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# APPLICATION OF PUSHOVER ANALYSIS ON REINFORCED CONCRETE BRIDGE MODEL

Part I – NUMERICAL MODELS

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# ABSTRACT

This report describes a non-linear static (pushover) analysis method for prestressed reinforced concrete structures that predicts behavior at all stages of loading, from the initial application of loads up to and beyond the collapse condition. A look insight into pushover methodology described in EC8, FEMA-273/356 and ATC-40 documents also is presented.

The nonlinear static (pushover) analysis method (NSP), developed here use "line elements" approach, and are based on the degree of refinement in representing the plastic yielding effects. The elasto-plastic behavior is modeled in two types: (1) *distributed plasticity model*, when it is modeled accounting for spread-of-plasticity effects in sections and along the beam-column element and (2) *plastic hinge*, when inelastic behavior is concentrated at plastic hinge locations. Both local (P- $\delta$ ) and global (P- $\Delta$ ) nonlinear geometrical effects are taken into account in analysis.

The method has been developed for the purpose of investigating the collapse behavior of a three span prestressed reinforced concrete bridge of 115 meters in total length that is to be built in the northeastern of Portugal over Alva River. Performance of this bridge using the nonlinear static method presented here in conjunction with iterative capacity spectrum method specified in the EC8 guidelines will be evaluated.

# 1. INTRODUCTION

Simplified approaches for the seismic evaluation of structures, which account for the inelastic behaviour, generally use the results of static collapse analysis to define the global inelastic performance of the structure. Currently, for this purpose, the nonlinear static procedure (NSP) or pushover analysis described in EC8, FEMA-273/356 (Building Seismic Safety Council, 1997; American Society of Civil Engineers, 2000), and ATC-40 (Applied Technology Council, 1996) documents, are used. Seismic demands are computed by nonlinear static analysis of the structure subjected to monotonically increasing lateral forces with an invariant height-wise distribution until a predetermined target displacement is reached.

Nonlinear static (pushover) analysis can provide an insight into the structural aspects, which control performance during severe earthquakes. The analysis provides data on the strength and ductility of the structure, which cannot be obtained by elastic analysis.

By pushover analysis, the base shear versus top displacement curve of the structure, usually called capacity curve, is obtained. To evaluate whether a structure is adequate to sustain a certain level of seismic loads, its capacity has to be compared with the requirements corresponding to a scenario event. The aforementioned comparison can be based on force or displacement.

In pushover analyses, both the force distribution and target displacement are based on a very restrictive assumptions, i.e. a time-independent displacement shape. Thus, it is in principle inaccurate for structures where higher mode effects are significant, and it may not detect the structural weaknesses that may be generated when the structure's dynamic characteristics change after the formation of the first local plastic mechanism. One practical possibility to partly overcome the limitations imposed by pushover analysis is to assume two or three different displacements shapes (load patterns), and to envelope the results [1], or using the adaptive force distribution that attempt to follow more closely the time-variant distributions of inertia forces [6].

Many methods were presented to apply the Nonlinear Static Procedure (NSP) to structures. Those methods can be listed as (1) the Capacity Spectrum Method (CSM) (ATC, 1996); (2) the Displacement Coefficient Method (DCM) (FEMA-273, 1997); (3) Modal Pushover Analysis (MPA) Chopra and Goel (2001). However, these methods were developed to apply the NSP for buildings only. Bridge researchers and engineers are currently investigating similar concepts and procedures to develop simplified procedures for performance-based seismic evaluation of bridges [11,12]. Few studies were performed to apply the NSP for bridges. In those studies, the CSM was implemented to estimate the demand (target displacement). CSM needs many iterations while the DCM, in general, needs no iterations.

In this study, the CSM in accordance with EC8 provisions and DCM stipulated in FEMA-273 will be implemented to estimate the target displacement and perform the pushover analysis. Also, the performance acceptance criteria proposed by EC8 and FEMA-273 (1997) will be implemented to evaluate the performance levels.

