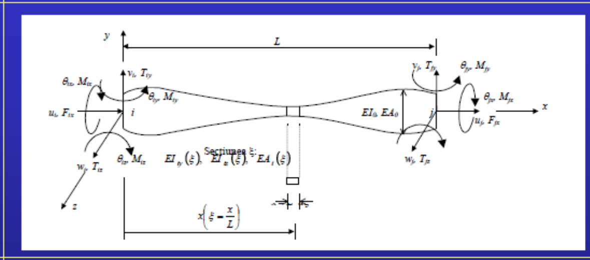


Fig. 2.1 Elemente liniare unidimensionale utilizate în analiza elasto-plastică. (a) Modelul plastificării concentrate; (b) Modelul plastificării distribuite.

Inelastic analysis models- Line elements

In the distributed plasticity models gradual yielding and spread of plasticity is allowed throughout cross-section and along the member length. There are two main approaches that have been used to model the gradual plastification of members in a second-order inelastic analysis, one based on the *displacement method* or finite element approach and the other based on the *force or flexibility method*. Because displacement based elements implicitly assumed linear curvatures along the element length, accuracy in this approach when material nonlinearity is taken into account can be obtained only using several elements in a single structural member, thus the computational effort is greatly enhanced and the method becomes prohibited computational in the case of large scale frame structures. On the other hand in the flexibility based approach *only one element* per physical member can be used to simulate the gradual spread of yielding throughout the volume of the members but the complexity of these methods derives from their implementation in a finite element analysis program and the inclusion of the element geometrical effects

Displacement -based (finite element)	Flexibility-based (proposed approach)
	
$\boxed{\mathbf{u}(x) = \mathbf{N}(x) \cdot \mathbf{d}}$	$\boxed{\mathbf{S}(x) = \mathbf{B}(x) \cdot \mathbf{E}}$
•Displacement shape functions •Influenced by the inelastic effects •Do not satisfy equilibrium •Many finite elements -**EXPENSIVE**	• Assumed force distribution •Satisfy equilibrium-distribution function is exact •Is not senzitive to the inelastic effects •One element per member -**EFFICIENT**

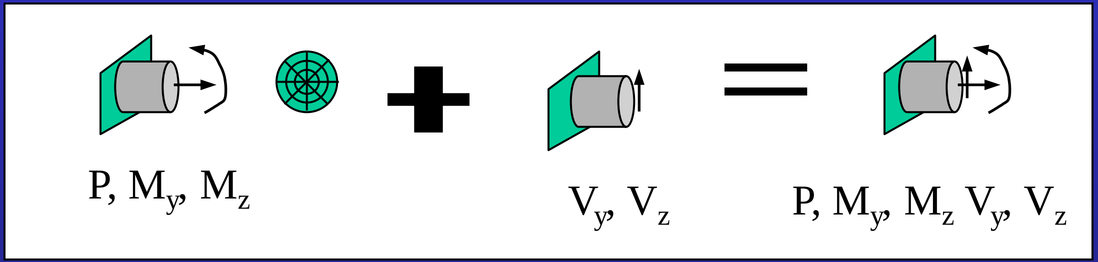
Distributed plasticity model

Fiber level

Nonlinear uniaxial stres-strain relationships

Yielding criteria (shear and normal stresses)

Cross-sectional level(**Axial and bending deformations coupled**, **Shear deformations uncoupled**)

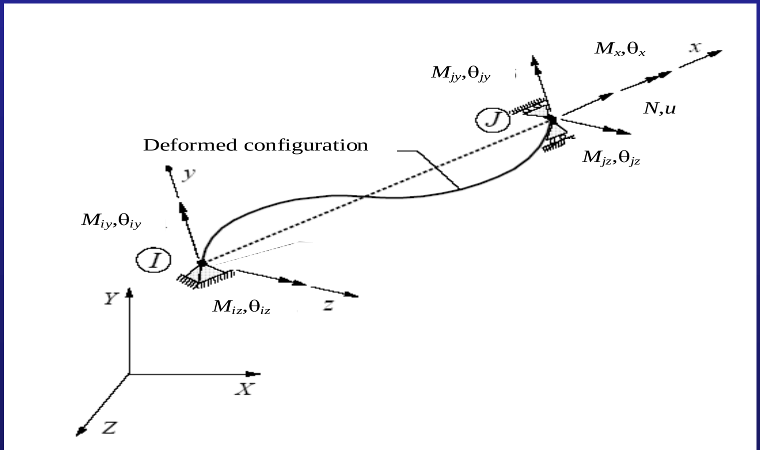


Element level (Euler-Timoshenko)

Equilibrium: $\mathbf{S}(x)=\mathbf{B}(x)$ E

Constitutive law: $\mathbf{k}_{\mathbf{sT}}\varphi=\mathbf{S}(x)$

Compatibility: $\mathbf{K}_{\mathrm{T}}\mathbf{u}_{\mathbf{e}}=\mathbf{E}$

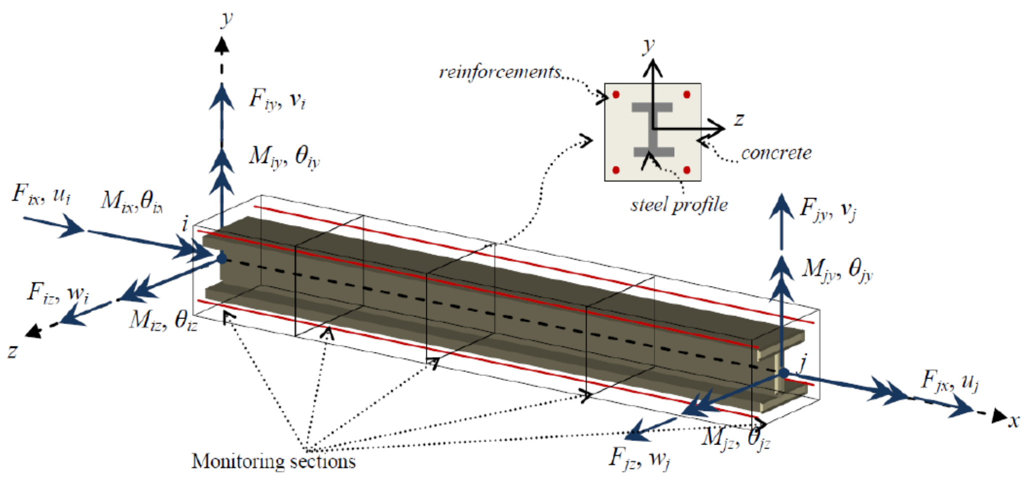


Mathematical formulation-Flexibility based formulation

Flexibility-based method is used to formulate the distributed plasticity model of a 3D frame element (12 DOF).

An element is represented by several cross sections (i.e. stations) that are located at the numerical integration scheme points. The spread of inelastic zones within an element is captured considering the variable section flexural EI_y and EI_z and axial EA rigidity along the member length, depending on the bending moments and axial force level, cross-sectional shape and nonlinear constitutive relationships. The elasto-plastic sectional rigidities are evaluated based on the iterative procedure already described.

Non-linear analysis by the stiffness method requires incremental loading, i.e. the inelastic behaviour is approximated by a series of elastic analysis. The element incremental flexibility matrix $\mathbf{f}_r$ which relates the end displacements to the actions and the elasto-plastic equivalent nodal forces transferred to the nodes, can be derived directly from energetic principles.



Second theorem of Castigliano

$$\Delta W = \frac{1}{2}\cdot \mathbf{u}_e^{\ T}\mathbf{k}_e\mathbf{u}_e = \frac{1}{2}\cdot \mathbf{p}^T\mathbf{u}_e = \frac{1}{2}\cdot \mathbf{p}^T\mathbf{k}_e^{-1}\mathbf{p}$$

The elasto-plastic equivalent nodal forces transferred to the nodes, from the member loads, will not be constant during the analysis, and will be dependent on the variable flexural rigidity along the member according with the process of gradual formation of plastic zones. The equivalent nodal forces will be computed in order to accommodate member lateral loads and the plastic strength surface requirements.

Mathematical formulation: Element analysis

Element force fileds (bending moment and shear forces)

$$
\begin{aligned}
M_{y(z)}(\xi) &= M_{iy(z)}(\xi-1) + M_{jy(z)}\xi \\
&\quad + \frac{L^2\xi(\xi-1)}{6}\left[q_{2y(z)}(\xi+1) - q_{1y(z)}(\xi-2)\right] \\
T_{y(z)}(\xi) &= \frac{\mathrm{d}M_{y(z)}(\xi)}{\mathrm{d}\xi} = \frac{M_{iy(z)} + M_{jy(z)}}{L} \\
&\quad + \frac{q_{2y(z)}L}{2}\left(\xi^2 - \frac{1}{3}\right) - \frac{q_{1y(z)}L}{2}\left[(\xi-1)^2 - \frac{1}{3}\right].
\end{aligned}
$$

Intrnal deformation enerrgy

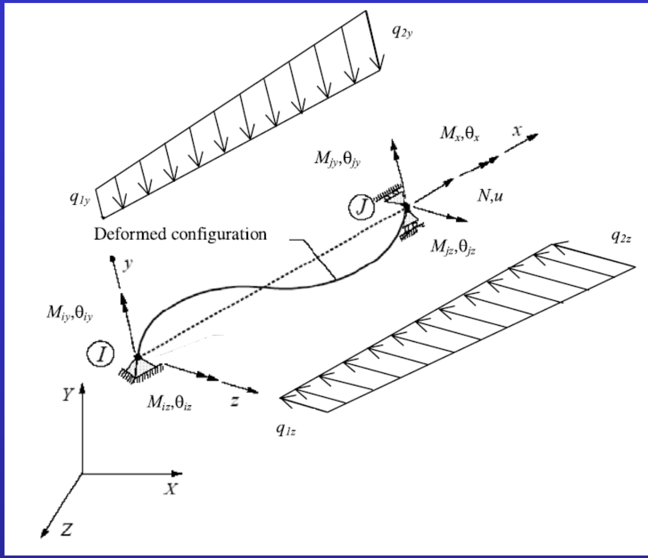
$$
\Delta W = \frac{1}{2}\cdot\int_0^L \frac{N^2}{EA_t(x)}dx + \frac{1}{2}\cdot\int_0^L \frac{\left(\frac{M_{jz}-M_{iz}}{L}x + M_{iz}\right)^2}{EI_{tz}(x)}dx + \frac{1}{2}\cdot\int_0^L \frac{\left(\frac{M_{jy}-M_{iy}}{L}x + M_{iy}\right)^2}{EI_{ty}(x)}dx +
$$

$$
+\frac{1}{2}\cdot\int_0^L \frac{\left(\frac{M_{jz}-M_{iz}}{L}\right)^2}{GA_z(x)}dx + \frac{1}{2}\cdot\int_0^L \frac{\left(\frac{M_{jy}-M_{iy}}{L}\right)^2}{GA_y(x)}dx + \frac{1}{2}\cdot\int_0^L \frac{M_x^2}{GI_t(x)}dx
$$

Second theorem of Castigliano

$$
\begin{bmatrix} u \\ \theta_{iy} \\ \theta_{jy} \\ \theta_{iz} \\ \theta_{jz} \\ \theta_x \end{bmatrix} = \begin{bmatrix} \frac{\partial \Delta W}{\partial N} \\ \frac{\partial \Delta W}{\partial M_{iy}} \\ \frac{\partial \Delta W}{\partial M_{jy}} \\ \frac{\partial \Delta W}{\partial M_{iz}} \\ \frac{\partial \Delta W}{\partial M_{jz}} \\ \frac{\partial \Delta W}{\partial M_x} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{1(3\times3)} & \mathbf{0}_{(3\times3)} \\ \mathbf{0}_{(3\times3)} & \mathbf{f}_{2(3\times3)} \end{bmatrix} \cdot \begin{bmatrix} N \\ M_{iy} \\ M_{jy} \\ M_{iz} \\ M_{jz} \\ M_x \end{bmatrix} + \begin{bmatrix} 0 \\ \delta_{iy} \\ \delta_{jy} \\ \delta_{iz} \\ \delta_{jz} \\ 0 \end{bmatrix}
$$

3D beam-column element in local system attached to the initially straight center line, with the rigid body modes removed.



$$
\Delta\mathbf{s}_r = \mathbf{k}_r \cdot \Delta\mathbf{u}_r - \mathbf{q}_r
$$

$$
\mathbf{k}_{r(6\times6)} = \mathbf{f}_r^{-1} = \begin{bmatrix} \mathbf{f}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{f}_2^{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{k}_{1(3\times3)} & \mathbf{0}_{(3\times3)} \\ \mathbf{0}_{(3\times3)} & \mathbf{k}_{2(3\times3)} \end{bmatrix}
$$

$$
\mathbf{q}_r = \mathbf{k}_r \boldsymbol{\delta}_r
$$

Mathematical formulation: element analysis

Flexibility matrix of the element with rigid body modes removed

$$\mathbf{f}_1 = \begin{bmatrix} L\int_0^1 \frac{1}{EA(\xi)}\mathrm{d}\xi & 0 & 0 \\ 0 & L\int_0^1 \frac{(\xi-1)^2}{EI_y(\xi)}\mathrm{d}\xi + \frac{1}{L}\int_0^1 \frac{\mathrm{d}\xi}{GA_y(\xi)} & L\int_0^1 \frac{\xi(\xi-1)}{EI_y(\xi)}\mathrm{d}\xi + \frac{1}{L}\int_0^1 \frac{\mathrm{d}\xi}{GA_y(\xi)} \\ 0 & L\int_0^1 \frac{\xi(\xi-1)}{EI_y(\xi)}\mathrm{d}\xi + \frac{1}{L}\int_0^1 \frac{\mathrm{d}\xi}{GA_y(\xi)} & L\int_0^1 \frac{\xi^2}{EI_y(\xi)}\mathrm{d}\xi + \frac{1}{L}\int_0^1 \frac{\mathrm{d}\xi}{GA_y(\xi)} \end{bmatrix}$$

$$\mathbf{f}_2 = \begin{bmatrix} L\int_0^1 \frac{(\xi-1)^2}{EI_z(\xi)}\mathrm{d}\xi + \frac{1}{L}\int_0^1 \frac{\mathrm{d}\xi}{GA_z(\xi)} & L\int_0^1 \frac{(\xi-1)^2}{EI_z(\xi)}\mathrm{d}\xi + \frac{1}{L}\int_0^1 \frac{\mathrm{d}\xi}{GA_z(\xi)} & 0 \\ L\int_0^1 \frac{(\xi-1)^2}{EI_z(\xi)}\mathrm{d}\xi + \frac{1}{L}\int_0^1 \frac{\mathrm{d}\xi}{GA_z(\xi)} & L\int_0^1 \frac{(\xi-1)^2}{EI_z(\xi)}\mathrm{d}\xi + \frac{1}{L}\int_0^1 \frac{\mathrm{d}\xi}{GA_z(\xi)} & 0 \\ 0 & 0 & L\int_0^1 \frac{\mathrm{d}\xi}{GI_t(\xi)} \end{bmatrix}$$

Correction coefficients evaluated using numerical quadratures

$$c_x = \int_0^1 \frac{1}{f_x(\xi)}\mathrm{d}\xi; \qquad c_{1y(z)} = 3\int_0^1 \frac{(1-\xi)^2}{f_{y(z)}(\xi)}\mathrm{d}\xi;$$

$$c_{2y(z)} = 3\int_0^1 \frac{\xi^2}{f_{y(z)}(\xi)}\mathrm{d}\xi; \qquad c_{3y(z)} = 6\int_0^1 \frac{\xi(1-\xi)}{f_{y(z)}(\xi)}\mathrm{d}\xi$$

The equivalent nodal displacements

$$\begin{aligned} \delta_{iy(z)} &= \int_0^1 \frac{L^3\xi(\xi-1)^2\left[q_{2y(z)}(\xi+1) - q_{1y(z)}(\xi-2)\right]}{6EI_{y(z)}(\xi)}\mathrm{d}\xi \\ &\quad + \int_0^1 \frac{q_{2z(y)}L\left(\xi^2-\frac{1}{3}\right) - q_{1z(y)}L\left[(\xi-1)^2-\frac{1}{3}\right]}{2GA_{y(z)}(\xi)}\mathrm{d}\xi \\ \delta_{iy(z)} &= \int_0^1 \frac{L^3\xi^2(\xi-1)\left[q_{2y(z)}(\xi+1) - q_{1y(z)}(\xi-2)\right]}{6EI_{y(z)}(\xi)}\mathrm{d}\xi \\ &\quad + \int_0^1 \frac{q_{2z(y)}L\left(\xi^2-\frac{1}{3}\right) - q_{1z(y)}L\left[(\xi-1)^2-\frac{1}{3}\right]}{2GA_{y(z)}(\xi)}\mathrm{d}\xi. \end{aligned}$$

Mathematical formulation: element analysis

STIFFNESS matrix of the element with rigid body modes removed

$$\mathbf{k}_{r(6\times 6)} = \mathbf{f}_r^{-1} = \begin{bmatrix} \mathbf{f}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{f}_2^{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{k}_{1(3\times 3)} & \mathbf{0}_{(3\times 3)} \\ \mathbf{0}_{(3\times 3)} & \mathbf{k}_{2(3\times 3)} \end{bmatrix}$$

$$\mathbf{q}_r = \mathbf{k}_r \delta_r$$

$$\mathbf{k}_1 = \begin{bmatrix} \frac{EA_0}{L}\cdot\lambda & 0 & 0 \\ 0 & \frac{4EI_{0y}}{L}\alpha_y & \frac{2EI_{0y}}{L}\beta_y \\ 0 & \frac{2EI_{0y}}{L}\beta_y & \frac{4EI_{0y}}{L}\gamma_y \end{bmatrix}$$

$$\mathbf{k}_2 = \begin{bmatrix} \frac{4EI_{0z}}{L}\alpha_z & \frac{2EI_{0z}}{L}\beta_z & 0 \\ \frac{2EI_{0z}}{L}\beta_z & \frac{4EI_{0z}}{L}\gamma_z & 0 \\ 0 & 0 & \frac{GI_t}{L} \end{bmatrix}$$

$$\lambda = \frac{1}{c_x}; \qquad \alpha_{y(z)} = \frac{3\left(c_{2y(z)} + \frac{t_{y(z)}}{4}\right)}{Z_{y(z)} + t_{y(z)}s_{y(z)}};$$

$$\beta_{y(z)} = \frac{3\left(c_{3y(z)} - \frac{t_{y(z)}}{2}\right)}{Z_{y(z)} + t_{y(z)}s_{y(z)}}; \qquad \gamma_{y(z)} = \frac{3\left(c_{1y(z)} + \frac{t_{y(z)}}{4}\right)}{Z_{y(z)} + t_{y(z)}s_{y(z)}}$$

