BASES OF ELECTROTECHNICS

~Theory of electric circuits ~

BASES OF ELECTROTECHNICS I.
Faculty of Electronics, Telecommunications and Information Technology
Specialization: IETTI
Academic year: 2023-2024

- Electric circuit theory and electromagnetic field theory are the two fundamental theories upon which all branches of electrical engineering are built.

The basic electric circuit theory course – an excellent starting point for a beginning student in electrical engineering education ☺

- Many branches of electrical engineering, such as energy production, electric machines, control, electronics, communications and instrumentation, are based on electric circuit theory.

The objectives of the Course

1. To present systematically the basic theory of the electric circuits
2. To introduce electrical components and the fundamental laws that govern the behavior of an electrical circuit in case of:
   - DC and AC circuits;
   - two-ports networks;
   - steady-state periodic non-sinusoidal regime;
   - transient regime of liniar circuits;
   - three-phase circuits;
   - transmission lines.

3. To practice specific methods of analysis.

➢ On successful completion of this course, students will be able to: analyze the operation of linear circuits in response to DC, sinusoidal, non-sinusoidal and transient waveforms.

Teaching details:

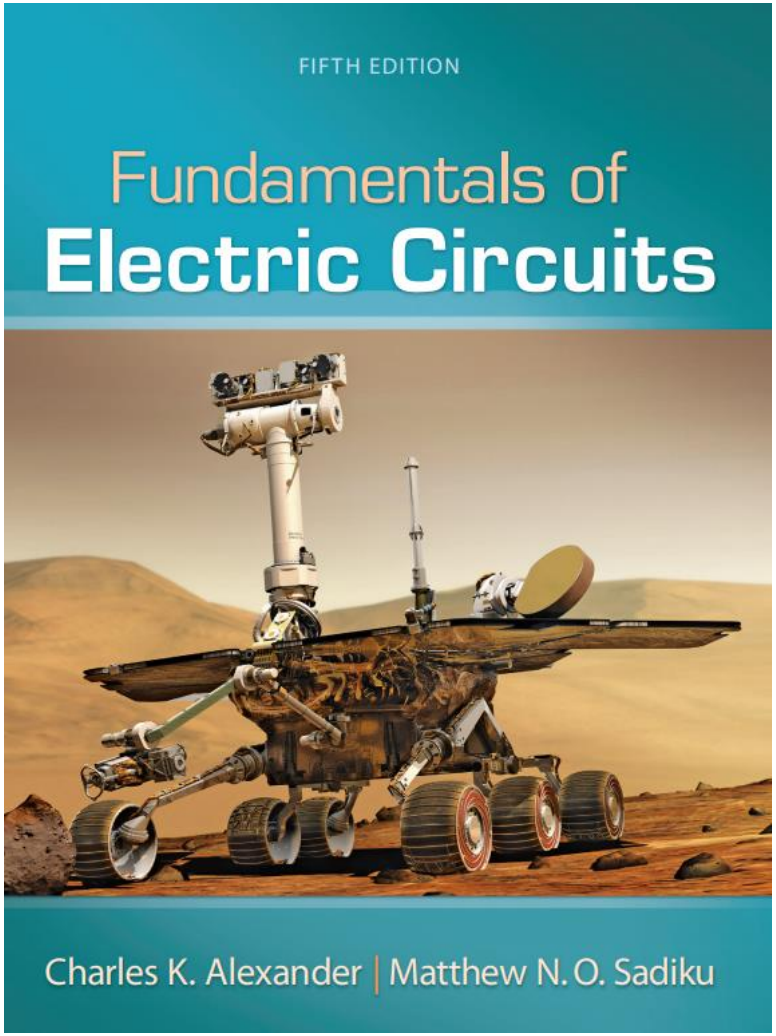
- 14 lectures
  (2 hours/lecture)
- 14 seminars
  (2 hours/seminar)

Assessment Details:

Exam, 2 hours, 100%

(ANSWER TO MULTIPLE CHOICE TEST + SOLVE PROBLEMS)

Number of ECTS credit points: 4



Charles K. Alexander

Department of Electrical and Computer Engineering

Cleveland State University

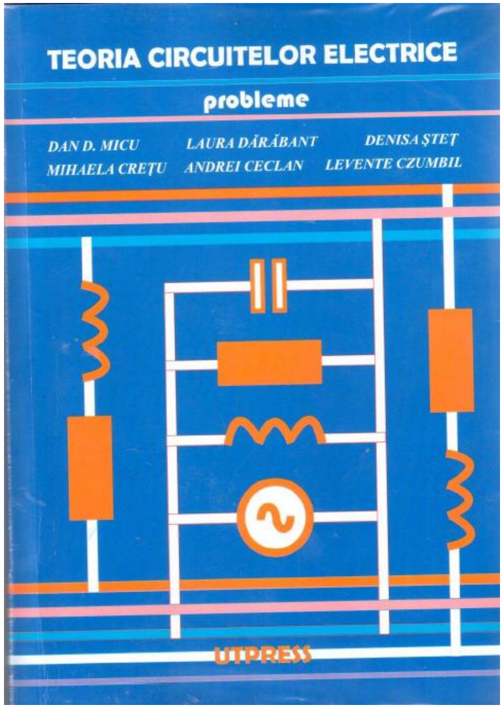
Matthew N. O. Sadiku

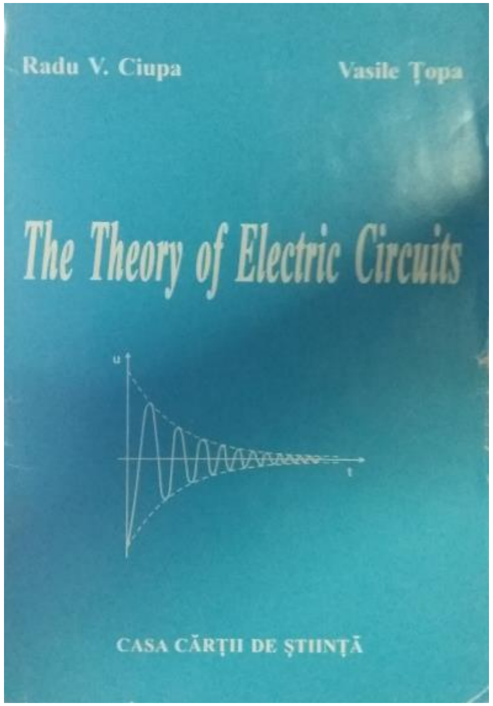
Department of Electrical Engineering

Prairie View A&M University

Reading and References

- ✓ Students are provided with set of notes: posted on TEAMS
- ✓ Students requiring extra material are recommended:





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ISBN 973-9204-98-8

Course content:

Chapter 1 - DC CIRCUITS *(recap from semester I)*

Chapter 2 – AC CIRCUITS

Chapter 3 – THREE PHASE CIRCUITS

Chapter 4 – TWO PORT NETWORKS

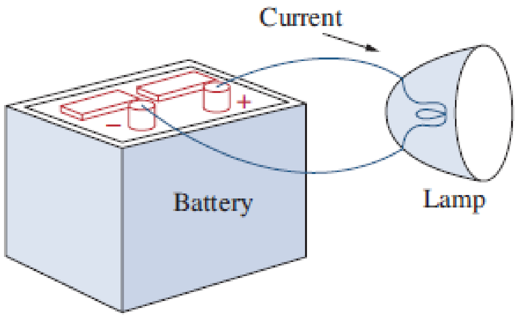
Chapter 5 – STEADY-STATE PERIODIC NON-SINUSOIDAL REGIME

Chapter 6 – TRANSIENT REGIME

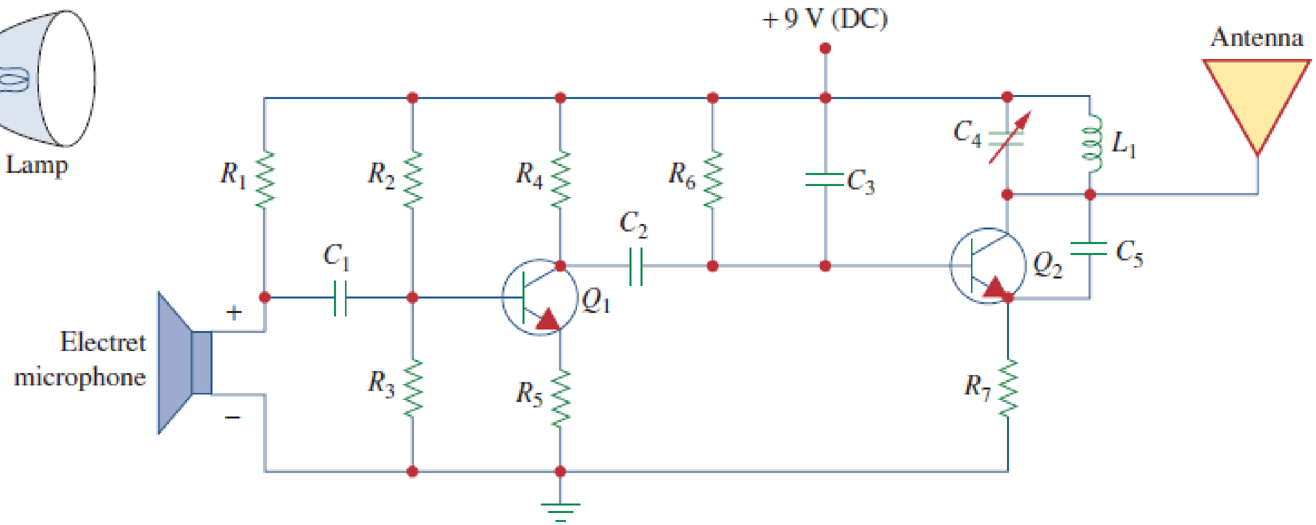
CHAPTER 1: DC Circuits

1. Basic concepts

- An **electrical circuit** is an interconnection of electrical elements



A simple electrical circuit



Electric circuit of a radio transmitter

Electric circuits are used in numerous electrical systems to accomplish different tasks. Our objectives in this course is not the study of various uses and applications of the circuits.

- **Rather, our major concern is the <u>analysis of the circuits</u> (study of the behavior of the circuit):**
- **How does it respond to a given input**
- **How do the interconnected elements and devices in the circuit interact etc.**

- International System of Units (SI) adopted by the General Conference on Weights and Measures in 1960

Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

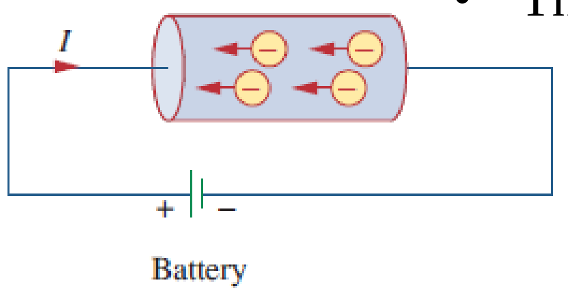
- The most basic quantity in an electric circuit is the electric charge.

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C)

The following points should be noted about electric charge:

1. The coulomb is a large unit for charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC.[1]
2. According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge $e = -1.602 \times 10^{-19}$ C.
3. The *law of conservation of charge* states that charge can neither be created nor destroyed, only transferred. Thus the algebraic sum of the electric charges in a system does not change.

- The motion of charges creates electric current



Electric current due to flow of electronic charge in conductor

- It is conventional to take the current flow as the movement of positive charges. That is, opposite to the flow of negative charges.
(This convention was introduced by Benjamin Franklin (1706-1790), the American scientist and inventor)

- Because the current in metallic conductors is due to negatively charged electrons, we will follow the universally accepted convention that current is the net flow of positive charges.

Electric current is the time rate of change of charge, measured in amperes (A)

$$i = \frac{dq}{dt}$$

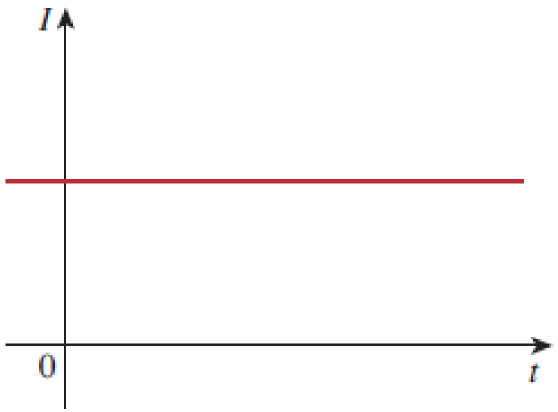
1 ampere = 1 coulomb/second

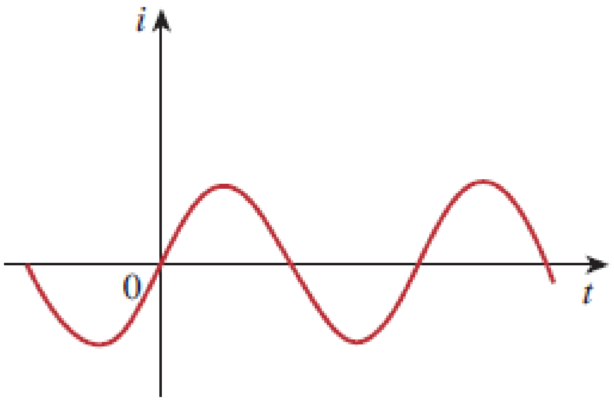
$$Q = \int_{t_0}^{t} i dt$$

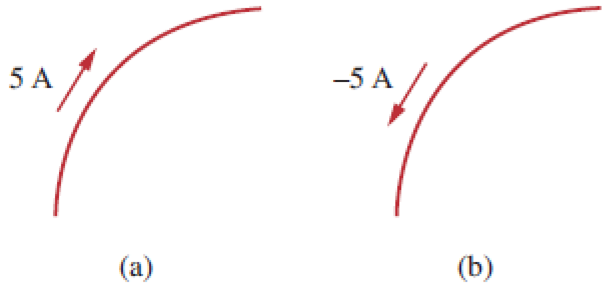
The charge transferred between time t_0 and t

A direct current (DC) is a current that remains constant with time.

An alternating current (AC) is a current that varies sinusoidally with time.







a) Positive current flow b) negative current flow

- The direction of current flow is conventionally taken as the direction of positive charge movement.

Historical

Andre-Marie Ampere (1775–1836), a French mathematician and physicist, laid the foundation of electrodynamics. He defined the electric current and developed a way to measure it in the 1820s.

Born in Lyons, France, Ampere at age 12 mastered Latin in a few weeks, as he was intensely interested in mathematics and many of the best mathematical works were in Latin. He was a brilliant scientist and a prolific writer. He formulated the laws of electromagnetics. He invented the electromagnet and the ammeter. The unit of electric current, the ampere, was named after him.

The Burndy Library Collection at The Huntington Library, San Marino, California.

EXAMPLE 1.1.

How much charge is represented by 4,600 electrons?

Solution:

Each electron has -1.602×10^{-19} C. Hence 4,600 electrons will have

$$-1.602 \times 10^{-19} \text{ C/electron} \times 4{,}600 \text{ electrons} = -7.369 \times 10^{-16} \text{ C}$$

PRACTICE PROBLEM 1.1.

Calculate the amount of charge represented by six million protons.

Answer: $+9.612 \times 10^{-13}$ C.

EXAMPLE 1.2.

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

PRACTICE PROBLEM 1.2.

If in Example 1.2, $q = (10 - 10e^{-2t})$ mC, find the current at $t = 1.0$ s.

Answer: 2.707 mA.

EXAMPLE 1.3.

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$Q = \int_{t=1}^{2} i\,dt = \int_{1}^{2} (3t^2 - t)\,dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5\ \text{C}$$

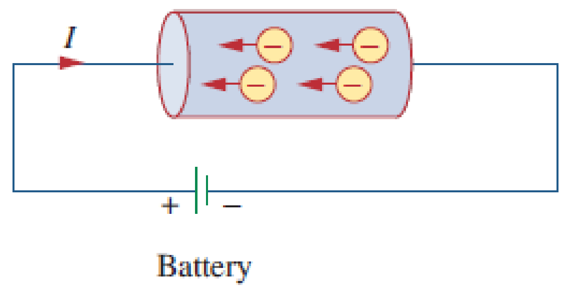
PRACTICE PROBLEM 1.3.

The current flowing through an element is

$$i = \begin{cases} 4\ \text{A}, & 0 < t < 1 \\ 4t^2\ \text{A}, & t > 1 \end{cases}$$

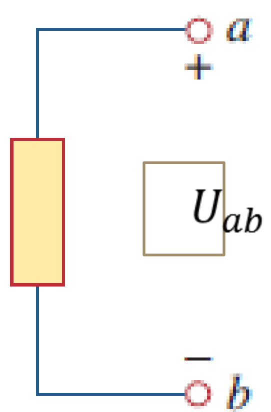
Calculate the charge entering the element from $t = 0$ to $t = 2$ s.

Answer: 13.333 C.



- To move the electron in a conductor in a particular direction requires some work or energy transfer: an external electromotive force (emf) also known as voltage or potential difference.

Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V)



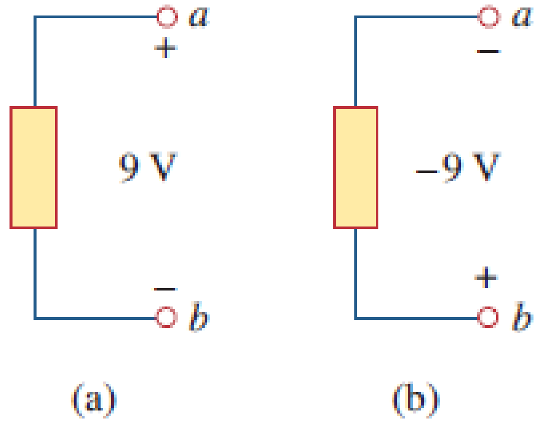
Polarity of voltage Uab

$$U_{ab} = \frac{dw}{dq}$$

- w is energy in joules (J)
- Q is charge in coulombs (C)

1 volt = 1 joule/coulomb = 1 newton-meter/coulomb

$$U_{ab} = -U_{ba}$$



Two equivalent representations of the same voltage v_{ab}: (a) Point *a* is 9 V above point *b*; (b) point *b* is −9 V above point *a*.

On the left: the voltage increases by 9V from the – sign to the + sign
On the right: the voltage decreases by 9V from the – sign t the + sign

A DC voltage: voltage that remains constant with time *(is represented by U).*

Is commonly produced by a battery.

An AC voltage: voltage that varies sinusoidally with time *(is represented by u).*

Is commonly produced by an electric generator.

Historical

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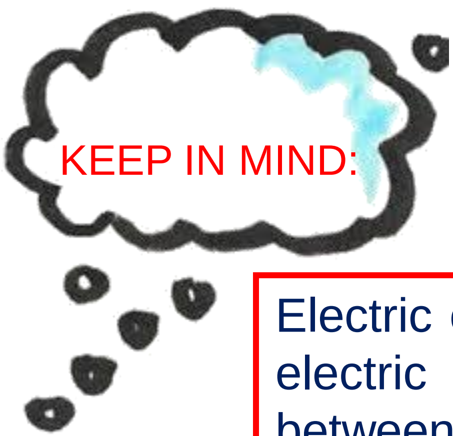
at The Huntington Library,
San Marino, California.

Alessandro Antonio Volta (1745–1827), an Italian physicist, invented the electric battery—which provided the first continuous flow of electricity—and the capacitor.

Born into a noble family in Como, Italy, Volta was performing electrical experiments at age 18. His invention of the battery in 1796 revolutionized the use of electricity. The publication of his work in 1800 marked the beginning of electric circuit theory. Volta received many honors during his lifetime. The unit of voltage or potential difference, the volt, was named in his honor.

- ✓ Current and voltage are the two basic variables in electric circuits.
- ✓ The common term *signal* is used for an electric quantity such as a current or a voltage (or even electromagnetic wave) when it is used for conveying information.
- ✓ Engineers prefer to call such variables signals rather than mathematical functions of time becuase of their importance in communications and other disciplines.



Electric current is always *through* an element and electric voltage is always *across* the element between two points.

Power is the time rate of expending or absorbing energy, measured in watts (W)

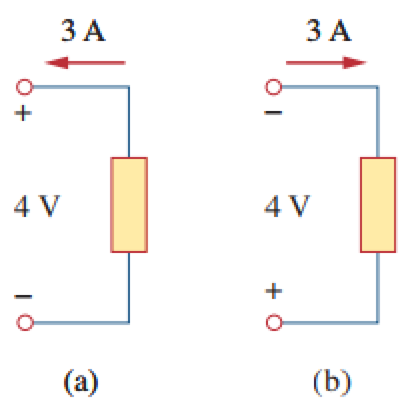
$$p = \frac{dw}{dt}$$

or

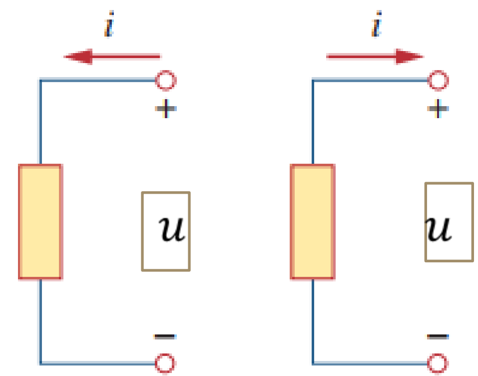
$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = u \cdot i \;\Rightarrow\; p = u \cdot i$$

- *p* is power in watts (W)
- *w* is energy in joules (J)
- *t* is time in seconds (s)

- The power *p* is a time-varying quantity called the *instantaneous power.*
- If *p* has a + sign, power is being delivered to or absorbed by the element.
- If *p* has a - sign, power is being supplied by the element.



Two cases of an element with a supplying power of 12 W: (a) $p = -4 \times 3 = -12$W, (b) $p = -4 \times 3 = -12$ W.



$p = +u \cdot i$ $\quad$ $p = -u \cdot i$

Reference polarities for power using the passive sign convention

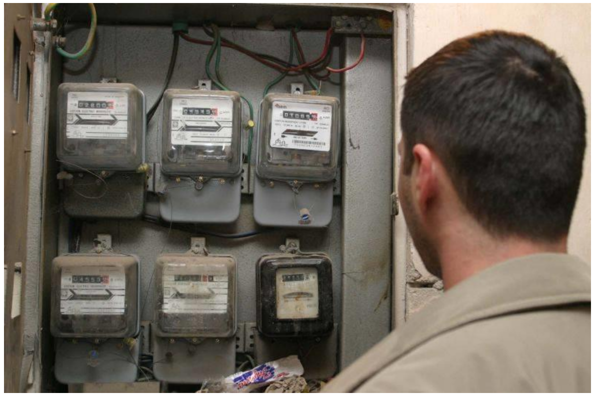
+Power absorbed = −Power supplied

$$\sum p = 0$$

Law of conservation of energy: **the algebraic sum of power in a circuit, at any instant of time, must be zero.**

$$w = \int_{t_0}^{t} p \cdot dt = \int_{t_0}^{t} ui \cdot dt$$

Energy **is the capacity to do work, measured in joules (J).**



The electric power utility companies measure energy in watts-hours (Wh), where:

$$1 \text{ Wh} = 3{,}600 \text{ J}$$

EXAMPLE 1.4.

An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:

The total charge is

$$\Delta q = i\,\Delta t = 2 \times 10 = 20\ \text{C}$$

The voltage drop is

$$u = \frac{\Delta w}{\Delta q} = \frac{2.3x10^3}{20} = 115V$$

PRACTICE PROBLEM 1.4.

To move charge q from point a to point b requires -30 J. Find the voltage drop u_{ab} if: (a) $q = 6$ C, (b) $q = -3$ C.

Answer: (a) -5 V, (b) 10 V.

EXAMPLE 1.5.

Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is

$$i = 5\cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3\,di/dt$.

Solution:

(a) The voltage is $v = 3i = 15\cos 60\pi t$; hence, the power is

$$p = vi = 75\cos^2 60\pi t \text{ W}$$

At $t = 3$ ms,

$$p = 75\cos^2(60\pi \times 3 \times 10^{-3}) = 75\cos^2 0.18\pi = 53.48 \text{ W}$$

(b) We find the voltage and the power as

$$v = 3\frac{di}{dt} = 3(-60\pi)5\sin 60\pi t = -900\pi\sin 60\pi t \text{ V}$$

$$p = vi = -4500\pi\sin 60\pi t\cos 60\pi t \text{ W}$$

At $t = 3$ ms,

$$p = -4500\pi\sin 0.18\pi\cos 0.18\pi \text{ W}$$
$$= -14137.167\sin 32.4^\circ\cos 32.4^\circ = -6.396 \text{ kW}$$

PRACTICE PROBLEM 1.5.

Find the power delivered to the element in Example 1.5 at $t = 5$ ms if the current remains the same but the voltage is: (a) $v = 2i$ V, (b) $v = \left(10 + 5\int_0^t i\,dt\right)$ V.

Answer: (a) 17.27 W, (b) 29.7 W.

EXAMPLE 1.6.

How much energy does a 100-W electric bulb consume in two hours?

Solution:

$$w = pt = 100\ (\text{W}) \times 2\ (\text{h}) \times 60\ (\text{min/h}) \times 60\ (\text{s/min})$$
$$= 720{,}000\ \text{J} = 720\ \text{kJ}$$

This is the same as

$$w = pt = 100\ \text{W} \times 2\ \text{h} = 200\ \text{Wh}$$

PRACTICE PROBLEM 1.6.

A stove element draws 15 A when connected to a 240-V line. How long does it take to consume 180 kJ?

Answer: 50 s.

There are two types of elements found in electric circuits:

- **PASSIVE ELEMENTS** which are not capable to generate energy (resistors, capacitors and inductors)
- **ACTIVE ELEMENTS** which are capable to generate energy (generators, batteries, operational amplifiers)

→ *Passive circuits and active circuits*

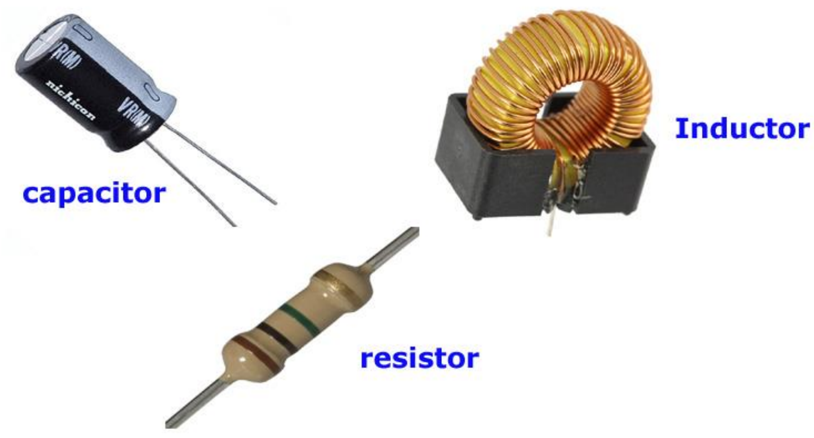
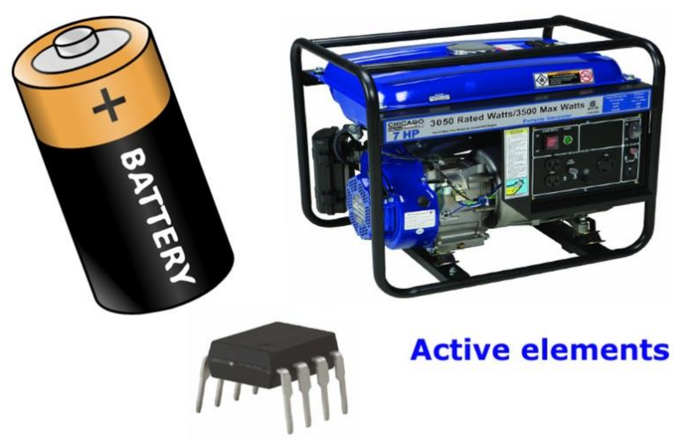


Figure: passive elements



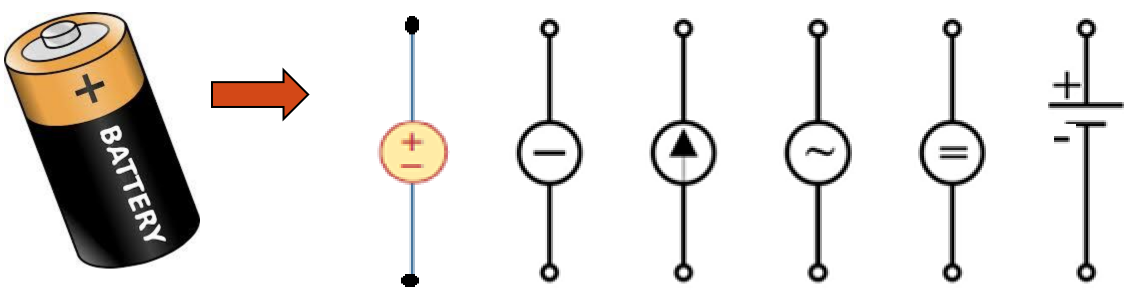
Source: [https://powerinception.com/]

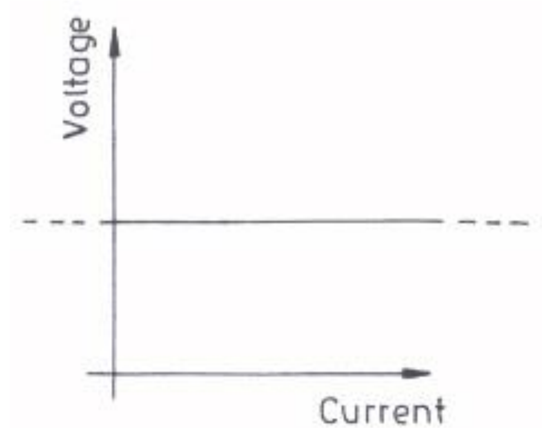
An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other elements.