The behavior of reinforced concrete (RC) structures may be highly inelastic when subjected to seismic loads. Therefore the global inelastic performance of the RC

structures will be dominated by the plastic yielding effects, and consequently the accuracy of the pushover analysis will be influenced by the ability of the analytical models to capture these effects. In general, analytical models for pushover analysis of frame structures may be categorized into two main types: (1) distributed plasticity (plastic zone) and (2) concentrated plasticity (plastic hinge). Although the plastic hinge approach has a clear computational advantage over the plastic zone method through simplicity in computation, this method is limited to its incapability to capture the more complex member behaviors that involve severe yielding under the combined actions of compression and bi-axial bending and buckling effects, which may significantly reduce the load-carrying capacity of the member of structure. It is believed that the distributed plasticity analysis is the best approach to solve the inelastic stability of reinforced concrete frames with the complex member behaviors. The nonlinear static (NSP) analysis method described here for reinforced concrete structures has been developed by adapting an existing procedure for second-order elasto-plastic analysis of three-dimensional semi-rigid steel structures [8]. This

method was chosen for adaptation because it has been in use for many years and has proven to be numerically stable, robust and computationally efficient. In the present work, the RC members are modeled as beam-column elements with finite joints, based on the updated Lagrangian formulation. The tapered members and variation of reinforcing bars along the length of the beam column is allowed. In order

to model the material non-linearity both concentrated and distributed plasticity methods will be performed. Both local (P- $\delta$ ) and global (P- $\Delta$ ) nonlinear geometrical effects are taken into account in analysis.



Figure 1. Typical component model

# 2. PRESENT METHOD OF ANALYSIS

The method, use "line elements" approach, and are based on the degree of refinement in representing the plastic yielding effects. The elasto-plastic behavior is modelled in two types : (1) *distributed plasticity model*, when it is modelled accounting for spread-of-plasticity effects in sections and along the beam-column element and (2)

*concentrated plasticity model*, when inelastic behaviour is concentrated at plastic hinge locations. Distributed plasticity models can be further distinguished by how they capture inelastic cross-section behavior. The cross-section stiffness may be modeled by explicit integration of stresses and strains over the cross-section area (e.g., as micro model formulation) or through calibrated parametric equations that represent force-generalized strain curvature response (e.g. macro model formulation). In either approach, tangent stiffness properties of the cross sections are integrated along the member length to yield member stiffness coefficients.

The geometrical nonlinear local effects (P- $\delta$ ) are taken into account in analysis, for each element, by the use of stability stiffness functions. Using and updated Lagrangian formulation, the global geometrical effects (P- $\Delta$ ) are considered updating the geometry of the structure at each load increment using the web plane vector approach.

The proposed model has been implemented in a simple incremental and incrementaliterative matrix structural-analysis program. At each load increment a modified constant arc-length method [7] is applied to compute the complete nonlinear loaddeformation path, including the ultimate load and post critical response.



Figure 2. Beam column elements (a) Concentrated plasticity approach (b) Distributed plasticity approach

#### 2.1 Concentrated plasticity model

In this approach, the effect of material yielding is "lumped" into a dimensionless plastic hinges (Figure 2a). Regions in the frame elements other than at the plastic hinges are assumed to behave elastically, and if the cross-section forces are less than cross-section plastic capacity, elastic behavior is assumed. When the steady-forces reach the yield surface, a plastic hinge is formed which follow the nonhardening plasticity flow rules. To develop the incremental elasto-plastic relations, following standard practices of the nonhardening plasticity flow theory, the incremental elasto-plastic stiffness matrix can be generated. The incremental equation can be written thus in the following form:

$$\Delta \mathbf{S} = \mathbf{K}_{ep} \cdot \Delta \mathbf{d} \tag{1}$$

where the elasto-plastic incremental stiffness matrix is:

$$\mathbf{K}_{ep} = \mathbf{K} - \frac{\mathbf{K} \cdot \mathbf{G} \cdot \mathbf{G}^T \cdot \mathbf{K}}{\mathbf{G}^T \cdot \mathbf{K} \cdot \mathbf{G}}$$
(2)

and  $\Delta S$  and  $\Delta d$  represents the incremental force and displacement vector at the ends of the beam-column element respectively, and **K** represents the standard elastic stiffness matrix of beam column element. The gradient vector **G** of the yielding surface  $\Gamma$  has the following expression (Fig. 3):

$$\mathbf{G} = \begin{bmatrix} \left\{ \frac{\partial \Gamma}{\partial \mathbf{S}_{i}} \right\} & \mathbf{0} \\ \mathbf{0} & \left\{ \frac{\partial \Gamma}{\partial \mathbf{S}_{j}} \right\} \end{bmatrix}, \quad \frac{\partial \Gamma}{\partial \mathbf{S}_{i(j)}} = \left\{ \frac{\partial \Gamma}{\partial N_{i(j)}} & \mathbf{0} & \frac{\partial \Gamma}{\partial M_{i(j),y}} & \frac{\partial \Gamma}{\partial M_{i(j),z}} & \mathbf{0} & \mathbf{0} \right\}^{\mathrm{T}}$$
(3)

It should be noted here that if any one or both ends are inside of the plastic surface, then the magnitude of plastic flow at that ends is zero, and that ends does note play role in the plastic deformations of the element. In other words, the rows and columns of the gradient matrices G are activated when the corresponding end forces of the member get to full plastic surface, and not before that.