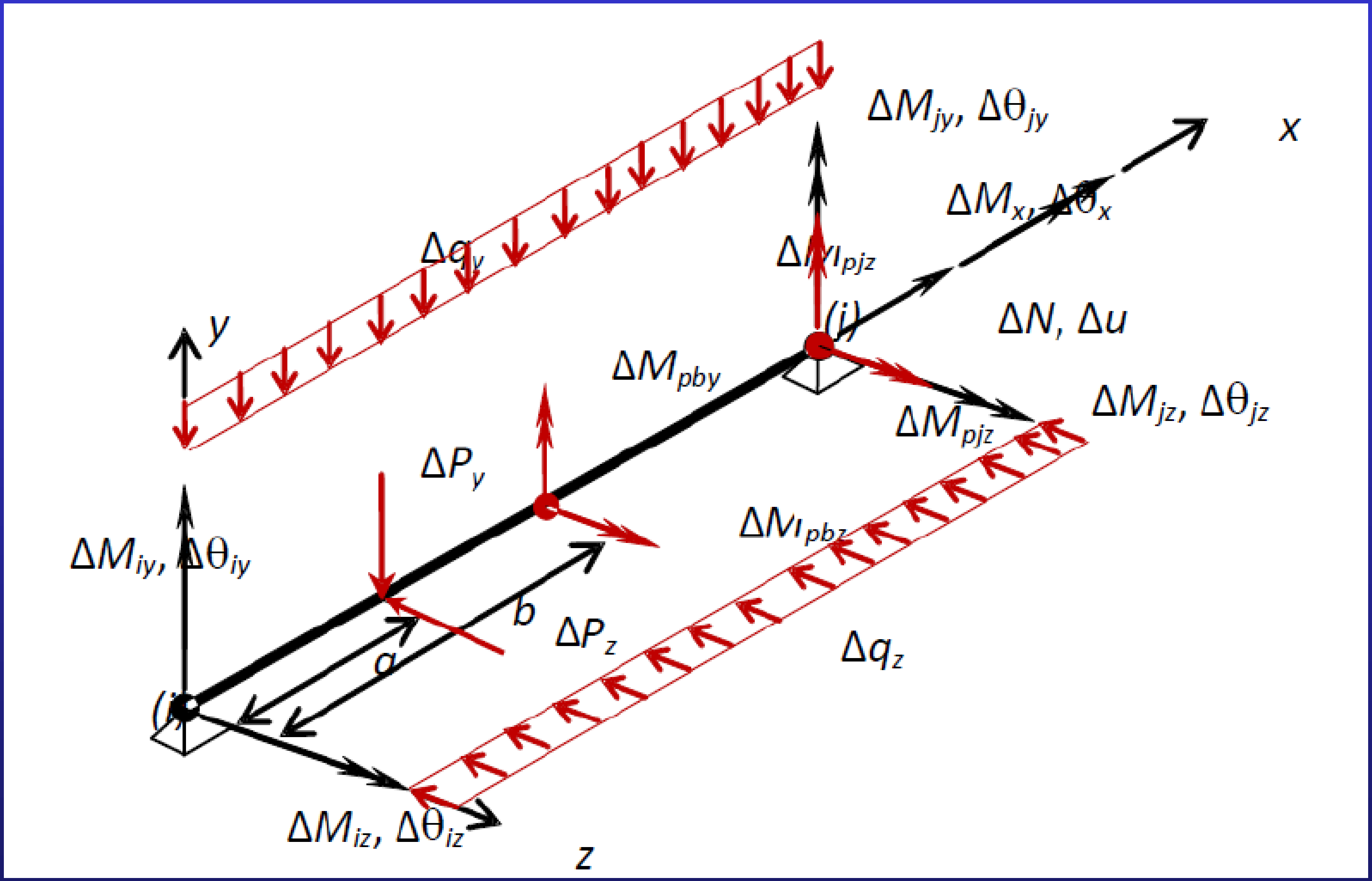
$$t_y = \frac{12EI_y}{GA_yL^2}; \qquad t_z = \frac{12EI_z}{GA_zL^2}$$

$$Z_{y(z)} = 4c_{1y(z)}c_{2y(z)} - c_{3y(z)}^2;$$

$$s_{y(z)} = c_{1y(z)} + c_{2y(z)} + c_{3y(z)};$$

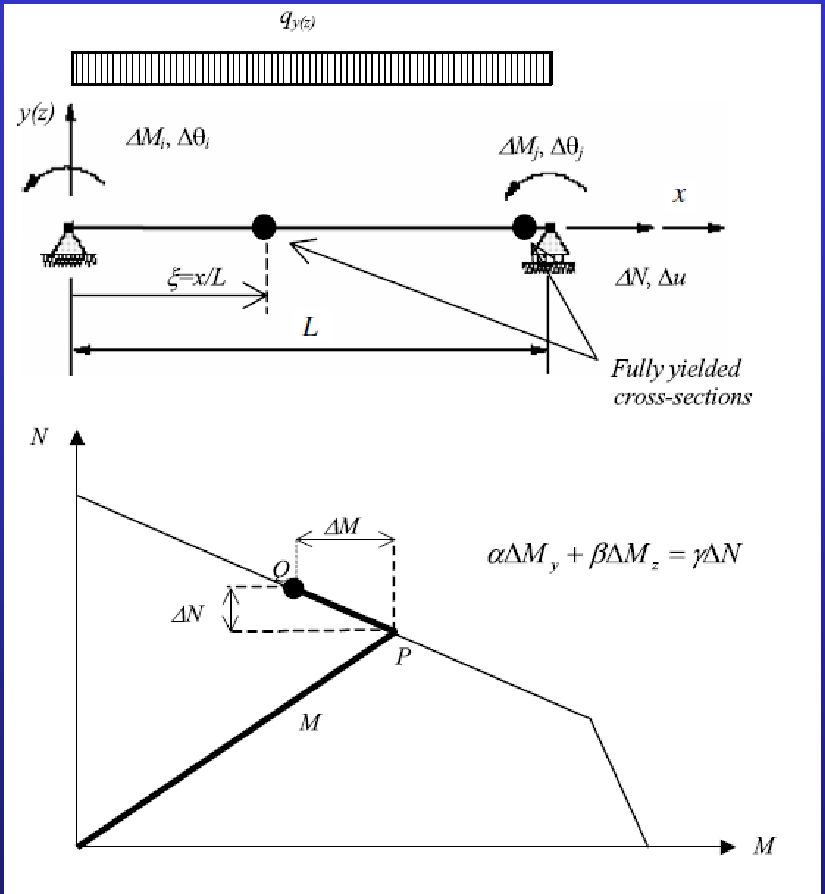
Mathematical formulation: element analysis

Lateral loads acting along the member length can be directly input into the analysis



Mathematical formulation: element analysis

The plastic surface requirements



If the state of forces at any cross-section along the element equals or exceeds the plastic section capacity the flexural stiffness approaches zero. Once the member forces get to the full plastic surface they are assumed to move on the plastic surface at the following loading step

$$\frac{\partial \Gamma}{\partial N} = \frac{1}{N_p} + \frac{8}{9M^0_{py}} \frac{\partial M_{iy}}{\partial N} + \frac{8}{9M^0_{pz}} \frac{\partial M_{iz}}{\partial N}$$

$$\cong \frac{1}{N_p} + \frac{8}{9M^0_{py}} \frac{\Delta M_{iy}}{\Delta N} + \frac{8}{9M^0_{pz}} \frac{\Delta M_{iz}}{\Delta N} = 0$$

$$\alpha = \frac{8}{9M^0_{py}}, \qquad \beta = \frac{8}{9M^0_{pz}}, \qquad \gamma = -\frac{1}{N^0_p}$$

$$\mathbf{k}_{r,ep} = \mathbf{T}_c \left(\mathbf{T}_c^{\mathrm{T}} \mathbf{f}_r \mathbf{T}_c\right)^{-1} \mathbf{T}_c^{\mathrm{T}}$$

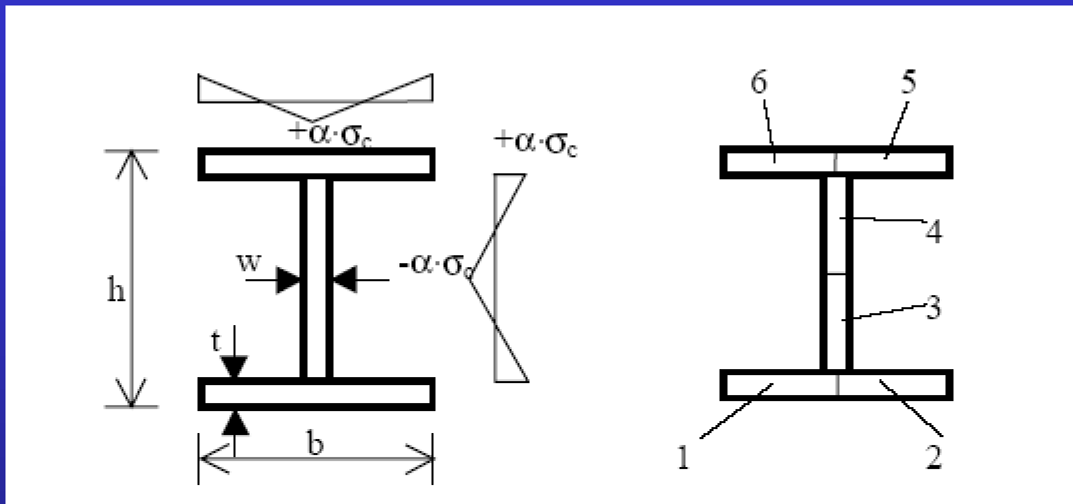
$$\Delta \mathbf{q}_{rp} = \mathbf{T}_c \left(\mathbf{T}_c^{\mathrm{T}} \mathbf{f}_r \mathbf{T}_c\right)^{-1} \mathbf{T}_c^{\mathrm{T}} \mathbf{f}_r \Delta \mathbf{q}_r + \Delta \hat{\mathbf{q}} - \mathbf{T}_c \left(\mathbf{T}_c^{\mathrm{T}} \mathbf{f}_r \mathbf{T}_c\right)^{-1} \mathbf{T}_c^{\mathrm{T}} \mathbf{f}_r \Delta \hat{\mathbf{q}}.$$

$$\mathbf{T}_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\gamma}{\alpha} & 0 & 0 & -\frac{\beta}{\alpha} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \text{for plastified section at } j\text{th end}$$

$$\mathbf{T}_c = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\gamma}{\alpha} & -\frac{\beta}{\alpha} & 0 \\ \frac{\gamma}{\alpha} & 0 & -\frac{\beta}{\alpha} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{for plastified sections at both end}$$

Mathematical formulation: element analysis

Residual stress effect: Cross-section discretization



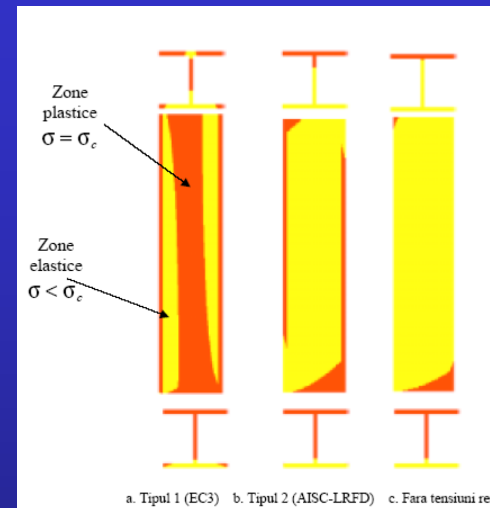


Fig.6.4 Dezvolatrea zonelor plastice în cele trei variante de considerare a tensiunilor reziduale

$$\varepsilon = \varepsilon_0 + \Phi_x \cdot y + \Phi_y \cdot x + \varepsilon_r$$

$$\boxed{\varepsilon_r = a_1 + a_2 x + a_3 y}$$

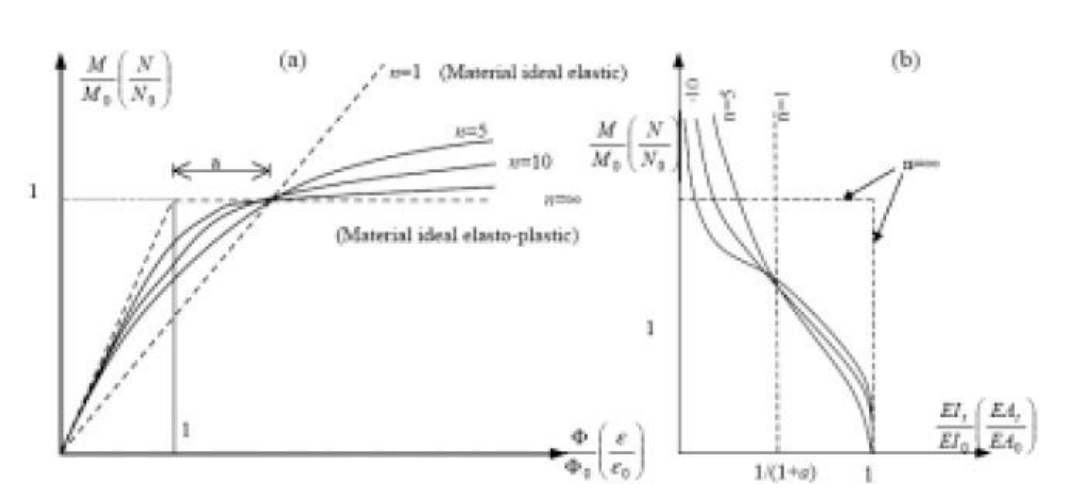
$$\boxed{\tan\theta = \frac{\Phi_y + a_2}{\Phi_x + a_3}}$$

Rotation angle of reference axes

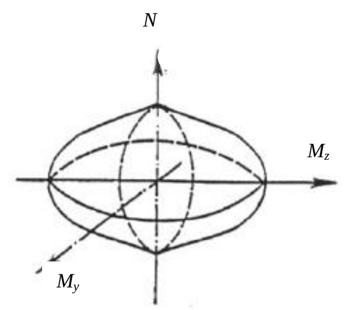
Mathematical formulation: element analysis: force strain cross section evaluation

Cross-section analysis: Macro model formulation

Ramberg-Osgood force-strain relationships (numerically calibrated)



Plastic interaction surface (Powel-Chen, Orbison, AISC-LRFD, etc)

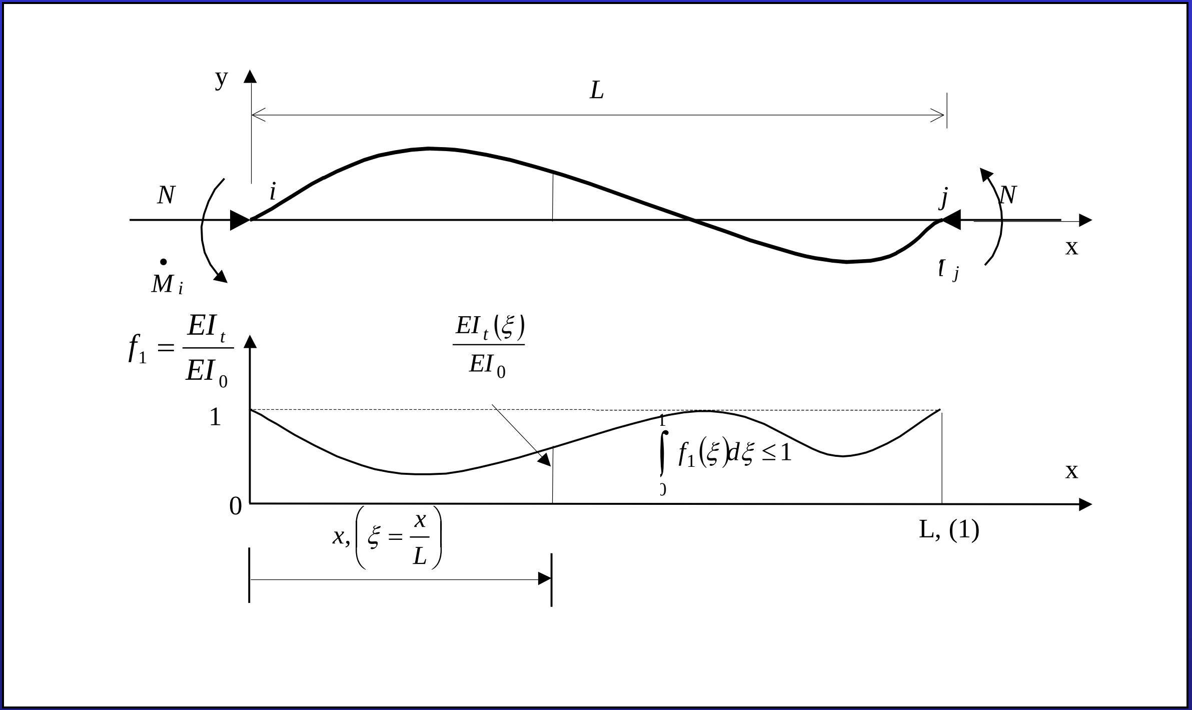


Powel-Chen, 1986

$$\sqrt{\left(\frac{M_y}{M_{py}}\right)^2+\left(\frac{M_z}{M_{pz}}\right)^2}+\left(\frac{N}{N_p}\right)^{\alpha}-1=0$$

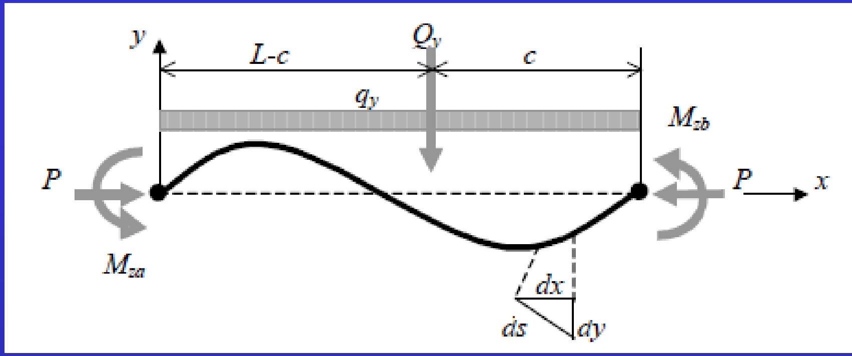
Mathematical formulation: element *analysis: second order effects*