Ideal → source has zero internal resistance

An ideal independent VOLTAGE source is an active element that delivers to the circuit whatever current is necessary to maintain its terminal voltage.

Element dipolar ideal, capabil să menţină între bornele sale o tensiune electrică independentă de curentul debitat



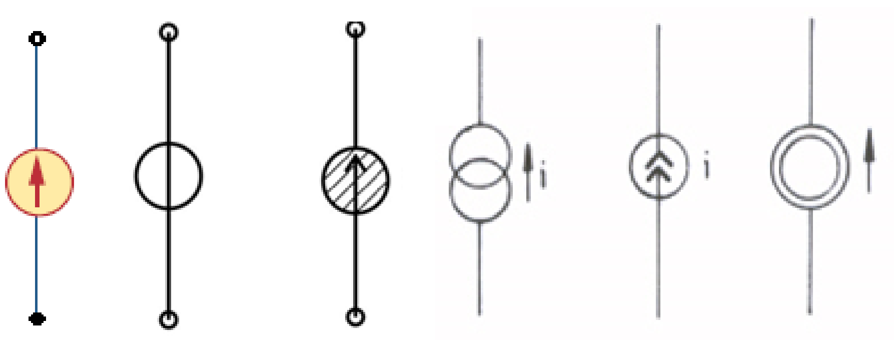


Voltage – current characteristic

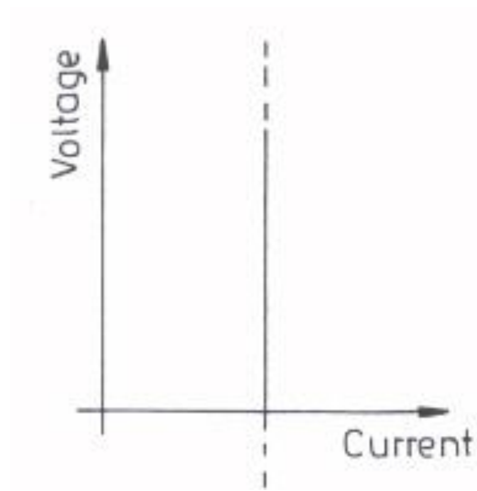
An ideal independent CURRENT source is an active element that provides a specified current completely independent of the voltage across the source.

Element dipolar ideal care debitează un curent de intensitate precizată independentă de tensiunea între bornele sale.

- The ideal current source delivers to the circuit whatever voltage is necessary to maintain the designated current.



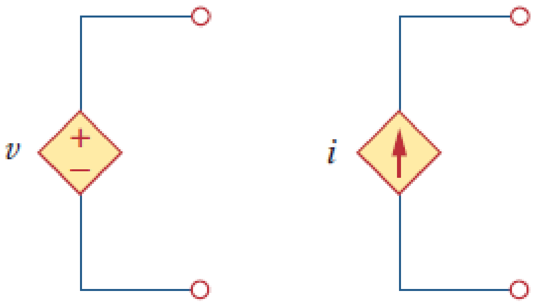
The arrows indicates the direction of current



Voltage – current characteristic

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

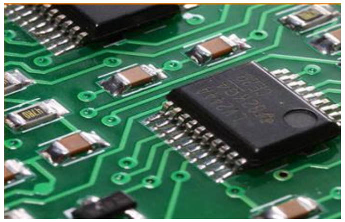
1. A voltage-controlled voltage source (VCVS).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS).

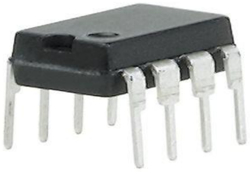


(a) (b)

Symbols for: (a) dependent voltage source, (b) dependent current source.

- Dependent sources are useful in modeling elements such us transistors, operational amplifiers and integrated circuits.





SUMMARY

1. An electric circuit consists of electrical elements connected together.
2. The International System of Units (SI) is the international measurement language, which enables engineers to communicate their results. From the seven principal units, the units of other physical quantities can be derived.
3. Current is the rate of charge flow past a given point in a given direction.

$$i = \frac{dq}{dt}$$

4. Voltage is the energy required to move 1 C of charge through an element.

$$u = \frac{dw}{dq}$$

5. Power is the energy supplied or absorbed per unit time. It is also the product of voltage and current.

$$p = \frac{dw}{dq} = u \cdot i$$

6. According to the passive sign convention, power assumes a positive sign when the current enters the positive polarity of the voltage across an element.
7. An ideal voltage source produces a specific potential difference across its terminals regardless of what is connected to it. An ideal current source produces a specific current through its terminals regardless of what is connected to it.
8. Voltage and current sources can be dependent or independent. A dependent source is one whose value depends on some other circuit variable.

REVIEW QUESTIONS

1.1 One millivolt is one millionth of a volt.

(a) True (b) False

1.2 The prefix *micro* stands for:

(a) 10^6 (b) 10^3 (c) 10^{-3} (d) 10^{-6}

1.3 The voltage 2,000,000 V can be expressed in powers of 10 as:

(a) 2 mV (b) 2 kV (c) 2 MV (d) 2 GV

1.4 A charge of 2 C flowing past a given point each second is a current of 2 A.

(a) True (b) False

1.5 The unit of current is:

(a) coulomb (b) ampere
(c) volt (d) joule

1.6 Voltage is measured in:

(a) watts (b) amperes
(c) volts (d) joules per second

1.7 A 4-A current charging a dielectric material will accumulate a charge of 24 C after 6 s.

(a) True (b) False

1.8 The voltage across a 1.1-kW toaster that produces a current of 10 A is:

(a) 11 kV (b) 1100 V (c) 110 V (d) 11 V

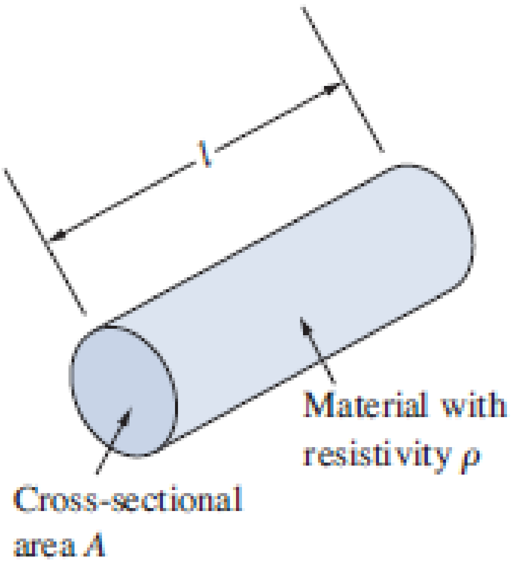
1.9 Which of these is not an electrical quantity?

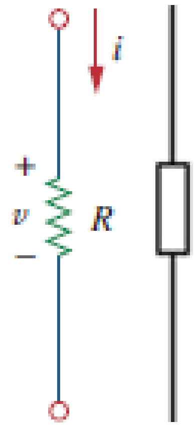
(a) charge (b) time (c) voltage
(d) current (e) power

Answers: 1.1b, 1.2d, 1.3c, 1.4a, 1.5b, 1.6c, 1.7a, 1.8c, 1.9b, 1.10d.

CHAPTER 1: DC CIRCUITS
2. BASIC LAWS

- Materials in general have the characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist current is kown as *resistance (R)*.





$$R = \rho \frac{l}{A}$$

- *ρ* is known as resistivity of the material (ohm/m)
- *A* cross section area of the material (m·m)
- *L* is the lenght of the material (m)

Resistivities of common materials.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

	1st	2nd	3rd	Multiplier		Tolerance	
0	Black	Black	Black	Black	1		
1	Brown	Brown	Brown	Brown	10^1	Brown	1%
2	Red	Red	Red	Red	10^2	Red	2%
3	Orange	Orange	Orange	Orange	10^3		
4	Yellow	Yellow	Yellow	Yellow	10^4		
5	Green	Green	Green	Green	10^5		
6	Blue	Blue	Blue	Blue	10^6		
7	Violet	Violet	Violet	Violet	10^7		
8	Grey	Grey	Grey	Grey	10^8		
9	White	White	White	White	10^9		
				Gold	0.1	Gold	5%
				Silver	0.01	Silver	10%

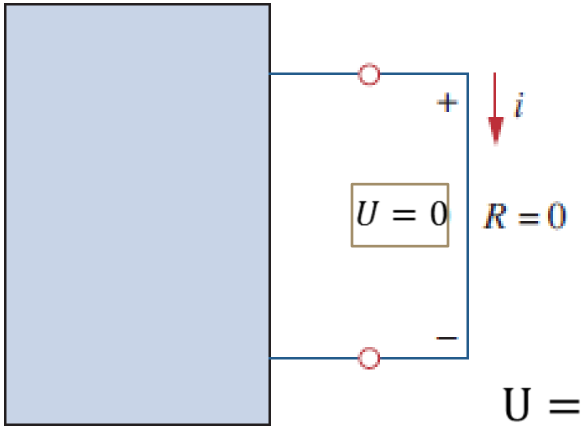
1st 2nd 3rd Multiplier Tolerance

Ohm's Law states that the voltage *V* across a resistor is directly proportional to the current *I* flowing through the resistor.

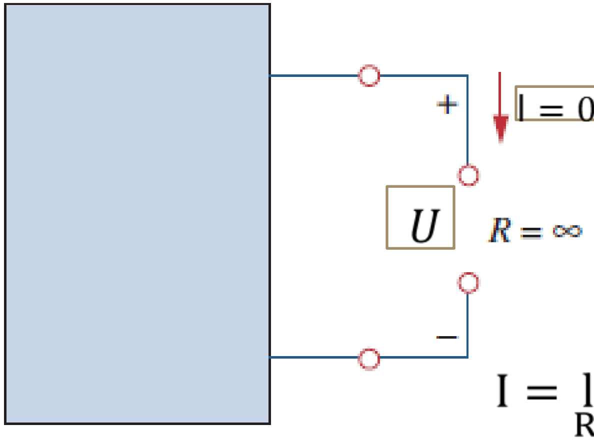
$$U = I \cdot R$$

$$R = \frac{U}{I} \quad 1\ \Omega = 1\ \text{V/A}$$

- Since the value of R can range from zero to infinity, it is important that we consider the two extreme possible values of R.



$$U = IR = 0$$

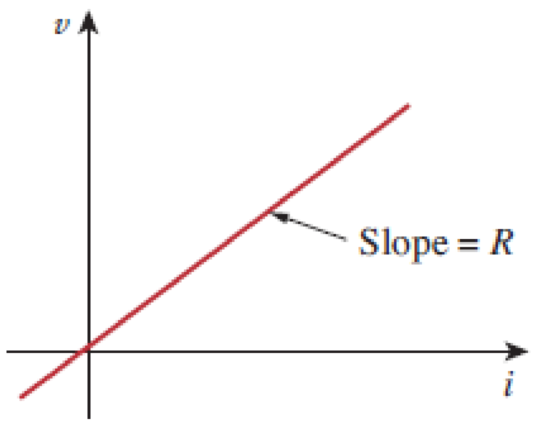


$$I = \lim_{R\to\infty} \frac{U}{R} = 0$$

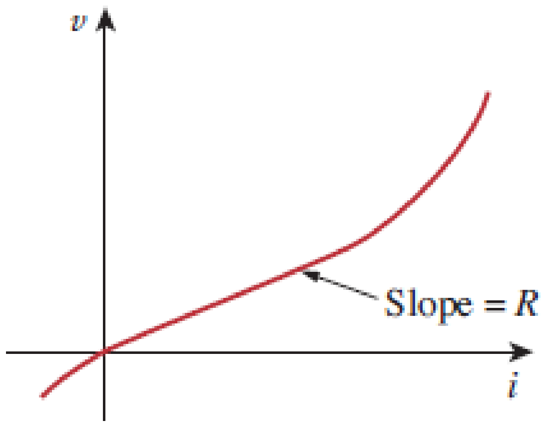
A short circuit is a circuit element with resistance approaching to zero.

An open circuit is a circuit element with resistance approaching infinity.

- A resistors that obeys Ohm's Law is kown as a linear resistor.



The current – voltage characteristic of a linear resistor



The current – voltage characteristic of a nonlinear resistor

- A resistors that not obeys Ohm's Law is kown as a nonlinear resistor (its resistance varies with current).

→ *Linear circuits and non-linear circuits*

Historical

Georg Simon Ohm (1787–1854), a German physicist, in 1826 experimentally determined the most basic law relating voltage and current for a resistor. Ohm's work was initially denied by critics.

Born of humble beginnings in Erlangen, Bavaria, Ohm threw himself into electrical research. His efforts resulted in his famous law. He was awarded the Copley Medal in 1841 by the Royal Society of London. In 1849, he was given the Professor of Physics chair by the University of Munich. To honor him, the unit of resistance was named the ohm.

Conductance G is the ability to conduct the flow of electric current; it is measured in siemens (S).

$$G = \frac{1}{R} = \frac{I}{U}$$

$$1S = \frac{1}{1\Omega} = \frac{1A}{1V}$$

- The power disipated by a resistor can be expressed in terms of R:

$$P = U \cdot I = I^2 R = \frac{U^2}{R}$$

- The power disipated by a resistor can be expressed in terms of G:

$$P = U \cdot I = U^2 G = \frac{I^2}{G}$$

EXAMPLE 2.1.

An electric iron draws 2 A at 120 V. Find its resistance.

Solution:

From Ohm's law,

$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

PRACTICE PROBLEM 2.1.

The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 15 Ω at 110 V?

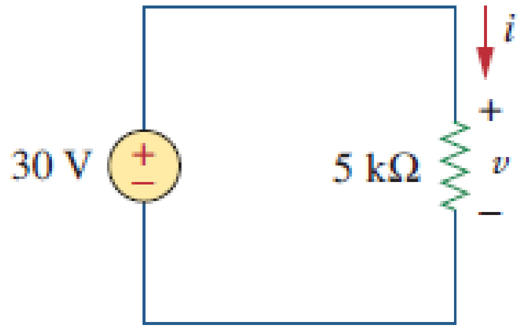
Answer: 7.333 A.

EXAMPLE 2.2.

In the circuit shown in Fig. [redacted], calculate the current i, the conductance G, and the power p.

Solution:

The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is



$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

We can calculate the power in various ways using either Eqs. (1.7), (2.10), or (2.11).

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

or

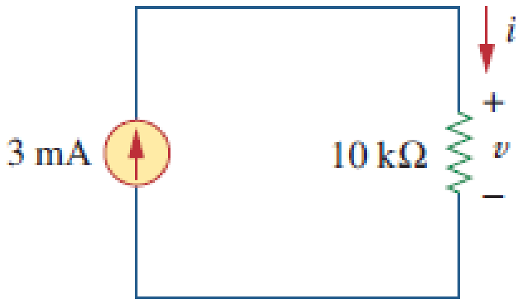
$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180 \text{ mW}$$

or

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180 \text{ mW}$$

PRACTICE PROBLEM 2.2.

For the circuit shown in Fig. calculate the voltage v, the conductance G, and the power p.

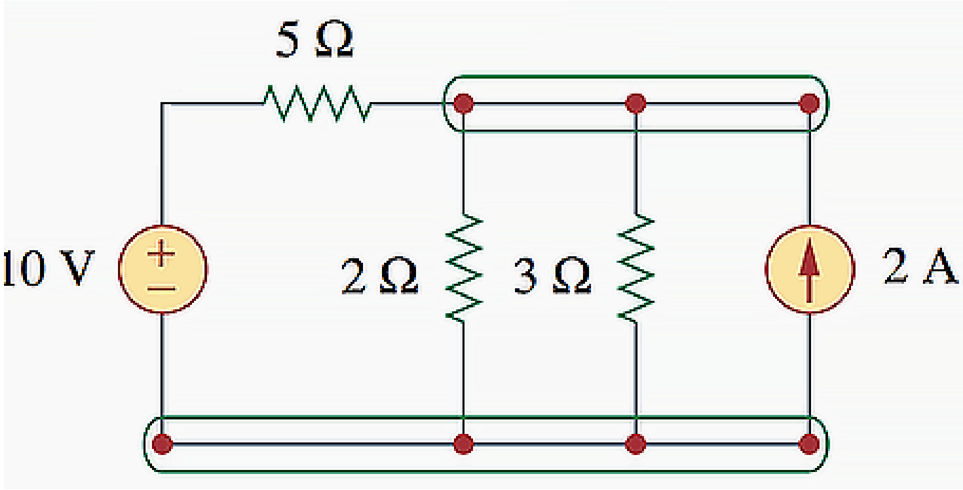


Answer: 30 V, 100 μS, 90 mW.

A node *(n)* is the point of connection between more than two branches.

A branch *(b)* is the connections between two nodes.

A loop *(l)* is any closed path in a circuit.



$$b = l + n - 1$$

b=4
l=3
n=2

Kirchhoff's current law (KCL) applies to the nodes of a network and states that the algebraic sum of the currents at a node is zero.

$$\sum_{n=1}^{N} I_n = 0$$

(there are *n – 1* independent nodes or equations)

- We shall assign positive polarity (+) to a current leaving a node, and negative polarity (-) to a current entering a node.

Kirchhoff's voltage law (KVL) the algebric sum of all voltages around a closed path (or loop) iz zero.

$$\sum_{m=1}^{M} U_m = 0$$

Sum of voltage drops = sum of voltage rise

(there are *l – n + 1* independent loops or equations)

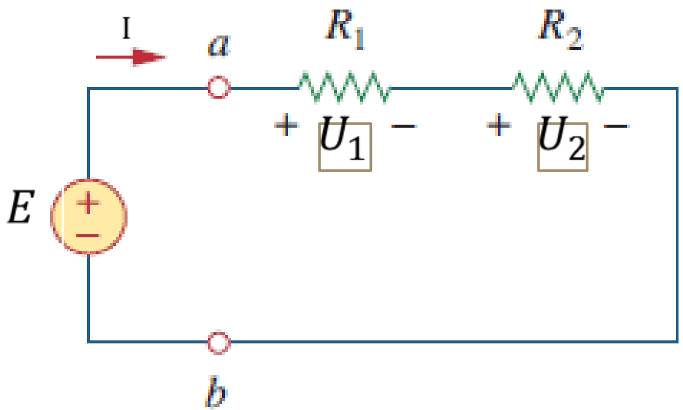
For Examples and Practice problems
using Ohm's Law, KCL and KVL, see SEMINAR 1

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

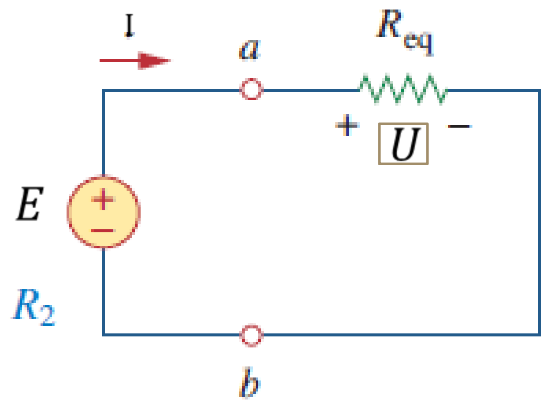
$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

$$U_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N}$$

Principle of voltage division



$$R_{eq} = R_1 + R_2$$



$$U = IR_{eq}$$

$$I = \frac{U}{R_1 + R_2}$$

$$U_1 = IR_1 \qquad U_2 = IR_2$$

$$-U + U + U_2 = 0$$

$$U = U_1 + U_2 = I(R_1 + R_2)$$

$$U_1 = \frac{R_1}{R_1 + R_2} U \qquad U_2 = \frac{R_2}{R_1 + R_2} U$$

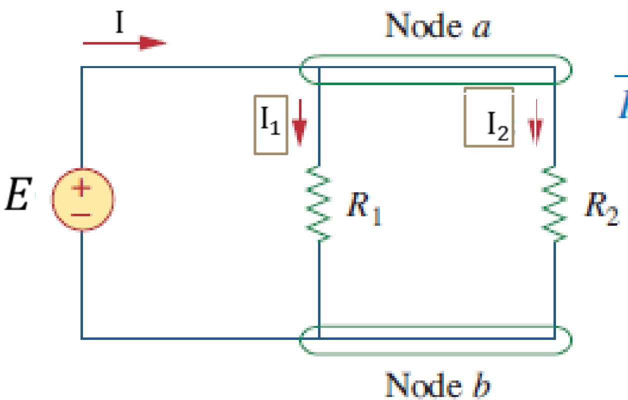
The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

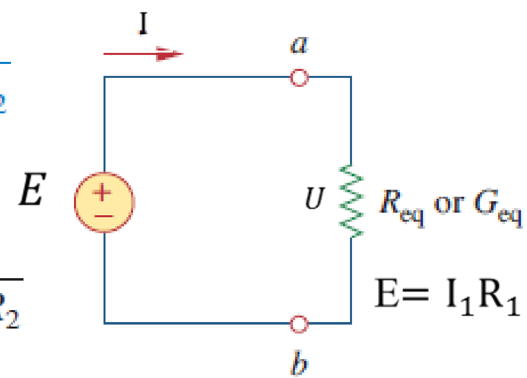
$$I_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} I$$

Principle of current division



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



$$E = I_1 R_1 = I_2 R_2$$

$$I_1 = \frac{E}{R_1} \qquad I_2 = \frac{E}{R_2}$$

$$I = \frac{E}{R_1} + \frac{E}{R_2} = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_{eq}}$$

$$I_1 = \frac{R_2 \cdot I}{R_1 + R_2} \qquad I_2 = \frac{R_1 \cdot I}{R_1 + R_2}$$

- This situasions arise in circuits analysis when the resistors are neither in parallel nor in series.
- Many circuits of the type shown in Fig.1 (a bridge network), can pe simplified by using three-terminal equivalent networks such: the wye (Y) or tee (T) network, and delta (Δ) or pi (Π) network.

- They are used in three-phase networks, electrical filters and matching networks

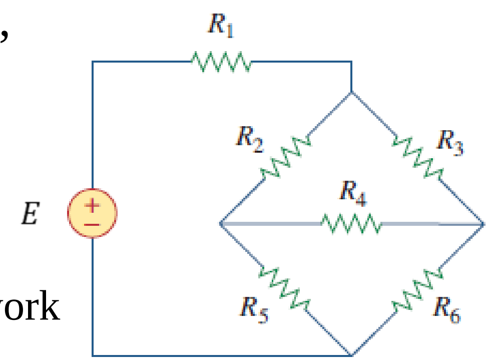
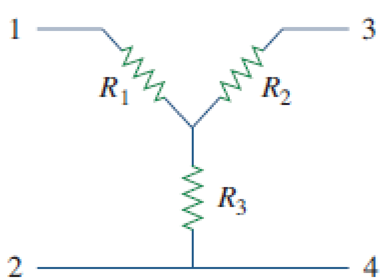
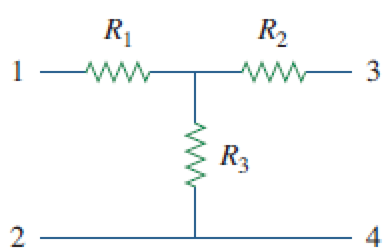


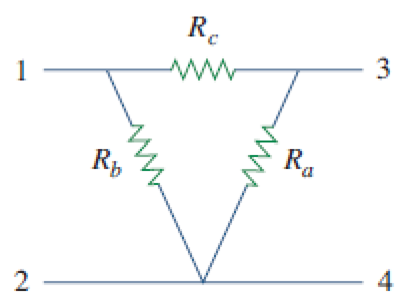
Fig.1 The bridge network



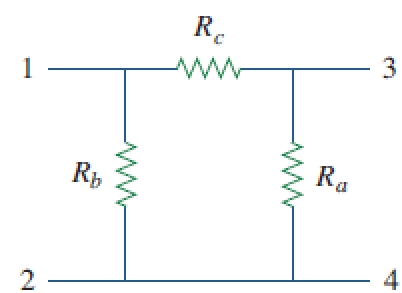
Wye (Y) network



Tee (T) network

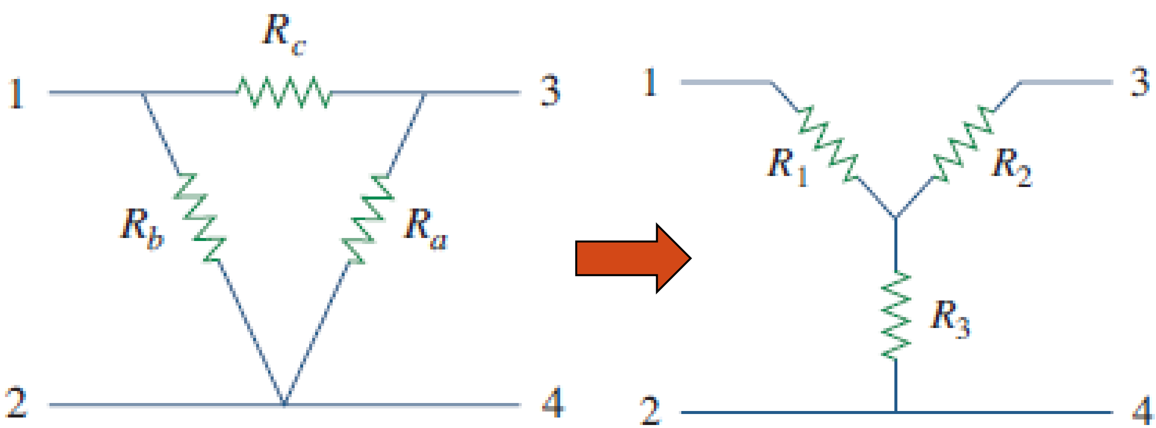


Delta (Δ) network



Pi (Π) network

➢ DELTA TO WYE CONVERSION



Delta (Δ) network

Wye (Y) network

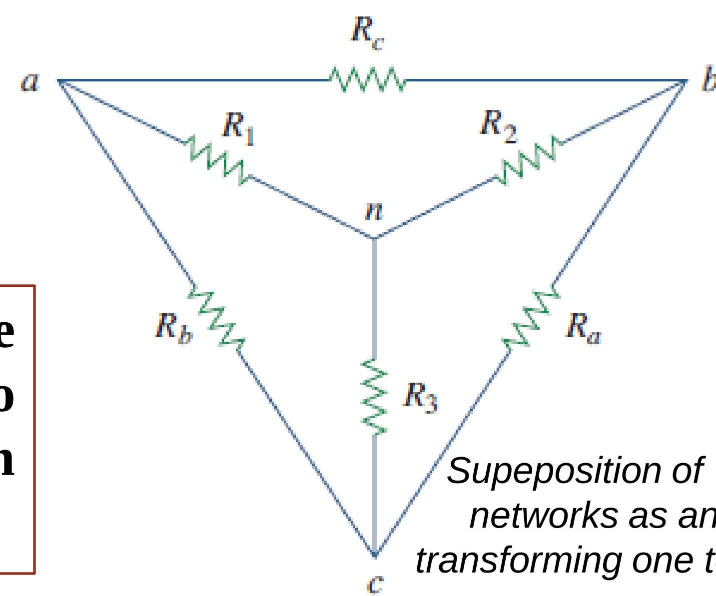
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

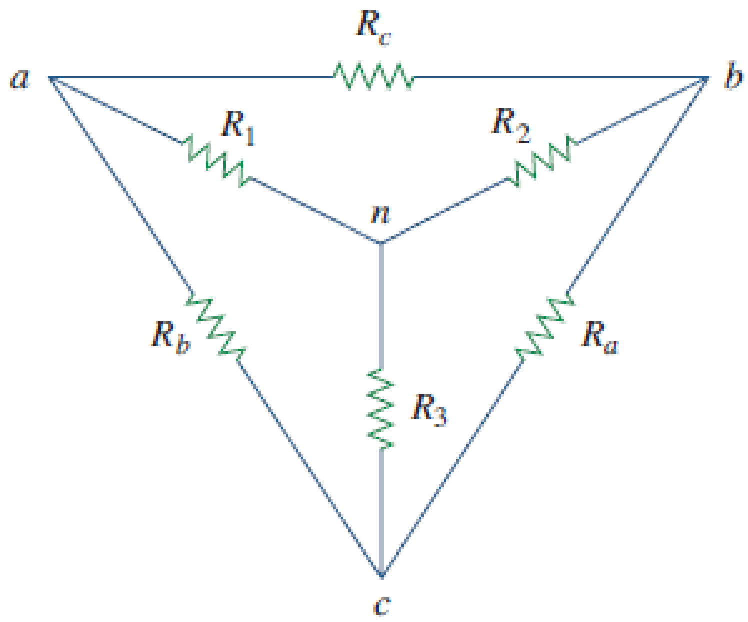
- We not need to memorize equations; to transform Δ in Y, we create an extra node *n* and follow this conversion rule:

Each resistor in the Y network is the product of the resistors in the two adiacent Δ branches, divided by the sum of the three Δ resistors.