The plastic hinge approach eliminates the integration process on the cross section and permits the use of fewer elements for each member, and hence greatly reduces the computing effort. However, the method has been shown to overestimate the limit load in the case of reinforced concrete structures, where spread of plasticity effects is very significant. Since significant cracking and yielding of the members are expected, with inelastic deformations propagating into the member, it is essential to consider a member model in which the effects of spread plasticity are incorporated.



Figure 3. Yield surface of cross section

#### 2.2 Distributed plasticity model

Flexibility-based method is used to formulate the distributed plasticity model of a 3D frame element (12 DOF) under the assumptions of the Timoshenko beam theory. An element is represented by several cross sections that are located at the numerical integration scheme points. The cross-sections are located at control points whose number and location depend the numerical integration scheme. In this work, the Gauss-Lobatto rule [7] for element quadrature is adopted. Though this rule has lower order of accuracy than customary Gauss-Legendre rule, it has integration points at each ends of the element, where the plastic deformations is important, and hence performs better in detecting yielding. The cross-section stiffness may be modeled by explicit integration of stresses and strains over the cross-section area (e.g., as micro model formulation) or through calibrated parametric equations that represent force-generalized strain curvature response (e.g. macro model formulation). In either approach, tangent stiffness properties of the cross sections are integrated along the member length to yield member stiffness coefficients [7].



Figure 4. Distributed plasticity model

#### 2.2.1 Macro model formulation

In this approach the gradual plastification of the cross section of each member are accounted for by smooth force-generalized strain curves, experimentally calibrated. In the present elasto-plastic frame analysis approach, gradual plastification through the cross-section subjected to combined action of axial force and biaxial bending moments may be described by moment-curvature-thrus (M- $\Phi$ -N), and moment-axial deformation-thrus (M- $\epsilon$ -N) Ramberg-Osgood type curves that are calibrated by numerical tests. Other simplified force-strain relationships can be taking into account in analysis including multi-linear force strain curves. The effect of axial forces on the plastic moments capacity of sections is considered by a standard strength interaction curves [EC4]. In [9] has been shown that the yield surface given by:

$$\Gamma(N, M_y, M_z) = \sqrt{\left(\frac{M_y}{M_{py}}\right)^2 + \left(\frac{M_z}{M_{pz}}\right)^2 + \left(\frac{N}{N_p}\right)^{1.6} - 1 = 0}$$
(4)

gives acceptable results in a wide range of practical domain. The effective sectional properties will be computed according with EC4.

#### 2.2.2 Micro model formulation

In this approach, each cross-section is subdivided into a number of fibbers where each fibber is under uniaxial state of stress. The discretization process is shown in figure 1 for the case of RC structural member. The cross-section stiffness is modeled by explicit integration of stresses and strains over the cross-section area. Then tangent stiffness properties of the cross sections are integrated along the member length to yield member stiffness coefficients.



Figure 5 Cross section characteristics

Consider the cross-section of a beam-column subjected to the action of the biaxial bending moments My and Mz and axial force N as shown in above figure. The strain  $\varepsilon_{ii}$  in arbitrary point of the section can be expressed as follows:

$$\mathcal{E}_{ij} = u + \Phi_y \cdot y_{ij} + \Phi_z \cdot z_{ij} \tag{5}$$

in which u is the strain produced by axial force,  $\Phi_y$  and  $\Phi_z$  are the curvature in the section about principal y and z axes respectively.

The equations of static equilibrium in the section are:

S

$$f_{N}(u, \Phi_{y}, \Phi_{z}) \equiv \sum_{i} \int_{\Omega_{i}} \sigma(\varepsilon) \cdot d\Omega_{i} - N = 0$$
  

$$f_{My}(u, \Phi_{y}, \Phi_{z}) \equiv \sum_{i} \int_{\Omega_{i}} \sigma(\varepsilon) \cdot y \cdot d\Omega_{i} - M_{y} = 0$$
  

$$f_{Mz}(u, \Phi_{y}, \Phi_{z}) \equiv \sum_{i} \int_{\Omega_{i}} \sigma(\varepsilon) \cdot z \cdot d\Omega_{i} - M_{z} = 0$$
(6)