The second-order effects (P-δ): Element level

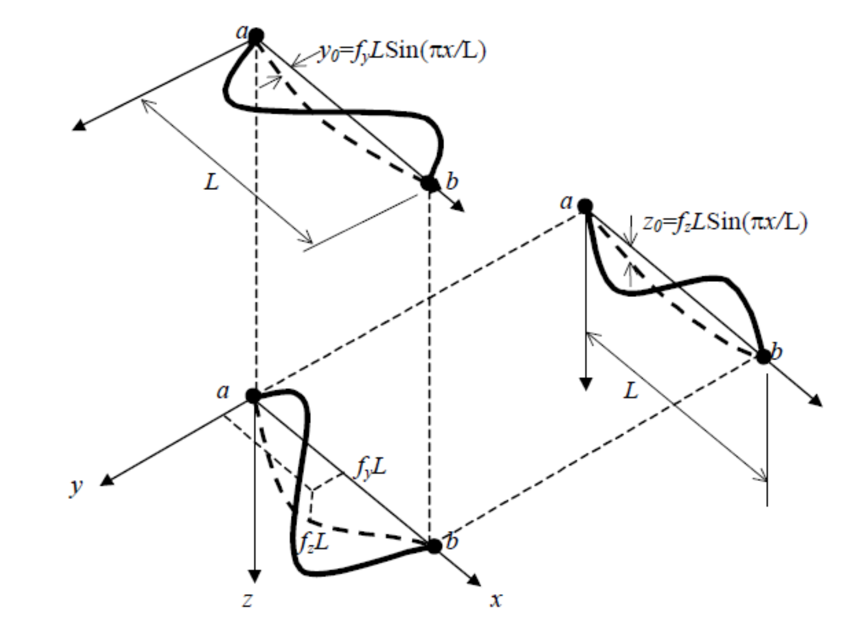


Inelastic stability stiffness functions (Tanegnt flexural rigidity)

Element second-order geometrical effects



$$\frac{d^2y}{dx^2}-\frac{d^2y_0}{dx^2}=\frac{d^2y_e}{dx^2}=-\frac{M_z}{EI_z}$$



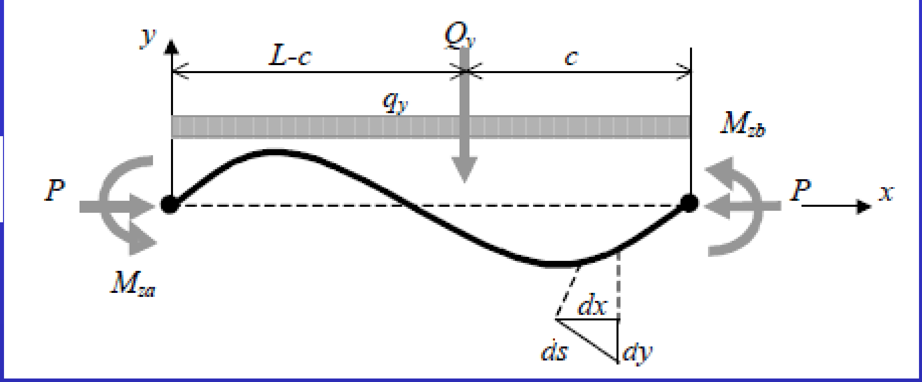
Element second-order geometrical effects

$$M_z = Py + \frac{1}{2}q_y x(L-x) + \frac{Q_y cx}{L} + M_{za}\left(\frac{x}{L}-1\right) + M_{zb}\frac{x}{L},\ \ 0 \le x \le L-c$$

$$M_z = Py + \frac{1}{2}q_y x(L-x) + Q_y\left(\frac{cx}{L} - x + L - c\right) + M_{za}\left(\frac{x}{L}-1\right) + M_{zb}\frac{x}{L},\ \ L-c \le x \le L$$

$$\frac{d^2y}{dx^2} - \frac{d^2y_0}{dx^2} = \frac{d^2y_e}{dx^2} = -\frac{M_z}{EI_z}$$

$$\frac{d^2y_e}{dx^2} + \alpha^2 y_e = -\frac{1}{EI_z}\begin{cases} P\left(f_y \sin\frac{\pi x}{L}\right)L + \frac{1}{2}q_y x(L-x) + \\ \quad + \frac{Q_y cx}{L} + M_{za}\left(\frac{x}{L}-1\right) + M_{zb}\frac{x}{L},\ \ 0 \le x \le L-c \\ P\left(f_y \sin\frac{\pi x}{L}\right)L + \frac{1}{2}q_y x(L-x) + \\ + Q_y\left(\frac{cx}{L} - x + L - c\right) + M_{za}\left(\frac{x}{L}-1\right) + M_{zb}\frac{x}{L},\ \ L-c \le x \le L \end{cases}$$



$$y_e = -\frac{1}{EI_z}\begin{cases} C_1 \sin\alpha x + C_2\cos\alpha x + \frac{\alpha^2 f_y L^3}{\pi^2 - \alpha^2 L^2}\sin\frac{\pi x}{L} - \\ -\frac{1}{P}\left[M_{za}\left(\frac{x}{L}-1\right) + \frac{1}{2}q_y x(L-x) + \frac{q_y}{\alpha^2} + \frac{Q_y cx}{L} + M_{zb}\frac{x}{L}\right],\ 0 \le x \le L-c \\ C_3 \sin\alpha x + C_4\cos\alpha x + \frac{\alpha^2 f_y L^3}{\pi^2 - \alpha^2 L^2}\sin\frac{\pi x}{L} - \\ -\frac{1}{P}\left[M_{za}\left(\frac{x}{L}-1\right) + \frac{1}{2}q_y x(L-x) + \frac{q_y}{\alpha^2} + \frac{Q_y(L-c)(L-x)}{L} + M_{zb}\frac{x}{L}\right], L-c \le x \le L \end{cases}$$

$$C_1 = \frac{1}{P\sin\alpha L}\left[M_{zb} + M_{za}\cos\alpha L + \frac{q_y}{\alpha^2}(1-\cos\alpha L) + \frac{Q_y}{\alpha}\sin\alpha c\right]$$

$$C_2 = \frac{1}{P}\left[\frac{q_y}{\alpha^2} - M_{za}\right]$$

$$C_3 = \frac{1}{P\sin\alpha L}\left[\frac{q_y}{\alpha^2} + M_{zb} - \cos\alpha L\left(\frac{q_y}{\alpha^2} - M_{za}\right) - \frac{Q_y}{\alpha}\cos\alpha L\sin\alpha(L-c)\right]$$

$$C_4 = \frac{1}{P}\left[\frac{q_y}{\alpha^2} - M_{za}\right] + \frac{Q_y}{\alpha P}\sin\alpha(L-c)$$

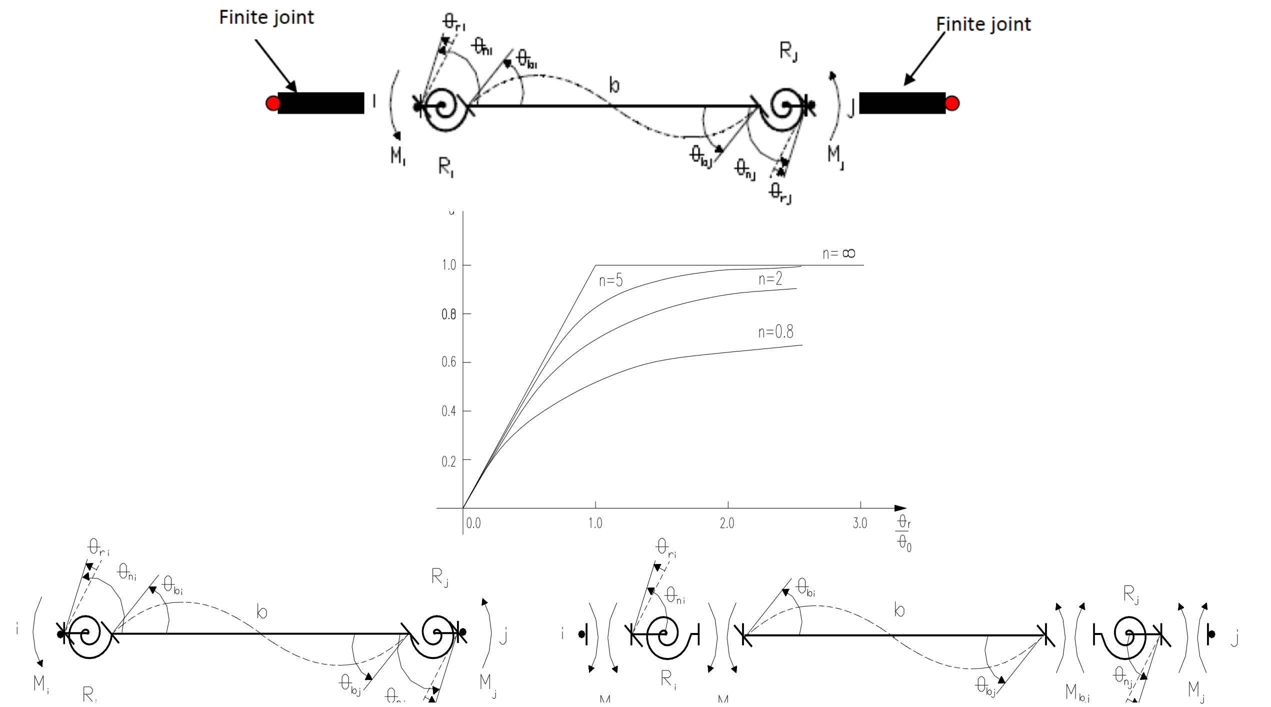
Mathematical formulation: *Semi-rigid connections*

Semi-rigid connections modelling

The behavior of the connection element in each principal bending direction (major and minor axis flexibility) is represented by a rotational dimensionless spring attached to the member end. We assume no coupling between different rotational degree of freedom at the connection. The connections could have either linear or nonlinear behavior. The effects of semi-rigid connections are included in the analysis adopting the model presented in the figure.

The element stiffness matrix and equivalent nodal looads are expressed by the following relations.

- Mi, Mj are the applied moments at nodes
- θn, θb are the node, respective end element rotation
- θr is the relative rotational angle
- Ri, Rj the initial connection stiffness

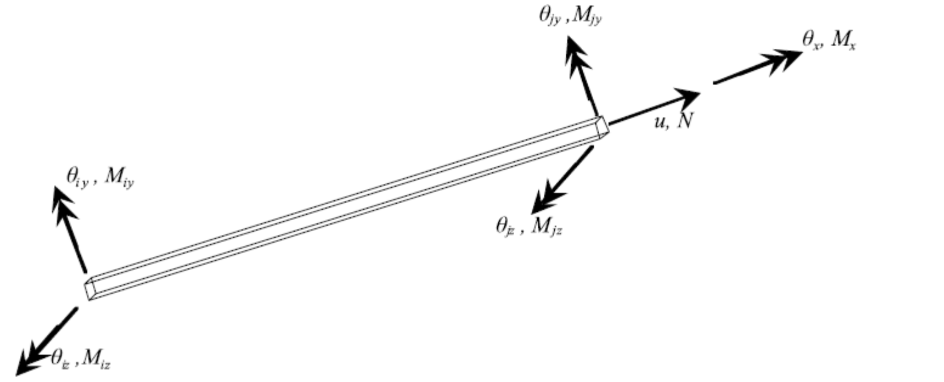


$$\mathbf{k}_{sem(4\times4)} = \left[\mathbf{k}_{r(4\times4)} - \mathbf{k}_{r(4\times4)}\left(\mathbf{k}_{r(4\times4)} + \mathbf{k}_{sc(4\times4)}\right)^{-1}\mathbf{k}_{r(4\times4)}\right]$$

$$\Delta\mathbf{q}_{sem(4\times1)} = \Delta\mathbf{q}_{eq(4\times1)} - \Delta\mathbf{q}_{eq(4\times1)}\mathbf{k}_{r(4\times4)}\left(\mathbf{k}_{r(4\times4)} + \mathbf{k}_{sc(4\times4)}\right)^{-1}$$

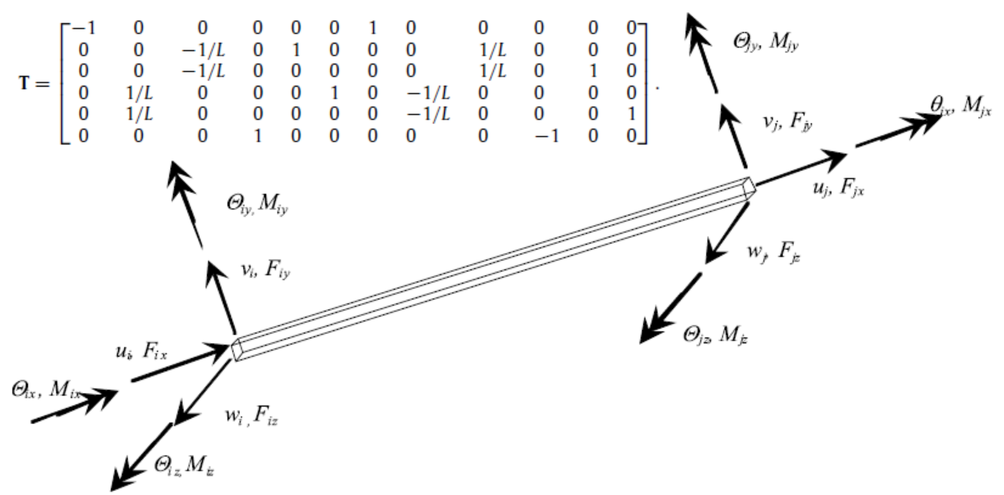
$$\mathbf{k}_{sc} = \mathrm{diag}(R_{iy}, R_{jy}, R_{iz}, R_{jz})$$

Transformation of stiffness matrix



(a) Natural element coordinates.

$$
\mathbf{T} = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1/L & 0 & 1 & 0 & 0 & 0 & 1/L & 0 & 0 & 0 \\
0 & 0 & -1/L & 0 & 0 & 0 & 0 & 0 & 1/L & 0 & 1 & 0 \\
0 & 1/L & 0 & 0 & 0 & 1 & 0 & -1/L & 0 & 0 & 0 & 0 \\
0 & 1/L & 0 & 0 & 0 & 0 & 0 & -1/L & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0
\end{bmatrix}.
$$

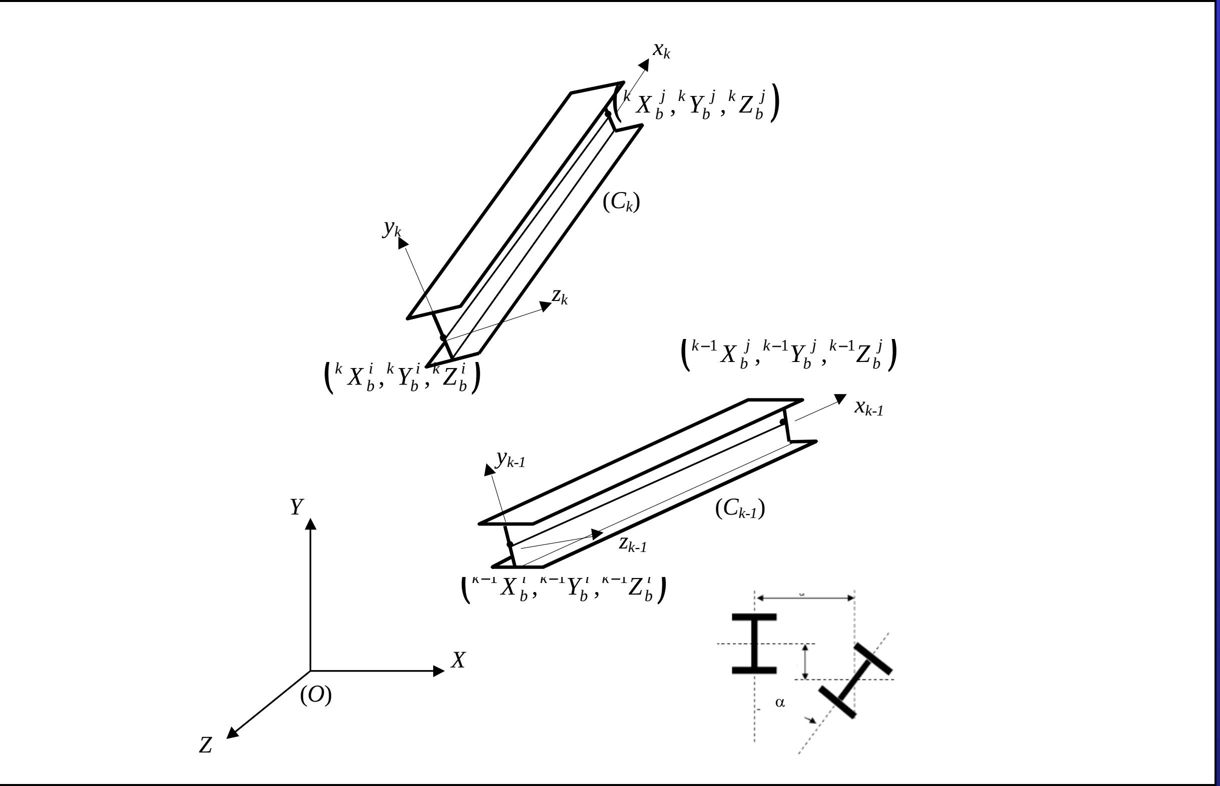


(b) Local element coordinates.

$$
\boxed{\begin{aligned}
\mathbf{k}_{l(12\times 12)} &= \mathbf{T}^{\mathrm{T}}_{(12\times 6)} \cdot \mathbf{k}_{r(6\times 6)} \cdot \mathbf{T}_{(6\times 12)} \\
\Delta\mathbf{q}_{eq(12\times 1)} &= \mathbf{T}^{\mathrm{T}}_{(12\times 6)} \cdot \Delta\mathbf{q}_{r,eq(6\times 1)} + \Delta\mathbf{q}_{(12\times 1)}.
\end{aligned}}
$$