Supeposition of Y and Δ networks as an aid in transforming one to another

➢ WYE TO DELTA CONVERSION



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

- We not need to memorize equations; to transform Δ in Y, we create an extra node *n* and follow this conversion rule:

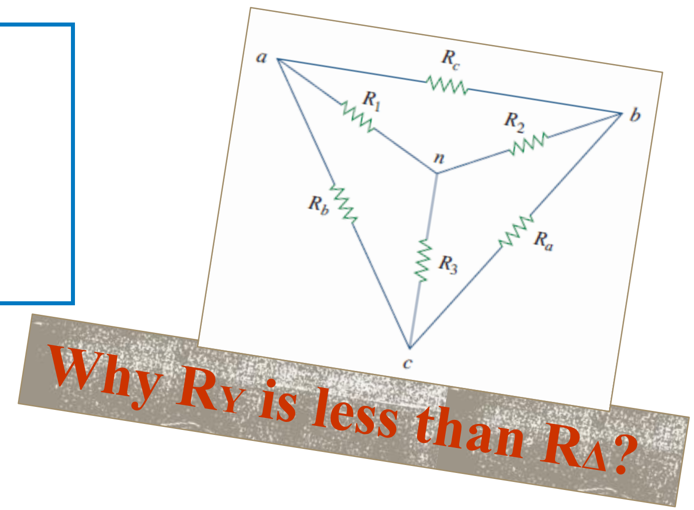
Each resistor in the Δ network is the sum of the all possible products of Y resistors taken two time, divided by the opposite Y resistor.

The Δ and Y networks are said to be balanced when:

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta$$

Under this conditions, conversion formula become:

$$R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y$$



KEEP IN MIND:

In making Δ/Y transformation, we do not take anything out of the circuit or put anything new. We are merely substituting different but mathematically equivalent three-terminal networks patterns to create a circuit in which resistors are either in series or in parallel, allowing us to calculate *Req* if necessary.

For Examples and Practice problems regarding calculation of R_{eq}, G_{eq} and Δ/Y conversion, see SEMINAR 1

SUMMARY

1. A resistor is a passive element in which the voltage v across it is directly proportional to the current i through it. That is, a resistor is a device that obeys Ohm's law,

$$v = iR$$

where R is the resistance of the resistor.

2. A short circuit is a resistor (a perfectly, conducting wire) with zero resistance ($R = 0$). An open circuit is a resistor with infinite resistance ($R = \infty$).
3. The conductance G of a resistor is the reciprocal of its resistance:

$$G = \frac{1}{R}$$

4. A branch is a single two-terminal element in an electric circuit. A node is the point of connection between two or more branches. A loop is a closed path in a circuit. The number of branches b, the number of nodes n, and the number of independent loops l in a network are related as

$$b = l + n - 1$$

SUMMARY

5. Kirchhoff's current law (KCL) states that the currents at any node algebraically sum to zero. In other words, the sum of the currents entering a node equals the sum of currents leaving the node.
6. Kirchhoff's voltage law (KVL) states that the voltages around a closed path algebraically sum to zero. In other words, the sum of voltage rises equals the sum of voltage drops.
7. Two elements are in series when they are connected sequentially, end to end. When elements are in series, the same current flows through them ($i_1 = i_2$). They are in parallel if they are connected to the same two nodes. Elements in parallel always have the same voltage across them ($v_1 = v_2$).
8. When two resistors $R_1\ (=1/G_1)$ and $R_2\ (=1/G_2)$ are in series, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are

$$R_{eq} = R_1 + R_2, \qquad G_{eq} = \frac{G_1 G_2}{G_1 + G_2}$$

SUMMARY

9. When two resistors $R_1\,(=1/G_1)$ and $R_2\,(=1/G_2)$ are in parallel, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}, \qquad G_{eq} = G_1 + G_2$$

10. The voltage division principle for two resistors in series is

$$v_1 = \frac{R_1}{R_1 + R_2}v, \qquad v_2 = \frac{R_2}{R_1 + R_2}v$$

11. The current division principle for two resistors in parallel is

$$i_1 = \frac{R_2}{R_1 + R_2}i, \qquad i_2 = \frac{R_1}{R_1 + R_2}i$$

12. The formulas for a delta-to-wye transformation are

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \qquad R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

SUMMARY

13. The formulas for a wye-to-delta transformation are

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}, \qquad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

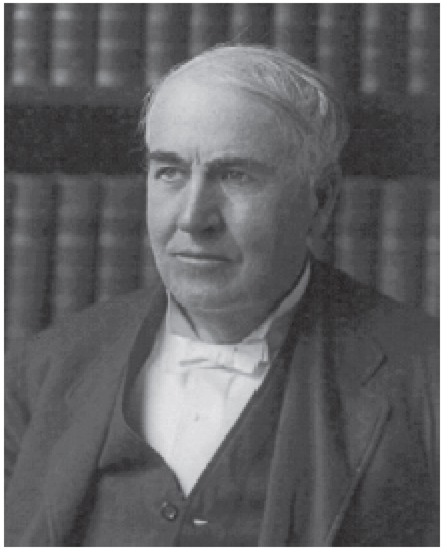
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

14. The basic laws covered in this chapter can be applied to the problems of electrical lighting and design of dc meters.

Historical

Thomas Alva Edison (1847–1931) was perhaps the greatest American inventor. He patented 1093 inventions, including such history-making inventions as the incandescent electric bulb, the phonograph, and the first commercial motion pictures.

Born in Milan, Ohio, the youngest of seven children, Edison received only three months of formal education because he hated school. He was home-schooled by his mother and quickly began to read on his own. In 1868, Edison read one of Faraday's books and found his calling. He moved to Menlo Park, New Jersey, in 1876, where he managed a well-staffed research laboratory. Most of his inventions came out of this laboratory. His laboratory served as a model for modern research organizations. Because of his diverse interests and the overwhelming number of his inventions and patents, Edison began to establish manufacturing companies for making the devices he invented. He designed the first electric power station to supply electric light. Formal electrical engineering education began in the mid-1880s with Edison as a role model and leader.



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REVIEW QUESTIONS

2.1 The reciprocal of resistance is:

(a) voltage (b) current
(c) conductance (d) coulombs

2.2 An electric heater draws 10 A from a 120-V line. The resistance of the heater is:

(a) 1200 Ω (b) 120 Ω
(c) 12 Ω (d) 1.2 Ω

2.3 The voltage drop across a 1.5-kW toaster that draws 12 A of current is:

(a) 18 kV (b) 125 V
(c) 120 V (d) 10.42 V

2.4 The maximum current that a 2W, 80 kΩ resistor can safely conduct is:

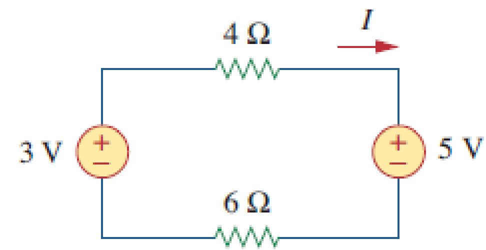
(a) 160 kA (b) 40 kA
(c) 5 mA (d) 25 μA

2.5 A network has 12 branches and 8 independent loops. How many nodes are there in the network?

(a) 19 (b) 17 (c) 5 (d) 4

2.6 The current I in the circuit of Fig. 2.63 is:

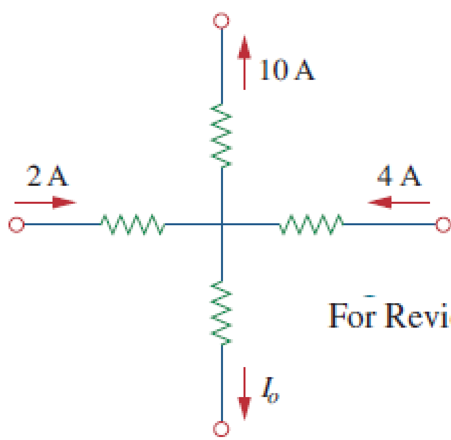
(a) −0.8 A (b) −0.2 A
(c) 0.2 A (d) 0.8 A



For Review Question 2.6.

2.7 The current I_o of Fig. 2.64 is:

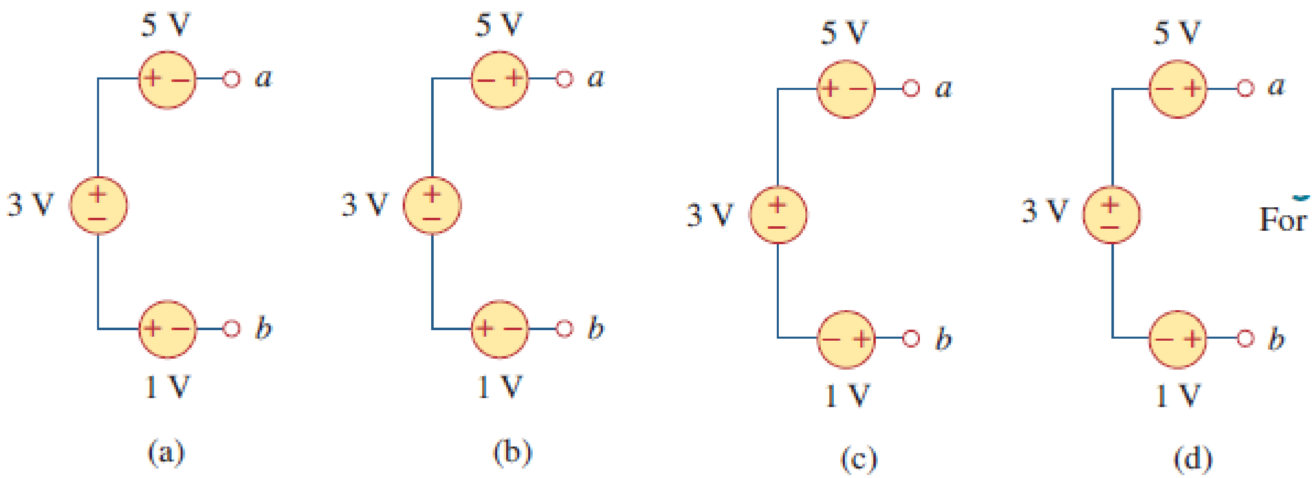
(a) −4 A (b) −2 A (c) 4 A (d) 16 A



For Review Question 2.7.

REVIEW QUESTIONS

2.9 Which of the circuits in Fig. 2.66 will give you $V_{ab} = 7$ V?



For Review Question 2.9.

2.10 In the circuit of Fig. 2.67, a decrease in R_3 leads to a decrease of, select all that apply:

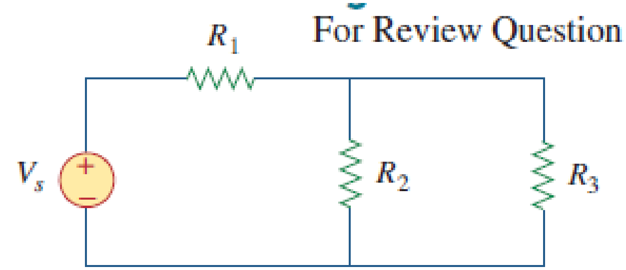
(a) current through R_3

(b) voltage across R_3

(c) voltage across R_1

(d) power dissipated in R_2

(e) none of the above



For Review Question 2.10.

Answers: 2.1c, 2.2c, 2.3b, 2.4c, 2.5c, 2.6b, 2.7a, 2.8d, 2.9d, 2.10b, d.

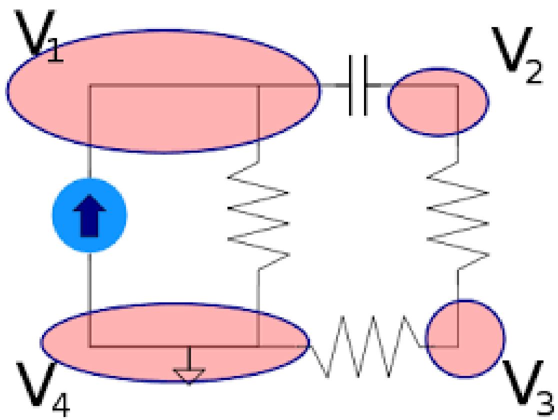
CHAPTER 1: DC CIRCUITS

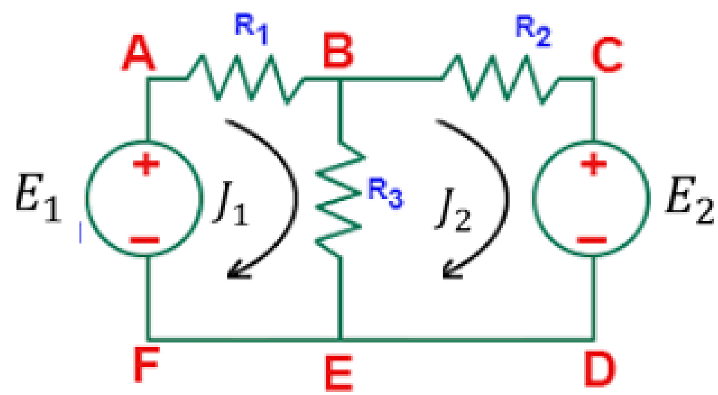
3. Methods of analysis

BASES OF ELECTROTECHNICS I.
Faculty of Electronics, Telecommunications and Information Technology
Specialization: IETTI
Academic year: 2023-2024

Content of this Subchapter:

1. Nodal Analysis
2. Mesh Analysis





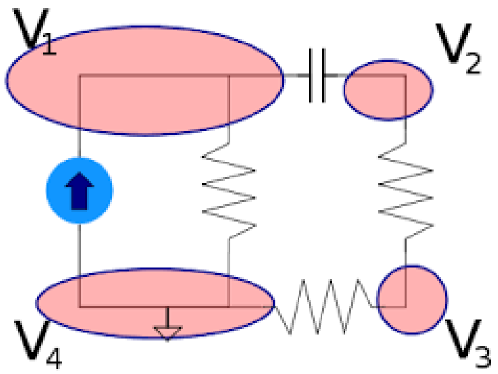
!!! With this two techniques, any linear circuit can be analyzed by obtaining a set of simultaneous equations that are then solved to obtain the required values of current or voltage.

Nodal Analysis

Nodal Analysis, Node-Voltage Analysis, or the Branch Current Method

- Nodal analysis provide a general procedure for analyzing circuits using node voltages as the circuit variables.

- Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.

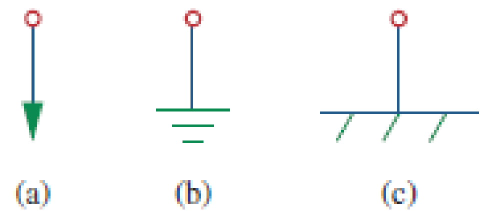


- Nodal analysis is possible when all the circuit elements' branch constitutive relations have a conductance representation. Nodal analysis produces a compact set of equations for the network, which can be solved by hand for "small circuits", or can be quickly solved using linear algebra by computer.

- Because of the compact system of equations, many circuit simulation programs (e.g. SPICE Module from ORCAD software) use nodal analysis as a basis.

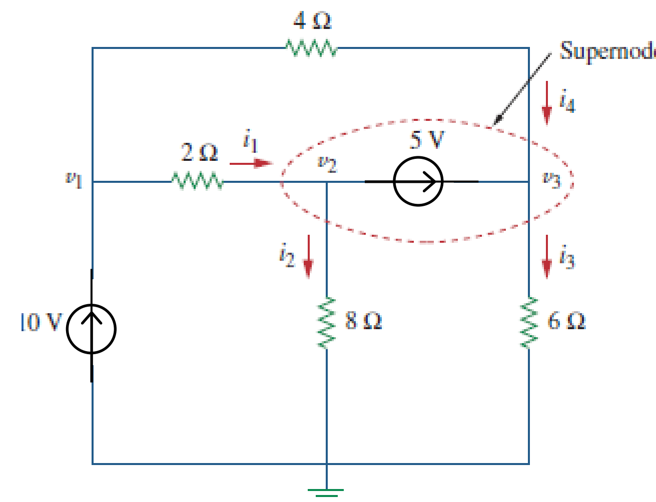
Steps to compute currents using Node Analysis:

1. Select a node as the reference node. Assign voltages V_1, V_2, . . . , $V_{(N-1)}$ to the remaining (N-1) nodes.



Common symbols for indicating a reference node, (a) common ground, (b) ground, (c) chassis ground.

- ✓ Take as reference node the node with most branches connecting to it.
- ✓ If a voltage source is connected between the reference node and a non-reference node, we simply set the voltage at the non-reference node equal to the voltage source.



2. Write the system of equations specific to the Node Analysis.

$$\begin{cases} G_{11}V_1 - G_{12}V_2 - \cdots - G_{1(N-1)}V_{(N-1)} = \sum\limits_{k\in node1} I_k + \sum\limits_{j\in node1} G_j E_j \\ -G_{21}V_1 + G_{22}V_2 - \cdots - G_{2(N-1)}V_{(N-1)} = \sum\limits_{k\in node2} I_k + \sum\limits_{j\in node2} G_j E_j \\ \ldots \\ -G_{(N-1)1}V_1 - G_{(N-1)2}V_2 - \cdots + G_{(N-1)(N-1)}V_{(N-1)} = \sum\limits_{k\in node(N-1)} I_k + \sum\limits_{j\in node(N-1)} G_j E_j \end{cases}$$

G_{aa} - the self-conductance of node *a*

G_{ab} - the mutual conductance between node *a* and node *b*

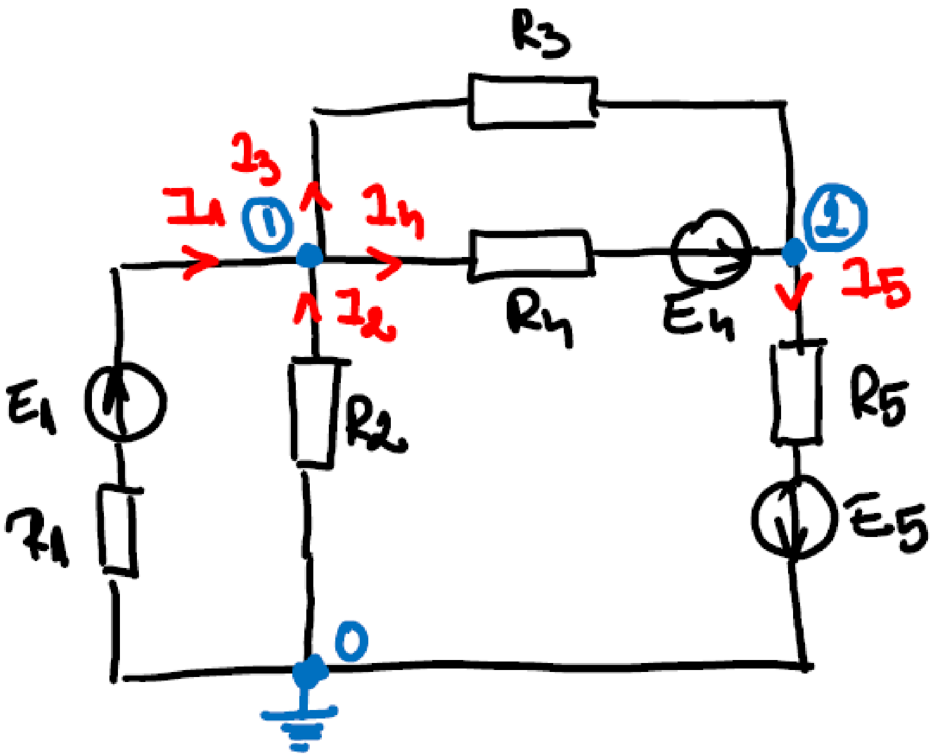
$\sum G_j E_j$ - sum of the currents due to current sources connected to node *k*

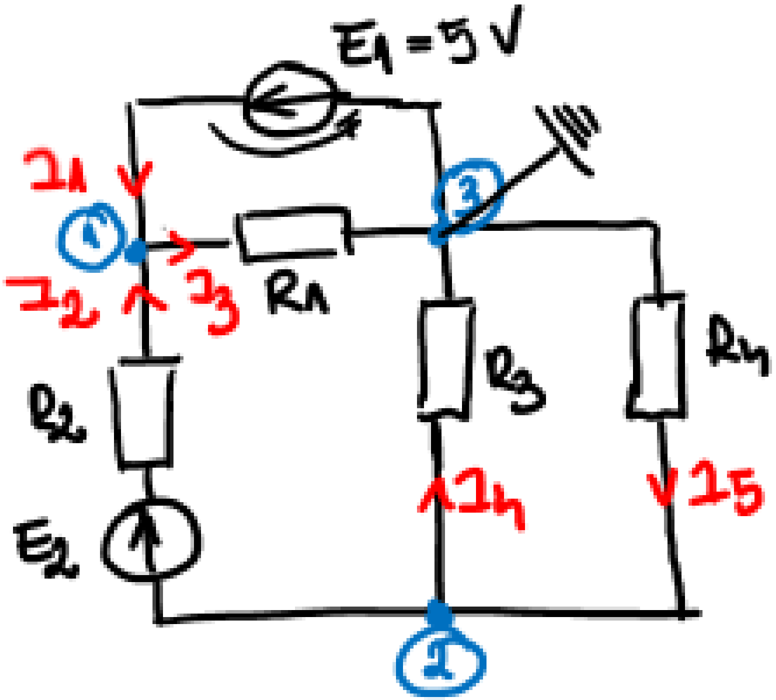
$\sum I_k$ - sum of the currents due to voltage sources connected to node *k*

(sign + for currents that enters node; sign – for currents that leaves node)

3. Compute the above system of equations in order to compute de Node Voltages.

4. Compute the currents by applying Omh's Law for each branch of the circuit.


R3
I3
I1
1
I4
2
I5
I2
R4
E4
E1
R2
R5
E5
0

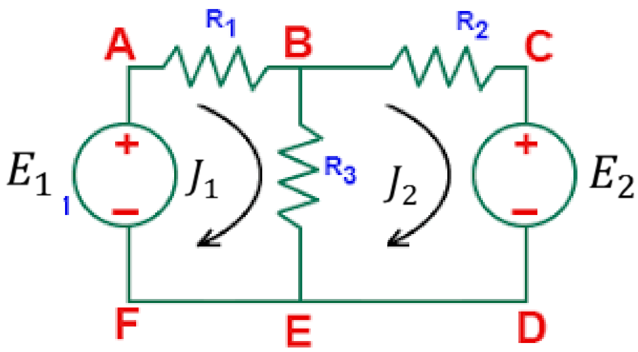

E1 = 5 V
I1
I2
I3
R1
R2
R3
R4
E2
I4
I5
1
2
3

Mesh Analysis

Loop Analysis, Mesh Current Method or Maxwell´s Circulating Currents Method

- Is a method that is used to solve planar circuits for the currents (and indirectly the voltages) at any place in the electrical circuit.

- Using mesh currents instead of directly applying KCL and KVL can greatly reduce the amount of calculation required.



This is because there are fewer mesh currents than there are physical branch currents.

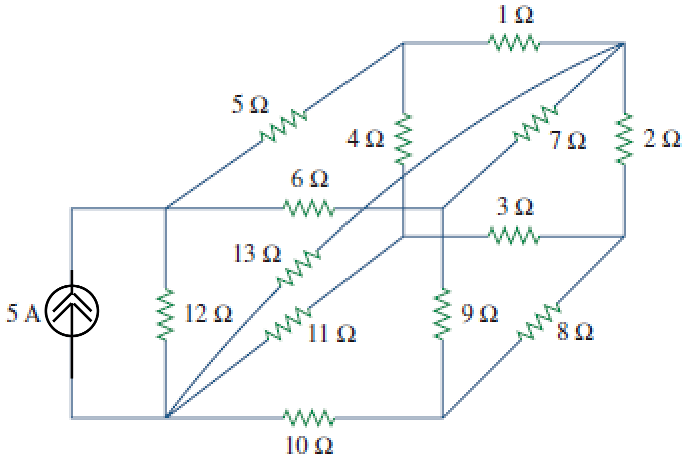
- Mesh Analysis is a very handy tool to compute current within electronic circuits. From knowing the current within each mesh (section), we can solve for voltage and power (watts) at each component.

Ex: Engineers and designers use this information to select correct parts that won't emit the magic white smoke when power is applied.

Magic smoke is a humorous name for the caustic smoke produced by burning out electronic circuits or components (usually by overheating, overdamping, or incorrect wiring configurations), which is held to contain the essence of the component's function. The smoke typically smells of burning plastic and other chemicals, and sometimes contains specks of sticky black ash. The color of the smoke depends on which component is overheating, but it is commonly white or grey. Simple overheating eventually results in component failure, but does not release smoke. Real smoke is almost always the result of incorrect wiring or a manufacturing failure in the component.

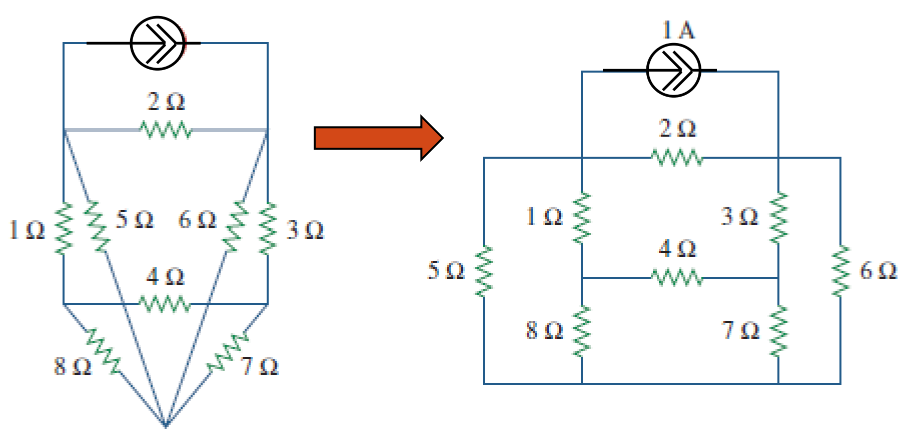
✓ Mesh Analysis is not as general as Nodal Analysis because is it only applicable to a circuit that is planar

Planar circuit = circuit that can be drawn on a plane surface with no wires crossing each other; otherwise is *nonplanar circuit*.



Example of a nonplanar circuit

- A circuit may have crossing branches and still be planar if it can be redraw such that it has no crossing branches.



Mesh = loop which does not contain any other loops within it.

Steps to compute currents using Mesh Analysis:

STEP_1: **Identify "meshes" within the circuit encompassing all components. Label all the internal loops with circulating currents (mesh currents) (J_1 , J_2 , …, J_N etc.).**

- The direction of the mesh current is arbitrary (clockwise or counterclockwise) and does not affect the validity of the solution.

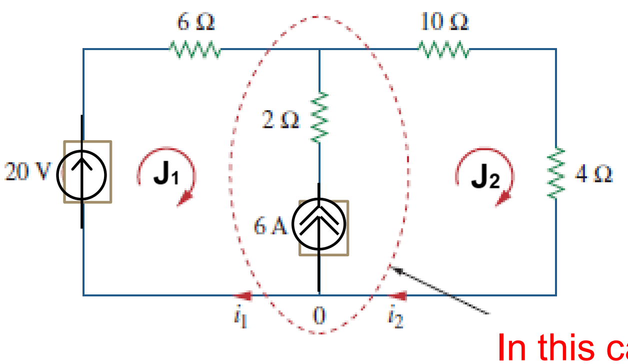
- ✓ As a general rule of thumb, only label inside loops in a clockwise direction with circulating currents as the aim is to cover all the elements of the circuit at least once.
- ✓ If the assumed direction of a mesh current is wrong, the answer for that current will have a negative value.

KEEP IN MIND:

Before you chose the mesh currents check in which of those 3 possible cases your circuit is fit:

- CASE 1: Circuit with no current sources
- CASE 2: When a current source exists only in one mesh
- CASE 3: When a current source exists between two meshes

Take into account in choosing mesh currents the rule of thumbs for each case.



(a)

In this case we must reconsider the meshes (CASE 3)

STEP_2: Write the system of equations specific to the Mesh Analysis.

$$\begin{cases} R_{11}J_1 + R_{12}J_2 + \cdots + R_{1N}J_N = \sum\limits_{k \in mesh1} E_k + \sum\limits_{j \in mesh1} R_j I_j \\ R_{21}J_1 + R_{22}J_2 + \cdots + R_{2N}J_N = \sum\limits_{k \in mesh2} E_k + \sum\limits_{j \in mesh2} R_j I_j \\ \cdots \\ R_{N1}J_1 + R_{N2}J_2 + \cdots + R_{NN}J_N = \sum\limits_{k \in meshN} E_k + \sum\limits_{j \in nodeN} R_j I_j \end{cases}$$

R_{NN} - self-resistance (the total resistance) of the Nth loop

R_{ab} - the mutual resistance between loop *a* and loop *b*

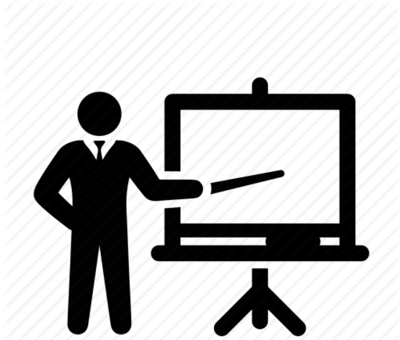
(with sign + if the mesh currents through the common resistance have the same direction, with sign – otherwise)

$\sum R_j I_j$ - sum of the sources contained by the mesh

$\sum E_k$ - sum of the voltages due to voltage sources contained by the mesh

STEP_3: Compute the above system of equations in order to compute de Mesh Currents.