The above nonlinear system of equations is solved numerically using Newton method, and results in three recurrence relationships to obtain the unknowns, curvatures about each principal axes and axial strain:

$$\mathbf{r}^{k+1} = \mathbf{r}^{k} + \left(\mathbf{J}^{k}\right)^{-1} \cdot \mathbf{S}$$
(7)

where:

$$\mathbf{r}^{k} = \begin{bmatrix} u^{k}, \Phi_{y}^{k}, \Phi_{z}^{k} \end{bmatrix}^{T}, \mathbf{r}^{k+1} = \begin{bmatrix} u^{k+1}, \Phi_{y}^{k+1}, \Phi_{z}^{k+1} \end{bmatrix}^{T}$$

$$\mathbf{S} = \begin{bmatrix} N - f_{N}(\mathbf{r}^{k}), M_{y} - f_{M_{y}}(\mathbf{r}^{k}), M_{z} - f_{M_{z}}(\mathbf{r}^{k}) \end{bmatrix},$$

$$\mathbf{J}^{k} = \begin{bmatrix} \frac{\partial f_{N}^{k}}{\partial u} & \frac{\partial f_{N}^{k}}{\partial \Phi_{y}} & \frac{\partial f_{N}^{k}}{\partial \Phi_{z}} \\ \frac{\partial f_{M_{y}}^{k}}{\partial u} & \frac{\partial f_{M_{y}}^{k}}{\partial \Phi_{y}} & \frac{\partial f_{M_{y}}^{k}}{\partial \Phi_{z}} \\ \frac{\partial f_{M_{z}}^{k}}{\partial u} & \frac{\partial f_{M_{z}}^{k}}{\partial \Phi_{y}} & \frac{\partial f_{M_{z}}^{k}}{\partial \Phi_{z}} \end{bmatrix}$$
(8)

In this way, the state of stress and strain in each point of cross-section at any stage of loading, can be obtained and then the tangent stiffness properties of cross sections are readily calculated. In situations where loading and unloading occur in a section, the strain history of each layer (point) is used to determine the current stress and hence the stress resultants. The surface integrals are computed numerically using the 2D Simpson rule. In this way, fully non-linear material behaviour can be considered, including cracking of the concrete in tension, tension stiffening, compressive softening of the concrete, and yielding, strain hardening or unloading of reinforcing steel.

#### **Material Models**

In the calculation of non-linear curvatures, the following constitutive relationship is used for concrete stress,  $f_c$ , in compression, as a function of concrete strain  $\varepsilon_c$ :

$$f_{c} = f_{co} \left[ \frac{2\varepsilon_{c}}{\varepsilon_{o1}} - \left( \frac{\varepsilon_{085}}{\varepsilon_{01}} \right)^{2} \right] \qquad \qquad for \quad \varepsilon_{c} \le \varepsilon_{01}$$

$$\tag{9}$$

$$f_{c} = f_{co} - (\varepsilon_{c} - \varepsilon_{ol}) \left( \frac{0.15 f_{co}}{\varepsilon_{085} - \varepsilon_{0l}} \right) \qquad \qquad for \quad \varepsilon_{0l} < \varepsilon_{c} \le \varepsilon_{cu} \tag{10}$$

Where,  $f'_{co}$  is the specified strength of concrete,  $\varepsilon_{01}$ , and  $\varepsilon_{085}$  are the strains at peak and 85% of the peak strength.

For concrete stress in tension, the following linear constitutive relationship is used:

$$f_c = \varepsilon_c E_c \qquad \qquad for \quad \varepsilon_c \le \varepsilon_{cr} \tag{11}$$

$$f_c = 0 \qquad \qquad for \quad \varepsilon_c > \varepsilon_{cr} \tag{12}$$

Where,  $E_c$  is the modulus of elasticity of concrete and is expressed as:

$$E_c = 5000\sqrt{f_{co}}$$
 (13)

 $\varepsilon_{cr}$  is the strain at cracking and is expressed by the following equation;

$$\varepsilon_{cr} = \frac{\sqrt{f_{co}}}{2E_c} \tag{14}$$

For the stress,  $f_s$ , in reinforcing steel, the following elasto-plastic stress-strain relationship is assumed.

$$f_s = \varepsilon_s E_s \qquad \qquad for \quad \varepsilon_s \le \varepsilon_y \tag{15}$$

$$f_s = f_y \qquad \qquad for \quad \varepsilon_s > \varepsilon_y \qquad (16)$$

The strain hardening part of the stress-strain relationship for steel may be take into account.