Global nonlinear geometrical effects

Geometry updating: Large deflections effect

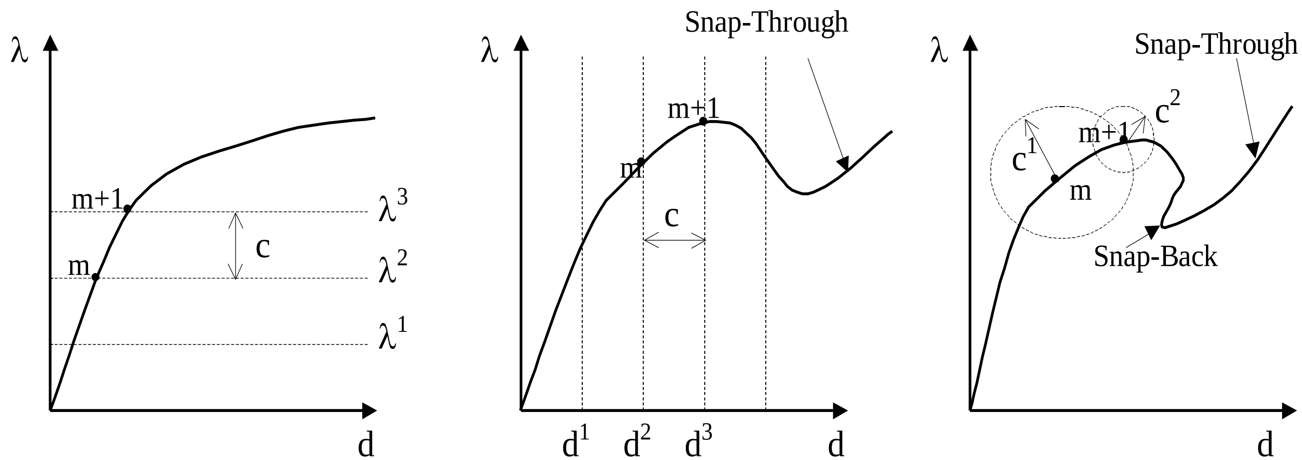


Updated Lagrangian formulation (UL)

Natural deformation approach (NDA)

Geometrical rigid body qualified stiffness matrix

Solution strategy



Control in incarcari

$$\lambda^{i+1} - \lambda^{i} = c$$

(a)

Control in deplasari

$$d^{i+1} - d^{i} = c$$

(b)

Control in lungimea de arc

$$\left(\lambda^{i+1} - \lambda^{i}\right)^2 + \left(d^{i+1} - d^{i}\right)^2 = c^2$$

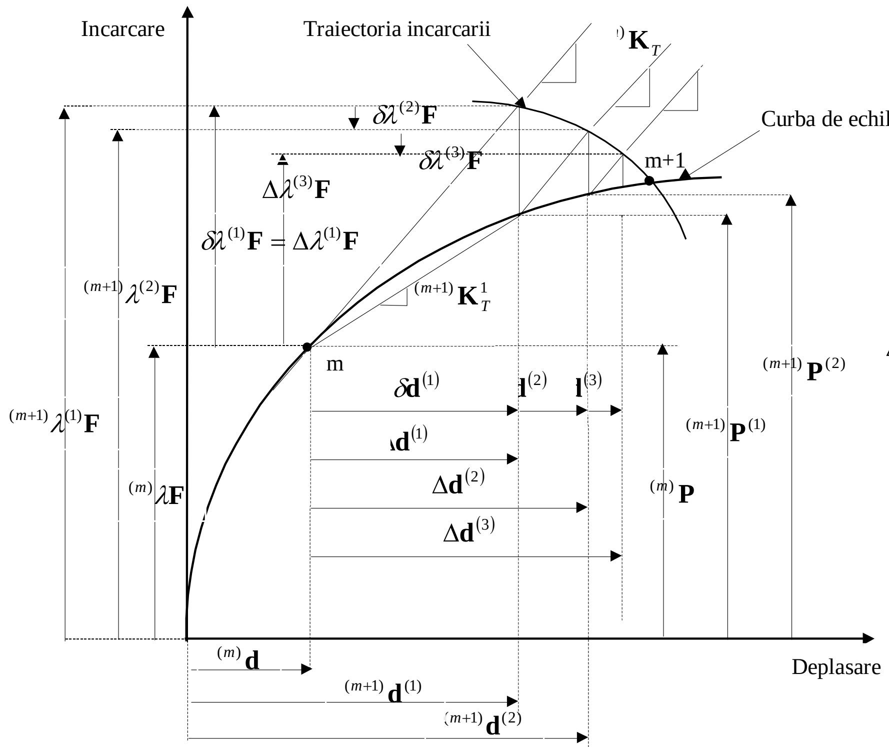
(c)

$$\mathbf{P}(\mathbf{d}) - \lambda \cdot \mathbf{F} = 0$$

$$\sum_{k=1}^{n} \beta_k \cdot \left(d_k^m - d_k^{m-1}\right)^2 + \beta_{n+1} \cdot \alpha^2 \cdot \left(\lambda^m - \lambda^{m-1}\right)^2 = c^2$$

Arc-length incremental iterative approach

Solution strategy



$$^{m}\mathbf{K}_T \cdot \delta\mathbf{d}^{(i)} = {}^{(m+1)}\lambda^{(i)} \cdot \mathbf{F} - {}^{(m+1)}\mathbf{P}^{(i-1)}$$

$$\Delta\mathbf{d}^{(i)} = \Delta\mathbf{d}^{(i-1)} + \delta\mathbf{d}^{(i)}$$

$$\Delta\mathbf{P}^{(i)} = \left({}^{m}\mathbf{K}_0^{(i)} + {}^{m}\mathbf{K}_3 + {}^{m}\mathbf{K}_1^{(i)} + {}^{m}\mathbf{K}_2^{(i)}\right) \cdot \Delta\mathbf{d}^{(i)}$$

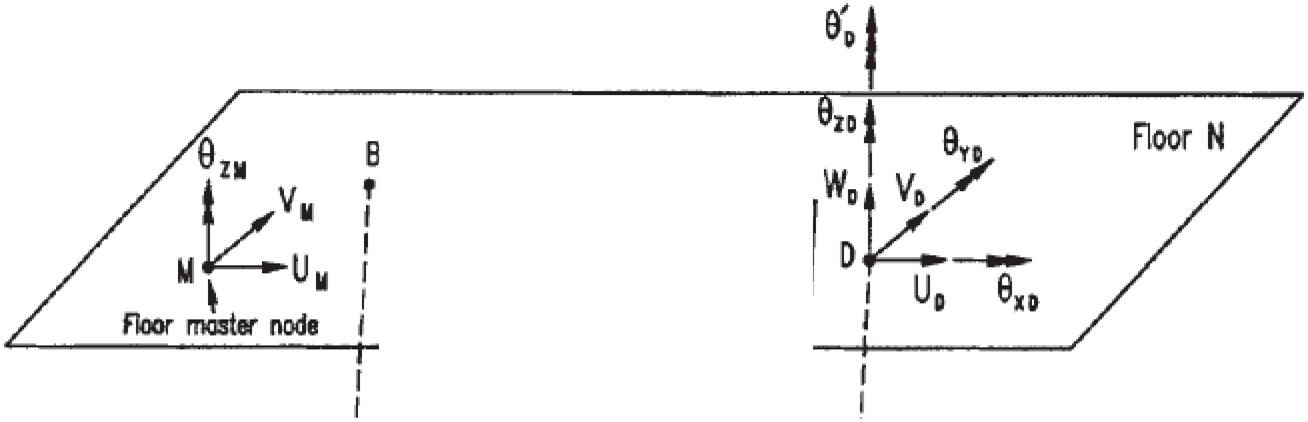
$$\Delta\lambda^{(i)} = \Delta\lambda^{(i-1)} + \delta\lambda^{(i)}$$

Predictor-corrector solution scheme

Arc-length incremental-iterative procedure

Proposed analysis procedure-Formulation

Rigid-floor diaphragm effect



Master-slave technique

homogenous constraints **Au**=0

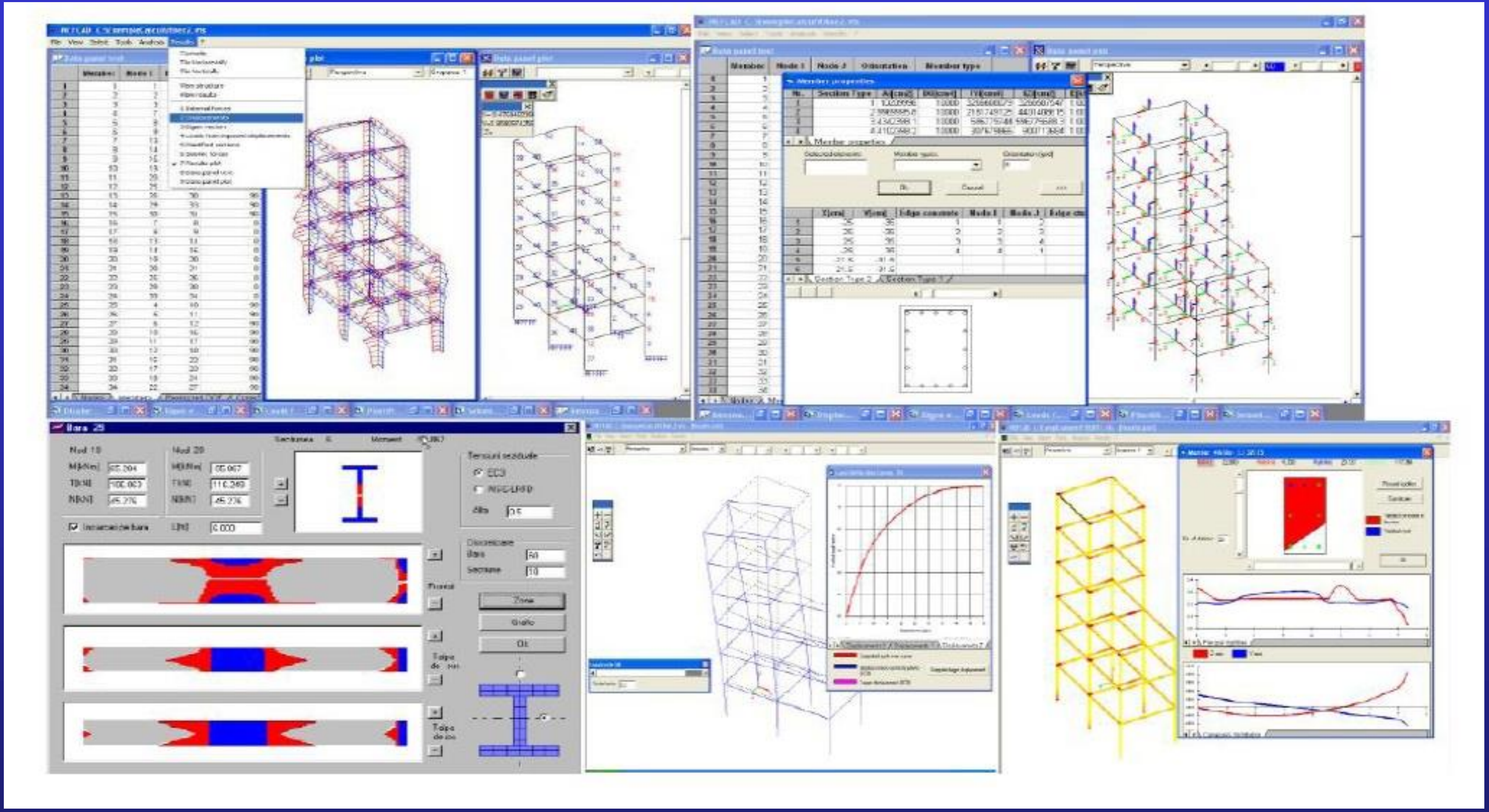
penalty augmented system

$$\left(\mathbf{K}_T + \mathbf{A}^T\mathbf{W}\mathbf{A}\right)\Delta\mathbf{U} = \Delta\mathbf{F}$$

Model Capabilities

- Large deflection and large rotations
- Geometrical local effects (P-δ) including bowing effect, shear deformations
- Concentrated and distributed plasticity (fiber and M-N-Φ aproaches)
- Consistency between linear and nonlinear models (one element/member)
- Local geometrical and material imperfections
- Flexible (semi-rigid) and finite joints
- Complete non-linear behavior (pre and post crtical response: snap-back and snap-through)

COMPUTER PROGRAM-NEFCAD



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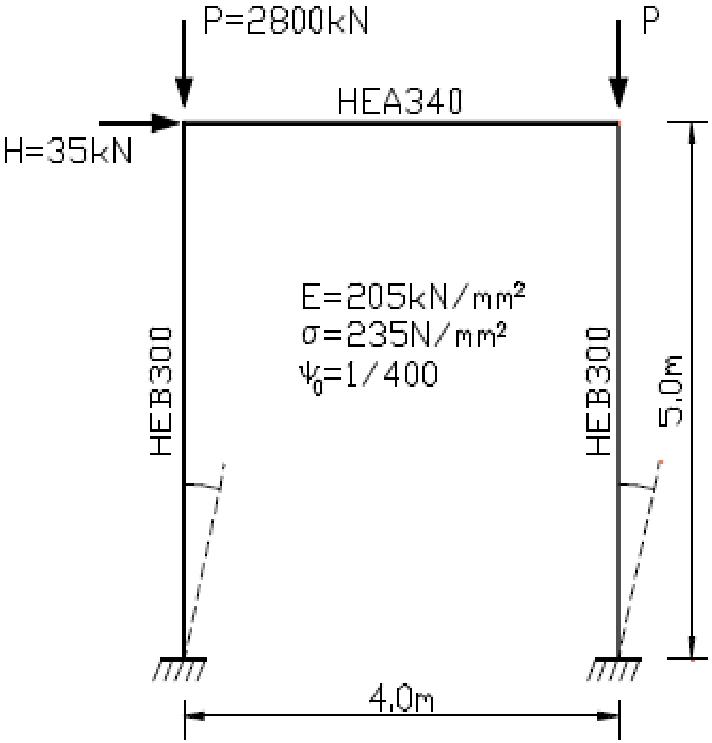
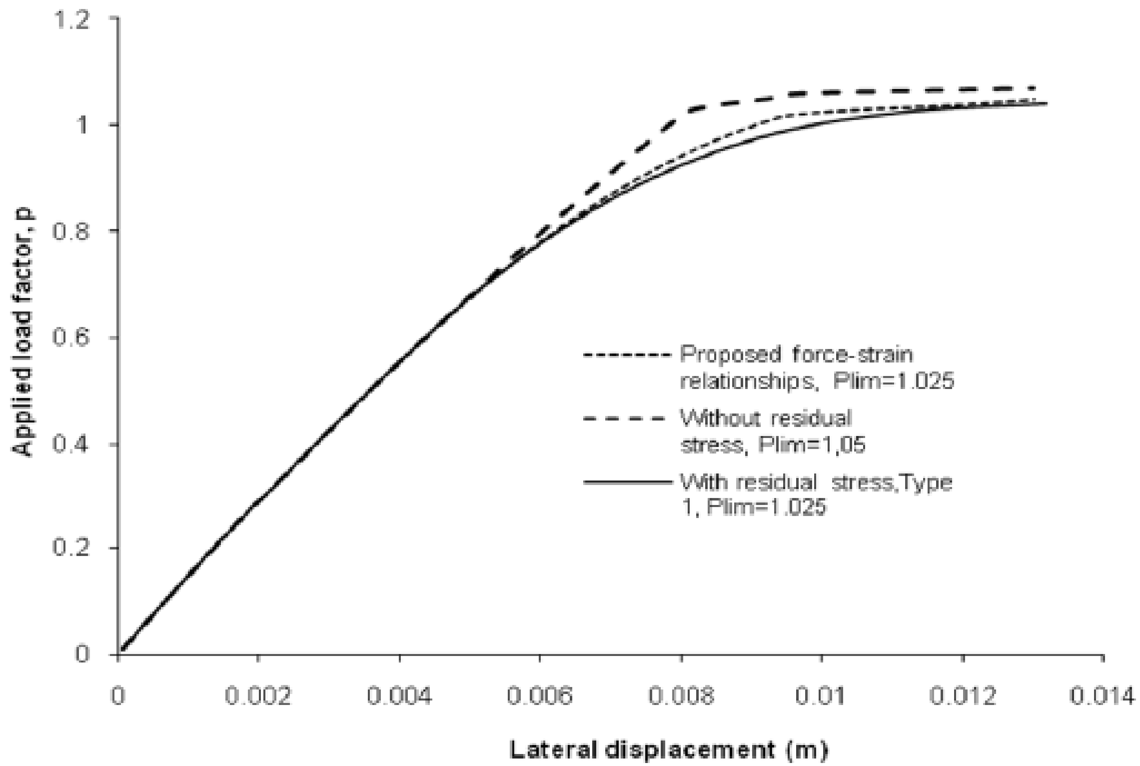
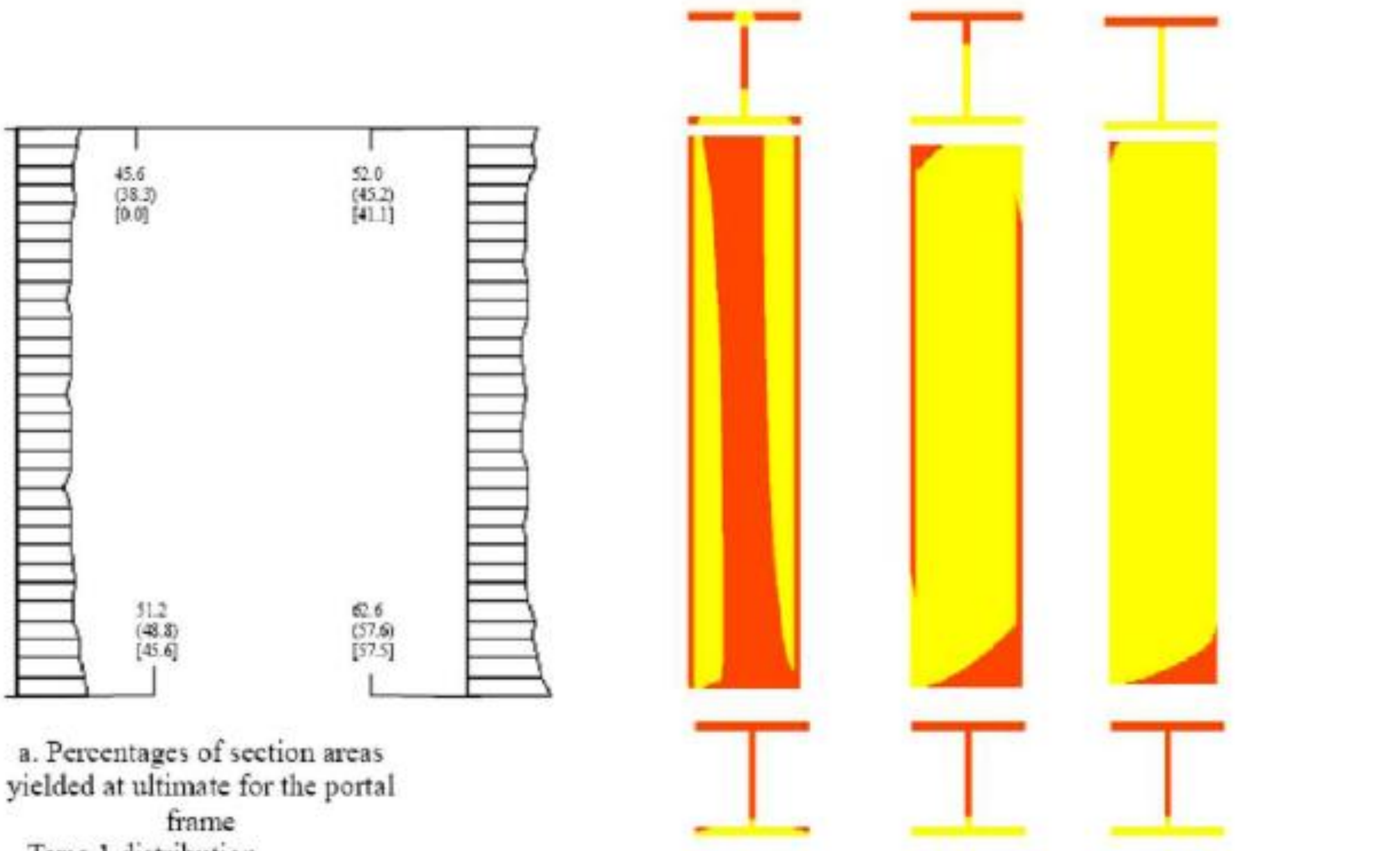
COMPUTATIONAL EXAMPLES