STEP_4: Compute the currents through branches based on Mesh (loop) Currents.



For Examples and Practice problems Mesh Analysis see SEMINAR 2

SUMMARY

1. Nodal analysis is the application of Kirchhoff's current law at the nonreference nodes. (It is applicable to both planar and nonplanar circuits.) We express the result in terms of the node voltages. Solving the simultaneous equations yields the node voltages.
2. A supernode consists of two nonreference nodes connected by a (dependent or independent) voltage source.
3. Mesh analysis is the application of Kirchhoff's voltage law around meshes in a planar circuit. We express the result in terms of mesh currents. Solving the simultaneous equations yields the mesh currents.
4. A supermesh consists of two meshes that have a (dependent or independent) current source in common.
5. Nodal analysis is normally used when a circuit has fewer node equations than mesh equations. Mesh analysis is normally used when a circuit has fewer mesh equations than node equations.

CHAPTER 1: DC CIRCUITS

4. Circuit theorems

Content of this Subchapter:

1. Superposition Theorem
2. Source Transformation
3. Thevenin's Theorem
4. Norton's Theorem
5. Milman's Theorem
6. Maximum Power Transfer Theorem

For the following analysis methods:

- *Superposition Theorem*
- *Thevenin's Theorem*
- *Norton's Theorem*

PLEASE CHECK THE COURSE FROM SEMESTER 1:

Passive Electronic Components and Circuits (PECC)

Superposition Theorem

Superposition Theorem is one of those strokes of genius that takes a complex subject and simplifies it in a way that makes perfect sense.

Superposition theorem states that for a linear system the response (voltage or current) in any branch of a bilateral linear circuit having more than one independent source equals the algebraic sum of the responses caused by each independent source acting alone, where all the other independent sources are replaced by their internal impedances.

- The theorem is applicable to linear networks (time varying or time invariant) consisting of independent sources, linear dependent sources, linear passive elements (resistors, inductors, capacitors) and linear transformers.

- Superposition works for voltage and current but not power. In other words, the sum of the powers of each source with the other sources turned off is not the real consumed power. To calculate power we first use superposition to find both current and voltage of each linear element and then calculate the sum of the multiplied voltages and currents.

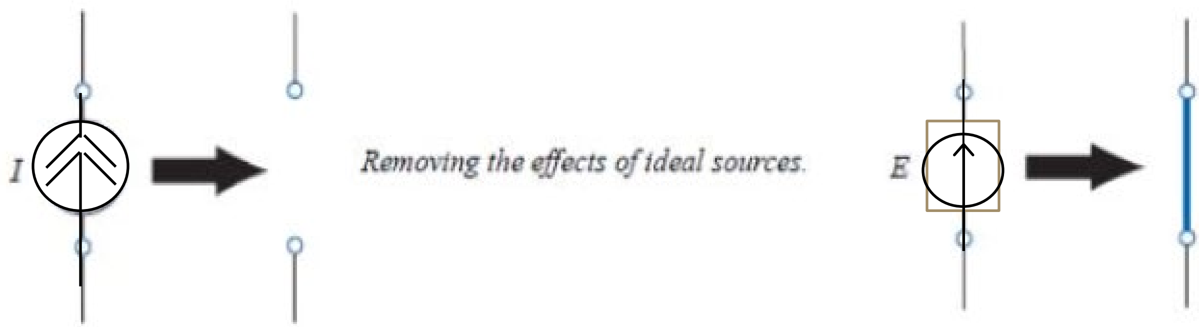
Steps to Apply Superposition Principle:

1. Turn off (set to zero) all independent sources except one by:

- Replacing all other independent voltage sources with a short circuit (thereby eliminating difference of potential i.e. U=0; internal impedance of ideal voltage source is zero (short circuit)).

- Replacing all other independent current sources with an open circuit (thereby eliminating current i.e. I=0; internal impedance of ideal current source is infinite (open circuit)).

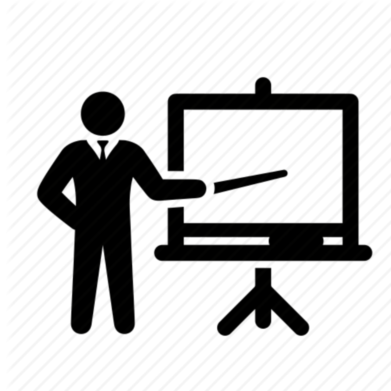
- Dependent sources are left intact because they are controlled by circuit variables.



2. Find the output (voltage or current) due to that active source using any techniques

3. Repeat steps 1 and 2 for each of the other independent source.

4. The total current through any portion of the circuit is equal to the algebraic sum of the currents produced by each independent source.



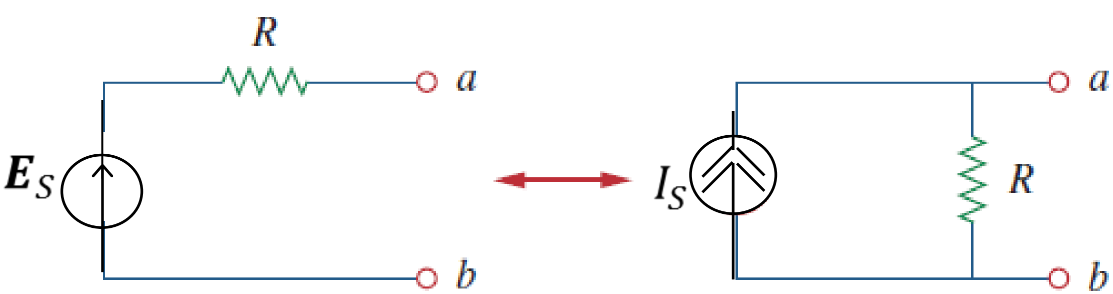
Access the below link to watch a video who explains with a numerical example Superposition Theorem:

https://www.youtube.com/watch?v=UwiaDe01s60

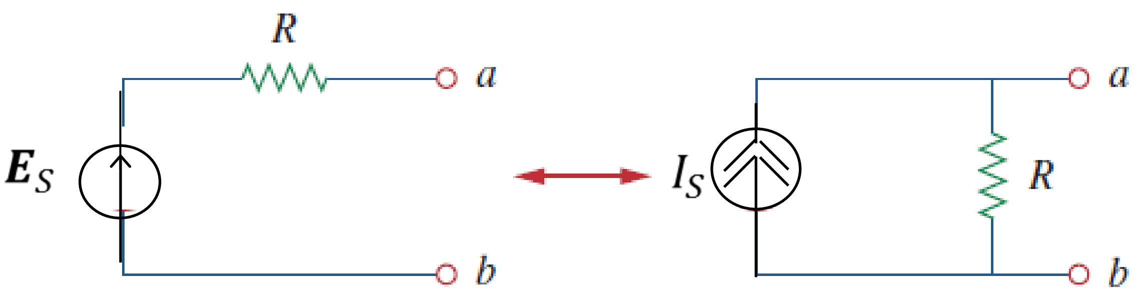
Source Transformation for Independent Sources

- When is applicable, source transformation is a powerful tool that allows circuit manipulations to ease circuit analysis.

A source transformation is the process of replacing a voltage source Us in series with a resistors R by a current source is in parallel with a resistor R, or vice versa.



A source transformation does not affect the remaining part of the circuit.



$$E_S = I_S \cdot R \quad \text{or} \quad I_S = \frac{E_S}{R}$$

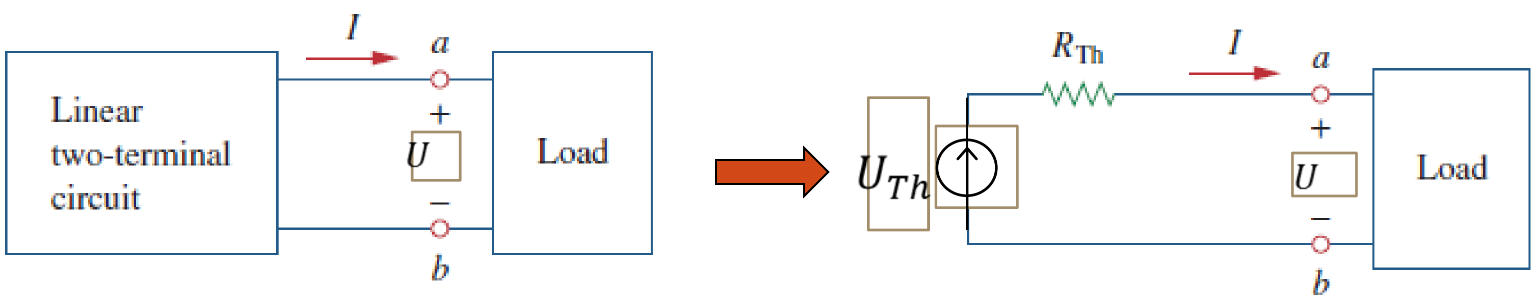
When dealing with source transformation, we should keep the following points in mind:

- The arrow of the current source is directed toward the positive terminal of the voltage source.
- Source transformation is not possible when R=0, which is the case with an ideal voltage source (for a practical, nonideal voltage source, R≠0).
- An ideal current source with R=∞ cannot be replaced by a finite voltage source.

Thevenin's Theorem

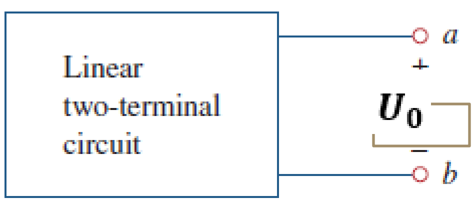
- Thevenin's Theorem is especially useful in analyzing power systems and other electronic circuits where one particular resistor in the circuit (called the "load" resistor) is subject to change, and re-calculation of the circuit is necessary with each trial value of load resistance, to determine voltage across it and current through it.

Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single voltage source and series resistance connected to a load.



Steps to Apply Thevenin’s Theorem:

1. Find the Thevenin source voltage by removing the load resistor from the original circuit and calculating voltage across the open connection points where the load resistor used to be.

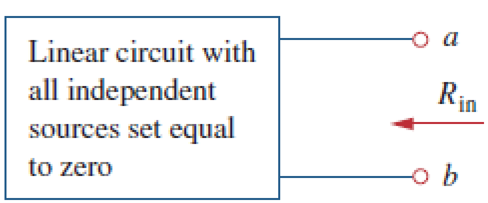


$U_{Th} = U_0$

$$U_{Th} = U_0$$

If terminals *a*-*b* are made open-circuited, no current flows, so that the open circuit voltage across the terminals *a*-*b* must be equal with the voltage source U_{TH} .

2. Find the Thevenin resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.



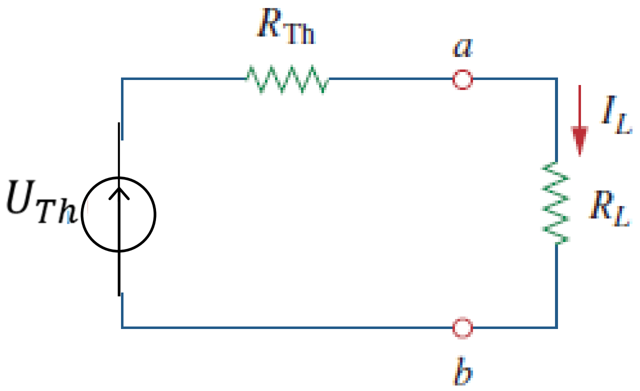
$R_{Th} = R_{in}$

$$R_{Th} = R_{in}$$

With the load disconnected and terminals *a*-*b* open-circuited, we turn-off all independent sources.

R_{TH} – is the input resistance at the terminals when the independent sources are turned off.

3. Draw the Thevenin equivalent circuit, with the Thevenin voltage source in series with the Thevenin resistance. The load resistor re-attaches between the two open points of the equivalent circuit

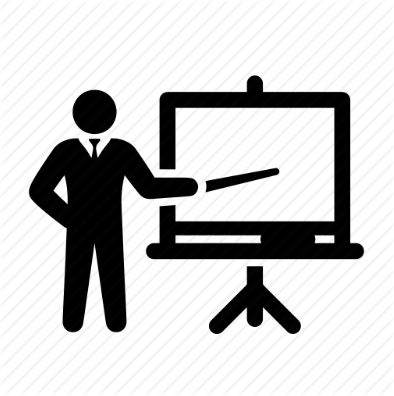


See in the figure that the Thevenin equivalent is a simple voltage divider.

2. Analyze voltage and current for the load resistor following the rules for series circuits.

$$I_L = \frac{U_{Th}}{R_{Th} + R_L}$$

$$U_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} U_{Th}$$



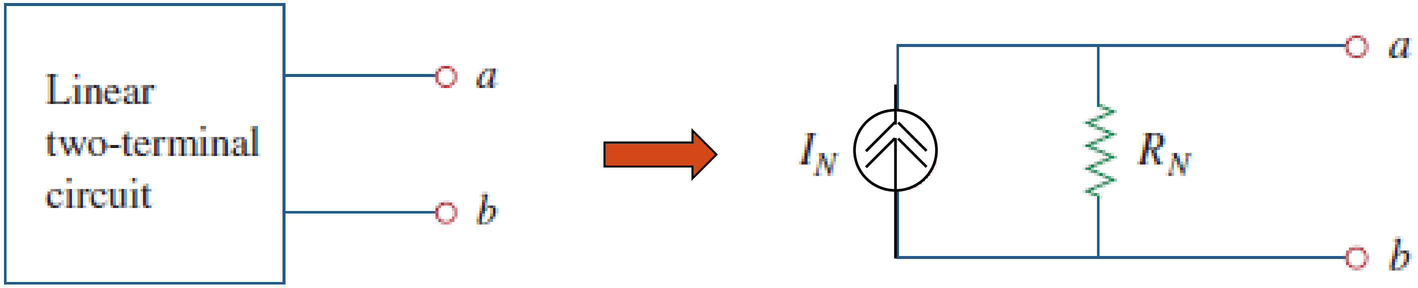
Access the below link to watch a video who explains with a numerical example Thevenin’s Theorem:

https://www.youtube.com/watch?v=Zqfi8SjmaBo

Norton's Theorem

- In 1936, about 43 years after Thevenin publish his theorem, E.L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single current source and parallel resistance connected to a load.



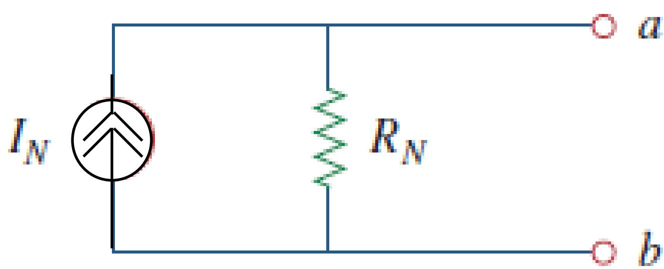
Steps to Apply Norton's Theorem

1. Find the Norton source current by removing the load resistor from the original circuit and calculating current through a short (wire) jumping across the open connection points where the load resistor used to be.

   As with Thevenin's Theorem, everything in the original circuit except the load resistance has been reduced to an equivalent circuit that is simpler to analyze. Also similar to Thevenin's Theorem are the steps used in Norton's Theorem to calculate the Norton source current (I_{Norton}) and Norton resistance (R_{Norton}).

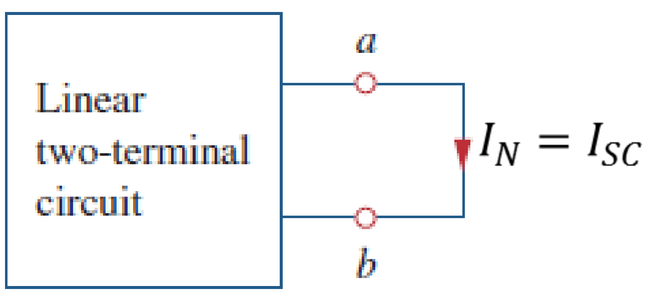
2. Find the Norton resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.

3. Draw the Norton equivalent circuit, with the Norton current source in parallel with the Norton resistance. The load resistor re-attaches between the two open points of the equivalent circuit.



$$R_N = R_{Th}$$

4. Analyze voltage and current for the load resistor following the rules for parallel circuits.



$$I_N = I_{SC}$$

- ✓ The Thevenin and Norton equivalent circuits are related by a source transformation which is often called Norton-Thevenin transformation.

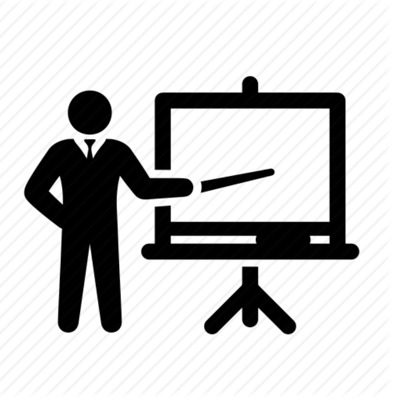
$$I_N = \frac{U_{Th}}{R_{Th}}$$

- ✓ Since V_{TH}, I_N and R_{TH} are related according above equation, to determine the Thevenin or Norton equivalent circuit requires that we find:
  - The open-circuit voltage U_0 across terminals *a* and *b*.
  - The short-circuit current I_{SC} at terminals *a* and *b*.
  - The equivalent or input resistance R_{in} at terminals *a* and *b* when all independent sources are turned off.

$$U_{Th} = U_0$$

$$I_N = I_{SC}$$

$$R_{Th} = \frac{U_0}{I_{sc}} = R_N$$



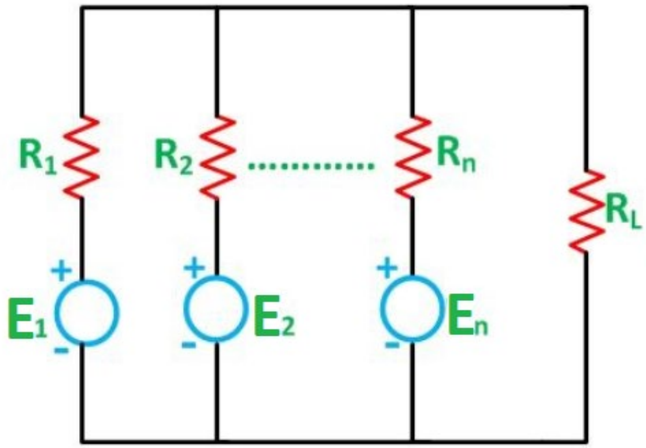
Access the below link to watch a video who explains with a numerical example Norton’s Theorem:

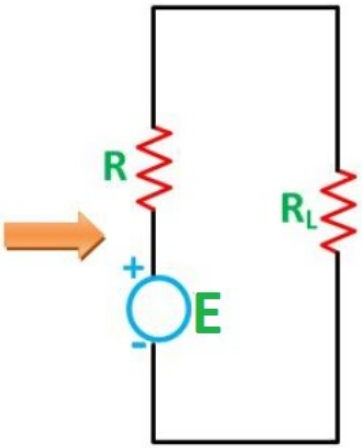
https://www.youtube.com/watch?v=bu4HR8b_QKI

Millman's Theorem

- A theorem which helps in simplifying electrical networks with a bunch of parallel branches.
- Can be used to find the potential difference between two points of a network which contains only parallel branches.

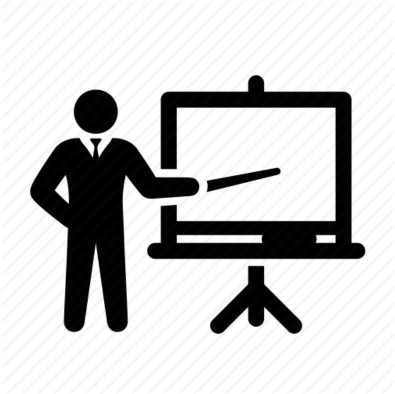
Millman's Theorem states that – when a number of voltage sources (E1, E2, E3…En) are in parallel having internal resistance (R1, R2, R3...Rn) respectively, the arrangement can replace by a single equivalent voltage source V in series with an equivalent series resistance R.





$$E = \frac{\sum_{k=1}^{n} E_k \cdot Y_k}{\sum_{k=1}^{n} Y_k}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + ,,,,,,, + G_n}$$



Access the below link to watch a video who explains with a numerical example Millman’s Theorem:

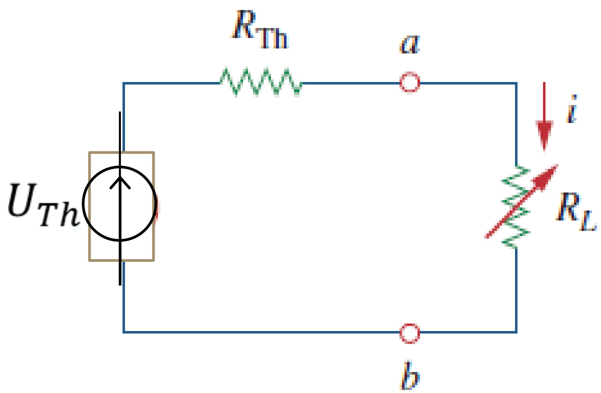
https://www.youtube.com/watch?v=8M1E3rn26Eg

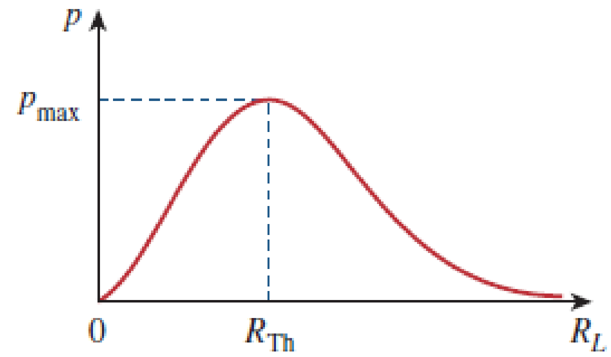
Maximum Power Transfer Theorem

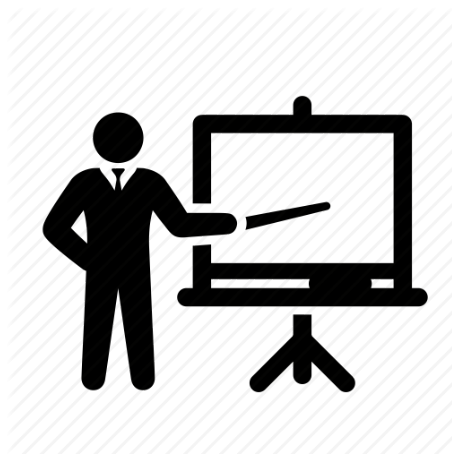
- The Maximum Power Transfer Theorem is not so much a means of analysis as it is an aid to system design.
- The theorem results in maximum power transfer, and not maximum efficiency. If the resistance of the load is made larger than the resistance of the source, then efficiency is higher, since a higher percentage of the source power is transferred to the load, but the magnitude of the load power is lower since the total circuit resistance goes up.

The maximum amount of power will be dissipated by a load resistance when that load resistance is equal to the Thevenin/Norton resistance of the network supplying the power

- This is essentially what is aimed for in radio transmitter design, where the antenna or transmission line "impedance" is matched to final power amplifier "impedance" for maximum radio frequency power output. Impedance, the overall opposition to AC and DC current, is very similar to resistance, and must be equal between source and load for the greatest amount of power to be transferred to the load. A load impedance that is too high will result in low power output. A load impedance that is too low will not only result in low power output, but possibly overheating of the amplifier due to the power dissipated in its internal (Thevenin or Norton) impedance.







https://www.youtube.com/watch?v=PCoyrvNnGU0

4.5 Maximum Power Transfer Theorem

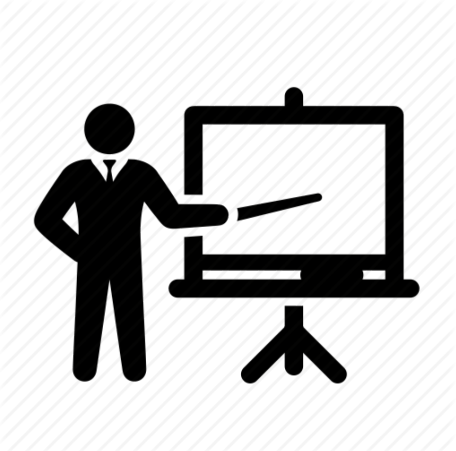
The Maximum Power Transfer Theorem is not:

- Maximum power transfer does not coincide with maximum efficiency. Application of The Maximum Power Transfer theorem to AC power distribution will not result in maximum or even high efficiency.

-The goal of high efficiency is more important for AC power distribution, which dictates a relatively low generator impedance compared to load impedance.

- Similar to AC power distribution, high fidelity audio amplifiers are designed for a relatively low output impedance and a relatively high speaker load impedance. As a ratio, “output impedance” : “load impedance” is known as *damping factor*, typically in the range of 100 to 1000.

- Maximum power transfer does not coincide with the goal of lowest noise. For example, the low-level radio frequency amplifier between the antenna and a radio receiver is often designed for lowest possible noise. This often requires a mismatch of the amplifier input impedance to the antenna as compared with that dictated by the maximum power transfer theorem.



For more Examples and Practice problems
using Superposition Theorem, Source Transformation,
Thevenin’s Theorem, Norton’s Theorem, Millman’s Theorem and Maximum Power Transfer Theorem

see SEMINAR 3

SUMMARY

1. A linear network consists of linear elements, linear dependent sources, and linear independent sources.
2. Network theorems are used to reduce a complex circuit to a simpler one, thereby making circuit analysis much simpler.
3. The superposition principle states that for a circuit having multiple independent sources, the voltage across (or current through) an element is equal to the algebraic sum of all the individual voltages (or currents) due to each independent source acting one at a time.
4. Source transformation is a procedure for transforming a voltage source in series with a resistor to a current source in parallel with a resistor, or vice versa.
5. Thevenin's and Norton's theorems allow us to isolate a portion of a network while the remaining portion of the network is replaced by an equivalent network. The Thevenin equivalent consists of a voltage source U_{Th} in series with a resistor R_{Th}, while the Norton equivalent consists of a current source I_N in parallel with a resistor R_N. The two theorems are related by source transformation.

$$R_N = R_{\text{Th}}, \qquad I_N = \frac{U_{Th}}{R_{Th}}$$

6. For a given Thevenin equivalent circuit, maximum power transfer occurs when $R_L = R_{Th}$; that is, when the load resistance is equal to the Thevenin resistance.
7. The maximum power transfer theorem states that the maximum power is delivered by a source to the load R_L when R_L is equal to R_{Th}, the Thevenin resistance at the terminals of the load.

REVIEW QUESTIONS

1 The current through a branch in a linear network is 2 A when the input source voltage is 10 V. If the voltage is reduced to 1 V and the polarity is reversed, the current through the branch is:

(a) −2 A (b) −0.2 A (c) 0.2 A
(d) 2 A (e) 20 A

2 For superposition, it is not required that only one independent source be considered at a time; any number of independent sources may be considered simultaneously.

(a) True (b) False

3 The superposition principle applies to power calculation.

(a) True (b) False

4 Refer to Fig. 1 . The Thevenin resistance at terminals a and b is:

(a) 25 Ω (b) 20 Ω
(c) 5 Ω (d) 4 Ω

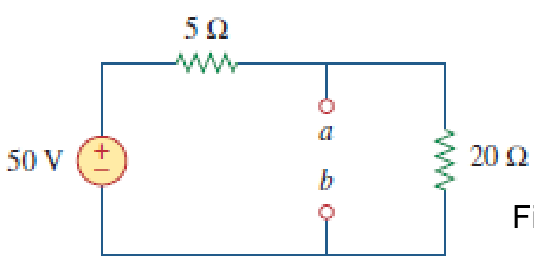


Fig. 1

5 The Thevenin voltage across terminals a and b of the circuit in Fig 1 is:

(a) 50 V (b) 40 V
(c) 20 V (d) 10 V

6 The Norton current at terminals a and b of the circuit in Fig. 4.67 is:

(a) 10 A (b) 2.5 A
(c) 2 A (d) 0 A

7 The Norton resistance R_N is exactly equal to the Thevenin resistance R_{Th}.

(a) True (b) False

8 Which pair of circuits in Fig. 2 are equivalent?

(a) a and b (b) b and d

(c) a and c (d) c and d

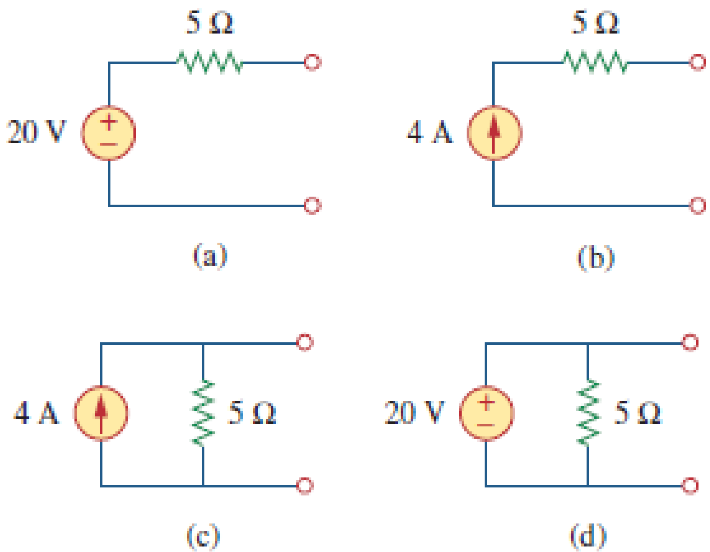


Fig. 2

.9 A load is connected to a network. At the terminals to which the load is connected, $R_{Th} = 10\ \Omega$ and $V_{Th} = 40$ V. The maximum possible power supplied to the load is:

(a) 160 W (b) 80 W

(c) 40 W (d) 1 W

.10 The source is supplying the maximum power to the load when the load resistance equals the source resistance.