## 2.3. Prestressed concrete effects

In order to treat prestressed concrete structures, a preliminary analysis is added to take account of the introduction of the prestressing force into the concrete-steel structure. In this preliminary analysis, the application of the prestressing force to each concrete element is considered, the case of post-tensioned elements which contain a curved prestressing cable is considered. If an analysis is undertaken of a practical structure with only the prestress acting, it is often found that cracking of the concrete is predicted. This is because the cable is designed to partially balance the stress due to external load. It is convenient therefore to analyze the initial state of the structure with the effects of both the prestressed and self-weight. If these effects are analyzed separately spurious non-linear effects are introduced because of cracking. Because the behavior in the post cracking stage is significantly non-linear, it is not possible to treat the two effects separately and superpose the results.

## 2.4. Analysis algorithm

The proposed model has been implemented in a simple incremental and incrementaliterative matrix structural-analysis program [7]. Also an adaptive load incrementation control has been implemented to perform the nonlinear elasto-plastic analysis. At each load increment a modified constant arc-length method is applied to compute the complete nonlinear load-deformation path, including the ultimate load and post critical response. The size of load increment is controlled by using the following criteria: (1) constraint on the maximum incremental displacement; (2) load increment control due to the formation of full plastic sections (plastic hinges); constraint of force point movement at plastic hinges.

Based on the analysis algorithm just described, an object-oriented computer program, NEFCAD 3D [7], has been developed to study the effects of material and geometric nonlinearities on the load-versus-deflection response for spatial reinforced concrete framed structures. It combines the structural analysis routine with a graphic routine, to display the final results. The interface allows for easy generation of data files, graphical representation of the data, and plotting of the deflected shape, bending moments, shear force and axial force diagrams, load-deflection curves for selected nodes, etc.



# Simplified flowchart of non-linear static analysis **3. PUSHOVER ANALYSIS METHODOLOGY**

A pushover analysis is performed by subjecting a structure to a monotonically increasing pattern of lateral forces, representing the inertial forces which would be experienced by the structure when subjected to ground shacking. Under incrementally increasing loads various structural elements yield sequentially. Consequently, at each event, the structure experiences a loss in stiffness. Using a pushover analysis, a characteristic nonlinear force-displacement relationship can be determined. In principle, any force and displacement can be chosen. Typically the first pushover load case is used to apply gravity load and then subsequent lateral pushover load cases are specified to start from the final conditions of the gravity. The selection of an appropriate lateral loads distribution is an important step within the pushover analysis. The non-linear static procedure in EC8 and FEMA-356 requires development of a pushover curve by first applying gravity loads followed by monotonically increasing lateral forces with a specified height-wise distribution. At least two force distributions must be considered. The first is to be selected from among the following: Fundamental (or first) mode distribution; Equivalent Lateral Force (ELF) distribution; and SRSS distribution. The second distribution is either the "Uniform" distribution or an adaptive distribution; the first is a ,uniform' pattern with lateral forces that a proportional to masses and the second pattern, varies with change in deflected shape of the structure as it yields. EC 8 gives two vertical distributions of lateral forces: (1) a ,uniform' pattern with lateral forces consisting with the lateral force distribution determined in elastic analysis. These force-distributions mentioned above are defined next:

- 1. Fundamental mode distribution:  $S_j = m_j \phi_{j1}$  where  $m_j$  is the mass and  $\phi_{j1}$  is the mode shape value at the *j*th floor;
- 2. Equivalent lateral force (ELF):  $S_j = m_j h_j^k$  where  $h_j$  is the height of the *j*th floor above the base and the exponent k=1 for fundamental mode period  $T_1 \le 0.5$  sec, k=2 for  $T_1 \le 2.5$  sec, and varies linearly in between;
- 3. SRSS distribution: S is defined by the lateral forces back-calculated from the story shears determined by response spectrum analysis of the structure, assumed to be linearly elastic;
- 4. Uniform distribution:  $S_i = m_i$
- 5. Modal distribution:  $S_j = \frac{m_j \phi_{j1}}{\sum m_j \phi_{j1}} S$  where  $S_j$  is the lateral force jth floor mj

is the mass assigned to floor  $j \phi_{j1}$  is the amplitude of the fundamental mode at level j, and S = base shear. This pattern may be used if more than 75% of the total mass participates in the fundamental mode of the direction under consideration (FEMA-273, 1997). The value of S in the previous equation can be taken as an optional value since the distribution of forces is important while the values are increased incrementally until reaching the prescribed target displacement or collapse.