COMPUTATIONAL EXAMPLES

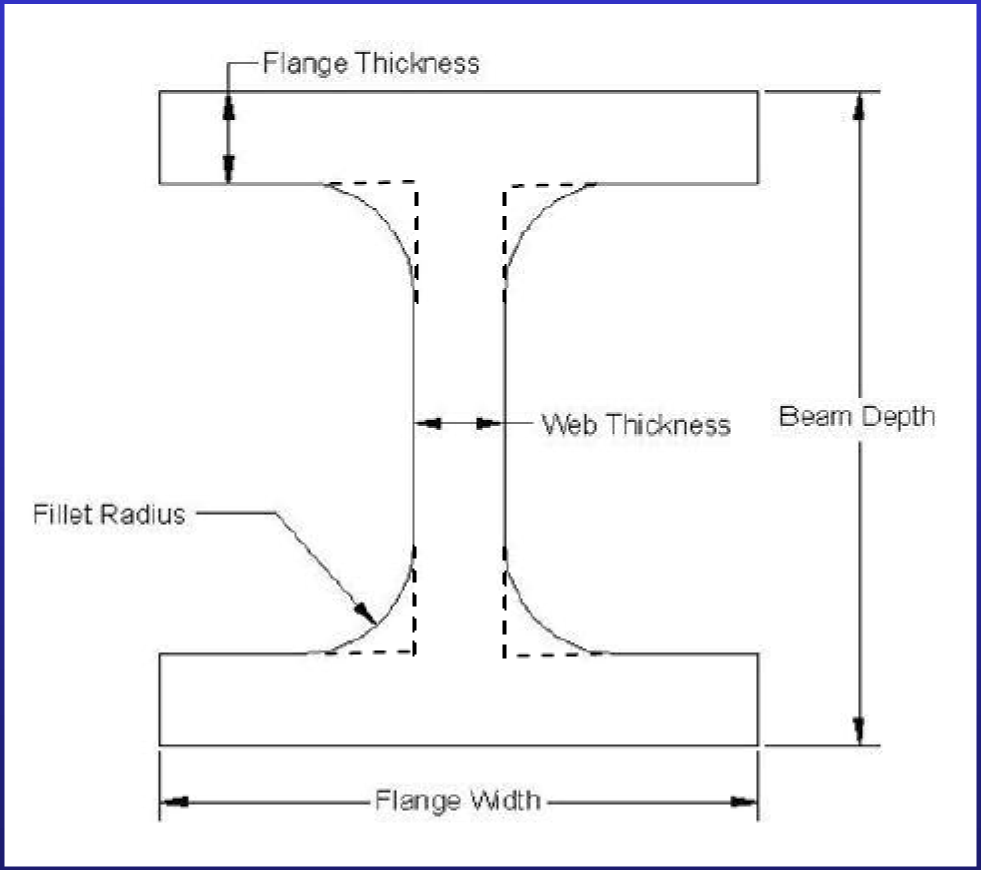


a. Percentages of section areas yielded at ultimate for the portal frame
Type 1 distribution.
(Type 2 distribution.)
[Without residual stress]

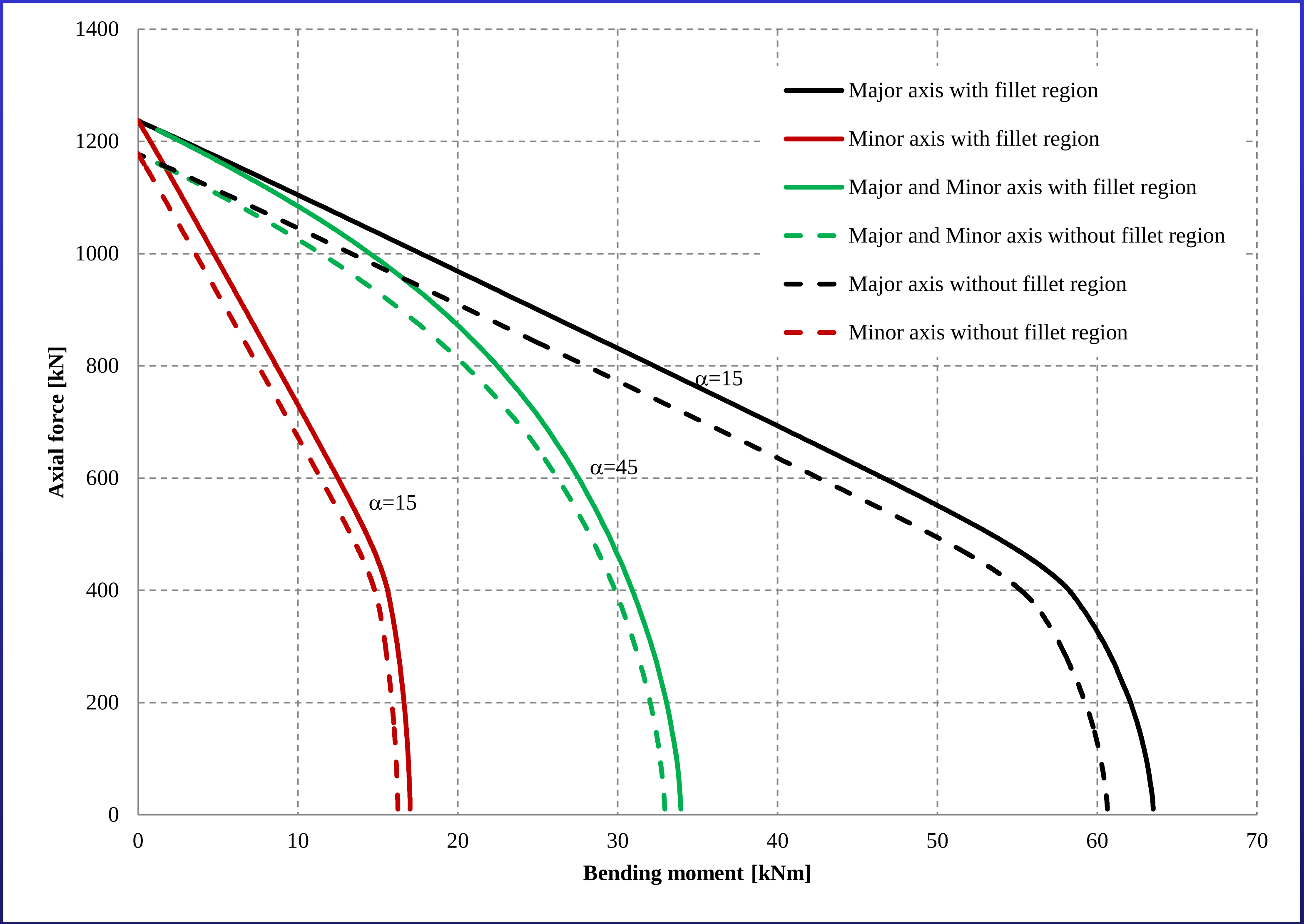
b. Type 1 c. Type 2 d. Without residual stress

COMPUTATIONAL EXAMPLES

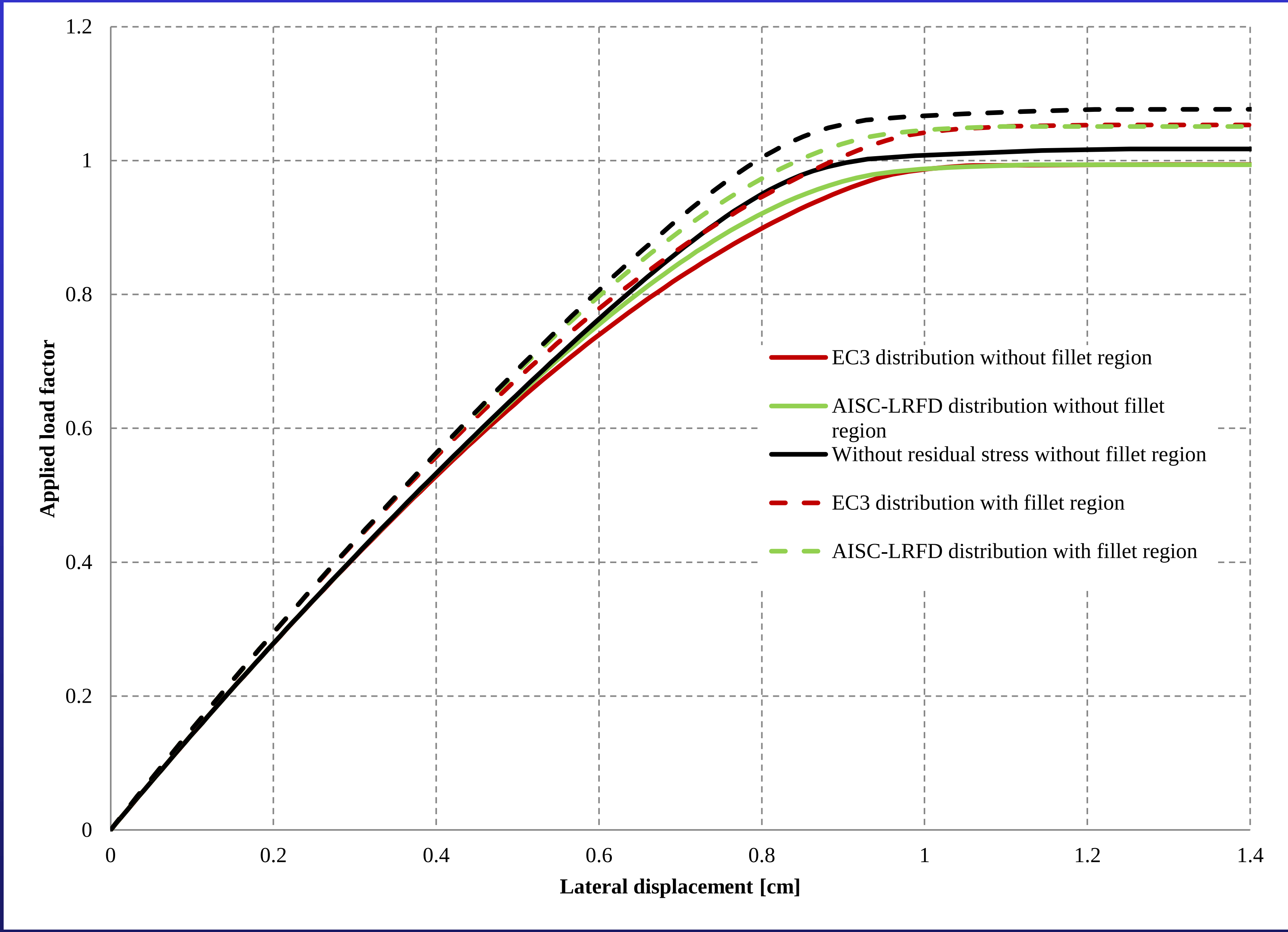
Fillet region: Cross-section discretization