(a) True (b) False

Answers: 4.1b, 4.2a, 4.3b, 4.4d, 4.5b, 4.6a, 4.7a, 4.8c, 4.9c, 4.10a.

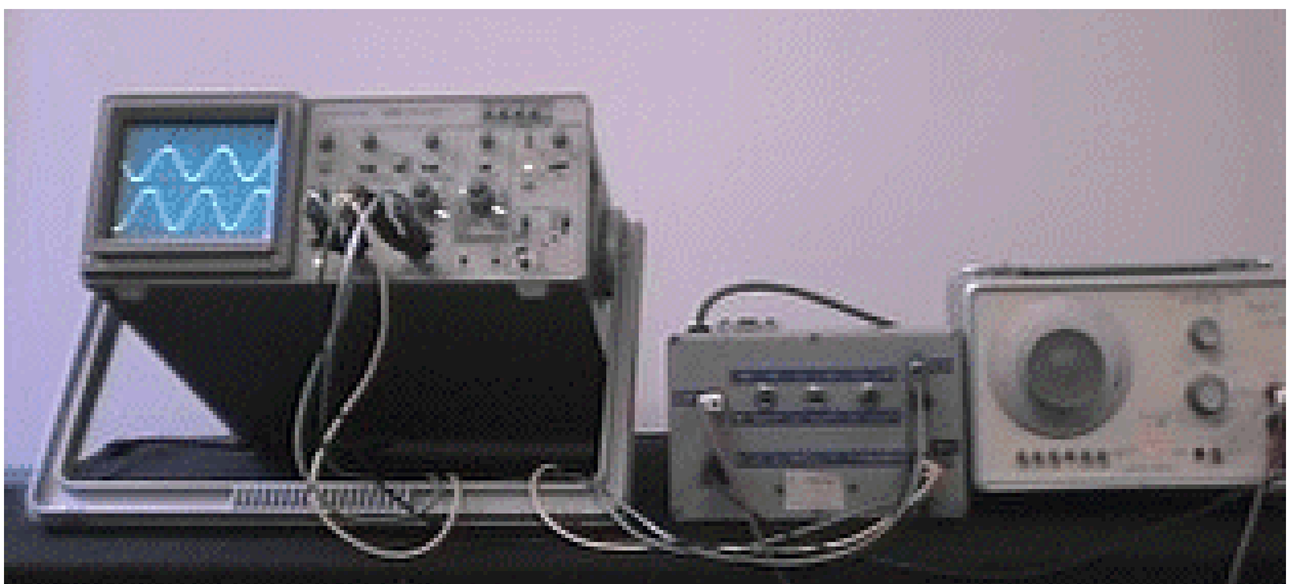
Chapter 2: AC Circuits

Sinusoids and phasors

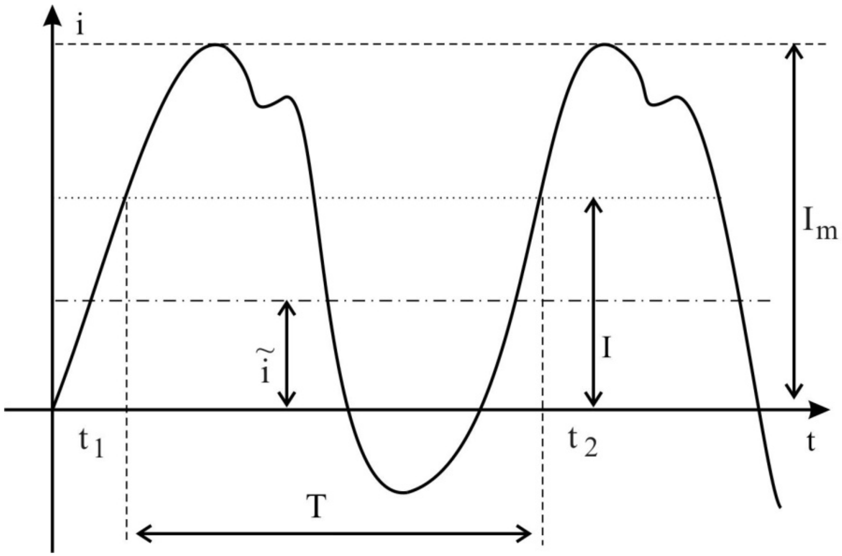
BASES OF ELECTROTECHNICS I.
Faculty of Electronics, Telecommunications and Information Technology
Specialization: IETTI
Academic year: 2023-2024

Content of this Subchapter:

1. Introduction in AC circuits
2. Capacitors
3. Inductors
4. RLC circuits
5. Power in sinusoidal regime
6. Phasors
7. Characterisation of linear circuits in complex plane



Generalities about TIME VARIABLE QUANTITIES



- ✓ **Instantaneous value:** the value of the current at any given instant of time: $i(t)$
- ✓ **Periodic function**: T – period

$$i(t) = i(t + kT)$$

- ✓ **Frequency:** f [Hz]:

$$f = \frac{1}{T} = \frac{\omega}{2\pi}; \omega = 2\pi f = \frac{2\pi}{T}; \omega T = 2\pi$$

- ✓ **Peak value (amplitude) – I_m ;**
- ✓ **Average (mean) value – I_{med} :**

$$I_{med} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i dt \qquad \tilde{i} = \frac{1}{T}\int_{t_1}^{t_1+T} i dt$$

- ✓ **Root mean square (effective, RMS) value – I :**

$$I = \sqrt{\frac{1}{T}\int_{t_1}^{t_1+T} i^2 dt} > 0$$

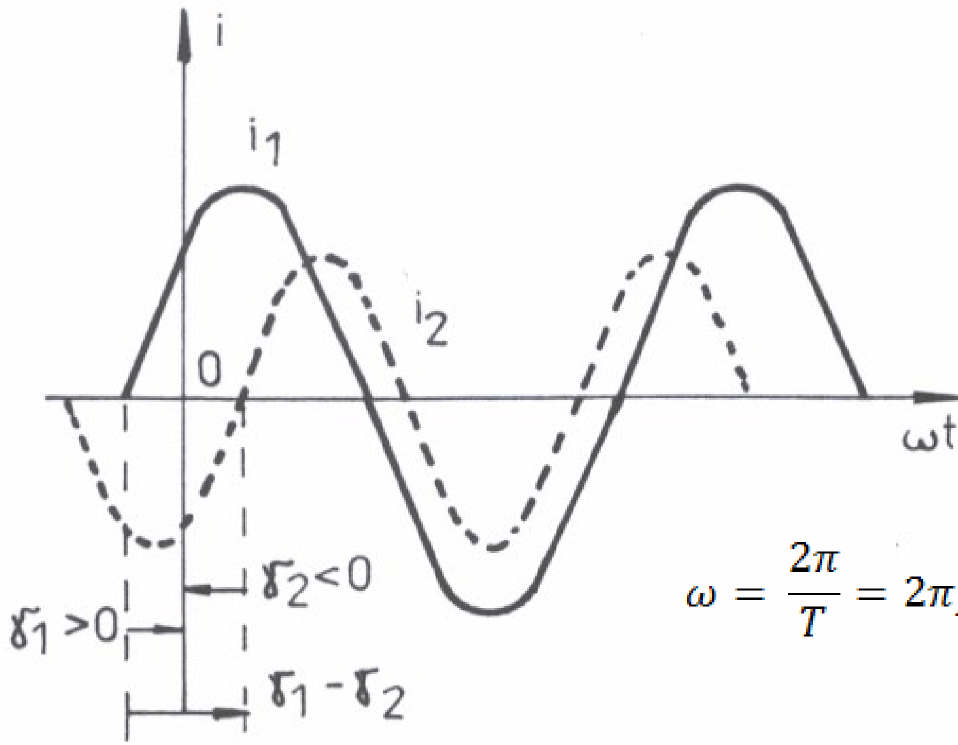
- **A sinusoid is a signal that has the form of the sine or cosine function.**

- A sinusoidal current is refered to an *alternating current (AC);*
- Such current reverses at regular time intervals and has alternately positive and negative values;
- Circuits driven by sinusoidal current or voltage sources are called *AC Circuits.*

We are interested in sinusoids because:

- Nature itself is characteristically sinusoidal (motion of pendulum, the vibrtaion of a string, the ripples on the ocean surface etc.
- A sinusoidal signal is easy to generate and transmit, it is the form of voltage generate throughout the world and supplied to homes, factories, laboratories, and so on.
- Through Fourier analysis, any practical periodic signal can be represented by a sum of sinusoids (play an important role in the analysis of periodic signals).
- A sinusois is easy to handle mathematically; the derivative and integral of a sinusoid are themselves sinusoids.

The sinusoid is an extremely important function in circuit analysis!!!



$$\omega = \frac{2\pi}{T} = 2\pi f$$

- the instantaneous value of a sinusoidal current:

$$i(t) = I_m \sin(\omega t + \gamma)$$

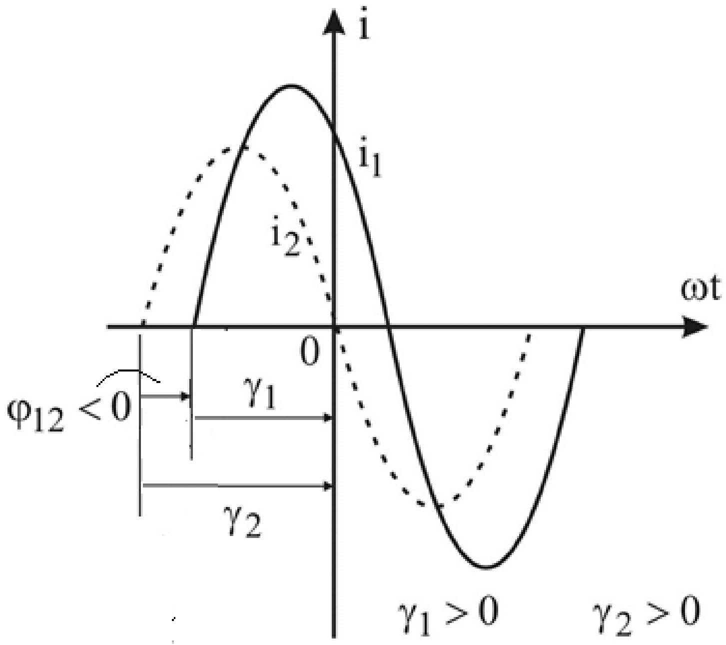
I_m - *peak value (amplitude)*

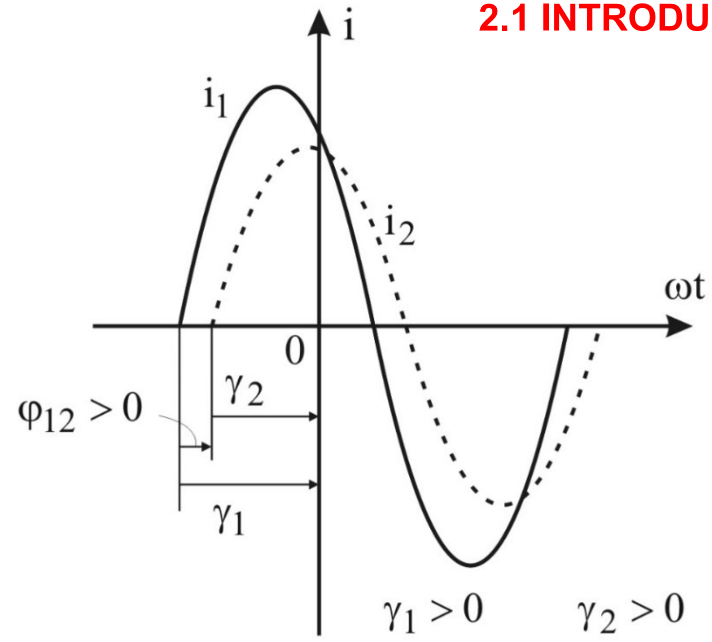
$\omega t + \gamma$ - *the argument*

γ - *the (initial) phase*

$\varphi_{12} = \gamma_1 - \gamma_2$ - *phase shift, phase displacement*

$$\varphi_{12} = (\omega t + \gamma_1) - (\omega t + \gamma_2) = \gamma_1 - \gamma_2 \neq 0$$





1) if $\varphi_{12} = \gamma_1 - \gamma_2 > 0$, i_1 *leads in phase the current* i_2

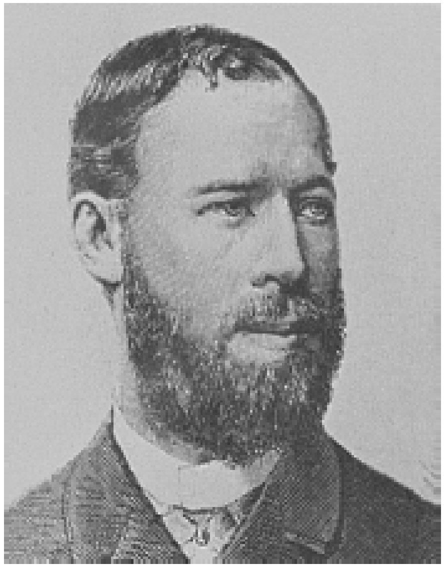
2) if $\varphi_{12} = \gamma_1 - \gamma_2 < 0$, i_1 *lags in phase behind the current* i_2

3) if $\varphi_{12} = \gamma_1 - \gamma_2 = 0$, i_1 *and* i_2 *are in phase*

4) if $\varphi_{12} = \gamma_1 - \gamma_2 = \pm\pi$, i_1 *and* i_2 *are in anti − phase*

5) if $\varphi_{12} = \gamma_1 - \gamma_2 = \pm\frac{\pi}{2}$, i_1 *and* i_2 *are in quadrature*

Historical



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Heinrich Rudorf Hertz (1857–1894), a German experimental physicist, demonstrated that electromagnetic waves obey the same fundamental laws as light. His work confirmed James Clerk Maxwell's celebrated 1864 theory and prediction that such waves existed.

Hertz was born into a prosperous family in Hamburg, Germany. He attended the University of Berlin and did his doctorate under the prominent physicist Hermann von Helmholtz. He became a professor at Karlsruhe, where he began his quest for electromagnetic waves. Hertz successfully generated and detected electromagnetic waves; he was the first to show that light is electromagnetic energy. In 1887, Hertz noted for the first time the photoelectric effect of electrons in a molecular structure. Although Hertz only lived to the age of 37, his discovery of electromagnetic waves paved the way for the practical use of such waves in radio, television, and other communication systems. The unit of frequency, the hertz, bears his name.

The average value (it cannot be used for sinusoids):

$$\tilde{i} = I_m \frac{1}{kT} \int_{t_1}^{t_1+kT} \sin(\omega t + \gamma) dt = \frac{I_m}{2\pi k} \left| -\cos(\omega t + \gamma) \right|_{t_1}^{t_1 + kT} =$$

$$= \frac{I_m}{2k\pi} [\cos(\omega t_1 + \gamma) - \cos(\omega t_1 + k\omega T + \gamma)] = 0$$

- instead:

$$I_{med} = \frac{2}{T} \int_0^{T/2} I_m \sin \omega t dt = \frac{2 I_m}{T\omega} \left| -\cos \omega t \right|_0^{T/2} = \frac{2}{\pi} I_m .$$

The RMS (effective) value :

$$I^2 = \frac{1}{T}\int_{t_1}^{t_1+T} i^2 dt = \frac{I_m^2}{T}\int_{t_1}^{t_1+T} \sin^2(\omega t + \gamma) dt = \frac{I_m^2}{2}$$

… it results: $I = \frac{I_m}{\sqrt{2}}$ or $I_m = \sqrt{2} I$

$$i = \sqrt{2} I \sin(\omega t + \gamma)$$

- *The root mean square (RMS) value of an alternating current is numerically equal to the magnitude of the steady direct current that would produce the same heating effect, in the same resistance, in the same period of time.*

The rms (effective) value:

$$t_1 = 0, \quad t_2 = T$$

$$I^2 = \frac{1}{T}\int_{t_1}^{t_1+T} i^2\,dt = \frac{1}{T}\int_0^T i^2(t)\cdot dt =$$

$$= \frac{1}{T}\int_0^T I_m^2 \sin^2(\omega t+\gamma)\,dt =$$

$$= \frac{I_m^2}{T}\int_0^T \frac{1}{2}\left[1-\cos 2(\omega t+\gamma)\right]dt =$$

$$2\sin^2 x = 1-\cos 2x \Rightarrow \sin^2 x = 1/2\,(1-\cos 2x)$$

$$= \frac{I_m^2}{2T}\left[\int_0^T dt - \int_0^T \cos 2(\omega t+\gamma)\right]$$

$$= \frac{I_m^2}{2T}\left[t\Big|_0^T - \sin 2(\omega t+\gamma)\Big|_0^T\right] \qquad \frac{2\pi}{T}\cdot T$$

$$= \frac{I_m^2}{2T}\left[T - \sin 2(\omega T+\gamma) + \sin 2\gamma\right] \qquad \sin 2\gamma$$

$$= \frac{I_m^2}{2T}\left(T - \sin(4\pi+2\gamma) + \sin 2\gamma\right) = \frac{I_m^2}{2T}\cdot T = \frac{I_m^2}{2}$$

$$-\sin(4\pi+2\gamma) + \sin 2\gamma = 0$$

$$\Rightarrow I = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = \sqrt{2}\,I$$

Mathematical operations with sinusoidal quantities :

- **The *multiplication* of a sinusoid with a scalar**:

$$i = ai_1 = aI_1\sqrt{2}\sin(\omega t + \gamma_1) = I\sqrt{2}\sin(\omega t + \gamma)$$

$$I = aI_1; \gamma = \gamma_1$$

- **The *addition* of a sinusoidal time function**:

$$i_1 = I_1\sqrt{2}\sin(\omega t + \gamma_1) \quad , \quad i_2 = I_2\sqrt{2}\sin(\omega t + \gamma_2)$$

$$i_1 + i_2 = i = I\sqrt{2}\sin(\omega t + \gamma).$$

$$I = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos(\gamma_1 - \gamma_2)}$$

$$tg\gamma = \frac{I_1\sin\gamma_1 + I_2\sin\gamma_2}{I_1\cos\gamma_1 + I_2\cos\gamma_2}$$

- **The *derivation* of a sinusoidal current**

$$i = I\sqrt{2}\sin(\omega t + \gamma)$$

$$\frac{di}{dt} = \omega I\sqrt{2}\cos(\omega t + \gamma) = \omega I\sqrt{2}\sin(\omega t + \gamma + \frac{\pi}{2}).$$

There results a sinusoidal current having the same frequency, having the amplitude multiplied by ω and which leads the initial current by $\pi/2$.

- **The *integration* of a sinusoidal current**

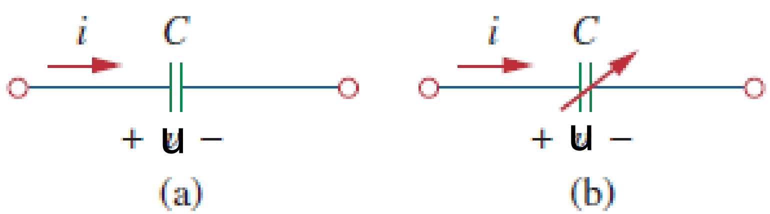
$$\int idt = -\frac{I\sqrt{2}}{\omega}\cos(\omega t + \gamma) = \frac{I\sqrt{2}}{\omega}\sin(\omega t + \gamma - \frac{\pi}{2}).$$

There results a sinusoidal current having the same frequency, having the amplitude divided by ω and which lags behind the initial current by $\pi/2$.

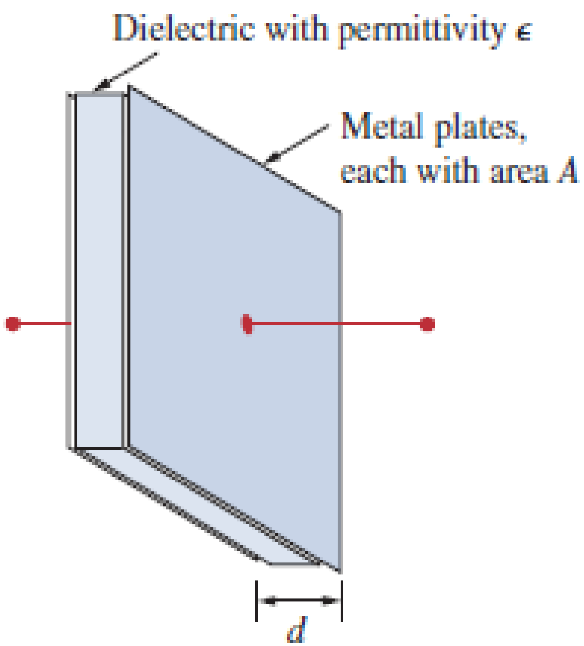
2.2. CAPACITORS

- In both digital and analog electronic circuits a capacitor is a fundamental element.
- It enables the filtering of signals and it provides a fundamental memory element.
- The capacitor is an element that stores energy in an electric field.

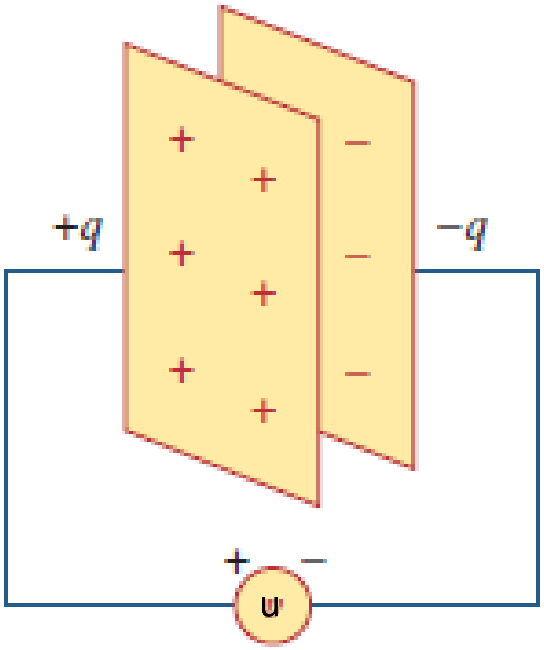
A capacitor consists of two conducting plates separated by an insulator (or dielectric).



Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.



- In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper or mica.

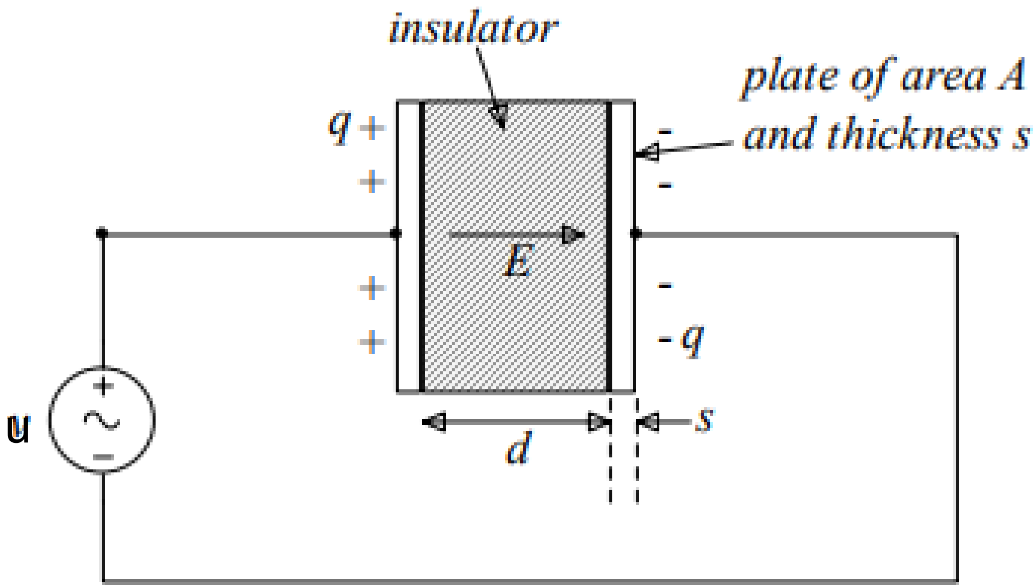


- When a voltage source is conected to the capacitor, the source deposits a positive charge *q* on one plate and a negative charge -*q* on the other. The capacitor is said to store the electric charge.

$$q = C \cdot u$$

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

1 farad = 1 coulomb/volt



If the plates have an area A and are separated by a distance d, the electric field generated across the plates is:

$$E = \frac{q}{\varepsilon A}$$

and the voltage across the capacitor plates is:

$$u = E \cdot d = \frac{qd}{\varepsilon A}$$

The current flowing into the capacitor is the rate of change of the charge across the capacitor plates i = dq/ dt:

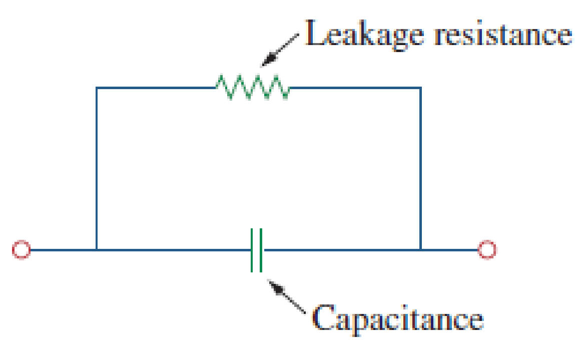
$$i = \frac{dq}{dt} = \frac{d}{dt}\left(\frac{\varepsilon A}{d}u\right) = \frac{\varepsilon A}{d} \cdot \frac{du}{dt} = C\frac{du}{dt}$$

The constant of proportionality *C* is referred to as the capacitance of the capacitor. It is a function of the geometric characteristics of the capacitor - plate separation *(d)* and plate area *(A)* - and by the permittivity *(ε)* of the dielectric material between the plates.

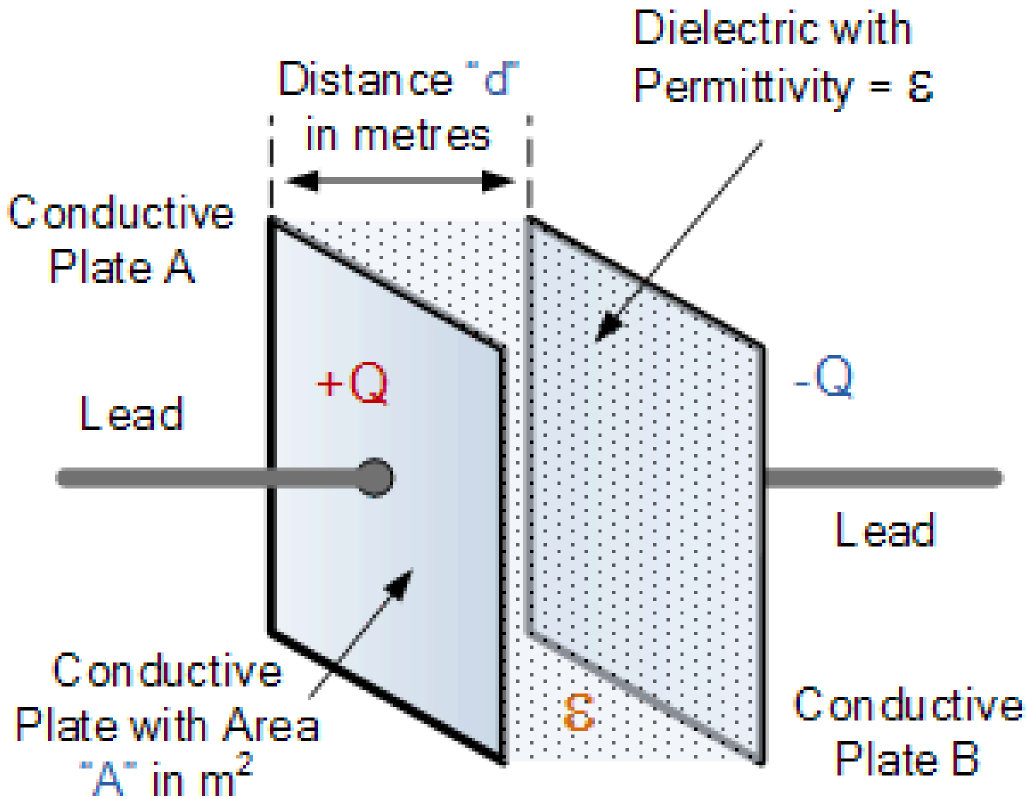
$$C = \frac{\varepsilon A}{d}$$

1. The surface area of the plates—the larger the area, the greater the capacitance.
2. The spacing between the plates—the smaller the spacing, the greater the capacitance.
3. The permittivity of the material—the higher the permittivity, the greater the capacitance.

$$C = \frac{\varepsilon A}{d}$$



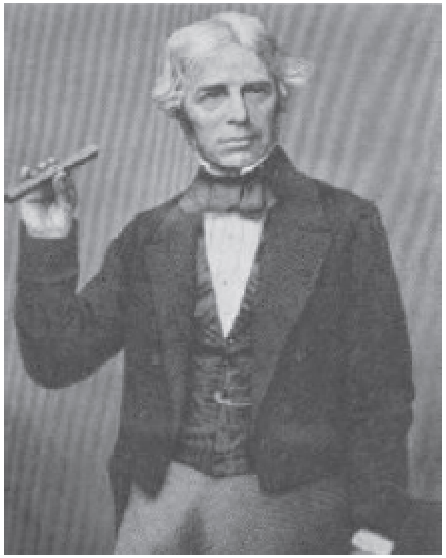
Circuit model of a nonideal capacitor.