The third load pattern, which is called the Spectral Pattern in this study, should be used when the higher mode effects are deemed to be important. This load pattern is based on modal forces combined using SRSS (Square Root of Sum of the Squares

method. It can be written as:  $S_j = \frac{m_j \delta_j}{\sum m_j \delta_j} S$  where Sj, mj, and S are the same as

defined for the Modal Pattern, and  $\delta_j$  is the displacement of node (floor) *j* resulted from response spectrum analysis of the structure (including a sufficient number of modes to capture 90% of the total mass), assumed to be linearly elastic. The appropriate ground motion spectrum should be used for the response spectrum analysis.

#### 3.1. Key elements of the pushover analysis on bridges

Due to the nature of bridges, which extend horizontally rather than buildings extending vertically, some considerations and modifications should be taken into consideration to render the NSP applicable for bridges. The modifications and considerations should concentrate on the following key elements:

1. Definition of the control node: Control node is the node used to monitor displacement of the structure. Its displacement versus the base-shear forms the capacity (pushover) curve of the structure.

2. Developing the pushover curve, which includes evaluation of the force distributions: To have a displacement similar or close to the actual displacement due to earthquake, it is important to use a force distribution equivalent to the expected distribution of the inertia forces. Different formats of force distributions along the structure are implemented in this study to represent the earthquake load intensity.

3. Estimation of the displacement demand: This is a key element when using the pushover analysis. The control node is pushed to reach the demand displacement, which represents the maximum expected displacement resulting from the earthquake intensity under consideration.

4. Evaluation of the performance level: Performance evaluation is the main objective of a performance-based design. A component or an action is considered satisfactory if it meets a prescribed performance level. For deformation-controlled actions the deformation demands are compared with the maximum permissible values for the component. For force-controlled actions the strength capacity is compared with the force demand. If either the force demand in force controlled elements or the deformation demand in deformation-controlled elements exceeds permissible values, then the element is deemed to violate the performance criteria.

# **3.2** Determination of the target displacement for nonlinear static (pushover) analysis

Simplified nonlinear methods for the seismic analysis of structures combines the pushover analysis of a multi-degree-of-freedom (MDOF) structures with the elastic or inelastic response spectrum analysis of an equivalent single-degree-of-freedom (SDOF) system. Examples of such an approach are the Capacity Spectrum Method (CSM) (ATC, 1996); Displacement Coefficient Method (DCM) (FEMA-273, 1997) and N2 method [5]. The main difference between these methods lies in the determination of the displacement demand. In FEMA 273, inelastic displacement demand is determined from elastic displacement demand using four modification factors, which take into account the transformation from MDOF to SDOF, nonlinear response, increase in displacement demand if hysteretic loops exhibit significant pinching and if the post-yield slope is negative. The determination of seismic demand in the capacity spectrum method used in ATC 40 is basically different. It is determined from equivalent elastic spectra, and equivalent damping and period are used in order to take into account the inelastic behavior of the structure. Moreover, in this method, demand quantities are obtained in an iterative way. The N2 method is in fact a variant of the capacity spectrum method based on inelastic spectra.

These methods are based on predefined invariant inertial force distributions and consequently cannot capture the contributions of higher modes to response or redistribution of inertia forces because of structural yielding and the associated changes in the vibration properties of the structure. Several methods were developed to overcome these limitations: adaptive force distributions [6] and more recently a modal pushover analysis procedure [1], which in a relatively simple format consider the contributions of higher modes of vibrations to nonlinear response.

The EC8 procedure to determination of the target displacement will be briefly described below. The target displacement is determined from the elastic response spectrum formulated in an acceleration-displacement (AD) format.

## **Summary of Procedure**

1. Compute the natural periods,  $T_n$ , and corresponding modes  $\Phi$ , for linearlyelastic vibration of the structure. Using the elastic spectra, base-shear forces are computed.

Assume two vertical distributions of lateral forces: uniform and modal pattern. A nonlinear static (pushover) analysis under constant gravity loads and monotonically increasing lateral forces is performed. In this way the base-shear-control point-displacement pushover curve is developed.



2. Idealize the pushover curve as a bilinear elasto-perfectly plastic forcedisplacement relationship. The yield force  $F_y^*$  is equal to the base shear force at the formation of the plastic mechanism. The initial stiffness of the idealized system is determined in such way that the areas under the actual and the idealized force-deformation curves are equal.