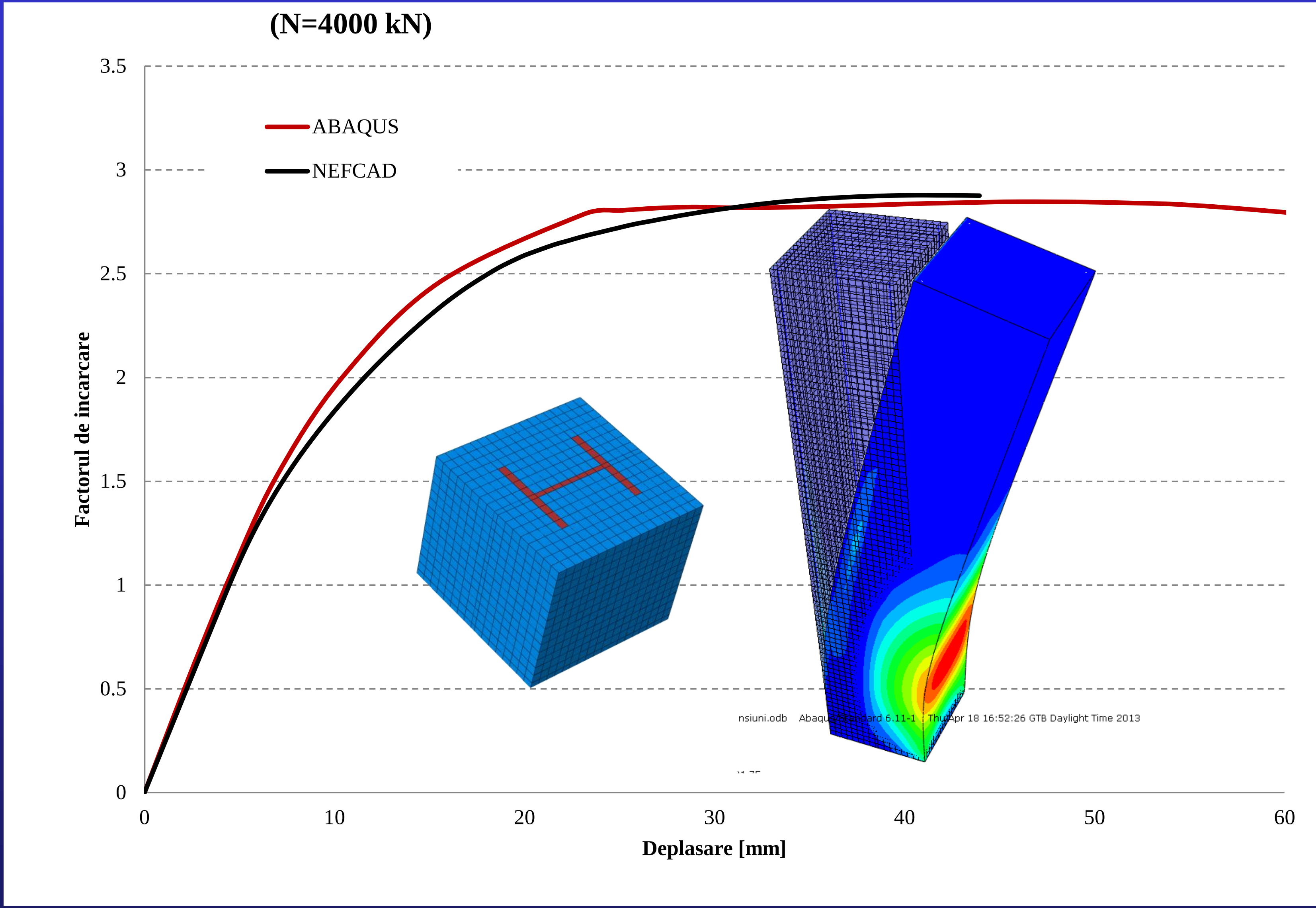
COMPUTATIONAL EXAMPLES



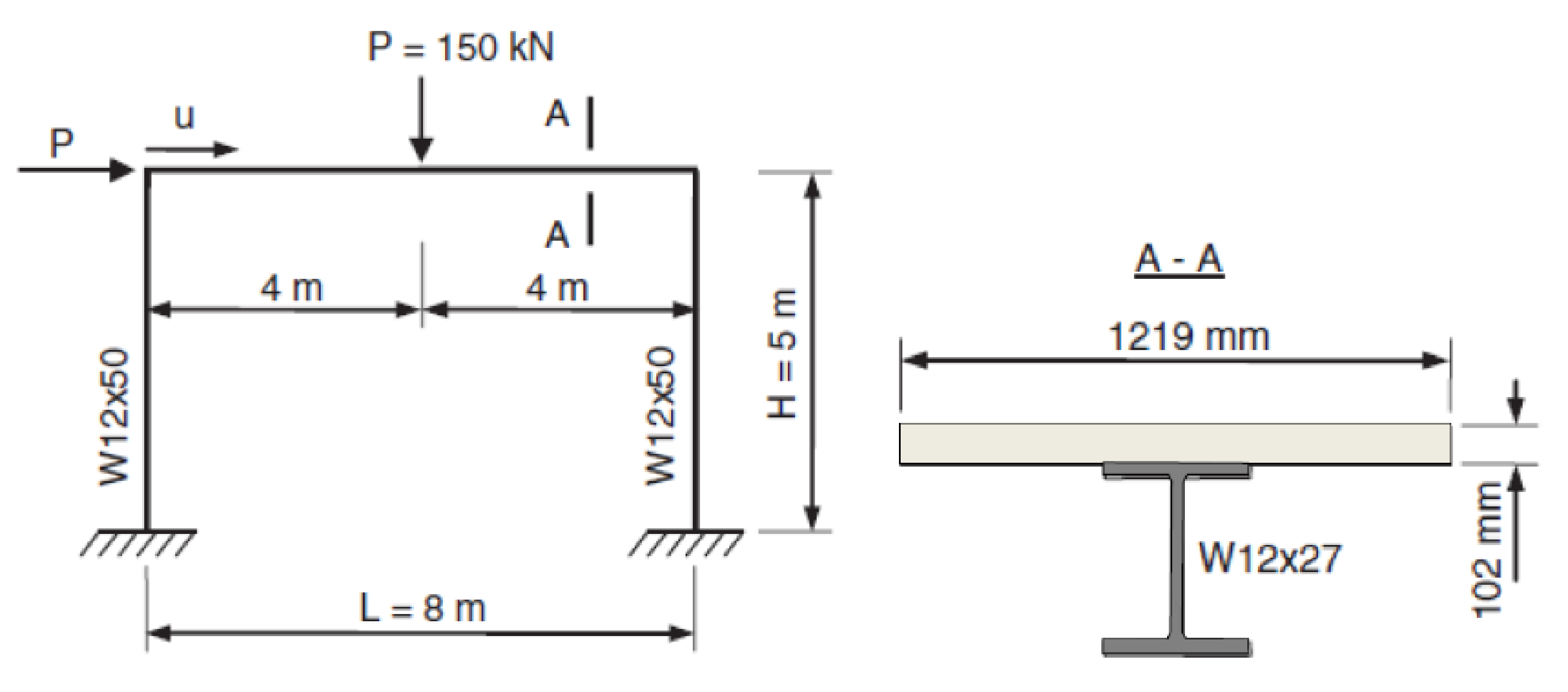
COMPUTATIONAL EXAMPLES



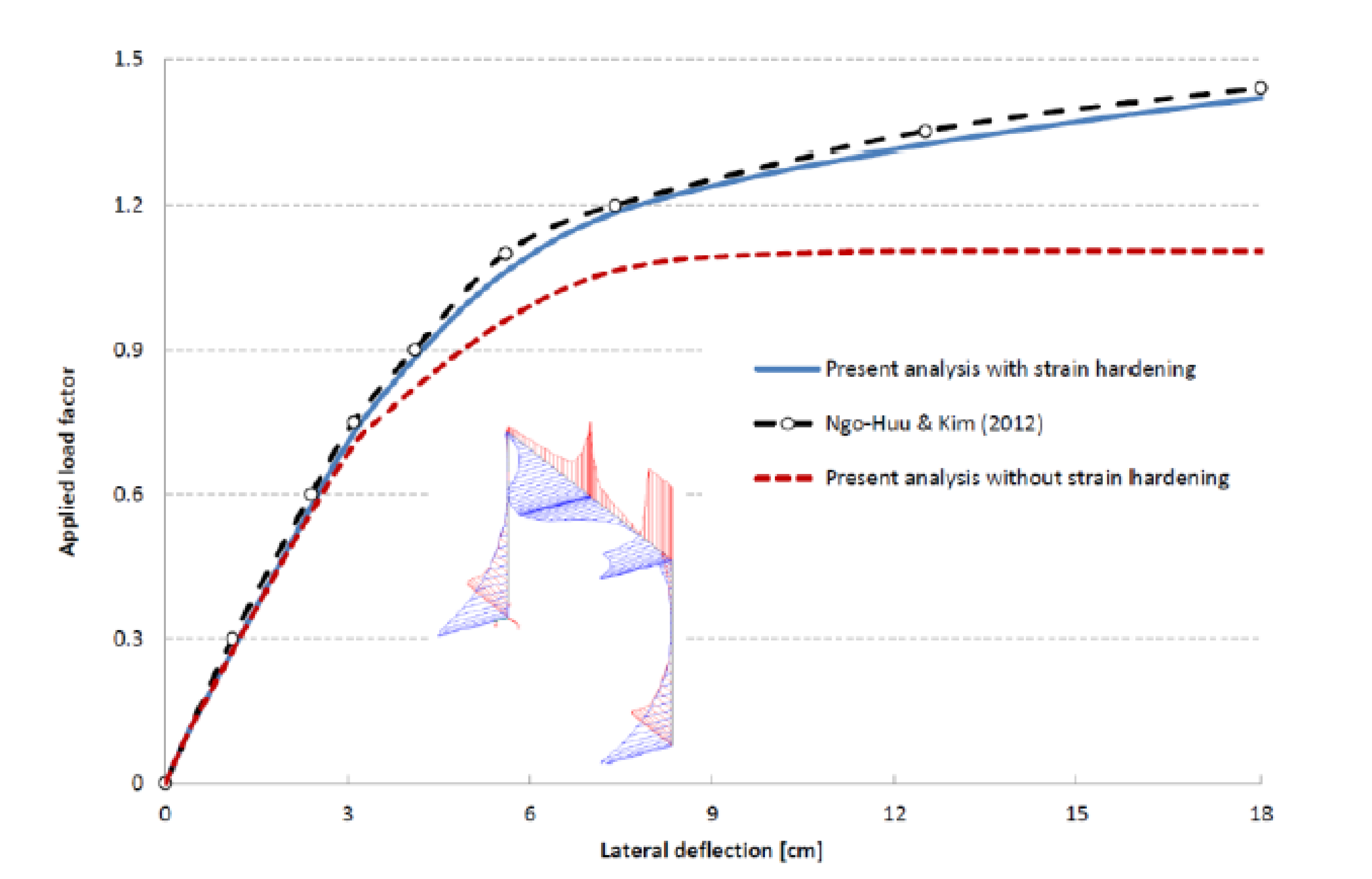
Computational examples: Example 1



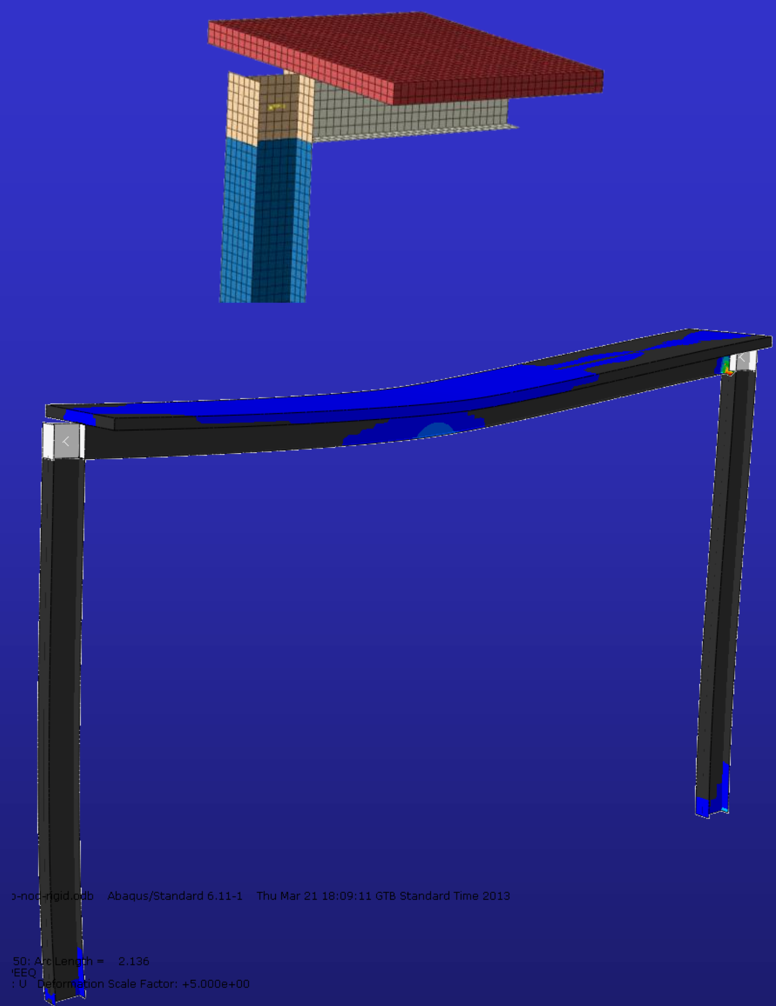
Computational examples: Example 2

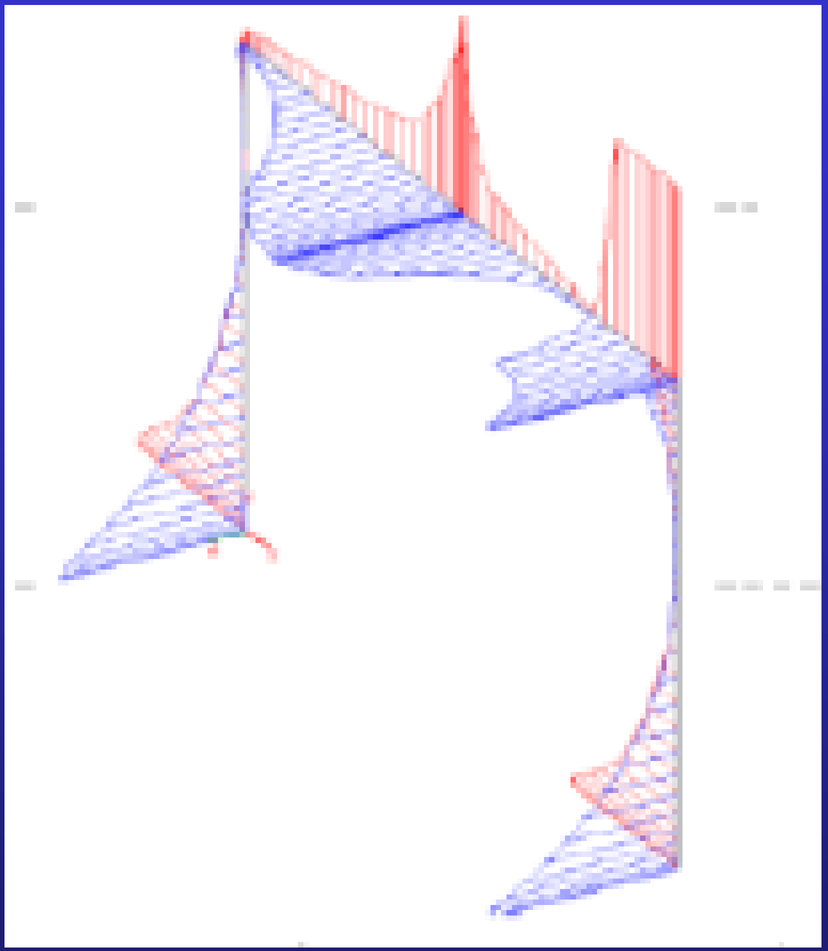


Computational examples: Example 2

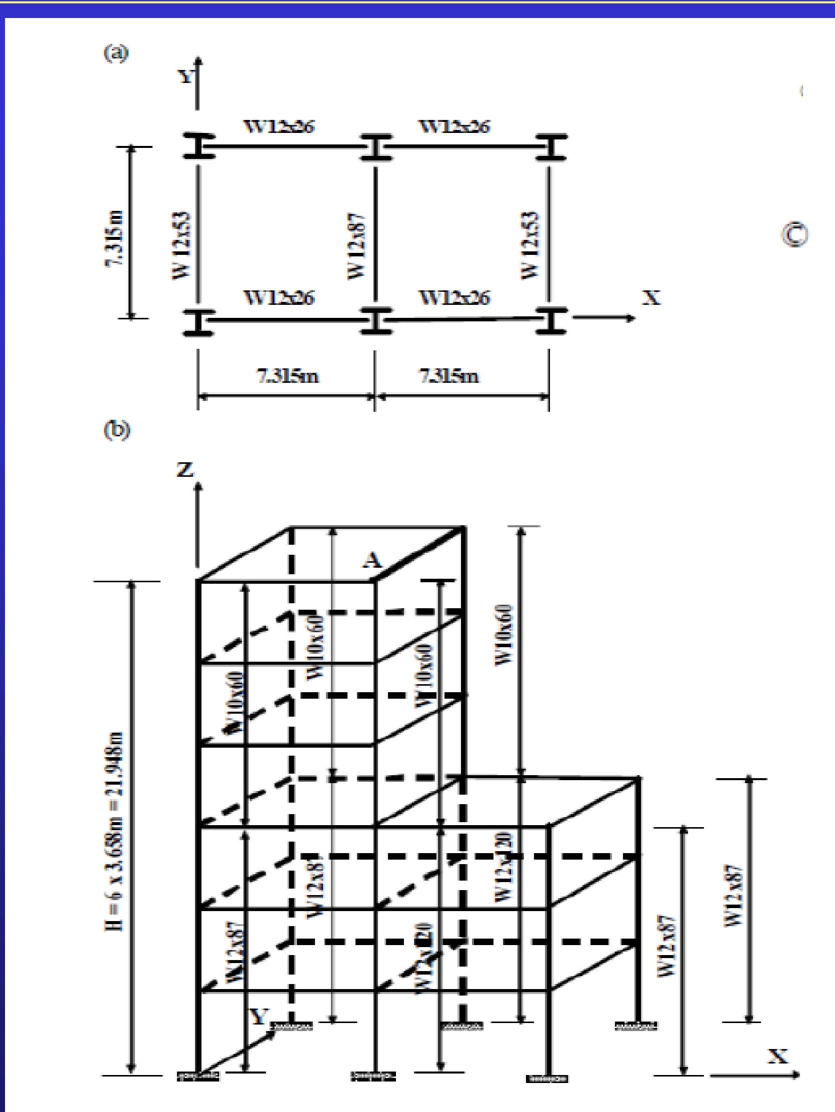


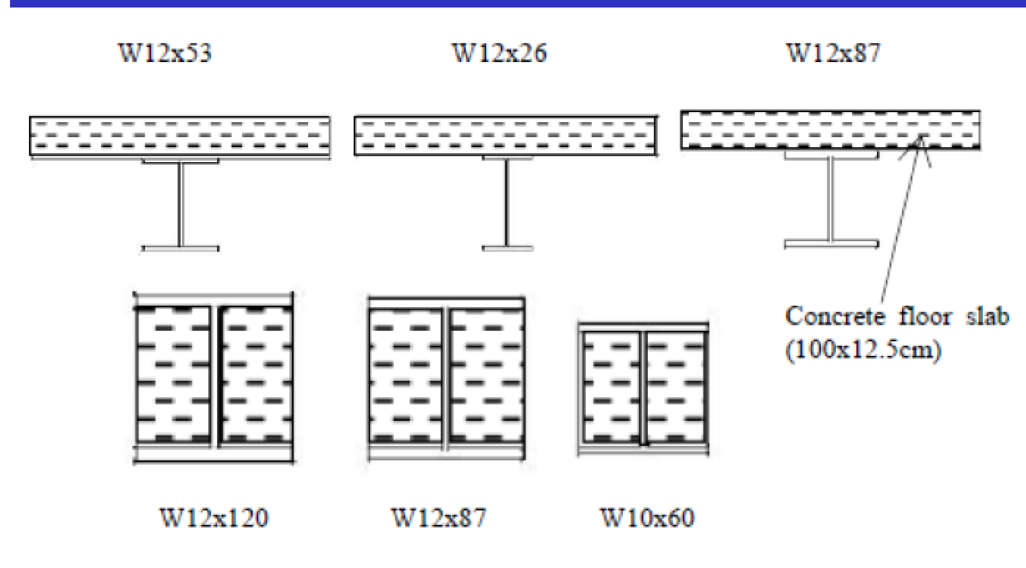
Computational examples: Example 2