Historical

Michael Faraday (1791–1867), an English chemist and physicist, was probably the greatest experimentalist who ever lived.

Born near London, Faraday realized his boyhood dream by working with the great chemist Sir Humphry Davy at the Royal Institution, where he worked for 54 years. He made several contributions in all areas of physical science and coined such words as electrolysis, anode, and cathode. His discovery of electromagnetic induction in 1831 was a major breakthrough in engineering because it provided a way of generating electricity. The electric motor and generator operate on this principle. The unit of capacitance, the farad, was named in his honor.



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2.2. CAPACITORS

Capacitors are commercially available in different values and types. Typically, capacitors have values in the picofarad (pF) to microfarad (μF) range. They are described by the dielectric material they are made of and by whether they are of fixed or variable type.

The current-voltage relationship of a capacitor is

$$i = C\frac{du}{dt}$$

$$u(t) = \frac{1}{C}\int_{-\infty}^{i} i(t)dt$$

$$u(t) = \frac{1}{C}\int_{t_0}^{t} i(t)dt + u(t_0)$$

$u(t_0) = q(t_0)/C$ is the voltage across the capacitor at time t_0.

Note that:

- for DC (constant in time) signals (du/dt = 0) the capacitor acts as an open circuit (i=0).
- the capacitor does not like voltage discontinuities since that would require that the current goes to infinity which is not physically possible.

EXAMPLE 2.1.

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
(b) Find the energy stored in the capacitor.

Solution:

(a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

(b) The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

PRACTICE PROBLEM 2.1.

What is the voltage across a 4.5-μF capacitor if the charge on one plate is 0.12 mC? How much energy is stored?

Answer: 26.67 A, 1.6 mJ.

EXAMPLE 2.2.

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:
By definition, the current is

$$i(t) = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(10 \cos 6000t)$$
$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A}$$

PRACTICE PROBLEM 2.2.

If a 10-μF capacitor is connected to a voltage source with

$$v(t) = 75 \sin 2000t \text{ V}$$

determine the current through the capacitor.

Answer: $1.5 \cos 2000t$ A.

EXAMPLE 2.2.

Determine the current through a 200-μF capacitor whose voltage is shown in Figure 1

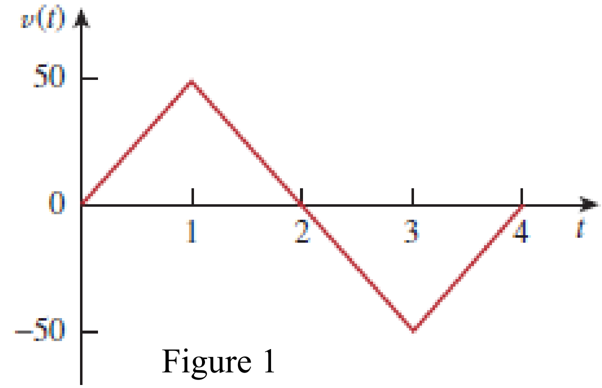


Figure 1

Solution:
The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Since $i = C\, dv/dt$ and $C = 200$ μF, we take the derivative of v to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Thus the current waveform is as shown in Figure 2 .

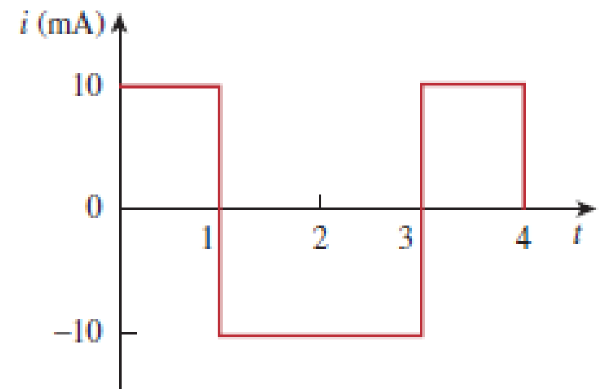


Figure 2

The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt}$$

The energy stored in the capacitor is therefore

$$w = \int_{-\infty}^{t} p(\tau)\,d\tau = C\int_{-\infty}^{t} v\frac{dv}{d\tau}d\tau = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

We note that $v(-\infty) = 0$, because the capacitor was uncharged at $t = -\infty$. Thus,

$$\boxed{w = \frac{1}{2}Cv^2}$$

$$q = Cv \quad \Rightarrow \quad w = \frac{q^2}{2C}$$

EXAMPLE 2.4.

Obtain the energy stored in each capacitor under dc conditions.

Solution:

Under dc conditions, we replace each capacitor with an open circuit, as shown in Figure 2 a). The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3+2+4}(6 \text{ mA}) = 2 \text{ mA}$$

Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4 \text{ V} \qquad v_2 = 4000i = 8 \text{ V}$$

and the energies stored in them are

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$

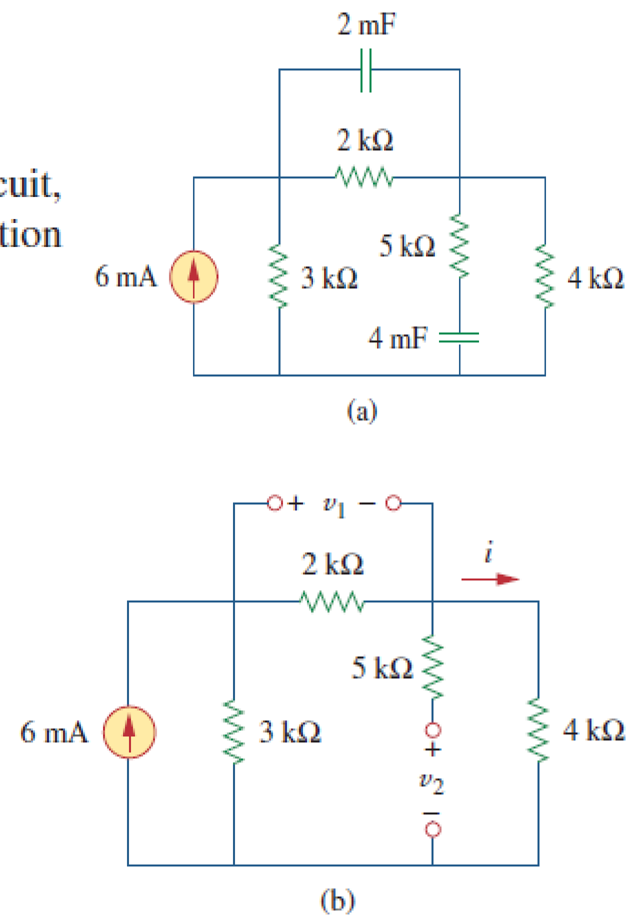
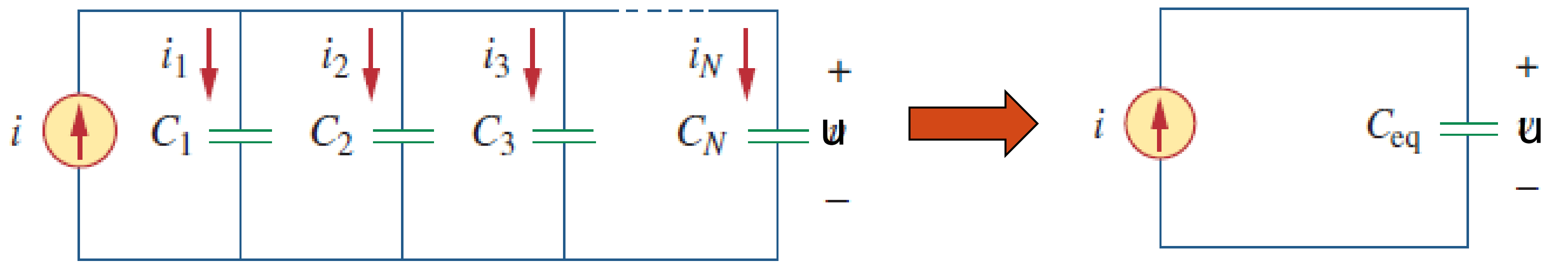


Figure 2

Parallel capacitors



$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

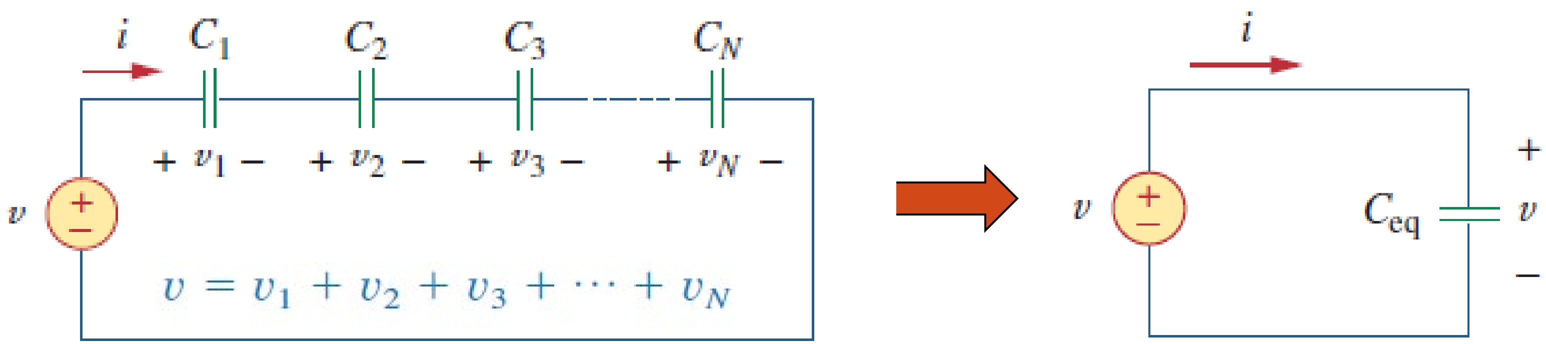
$$i = C_1\frac{du}{dt} + C_2\frac{du}{dt} + C_3\frac{du}{dt} + \cdots + C_n\frac{du}{dt} =$$

$$= \left(\sum_{k=1}^{N} C_k\right)\frac{du}{dt} = C_{eq}\frac{du}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

The equivalent capacitance of *N* parallel-conected capacitors is the sum of the individual capacitances.

Series capacitors



$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)\,d\tau + v_2(t_0)$$

$$+ \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)\,d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + v_2(t_0)$$

$$+ \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

The equivalent capacitance of *N* series-conected capacitors is the reciprocal of the sum of the reciprocal of the individual capacitances.

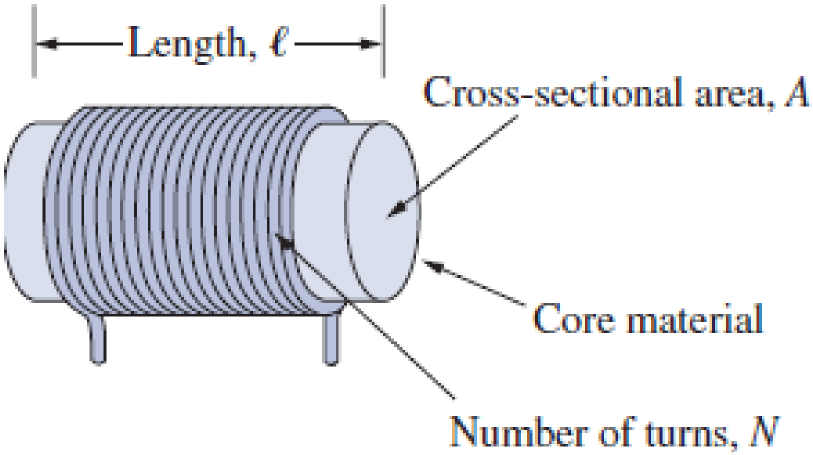
In case of 2 capacitors (N=2)

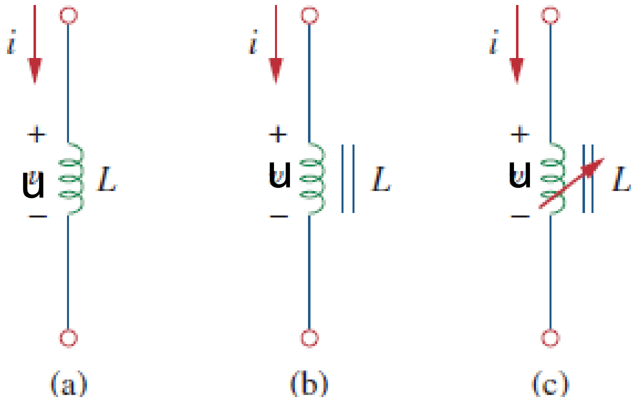
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \Rightarrow \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

2.3. INDUCTORS

- Is a passive element designed to store energy in magnetic field.
- Inductors are used in numerous applications in electronic and power system (power supplies, transformers, radios, TVs, radars, electric motors etc.)

An inductor consists of a coil of conducting wave.





Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.

- Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhence the inductive effect, a practical inductor is usually formed into cylindrical coil with many turns of conducting wire.

$$u = L\frac{di}{dt}$$

- If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current.
- **L** is the constant of proportionality called **inductance** of the inductor

Inductance is the property whereby an inductor exhibit opposition to the change of current through it, measured in henrys (H).

1 volt-second per ampere.

$$L = \frac{N^2 \mu A}{\ell}$$

- N is number of turns
- l is the lenght
- A is the cross-sectional area
- μ is the permeability of the core

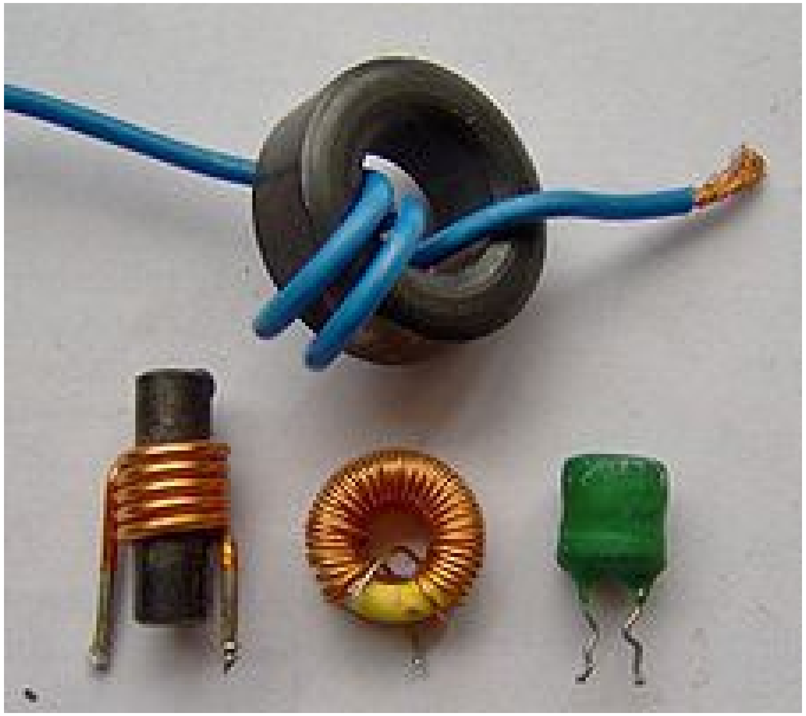
inductance can be increased by increasing the number of turns of coil, using material with higher permeability as the core, increasing the cross-sectional area, or reducing the length of the coil.

Historical

Joseph Henry (1797–1878), an American physicist, discovered inductance and constructed an electric motor.

Born in Albany, New York, Henry graduated from Albany Academy and taught philosophy at Princeton University from 1832 to 1846. He was the first secretary of the Smithsonian Institution. He conducted several experiments on electromagnetism and developed powerful electromagnets that could lift objects weighing thousands of pounds. Interestingly, Joseph Henry discovered electromagnetic induction before Faraday but failed to publish his findings. The unit of inductance, the henry, was named after him.

NOAA's People Collection



Like capacitors, commercially available inductors come in different values and types. Typical practical inductors have inductance values ranging from a few microhenrys (μH), as in communication systems, to tens of henrys (H) as in power systems. Inductors may be fixed or variable. The core may be made of iron, steel, plastic, or air. The terms *coil* and *choke* are also used for inductors.

$$di = \frac{1}{L}u$$

$$i(t) = \frac{1}{L}\int_{-\infty}^{i} u(t)dt$$

$$\Rightarrow \quad i(t) = \frac{1}{L}\int_{t_0}^{t} u(t)dt + i(t_0)$$

The power delivered to the inductor:

$$p = ui = \left(L\frac{di}{dt}\right)i$$

The energy stored in the inductor:

$$w = \int_{-\infty}^{i} p(t)dt = L\int_{-\infty}^{i} \frac{di}{dt} idt =$$

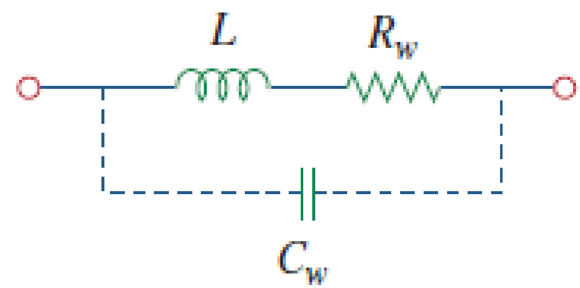
$$= L\int_{-\infty}^{i} idt = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

Since $i(-\infty) = 0$ $\Rightarrow$

$$w = \frac{1}{2}Li^2$$

Note that:
- for DC (constant in time) signals the inductor acts as an short-circuit (u=0).
- the current through an inductor cannot change instantaneously.
- like the ideal capacitor, the ideal inductor does not dissipate energy.

Circuit model for a practical inductor.



A practical, nonideal inductor has a significant resistive component, as shown in Fig. This is due to the fact that the inductor is made of a conducting material such as copper, which has some resistance. This resistance is called the *winding resistance* R_w, and it appears in series with the inductance of the inductor. The presence of R_w makes it both an energy storage device and an energy dissipation device. Since R_w is usually very small, it is ignored in most cases. The nonideal inductor also has a *winding capacitance* C_w due to the capacitive coupling between the conducting coils. C_w is very small and can be ignored in most cases, except at high frequencies.

EXAMPLE 2.2.

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:

Since $v = L\,di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t)\text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t}\text{ J}$$

EXAMPLE 2.6.

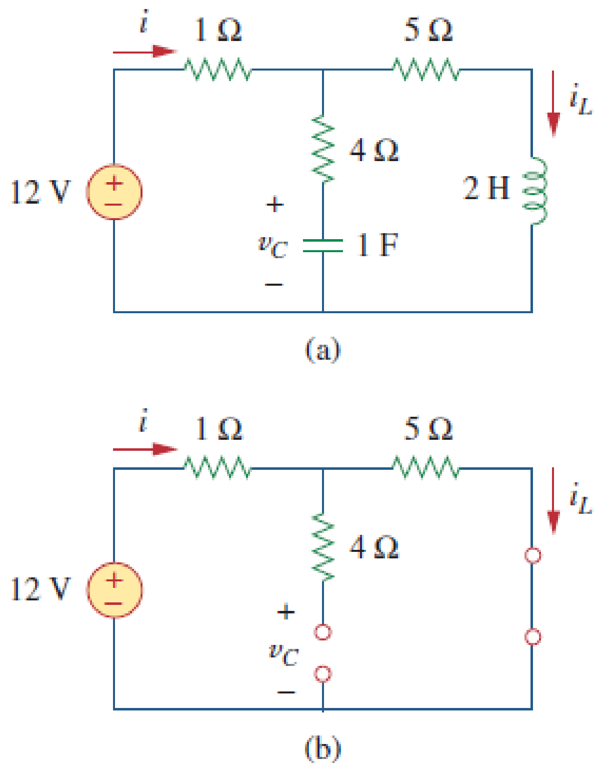


Figure 6.27
For Example 6.10.

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.

Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit, as in Fig. 6.27(b). It is evident from Fig. 6.27(b) that

$$i = i_L = \frac{12}{1 + 5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

$$v_C = 5i = 10 \text{ V}$$

(b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

EXAMPLE 2.7.

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

Solution:

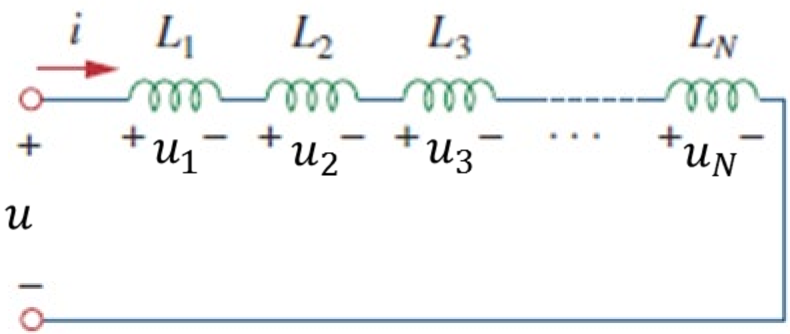
Since $i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$ and $L = 5$ H,

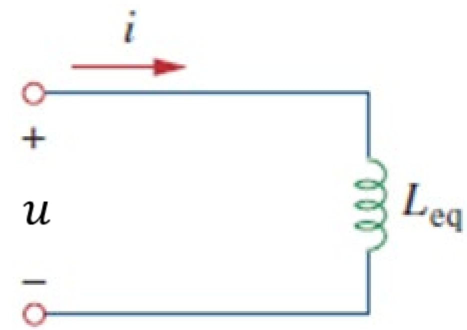
$$i = \frac{1}{5}\int_0^t 30t^2\,dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

The power $p = vi = 60t^5$, and the energy stored is then

$$w = \int p\,dt = \int_0^5 60t^5\,dt = 60\frac{t^6}{6}\bigg|_0^5 = 156.25 \text{ kJ}$$

Series inductors





$$u = u_1 + u_2 + u_3 + \cdots + u_N$$

$$u = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt} =$$

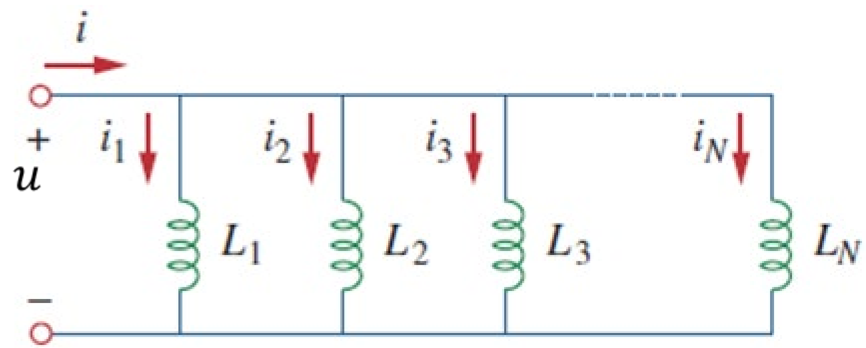
$$= (L_1 + L_2 + L_3 + \cdots + L_N)\frac{di}{dt} =$$

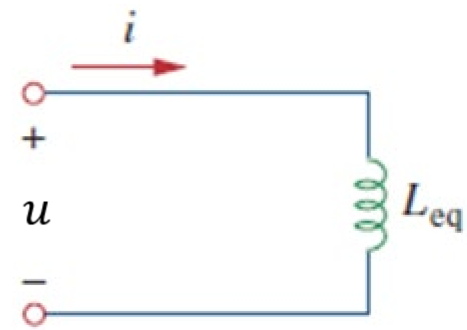
$$= \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

The equivalent inductance of *N* series-conected inductors is the sum of the individual inductances.

Series and parallel inductors





$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

$$i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0) + \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0)$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)$$

$$= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

The equivalent inductance of *N* parallel-conected inductors is the reciprocal of the sum of the reciprocal of the individual inductances.

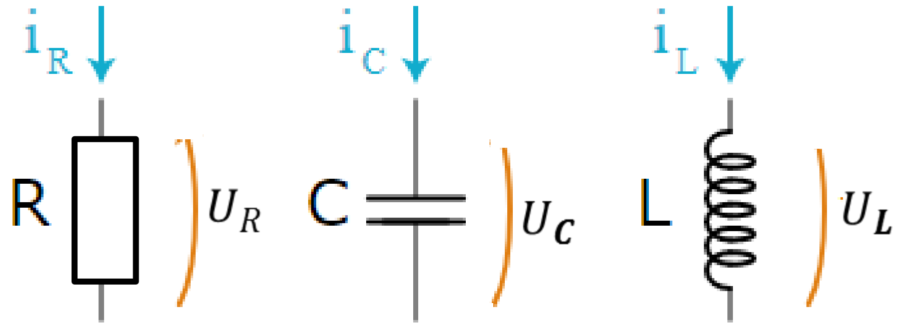
In case of 2 inductors (N=2)

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \qquad \text{or} \qquad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

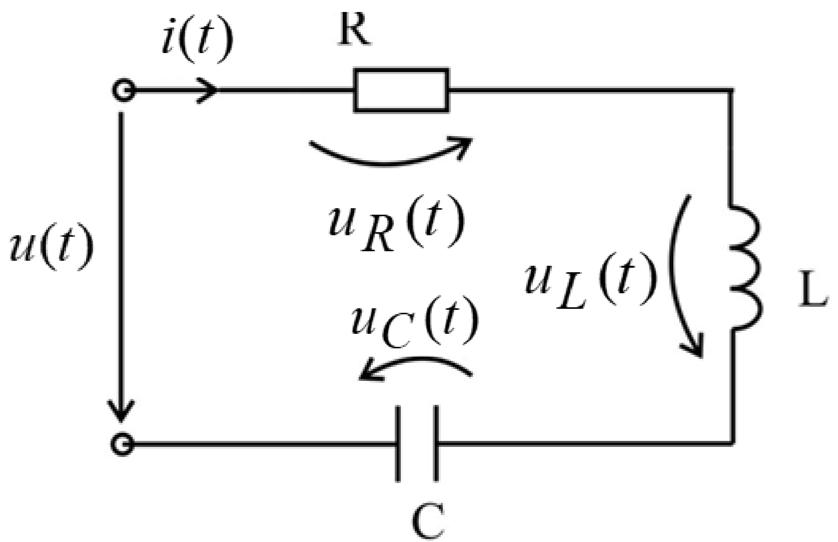
Important characteristics of the basic elements:

Relation	Resistor (R)	Inductor (L)	Capacitor (C)
$u-i$:	$u = iR$	$u = L\frac{di}{dt}$	$u(t) = \frac{1}{C}\int_{t_0}^{t} i(t)dt$
$i-u$:	$i = u/R$	$i(t) = \frac{1}{L}\int_{t_0}^{i} u(t)dt$	$i = C\frac{du}{dt}$
p or w:	$p = i^2R = \frac{u^2}{R}$	$w = \frac{1}{2}Cu^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$	$C_{eq} = C_1 + C_2$
At DC:	same	short circuit	open circuit
Circuit variable that cannot change abruptly:	Not applicable	i	u

In contrast to a *resistor*, which spends or dissipates energy ireversibly, an *inductor* or *capacitor*, stores or releases energy.