Figure 6 Idealized elasto-perfectly plastic pushover curve

Based on this assumption, the yield displacement of the idealized SDOF system  $d_v^*$  is given by:

$$d_{y}^{*} = 2 \left( d_{m}^{*} - \frac{E_{m}^{*}}{F_{y}^{*}} \right)$$
(17)

where  $E_m^*$  is the actual deformation energy up to the formation of the plastic mechanism, and  $d_m^*$  is the displacement of the control point at the yield force corresponding formation of plastic mechanism.

3. Determination of the target displacement for the equivalent SDOF.

For the determination of the target displacement  $d_t^*$  for structures in the shortperiod range and for structures in the medium and long-period ranges an iterative method is indicated:

3.1. Determination of the period of the idealized equivalent SDOF sytem:

$$T^{*} = 2\pi \sqrt{\frac{m^{*}d_{y}^{*}}{F_{y}^{*}}}$$
(18)

where  $d_y^*$  for first step of iteration is computed with equation (17), and  $m^* = \sum m_i \Phi_i$  is the mass of an equivalent SDOF system ( $m_i$  is the mass in the ith floor, and  $\Phi_i$  is the normalized mode shape value in such a way that  $\Phi_n = 1$ , where *n* is control node).

3.2. Determination of the target displacement of the structure with period  $T^*$  and unlimited elastic behavior:

$$d_{et}^* = S_e \left( T^* \right) \left[ \frac{T^*}{2\pi} \right]^2$$

where  $S_e(T^*)$  is the elastic acceleration response spectrum at the period  $T^*$ 

3.3 Determination of the target displacement  $d_t^*$  for structures in the shortperiod range and for structures in the medium and long-period ranges:



3.4. If the target displacement determined in the 3.3 step is much different from the displacement  $d_m^*$  (i.e. displacement of the control point corresponding formation of plastic mechanism, see figure 6) the steps 3.1, 3.2 and 3.3 are repeated by replacing the  $d_m^*$  with  $d_t^*$  and corresponding  $F_v^*$ .

 Determination of the target displacement for the MDOF system. The target displacement correspond to the MDOF system corresponds to the control node is given by:

$$D_t = \Gamma d_t^*$$

where the transformation factor is given by  $\Gamma = \frac{m^*}{\sum m_i \Phi_i^2}$ .

When using non-linear static (pushover) analysis and applying a spatial model, the combination of the effects of the components of the seismic actions should be applied, considering as  $E_{Edx}$  the forces and deformations due to the application of the target displacement in the x direction and as  $E_{Edy}$  the forces and deformations due to the applications due to the application of the target displacement in the y direction. The internal forces resulting from the combination shall not exceed the corresponding capacities.

#### 3.3. Global and local seismic demand for the MDOF model.

The local seismic demand, story drifts, joint rotations etc, can be determined by a pushover analysis. Under monotonically increasing lateral loads with a fixed pattern, the structure is pushed to its target displacement  $D_t$ . It is assumed that the distribution of deformations throughout the structure in the nonlinear static analysis approximately corresponds to that which would be obtained in the dynamic analyses. Note that  $D_t$  represents a mean value for the applied earthquake loading, and that there is a considerable scatter about the mean. Consequently, it is appropriate to investigate likely building performance under extreme load conditions that exceed the design value. This can be achieved by increasing the value of the target displacement. In EC8 it is recommended to carry out the analysis to at least 150% of the calculated top displacement. Expected performance can assessed by comparing the seismic demands with the capacities for the relevant performance level. Global performance can be visualized by comparing displacement capacity and demand.

## 4. CASE STUDY

A three span prestressed reinforced concrete bridge, which is to be built in the northeastern of Portugal over Alva River, is chosen as a case study. The total length of the bridge is 115 m with spans of 35, 45, and 35 m. Figure 7 shows the plan and the elevation of the bridge. The geometrical and mechanical characteristics of the bridge and the relevant cross-sections are presented in the technical plants in Appendix B. Also the profile of the prestressing cable and the effective prestressed force applied in superstructure of the bridge, can be found there.