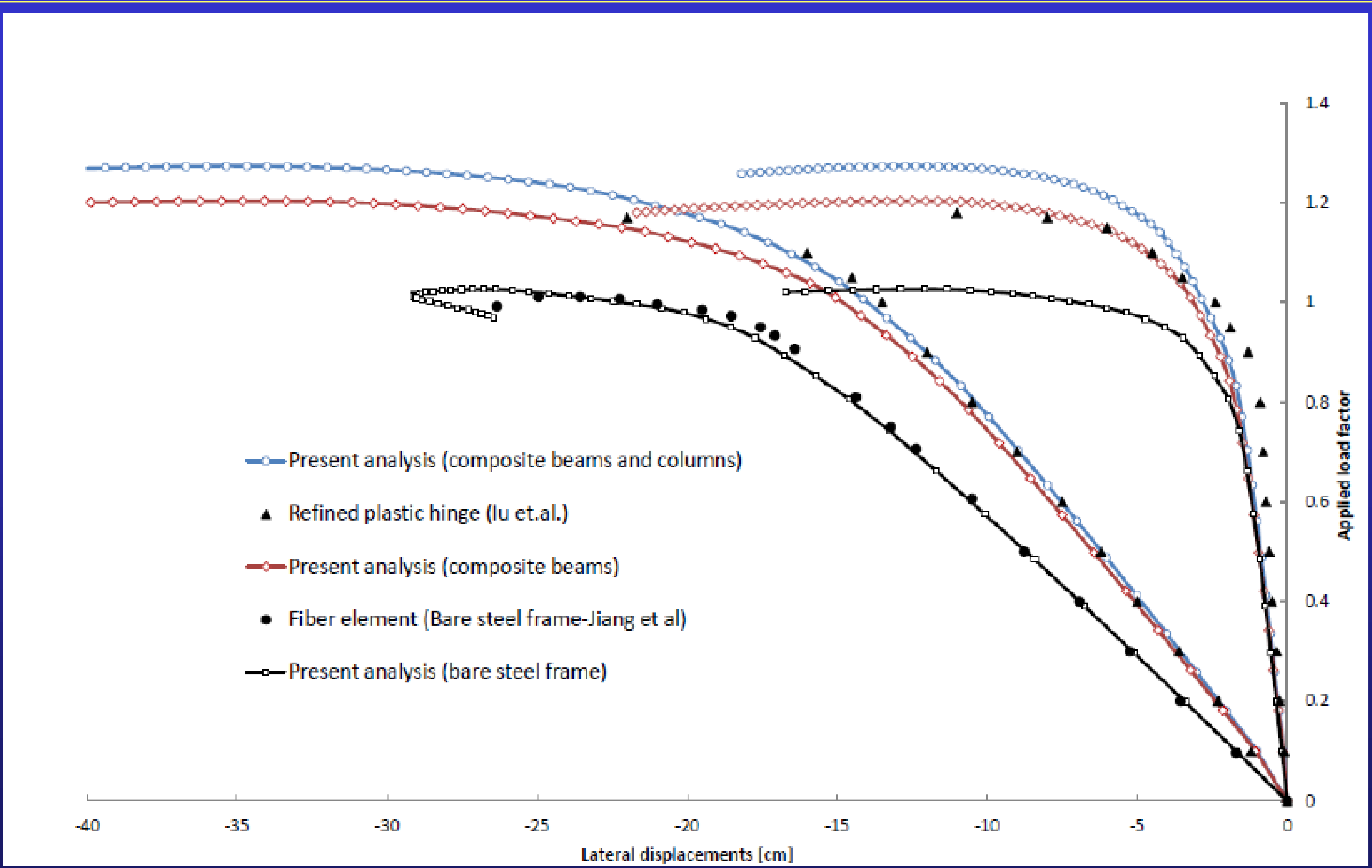


Computational examples: Example 3

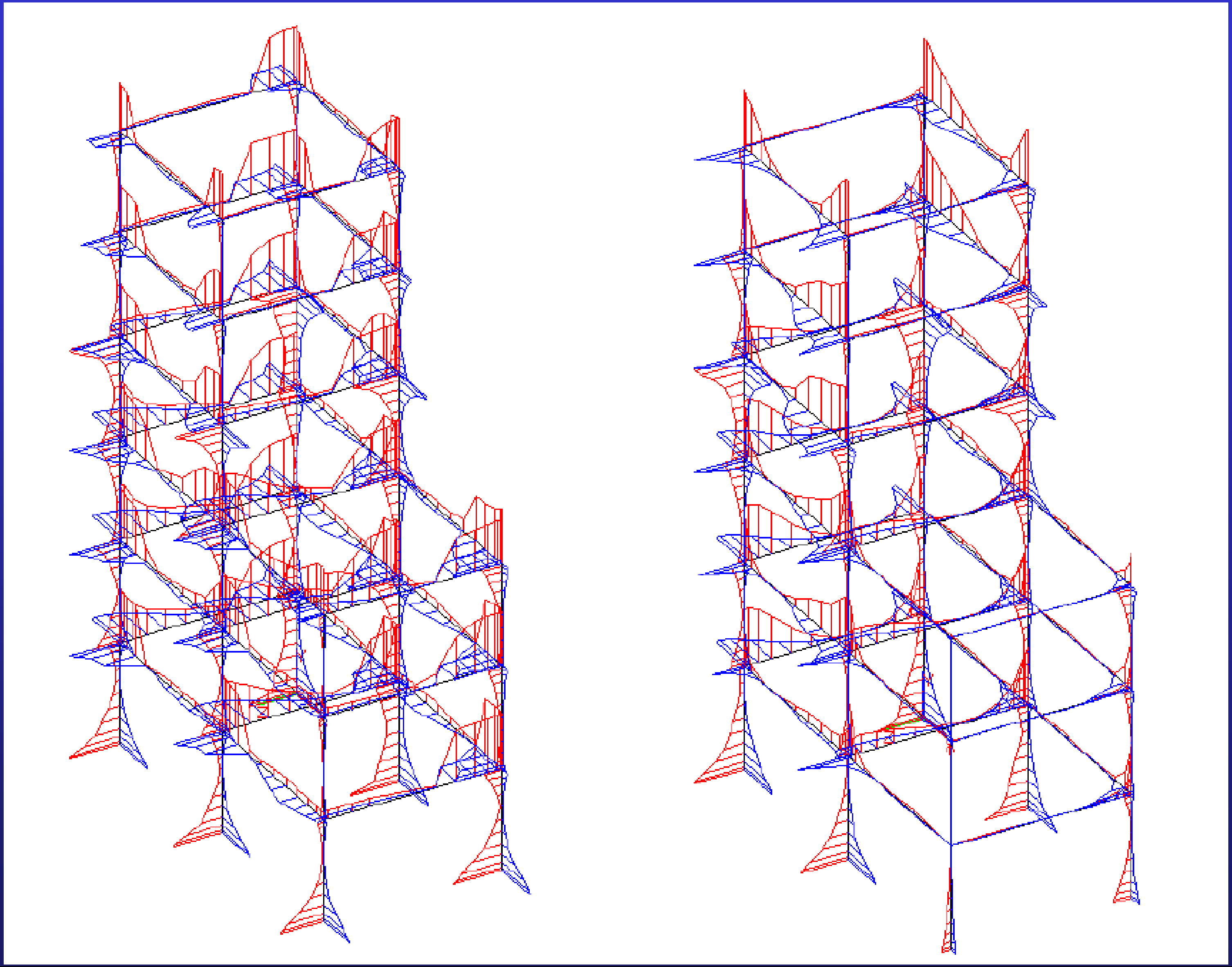




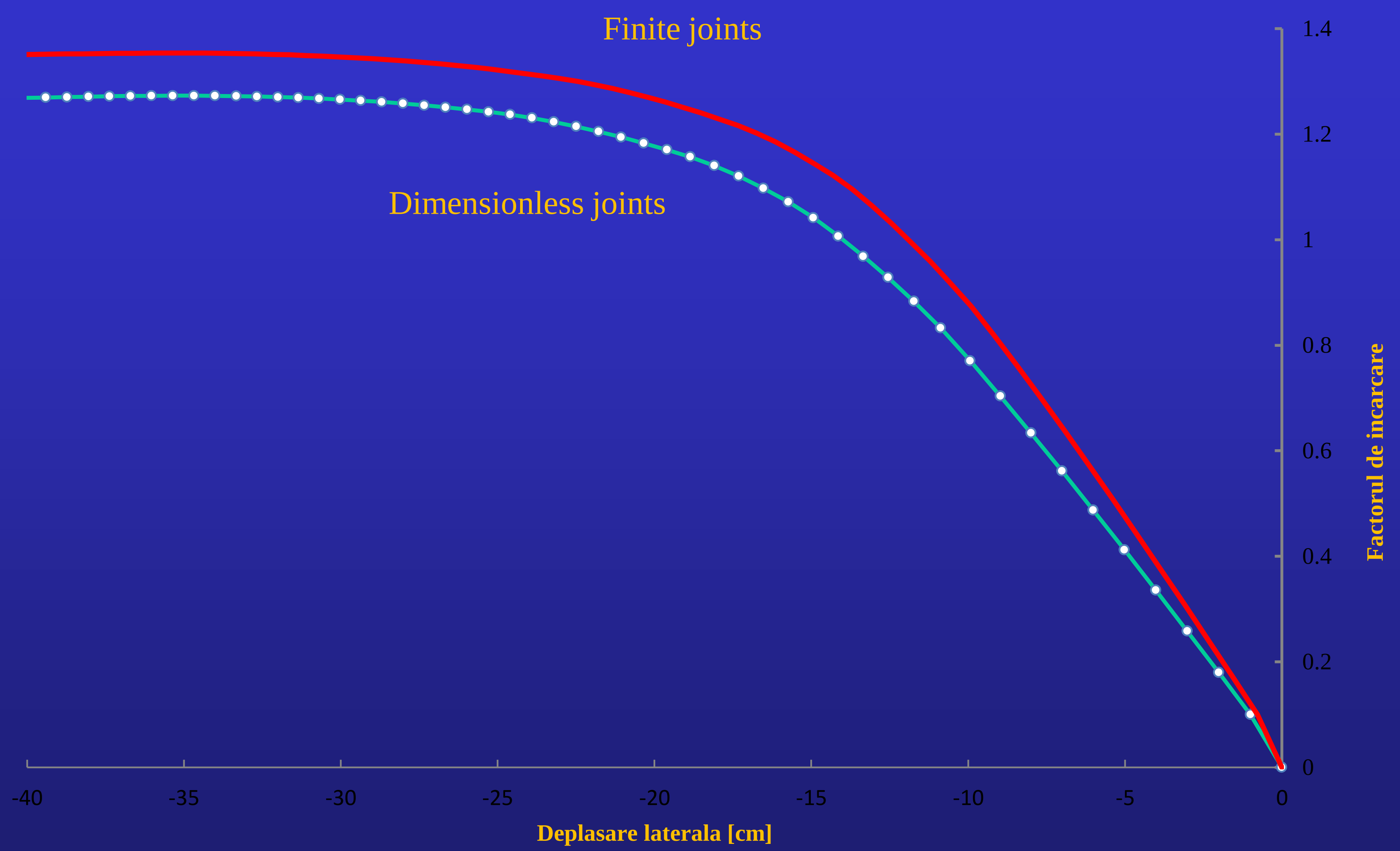
Computational examples: Example 3



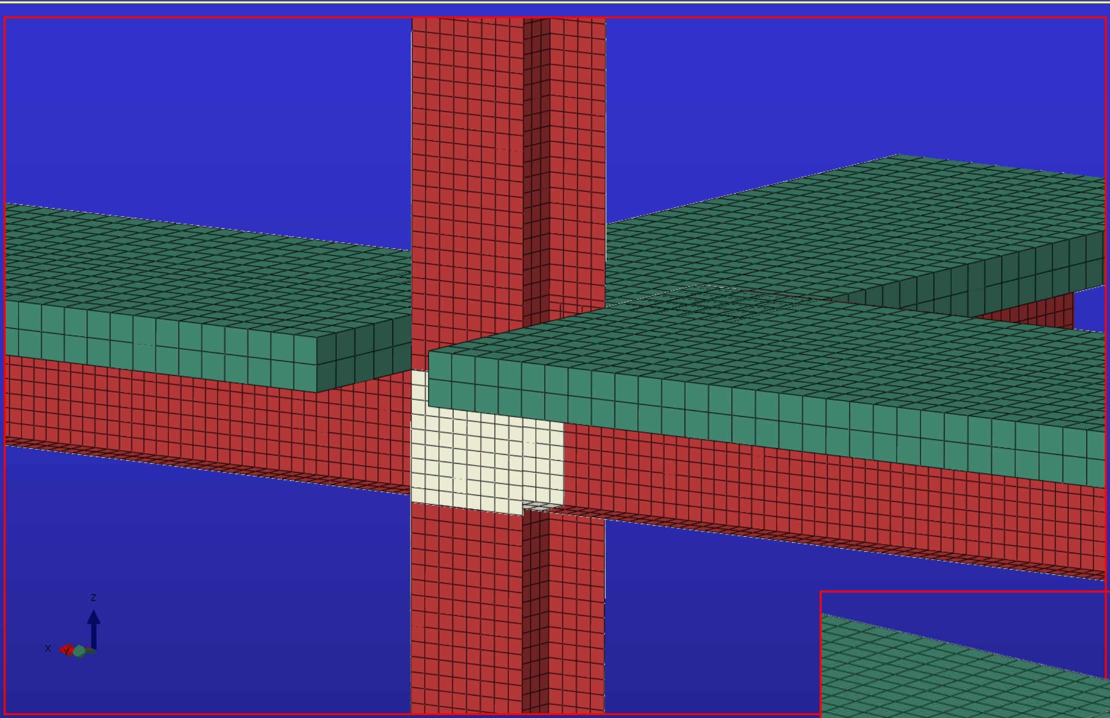
Computational examples: Example 3

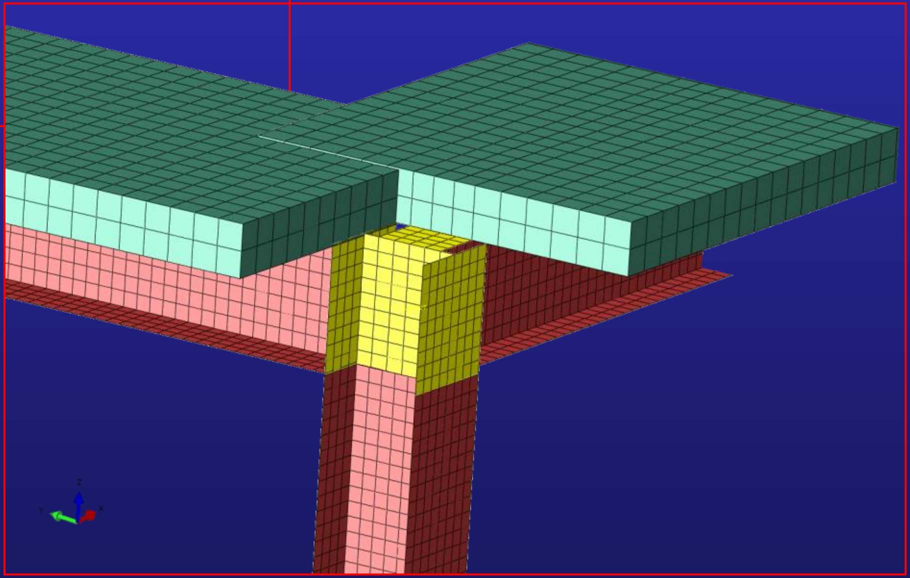


Computational examples: Example 3

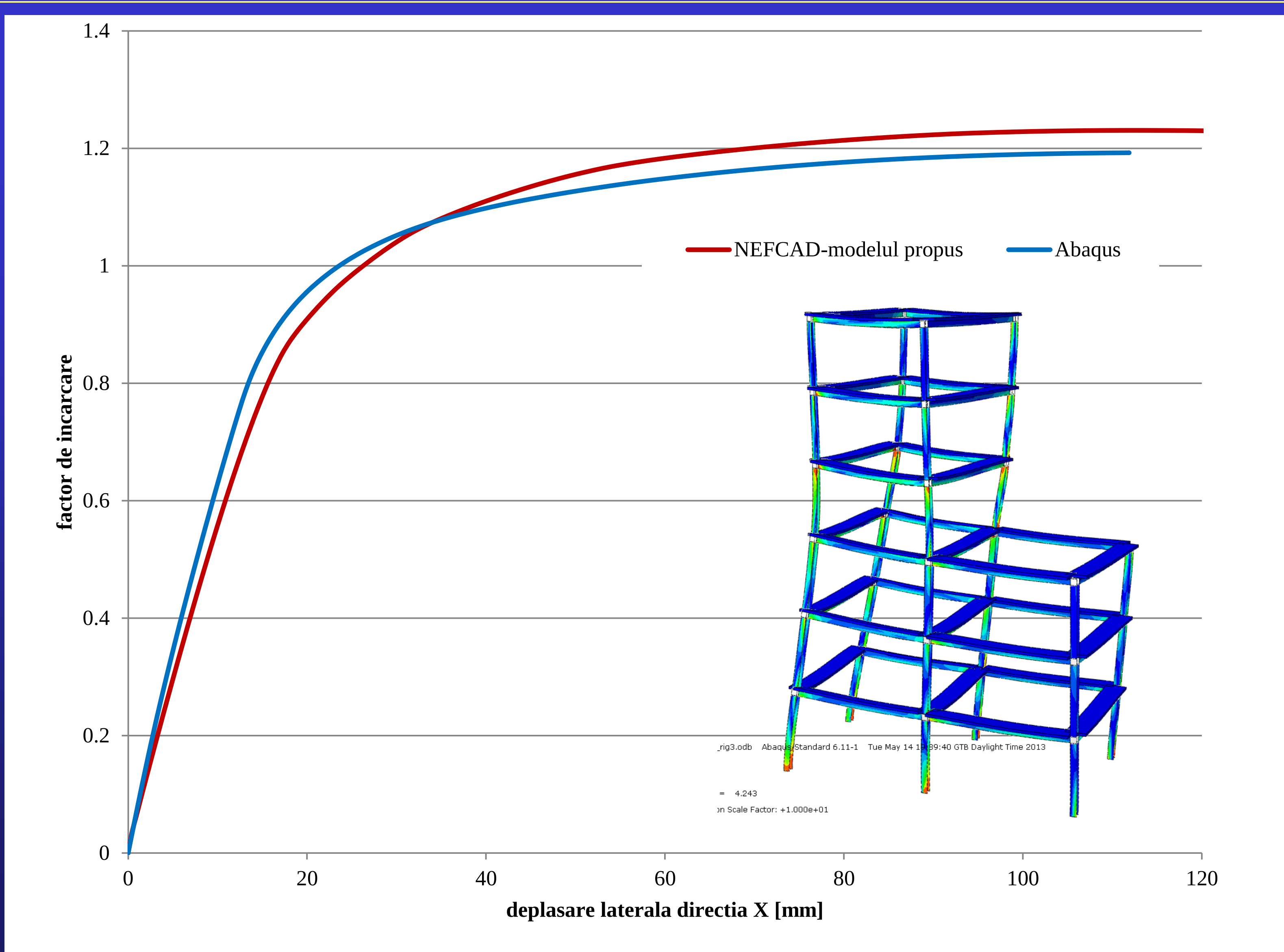


Computational examples: Example 3

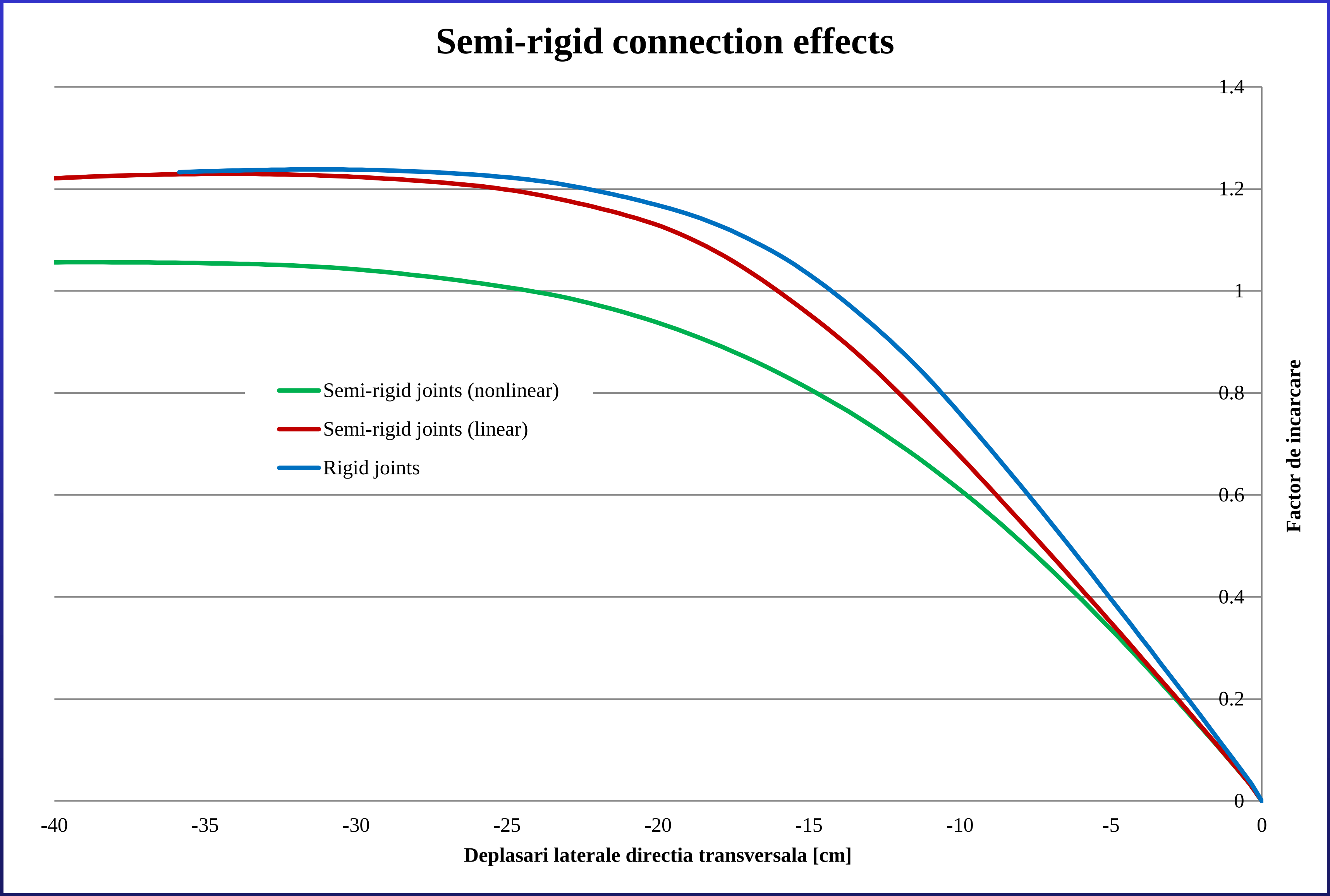




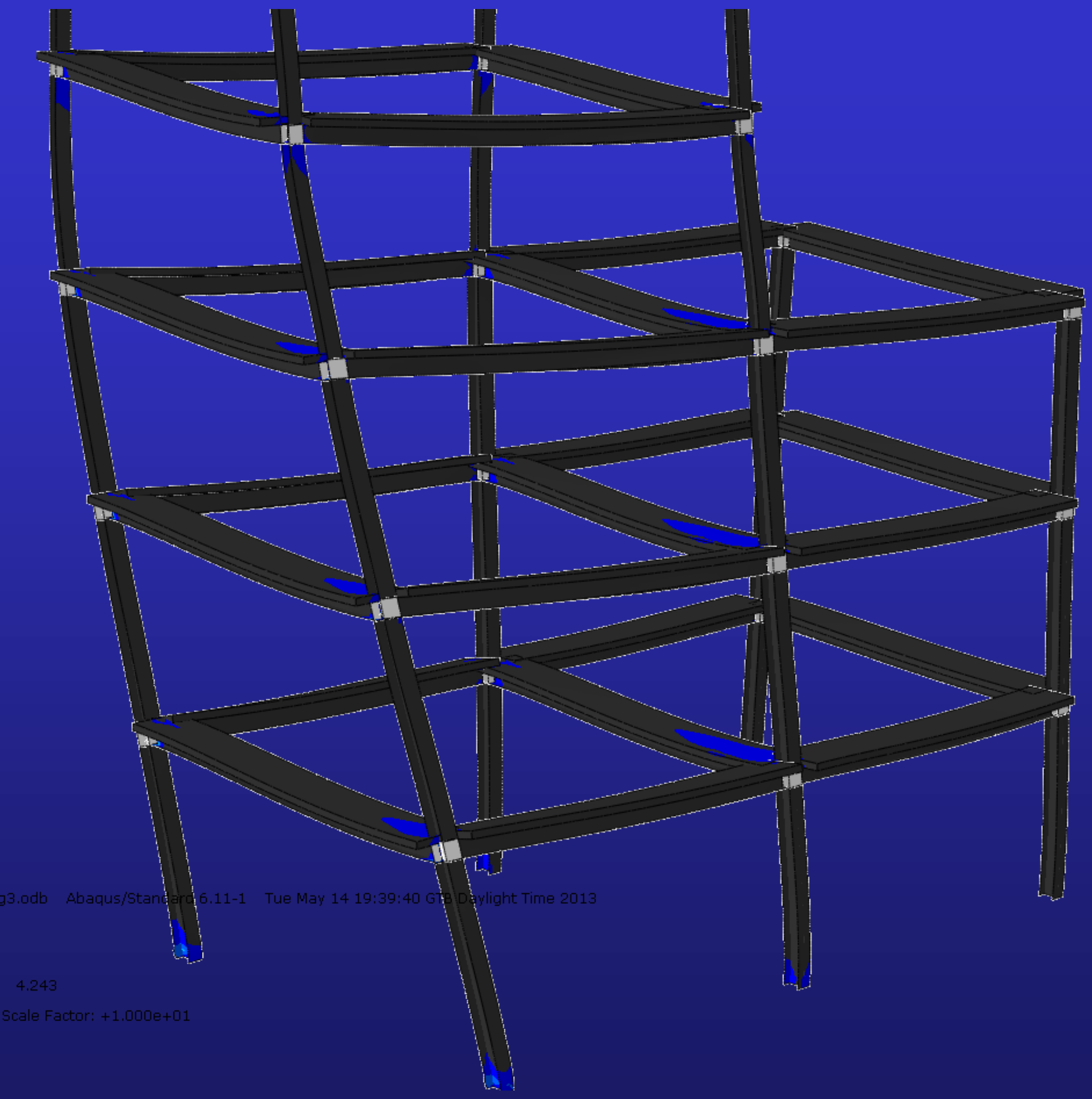