➢ R, L, C series circuit



$$u(t) = U\sqrt{2}\sin(\omega t + \beta)$$

$$i(t) = I\sqrt{2}\sin(\omega t + \gamma)$$

where I and γ are unknown.

$$u(t) = u_R(t) + u_L(t) + u_C(t)$$

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = U\sqrt{2}\sin(\omega t + \beta)$$

-the phase displacement between a voltage and the associated current

$$\varphi = \beta - \gamma$$

-There are two possibilities:

$$u(t) = U\sqrt{2}\sin(\omega t + \beta)$$

$$i(t) = I\sqrt{2}\sin(\omega t + \gamma)$$

$$\varphi = \beta - \gamma$$

a) If $\beta = 0$ then $\gamma = -\varphi$

$$\begin{cases} u(t) = U\sqrt{2}\sin\omega t \\ i(t) = I\sqrt{2}\sin(\omega t - \varphi) \end{cases}$$

b) If $\gamma = 0$ then $\beta = \varphi$

$$\begin{cases} u(t) = U\sqrt{2}\sin(\omega t + \varphi) \\ i(t) = I\sqrt{2}\sin\omega t \end{cases}$$

Remark: solving the problems, we will choose as having the initial phase angle equal to zero the quantity (current or voltage) which is more often encountered in the functioning equation of the circuit.

$$u(t) = U\sqrt{2}\sin(\omega t + \beta)$$

$$i(t) = I\sqrt{2}\sin(\omega t + \gamma)$$

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = U\sqrt{2}\sin(\omega t + \beta)$$

$$RI\sqrt{2}\sin\omega t + \omega LI\sqrt{2}\cos\omega t - \frac{1}{\omega C}I\sqrt{2}\cos\omega t =$$

$$= U\sqrt{2}\sin\omega t\cos\varphi + U\sqrt{2}\cos\omega t\sin\varphi$$

By identification:

$$\begin{cases} U\cos\varphi = IR \\ U\sin\varphi = I(\omega L - \dfrac{1}{\omega C}) \end{cases}$$

$U\cos\varphi$ is called the active component of the voltage,

$U\sin\varphi$ is called the reactive component of the voltage

$$I = \frac{U}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

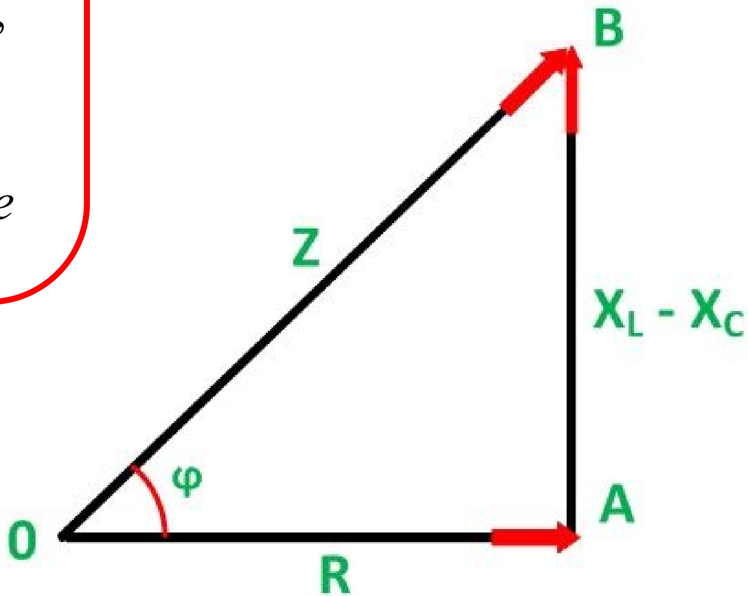
$$\varphi = \beta - \gamma = arctg\frac{\omega L - \frac{1}{\omega C}}{R}$$

where: $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$ is the *impedance* of the circuit

$\omega L - \frac{1}{\omega C} = X$ is the *reactance*,

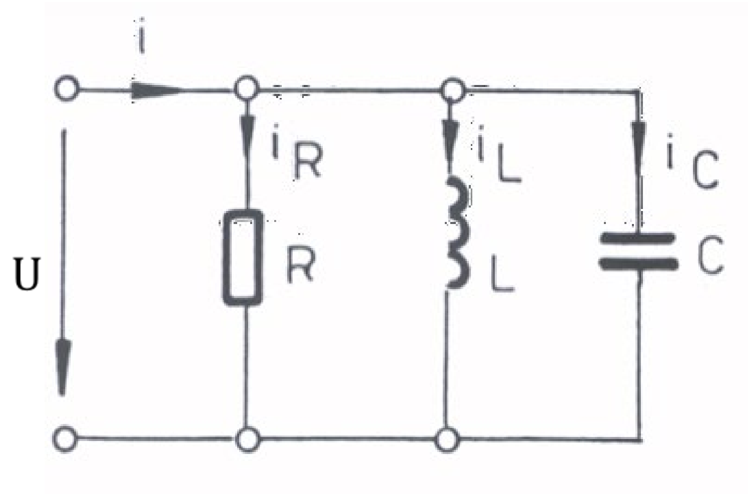
$\omega L = X_L$ is the *inductive reactance*,

$\frac{1}{\omega C} = X_C$ is the *capacitive reactance*



The *impedance triangle*: $R^2 + X^2 = Z^2$

R, L, C parallel circuit



$$\begin{cases} u(t) = U\sqrt{2}\sin\omega\cdot t \\ i(t) = I\sqrt{2}\sin(\omega\cdot t-\varphi) \end{cases}$$

where I and φ are unknown.

$$i(t) = i_R(t) + i_L(t) + i_C(t)$$

$$G\cdot u(t) + \frac{1}{L}\int u(t)dt + C\frac{du(t)}{dt} = I\sqrt{2}\sin(\omega\cdot t-\varphi)$$

$$GU\sin\omega\cdot t - \frac{U}{\omega\cdot L}\cos\omega\cdot t + \omega\cdot CU\cos\omega\cdot t \equiv$$

$$= I\sin\omega\cdot t\cos\varphi - I\sin\varphi\cos\omega\cdot t$$

By identification: $\begin{cases} I\cos\varphi = GU \\ I\sin\varphi = V\left(\dfrac{1}{\omega\cdot L} - \omega\cdot C\right) \end{cases}$

$I \cos \varphi$ - is called the active component of the current

$I \sin \varphi$ - is called the reactive component of the current

$$I = U\sqrt{G^2 + \left(\frac{1}{\omega\cdot L} - \omega\cdot C\right)^2}$$

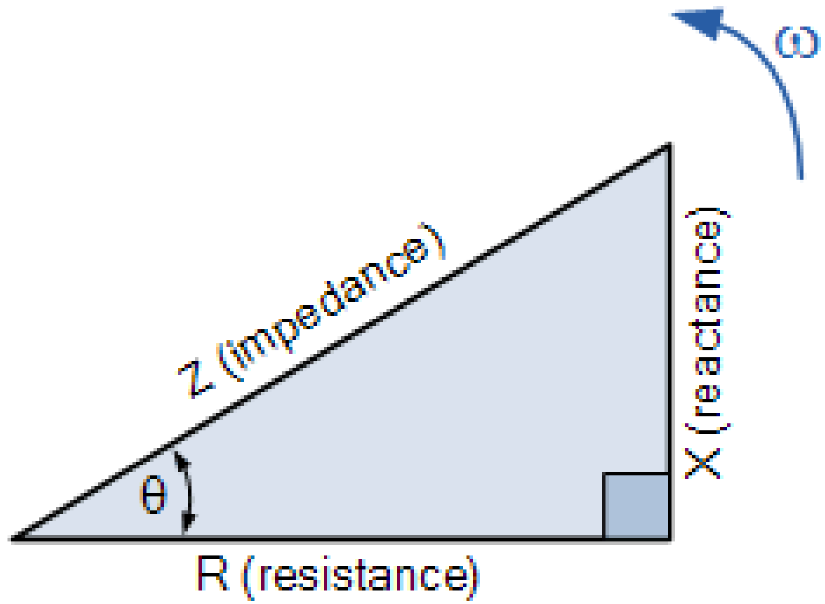
$$\varphi = \arctan\frac{\dfrac{1}{\omega\cdot L} - \omega\cdot C}{G}$$

$$\sqrt{G^2 + \left(\frac{1}{\omega\cdot L} - \omega\cdot C\right)^2} = Y$$

is the *admittance* of the circuit,

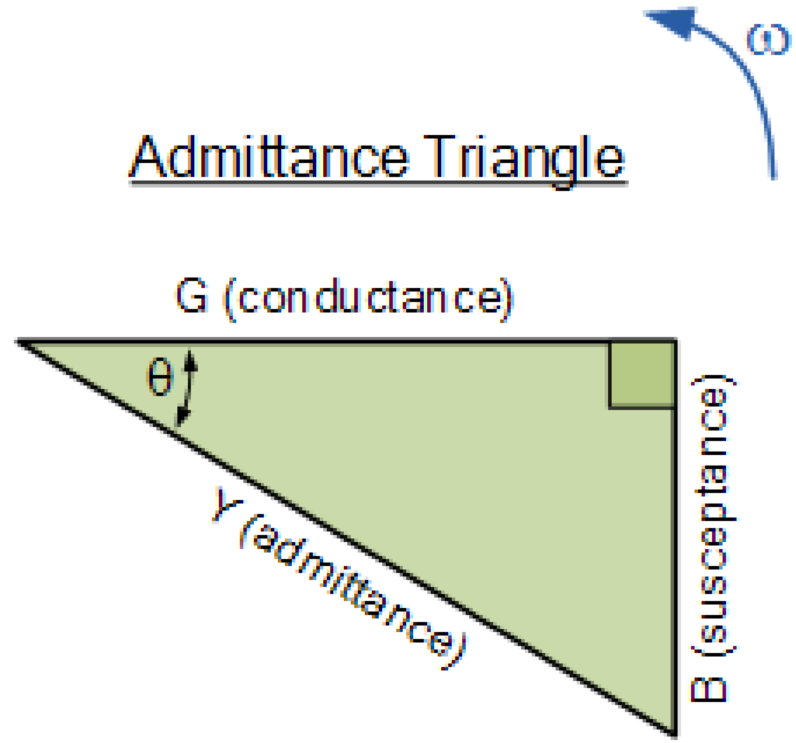
$$\frac{1}{\omega\cdot L} - \omega\cdot C = B$$

is the *susceptance*.



Impedance Triangle

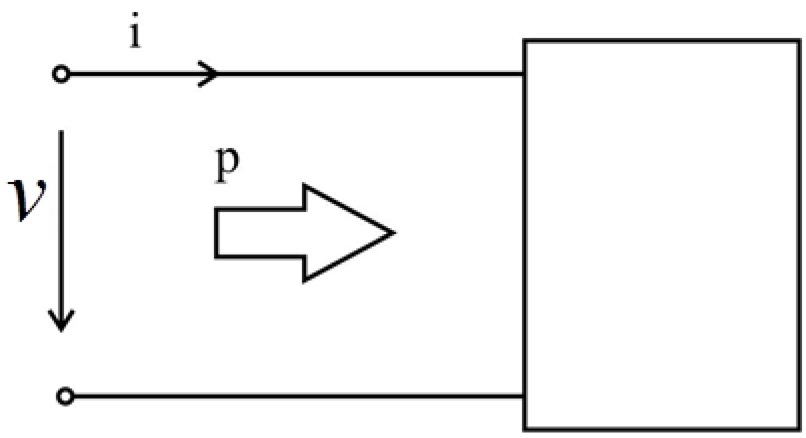
Admittance Triangle



Obs: Admittance, Y, is the reciprocal of impedance Z.

$$\left(Y^2 = G^2 + B^2\right)$$

POWER IN SINUSOIDAL REGIME



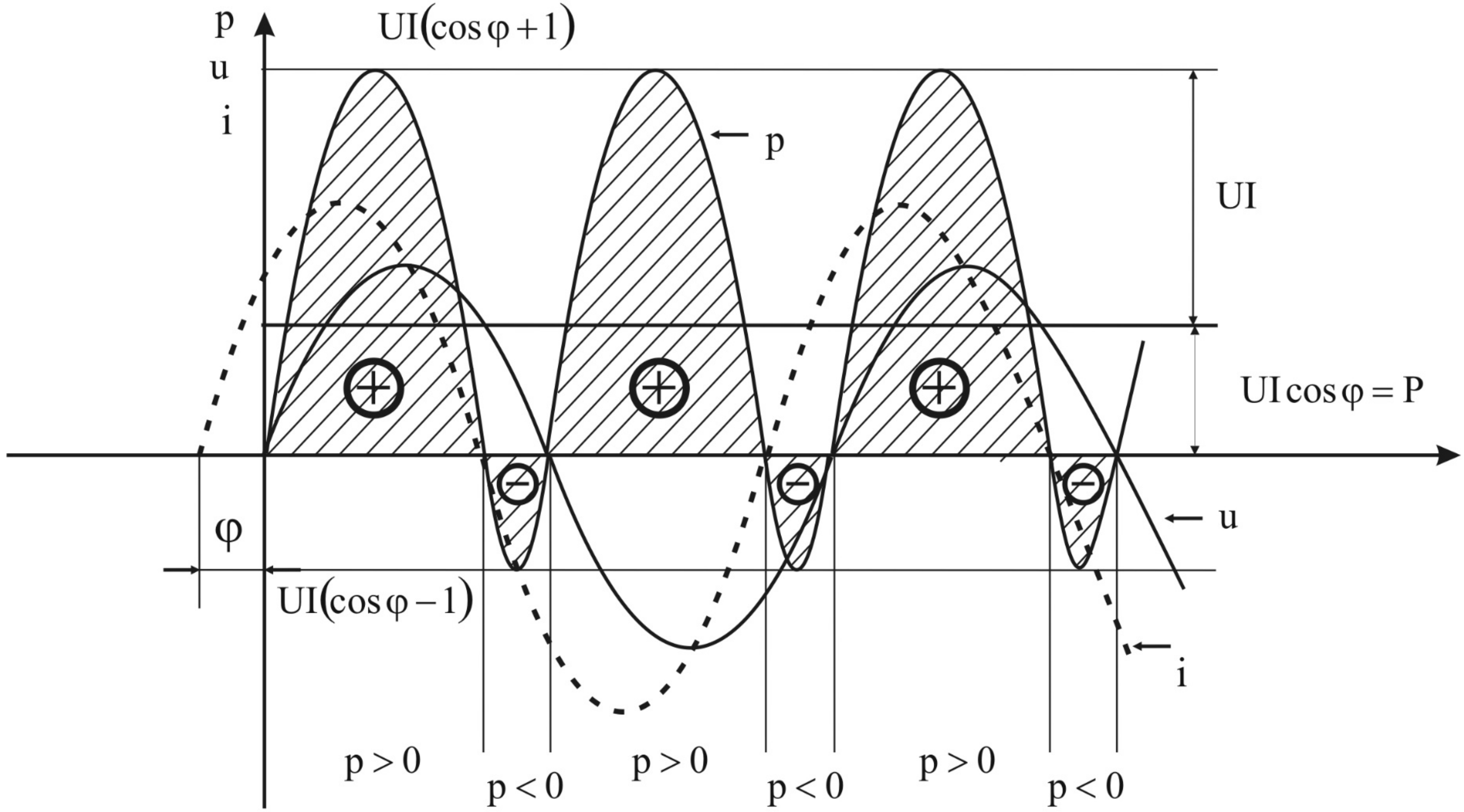
$$\begin{cases} u(t) = U\sqrt{2}\sin\omega\cdot t \\ i(t) = I\sqrt{2}\sin(\omega\cdot t - \varphi) \end{cases}$$

1) The instantaneous power:

$$p(t) = u(t)\cdot i(t)$$

$$p = 2UI\sin\omega t\sin(\omega t - \varphi) = UI\cos\varphi - UI\cos(2\omega t - \varphi)$$

- The instantaneous power is the sum of a *fundamental component* and a *harmonic component* whose angular frequency is twice the angular frequency of the voltage and current.



The meaning of a negative instantaneous power is that the circuit returns power to the source. This happens when the circuit contains reactive (or energy-storing) elements.

2) The active power - is the average power over a cycle:

$$P = \frac{1}{T}\int_0^T p dt = \frac{1}{T}\int_0^T UI\cos\varphi dt - \frac{1}{T}\int_0^T UI\cos(2\omega t + \varphi)dt = UI\cos\varphi$$

$P = UI \cos \varphi$ [W]

3) The total or apparent power :

$S = UI$ [VA]

- the power factor $k = \frac{P}{S} = cos\varphi$

4) The reactive power :

$Q = UI\sin\varphi$ [VAr]

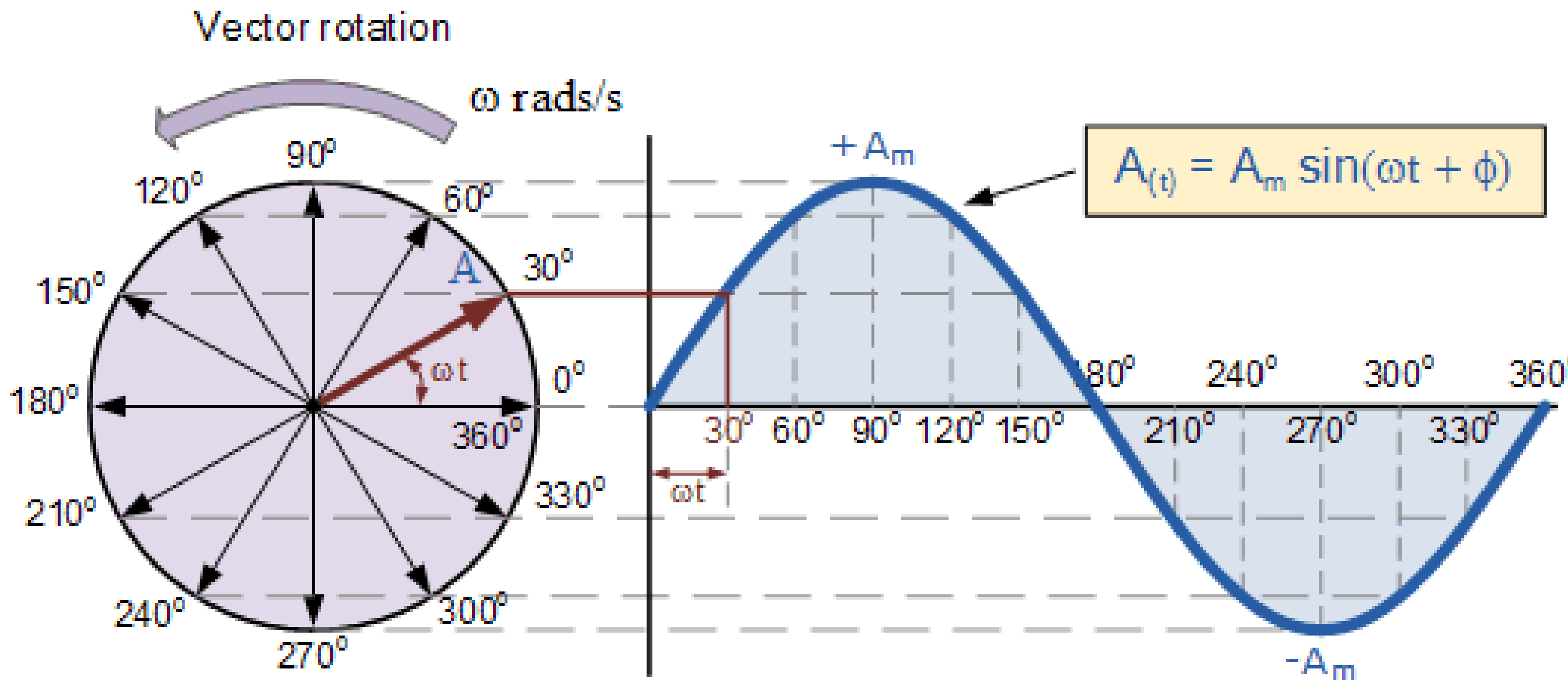
$$S = \sqrt{P^2 + Q^2}$$

PHASORS

REPRESENTATION OF SINUSOIDAL TIME FUNCTIONS BY
VECTORS AND COMPLEX NUMBERS

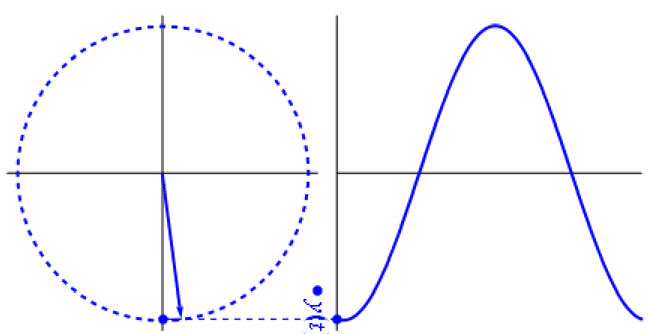
- **A phasor is a rotating vector representing a quantity, by means of a line rotating about a point in a plane, the magnitude of the quantity being proportional to the length of the line and the phase of the quantity being equal to the angle between the line and a reference line**

- Sinusoids are easily expressed in terms of phasors, which are more convenient to work with that sine and cosine functions.
- The origin of the term phasor rightfully suggests that a (diagrammatic) calculus somewhat similar to that possible for vectors is possible for phasors as well.
- An important additional feature of the phasor transform is that differentiation and integration of sinusoidal signals (having constant amplitude, period and phase) corresponds to simple algebraic operations on the phasors; the phasor transform thus allows the analysis (calculation) of the AC steady state of RLC circuits by solving simple algebraic equations (with complex coefficients) in the phasor domain instead of solving differential equations (with real coefficients) in the time domain.
- The originator of the phasor transform was Charles Proteus Steinmetz working at General Electric in the late 19th century.



Rotating Phasor

Sinusoidal Waveform in the Time Domain



The idea of phasor representation is based on Euler's identity. In general,

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi \qquad \text{(Eq. 1)}$$

which shows that we may regard $\cos\phi$ and $\sin\phi$ as the real and imaginary parts of $e^{j\phi}$; we may write

$$\cos\phi = \mathrm{Re}(e^{j\phi}) \qquad \text{(Eq. 2a)}$$

$$\sin\phi = \mathrm{Im}(e^{j\phi}) \qquad \text{(Eq. 2b)}$$

where Re and Im stand for the *real part of* and the *imaginary part of*. Given a sinusoid $v(t) = V_m\cos(\omega t + \phi)$, we use (Eq. 2a) to express $v(t)$ as

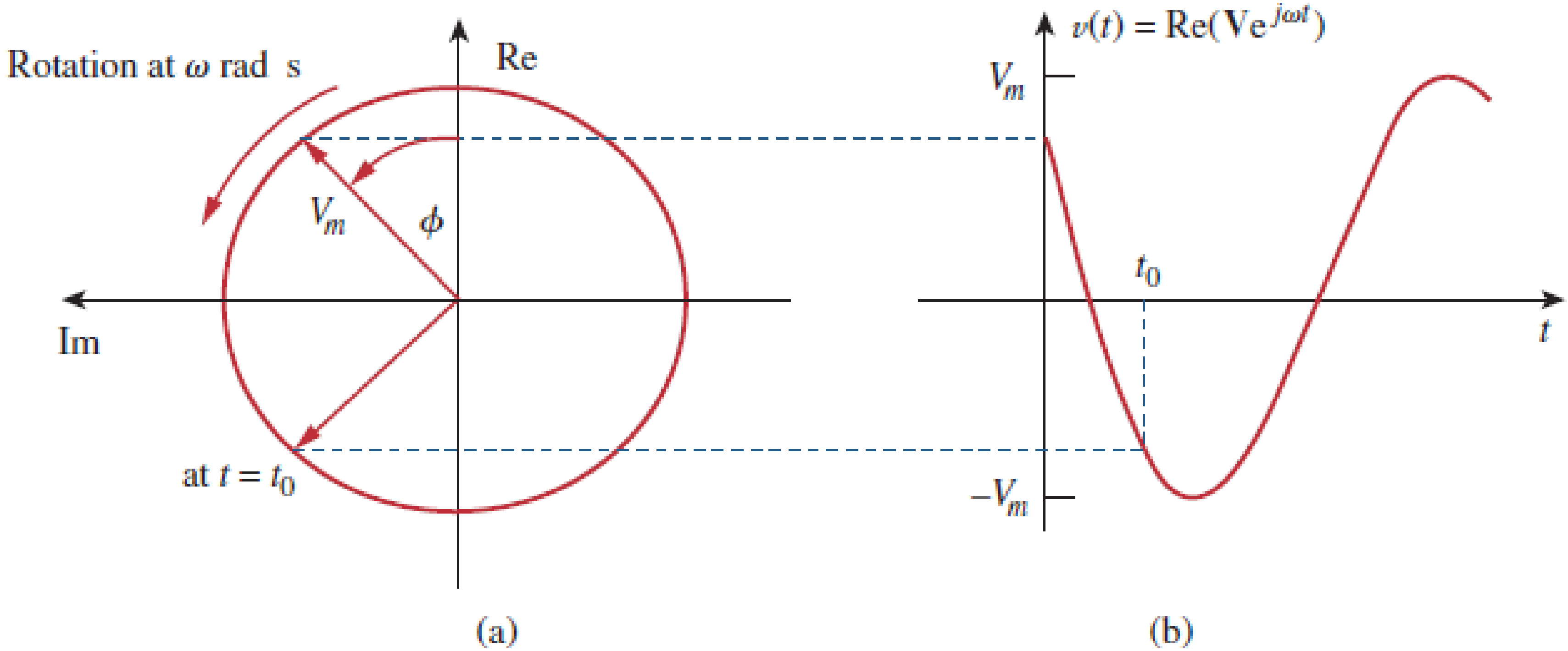
$$v(t) = V_m\cos(\omega t + \phi) = \mathrm{Re}(V_m e^{j(\omega t + \phi)}) \qquad \text{(Eq. 3)}$$

or

$$v(t) = \mathrm{Re}(V_m e^{j\phi} e^{j\omega t})$$

$$v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t})$$

$$\mathbf{V} = V_m e^{j\phi} = V_m\angle\phi$$



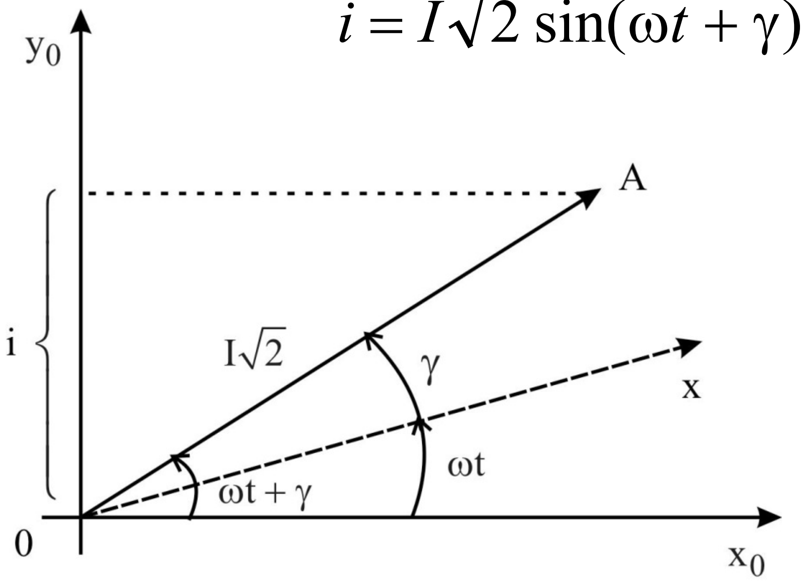
Representation of $\mathbf{V}e^{j\omega t}$: (a) sinor rotating counterclockwise, (b) its projection on the real axis, as a function of time.

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \underline{\mathbf{V}} = V_m \underline{/\phi}$$

(Time-domain representation) (Phasor-domain representation)

A) Geometrical (phasorial) representation.

$$i = I\sqrt{2}\sin(\omega t + \gamma)$$



$$OA = I\sqrt{2}$$

$$\hat{AOx_0} = \omega t + \gamma$$

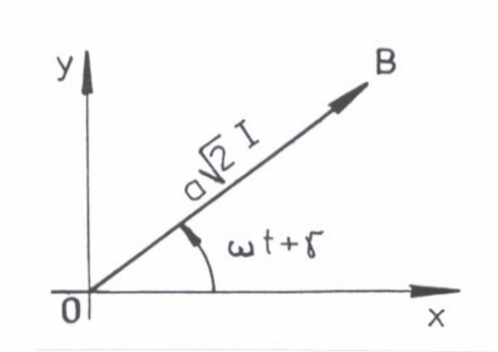
Vector:

- its projection onto the vertical axis *Oy* represents the instantaneous value of *i*(*t*) to the scale chosen.
- *Ox* axis is called *reference axis*.

The phasor is not an electric current (voltage), it is only a symbol for it.