#### 4.1. Numerical model

The structural analysis program NEFCAD [7] will be used to perform analyses. Geometric nonlinearity through considering local (P- $\delta$ ) and global (P- $\Delta$ ) effects will be applied to this bridge in addition to material nonlinearity. The elasto-plastic behavior is modeled in two types : (1) distributed plasticity model, when it is modeled accounting for spread-of-plasticity effects in sections and along the beam-column element and (2) plastic hinge, when inelastic behavior is concentrated at plastic hinge locations. The proposed method can capture the spreading of plasticity along the members with computational efficiency and the necessary degree of accuracy, usually only one element per physical member is necessary to analyze. Therefore the nodes, in numerical model, will be placed only at the ends of physical members or where occur changes in cross-sections properties (Figure 8). The superstructure has been modeled with three elements per span (3D frame elements) and the work lines of the elements are located along the centroid of the superstructure.



Figure 8 Structural model of the bridge

In the concentrated plasticity model, determination of the moment of inertia and torsional stiffness of the superstructure are based on uncraked cross sectional properties because the superstructure is expected to respond linearly to seismic loadings. The moment of inertia for columns will be calculated based on the cracked section using moment-curvature (M- $\Phi$ ) curve.

The abutments will be modeled with an equivalent spring stiffness, as shown in above figure 8. A rigid link is used to model the connection between the columns top.

In order to take into account the non-uniformly distributed force of prestressed cable to the concrete elements, the spans will be further subdivided in a few segments along the physical members and the cable is considered to be straight within each segment.



(a) Element with curved cable



(b) Piece-wise linear approximation with kinks at the junctures between segments



(a) Inclined local forces at

(b) Vertical and horizontal

(c) resultants at junctures





Figure 10. Pushover analysis flowchart

Statically, the distributed load acting on the concrete in Figure 9 is being represented approximately by a series of small inclined discrete forces acting at the junctures between elements. The real cable at any point is typically eccentric to the centroidal axis and inclined and curved. This produces a uniformly distributed force on the

concrete segment under consideration, which is perpendicular to the cable. The force applied at each juncture thus has a vertical and a horizontal component. Furthermore, to allow for the eccentricity of the real cable, a moment also acts at the juncture, as shown in Figure 9. The system of vertical and horizontal forces and moments acting at each juncture along the element is statically equivalent to the continuous force system being applied to the element by the prestressed cable.

The simplified flowchart of the main steps performed in the present approach is depicted in the figure 10.

# 5. CONCLUSIONS

A non-linear static (pushover) analysis (NSP) method for reinforced concrete structures that predicts behavior at all stages of loading, from the initial application of loads up to and beyond the collapse condition is presented. The method, use "line elements" approach, and are based on the degree of refinement in representing the plastic yielding effects, both concentrated and distributed plasticity models are presented. Applicability of the non-linear static procedure, described here, to bridges is investigated in this study using the capacity spectrum method, which was presented by EC8 (2003). A three span bridge was presented and described as a case study. Target displacement, base shear and deformation of plastic zones (hinges) obtained from this procedure will be compared with the corresponding values resulting from the more accurate but more time consumption, nonlinear time history analysis, in framework of a future work.

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NEFCAD-3D Non-Linear Elasto-Plastic Analysis

NEFCAD is a non-linear elasto-plastic analysis computer program for frame structures. NEFCAD consists of a control program that manages the database, an analysis and design engine and a graphical user interface (GUI) for user data input.

NEFCAD capabilities include:

- Geometric and material non-linear behaviour
- Member buckling
- Joint flexibility and joint plasticity
- Strain hardening and residual stress
- Non-uniform members and finite joints
- One element per physical member to simulate distributed plasticity.
- Incremental iterative methods (load and arclength control).
- Adaptive load incrementation



Distribution of plastic zone along the member length



Push-over analysis: Plot indicating plastic hinges developed

NEFCAD uses an event-to-event load incrementation strategy coupled with an equilibrium error correcting constant arc-length algorithm to solve for geometric and material nonliniarities associated with the ultimate load capacity of a structure. The size of load increment is controlled by using the following criteria: (1) constraint on the maximum incremental displacement; (2) load increment control due to the formation of full plastic sections (plastic hinges); constraint of force point movement at plastic hinges.

Load Combinations allow for combining the non-push load cases together with load factors before the push analysis is performed to capture effects of the structure's initial state of stress. Both concentrated and distributed plasticity models are allowed. The cross-section stiffness may be modeled by explicit integration of stresses and strains over the cross-section area or through calibrated parametric equations that represent force-generalized strain curvature response.



Graphically, during analysis, the user can observe clearly the structural deformation and the progression of plasticity as the load is incrementally applied to show the collapse mechanism, load-deflection curves, and various limit states of yielding. Element reports show local diagrams, plastic rotations, displaced shapes, and relevant member data.

Plot showing spread of plasticity through cross-section and along the beam column element.