Computational examples: Example 3

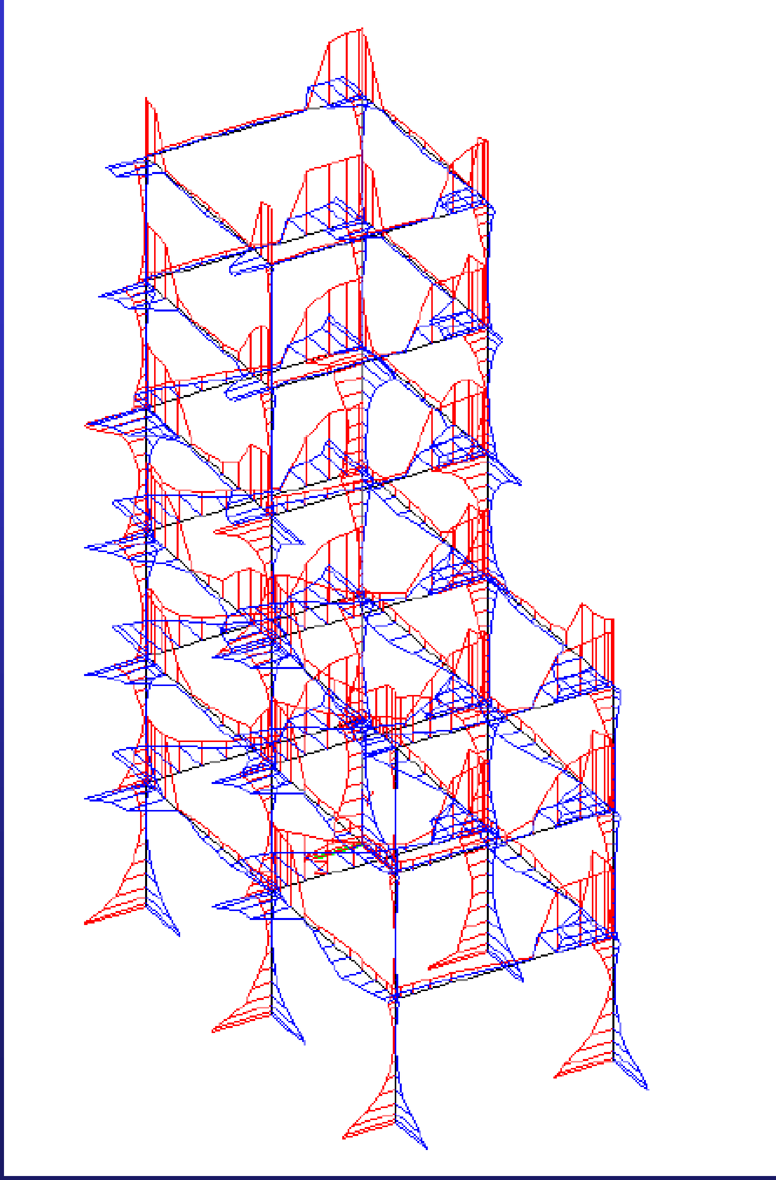


Computational examples: Example 3

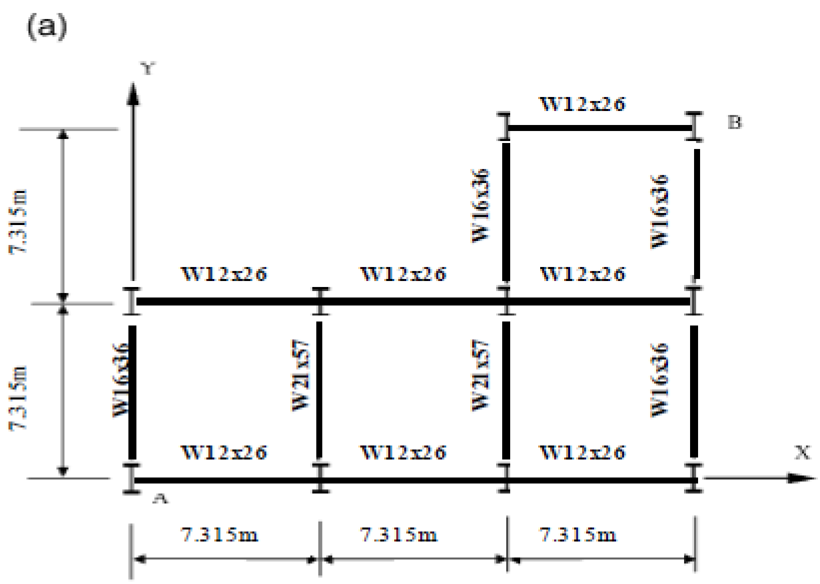


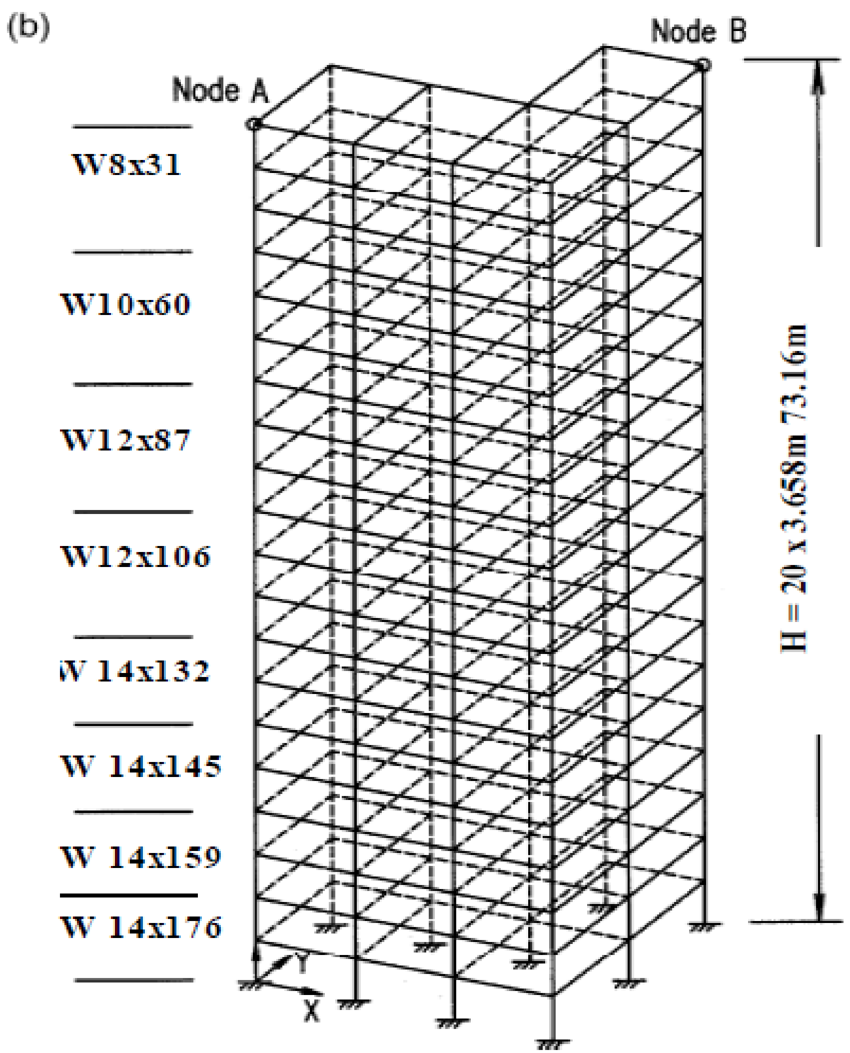
Computational examples: Example 3



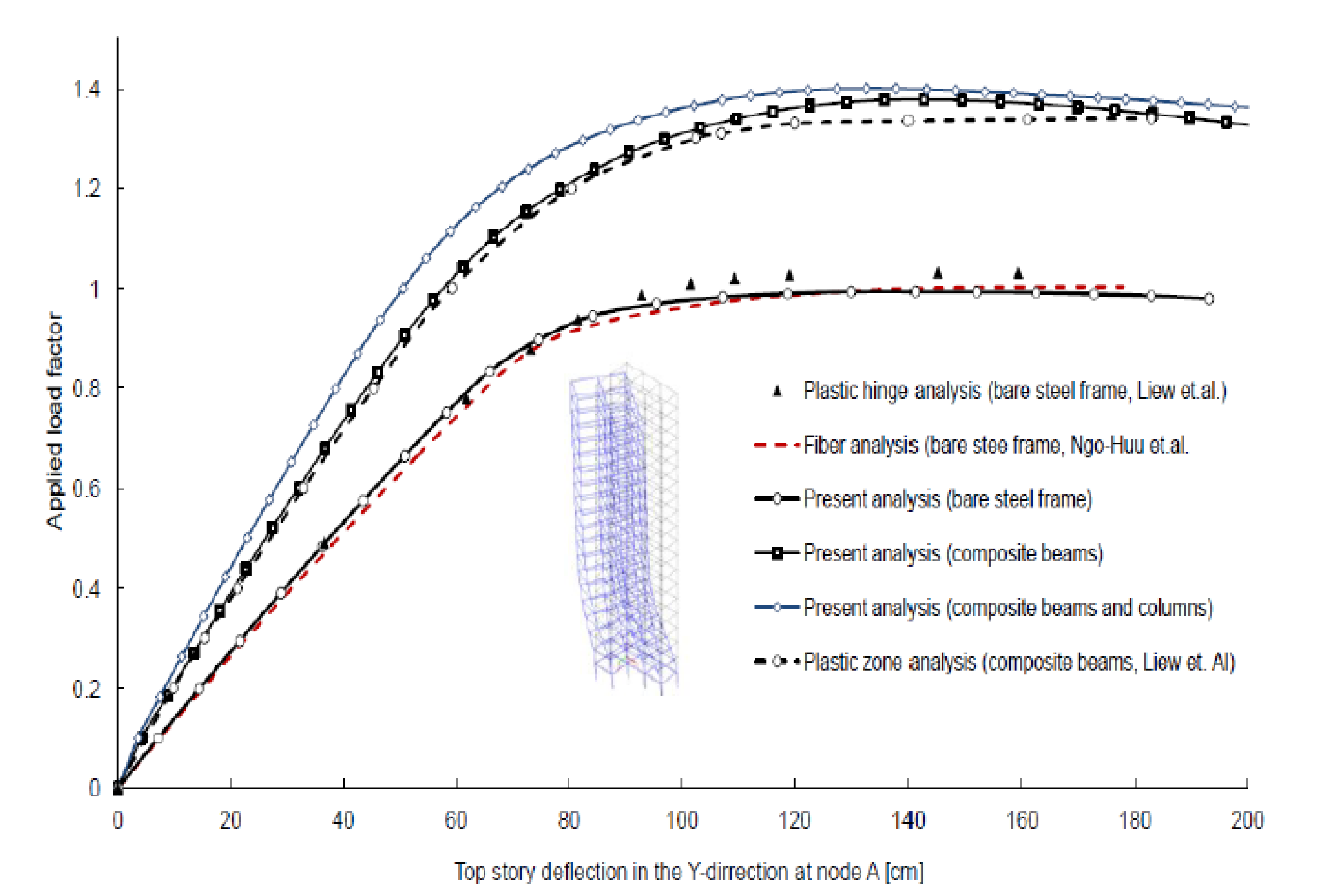


Computational examples: Example 4

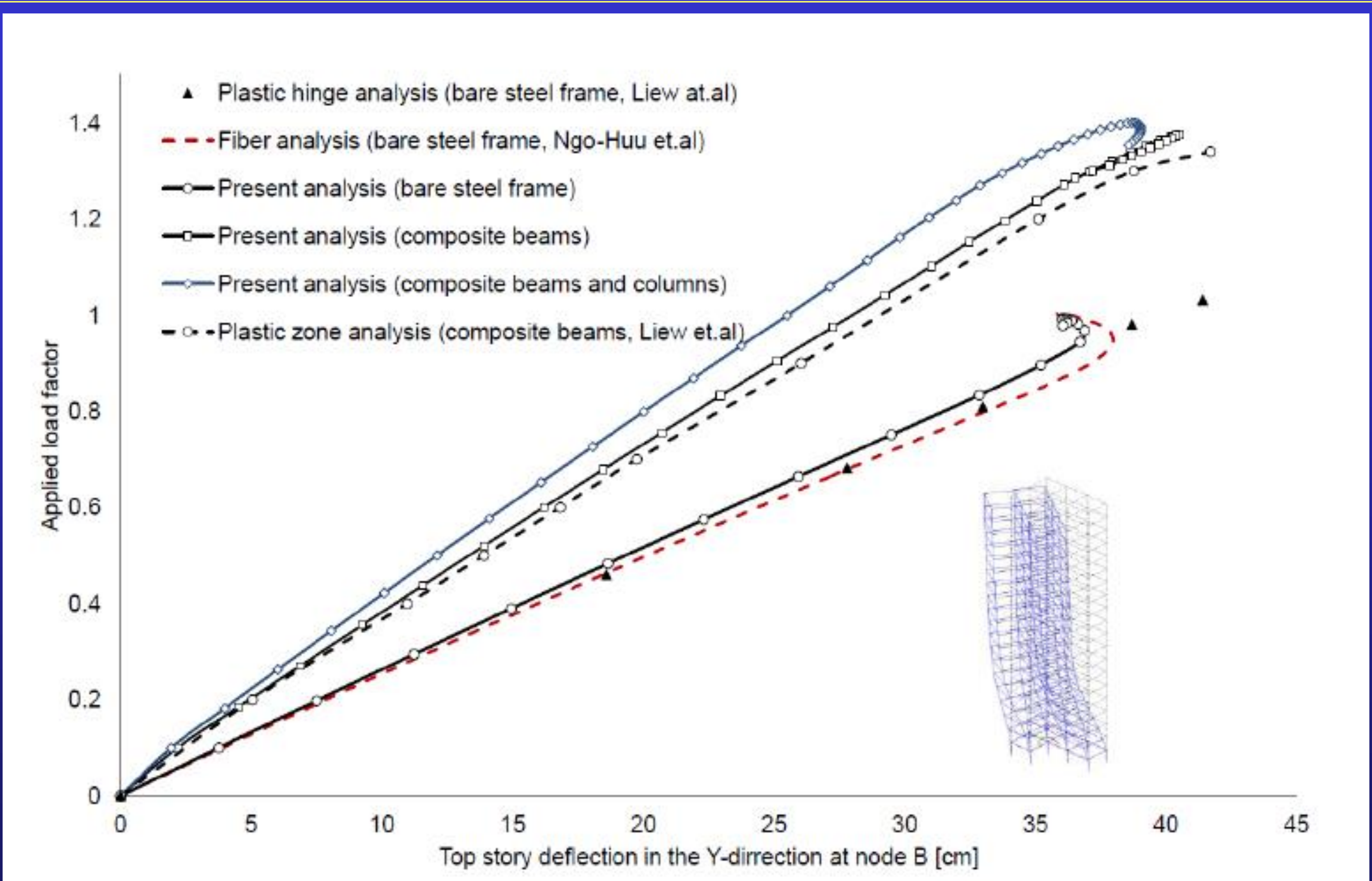




Computational examples: Example 4



Computational examples: Example 4



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