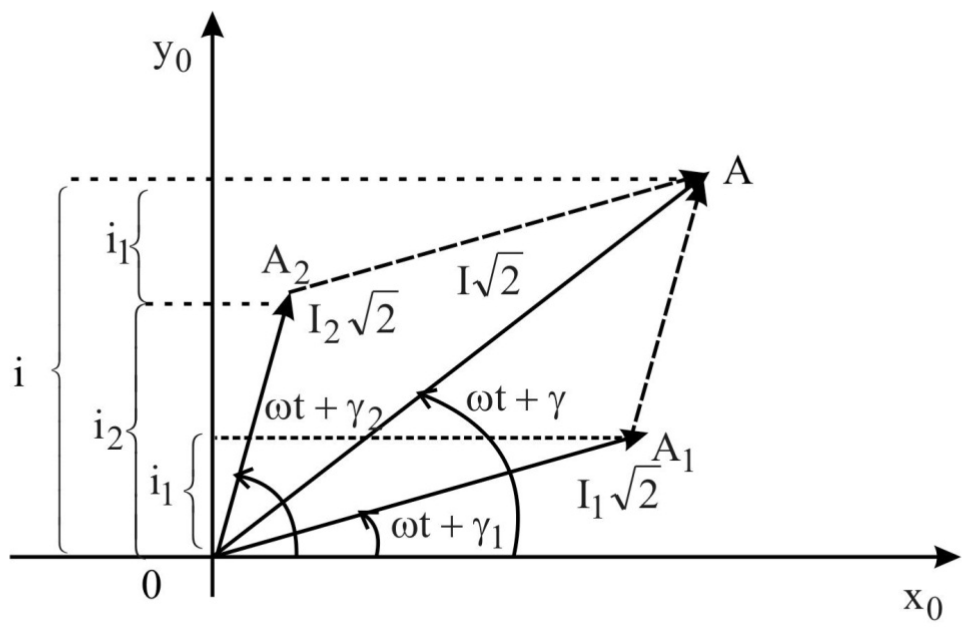
Mathematical operations in phasorial representation.

a) The *multiplication of a sinusoid by a scalar* „a":

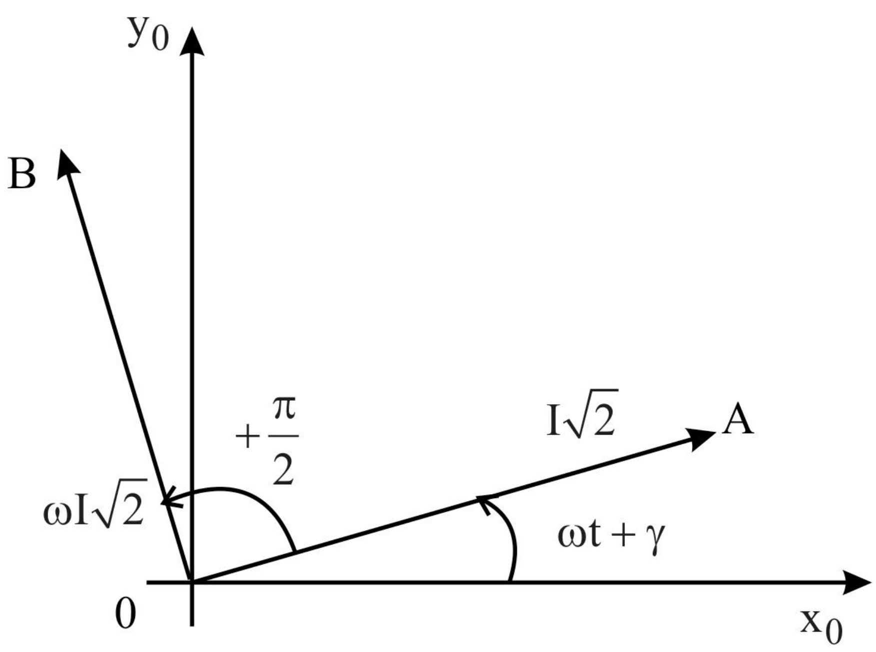


The resulting phasor has its peak value „a" times bigger and the same phase angle.

b) The *addition* : $i_1 + i_2 \rightleftarrows \overline{OA}_1 + \overline{OA}_2 = I_1\sqrt{2}\underline{/\ \omega t + \gamma_1} + I_2\sqrt{2}\underline{/\ \omega t + \gamma_2}$



c) The *derivation* :



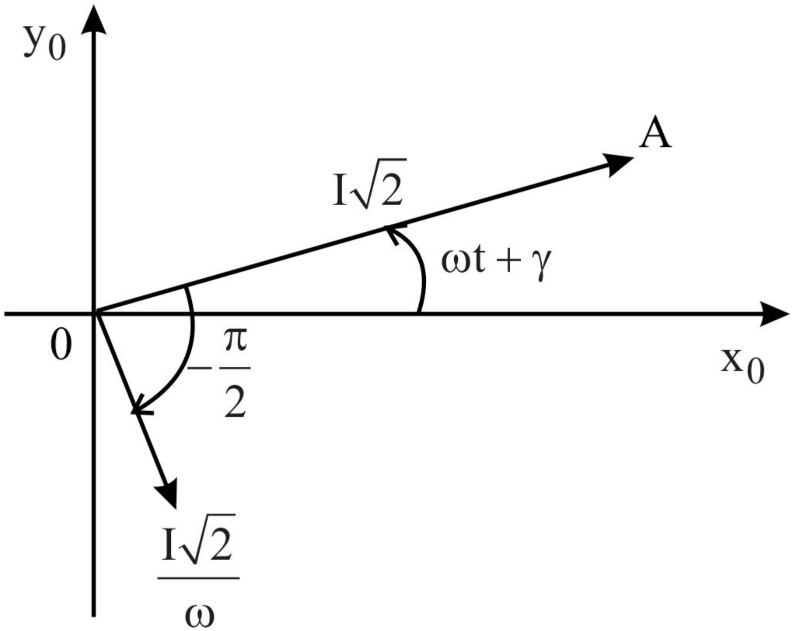
$$\frac{di}{dt} = \omega\sqrt{2}I\cos(\omega t + \gamma)$$

$$\frac{di}{dt} = \omega\sqrt{2}I\sin\left(\omega t + \gamma + \frac{\pi}{2}\right)$$

$$\frac{di}{dt} \rightleftarrows \omega I\sqrt{2}\,\angle\,\omega t + \gamma + \frac{\pi}{2}$$

The *derivation* of a sinusoid corresponds to the multiplication of the peak value by ω and counter-clockwise rotation of the phasor with $\pi/2$

d) The *integration*:



$$\int i dt = \frac{\sqrt{2}I}{\omega}\sin\left(\omega t + \gamma - \frac{\pi}{2}\right)$$

$$\rightleftarrows \frac{I}{\omega}\sqrt{2}\,\underline{/\ \omega t + \gamma - \frac{\pi}{2}}$$

The *integration* of a sinusoid corresponds to the division of the peak value by *ω* and clockwise rotation of the phasor with π/2

Important remark: in practical work, the Argand (phasorial) diagram is simplified by omitting the axes.

REVIEW: COMPLEX NUMBERS

A complex number z may be written in *rectangular form* as

$$z = x + jy \tag{B.1}$$

where $j = \sqrt{-1}$; x is the *real part* of z while y is the *imaginary part* of z; that is,

$$x = \text{Re}(z), \qquad y = \text{Im}(z) \tag{B.2}$$

The complex number z is shown plotted in the complex plane in Fig. B.1. Since $j = \sqrt{-1}$,

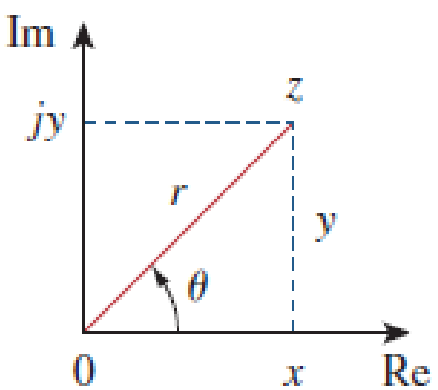


Figure B.1
Graphical representation of a complex number.

$$\begin{aligned}
\frac{1}{j} &= -j \\
j^2 &= -1 \\
j^3 &= j \cdot j^2 = -j \\
j^4 &= j^2 \cdot j^2 = 1 \\
j^5 &= j \cdot j^4 = j \\
&\vdots \\
j^{n+4} &= j^n
\end{aligned} \tag{B.3}$$

A second way of representing the complex number z is by specifying its magnitude r and the angle θ it makes with the real axis, as Fig. B.1 shows. This is known as the *polar form*. It is given by

$$z = |z|\underline{/\theta} = r\underline{/\theta} \tag{B.4}$$

where

$$r = \sqrt{x^2 + y^2}, \qquad \theta = \tan^{-1}\frac{y}{x} \tag{B.5a}$$

or

$$x = r\cos\theta, \qquad y = r\sin\theta \tag{B.5b}$$

that is,

$$z = x + jy = r\underline{/\theta} = r\cos\theta + jr\sin\theta \tag{B.6}$$

In converting from rectangular to polar form using Eq. (B.5), we must exercise care in determining the correct value of θ. These are the four possibilities:

$$\begin{aligned}
z &= x + jy, & \theta &= \tan^{-1}\frac{y}{x} & &\text{(1st Quadrant)} \\
z &= -x + jy, & \theta &= 180^\circ - \tan^{-1}\frac{y}{x} & &\text{(2nd Quadrant)} \\
z &= -x - jy, & \theta &= 180^\circ + \tan^{-1}\frac{y}{x} & &\text{(3rd Quadrant)} \\
z &= x - jy, & \theta &= 360^\circ - \tan^{-1}\frac{y}{x} & &\text{(4th Quadrant)}
\end{aligned} \qquad \textbf{(B.7)}$$

assuming that x and y are positive.

The third way of representing the complex z is the *exponential form*:

$$z = re^{j\theta} \tag{B.8}$$

This is almost the same as the polar form, because we use the same magnitude r and the angle θ.

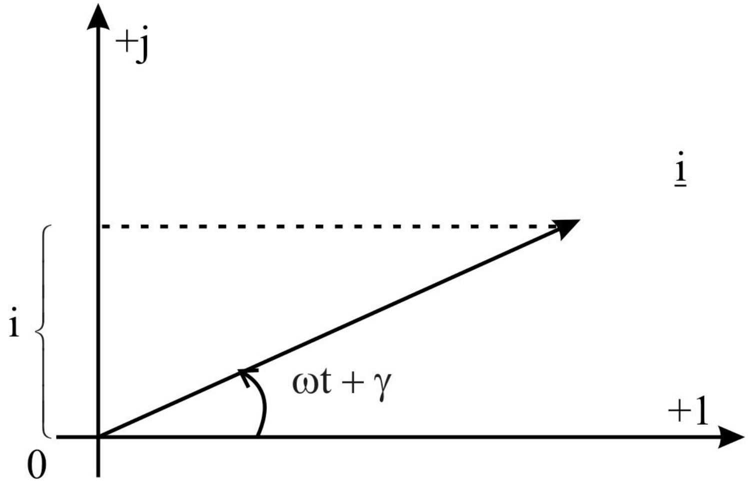
The three forms of representing a complex number are summarized as follows.

$z = x + jy,$	$(x = r\cos\theta, y = r\sin\theta)$	Rectangular form
$z = r\angle\theta,$	$\left(r = \sqrt{x^2 + y^2}, \theta = \tan^{-1}\frac{y}{x}\right)$	Polar form
$z = re^{j\theta},$	$\left(r = \sqrt{x^2 + y^2}, \theta = \tan^{-1}\frac{y}{x}\right)$	Exponential form

(B.9)

The first two forms are related by Eqs. (B.5) and (B.6). In Section B.3 we will derive Euler's formula, which proves that the third form is also equivalent to the first two.

B) Analytical (or complex) representation.



$$i = I\sqrt{2}\sin(\omega t + \gamma)$$

- The complex time function (*complex instantaneous value*):

$$\underline{i} = I\sqrt{2}e^{j(\omega t+\gamma)}$$

$$i = I\sqrt{2}\sin(\omega t + \gamma) \rightleftarrows \underline{i} = I\sqrt{2}e^{j(\omega t+\gamma)}$$

$$\underline{i} = \sqrt{2}I\cos(\omega t + \gamma) + j\sqrt{2}I\sin(\omega t + \gamma),$$

$$i = \Im_m(\underline{i}) = \Im_m[I\sqrt{2}e^{j(\omega t+\gamma)}]$$

Mathematical operations in complex representation.

a) The *multiplication by a scalar* „a”:

$$a \cdot i = aI\sqrt{2}\sin(\omega t + \gamma) \leftrightarrow aI\sqrt{2}e^{j(\omega t+\gamma)} = a \cdot \underline{i}$$

b) The *addition* : $i_1 + i_2 \rightleftarrows \underline{i}_1 + \underline{i}_2$

Adding sinusoids of the same frequency is equivalent to adding their corresponding phasors.

c) The *derivation*: $\dfrac{di}{dt} \rightleftarrows j\omega\underline{i}$

Differentiating a sinusoid is equivalent to multiplying its corresponding phasor by $j\omega$.

demonstration:

$$\frac{di}{dt} = \omega I\sqrt{2}\cos(\omega t + \gamma) = \omega I\sqrt{2}\sin(\omega t + \gamma + \frac{\pi}{2})$$

where:

$$\omega I\sqrt{2}e^{j(\omega t+\gamma+\frac{\pi}{2})} = \omega I\sqrt{2}e^{j(\omega t+\gamma)} \cdot e^{j\frac{\pi}{2}} = j\omega I\sqrt{2}e^{j(\omega t+\gamma)} = j\omega \cdot \underline{i},$$

$$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = j$$

d) The *integration* : $\int i dt \rightleftarrows \frac{1}{j\omega} \cdot \underline{i}$

Integrating a sinusoid is equivalent to dividing its corresponding phasor by $j\omega$.

$$\int i dt = -\frac{I\sqrt{2}}{\omega}\cos(\omega t + \gamma) = \frac{I\sqrt{2}}{\omega}\sin(\omega t + \gamma - \frac{\pi}{2}) \quad \leftrightarrow \quad \frac{I\sqrt{2}}{\omega} e^{j(\omega t + \gamma - \frac{\pi}{2})} =$$

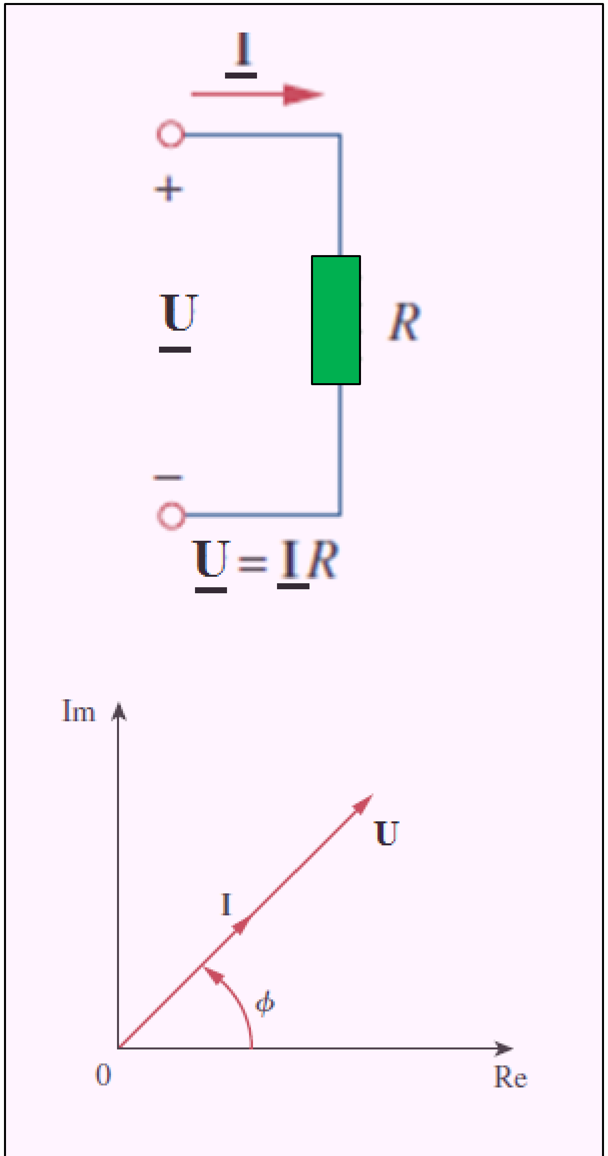
$$= \frac{I\sqrt{2}}{\omega} e^{j(\omega t + \gamma)} \cdot e^{-j\frac{\pi}{2}} = \frac{1}{j\omega} \cdot I\sqrt{2} e^{j(\omega t + \gamma)} = \frac{1}{j\omega} \cdot \underline{i}$$

where: $e^{-j\frac{\pi}{2}} = \cos\left(-\frac{\pi}{2}\right) + j\sin\left(-\frac{\pi}{2}\right) = -j = \frac{1}{j}$

The simplified complex representation:

$$i = I\sqrt{2}\sin(\omega t + \gamma) \rightleftarrows \underline{I} = I e^{j\gamma}$$

It is called *complex effective value*.



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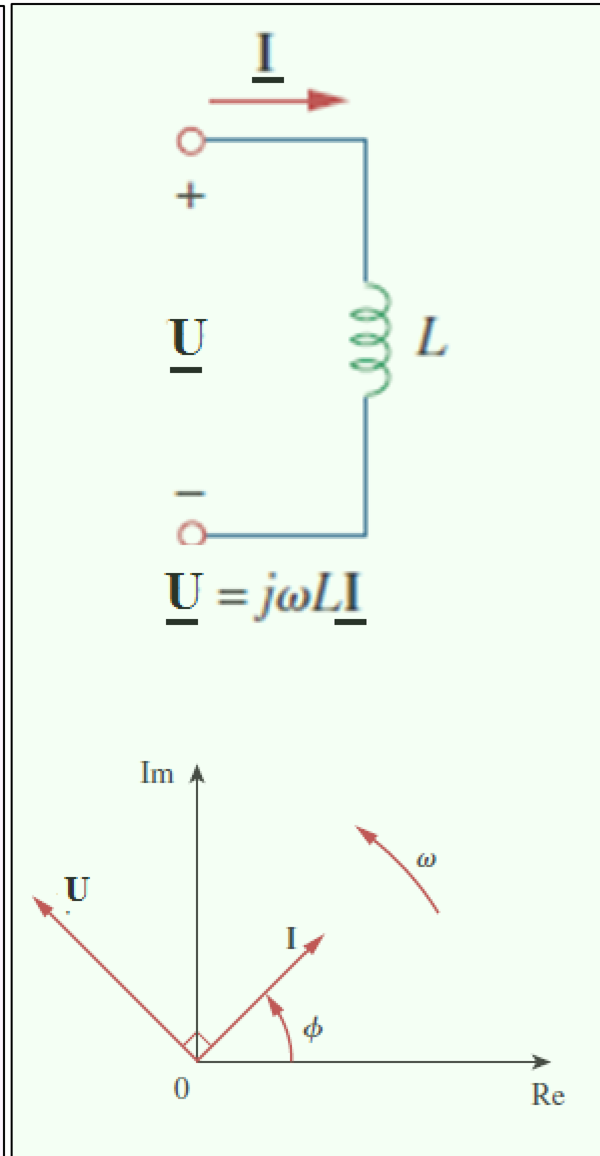

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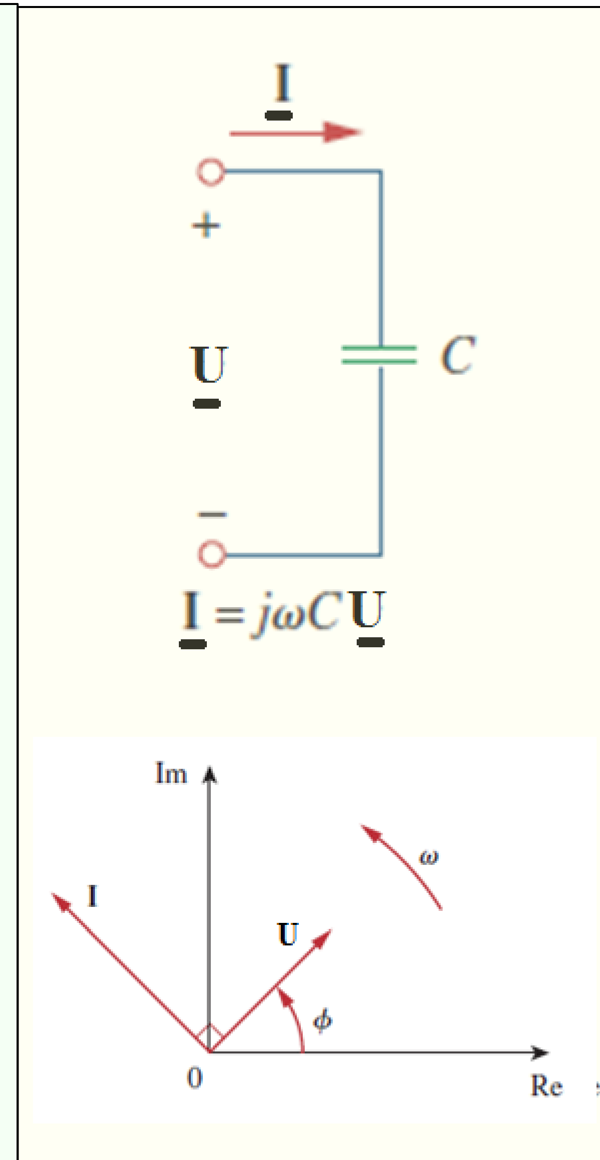

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I = jωCU
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Re

2.7. CHARACTERISATION OF LINEAR CIRCUITS IN COMPLEX PLANE

$$u = U\sqrt{2}\sin(\omega t + \beta) \qquad \underline{U} = U \cdot e^{j\beta}$$

$$i = I\sqrt{2}\sin(\omega t + \gamma) \qquad \underline{I} = I \cdot e^{j\gamma}$$

2.7.1. The complex impedance

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = f_1(\omega, R, L, C, \ldots)$$

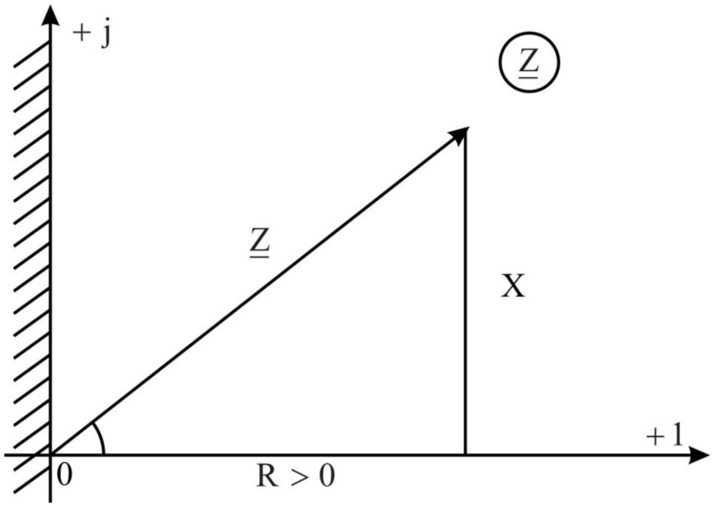
$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U}{I}e^{j(\beta-\gamma)} = \frac{U}{I}\cos(\beta-\gamma) + j\frac{U}{I}\sin(\beta-\gamma)$$

$$\underline{Z} = Ze^{j\varphi} = Z\cos\varphi + jZ\sin\varphi = R + jX$$

Where: Z is the impedance

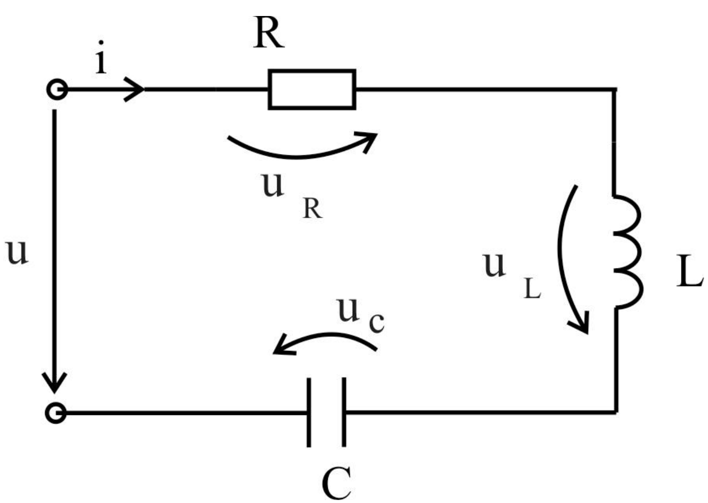
$R = Z\cos\varphi$ is the resistive or in-phase component,

$X = Z\sin\varphi$ is the reactive or quadrature component.



$$Z = \sqrt{R^2 + X^2}$$

$$\varphi = \arctan \frac{X}{R}$$



For the RLC series circuit:

$$\underline{Z} = R + jX = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{Ue^{j\beta}}{Ze^{j\varphi}} = \frac{U}{Z}e^{j(\beta-\varphi)}$$

$$i = \mathrm{Im}[\sqrt{2}\,\underline{I}\cdot e^{j\omega t}] = \frac{U}{Z}\sqrt{2}\sin(\omega t + \beta - \varphi)$$

2.7.2. The complex admittance

$$\underline{Y} = \frac{\underline{I}}{\underline{U}} = \frac{1}{\underline{Z}} = g_1(\omega; R, L, C, \ldots)$$

$$\underline{Y} = \frac{I \cdot e^{j\gamma}}{U \cdot e^{j\beta}} = \frac{I}{U} e^{-j(\beta-\gamma)} = \frac{I}{U}\cos(\beta-\gamma) - j \cdot \frac{I}{U}\sin(\beta-\gamma)$$

$$\underline{Y} = Ye^{-j\varphi} = Y\cos\varphi - jY\sin\varphi = G - jB$$

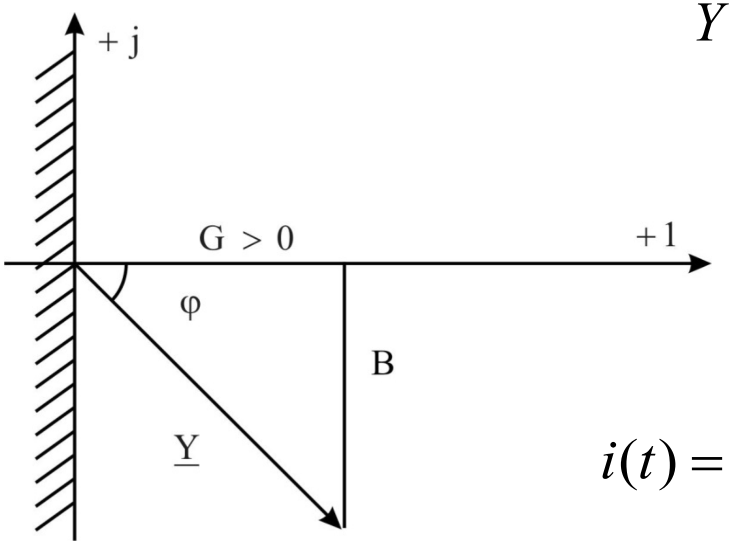
$G = Y\cos\varphi$ is the conductance (the real part of $\underline{Y}$),

$B = Y\sin\varphi$ is the susceptance (the imaginary part of $\underline{Y}$).

$$Y = \sqrt{G^2 + B^2}$$

- it should be noted that $G = \dfrac{R}{Z^2}$ and $B = \dfrac{X}{Z^2}$ because:

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} = G - jB$$



$Y = \sqrt{G^2 + B^2}$ $\longrightarrow$ the admittance triangle

- the current:

$$\underline{I} = \underline{U} \cdot \underline{Y} = \underline{U} Y e^{-j\varphi} = UYe^{j(\beta - \varphi)}$$

$$i(t) = \mathrm{Im}[\sqrt{2}\,\underline{I} e^{j\omega t}] = UY\sqrt{2}\sin(\omega t + \beta - \varphi)$$

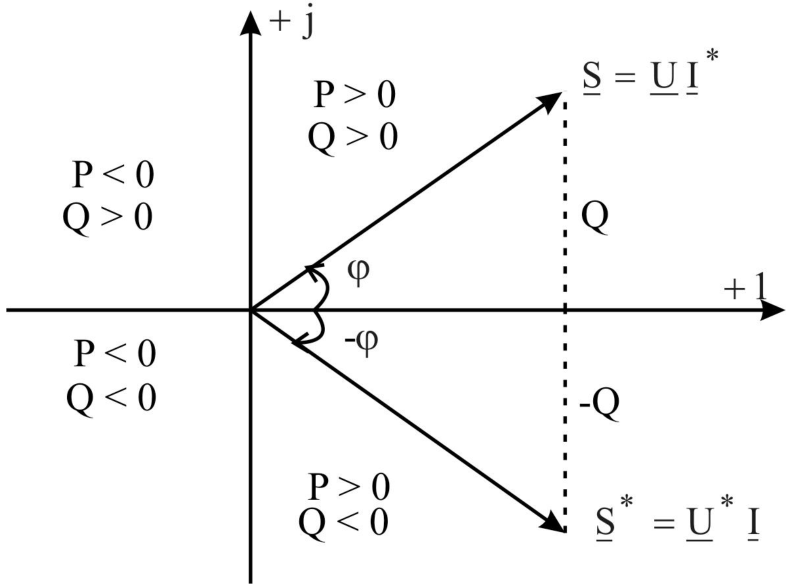
2.7.3. The complex power

Let $\underline{U} = Ue^{j\beta}$ and $\underline{I} = Ie^{j\gamma}$

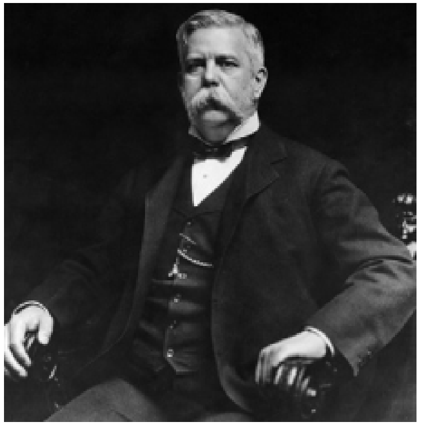
- the complex power: $\underline{S} = \underline{U}\,\underline{I}^*$

$$\underline{S} = Ue^{j\beta} Ie^{-j\gamma} = UIe^{j(\beta-\gamma)} = UIe^{j\varphi} = Se^{j\varphi} = UI(\cos\varphi + j\sin\varphi) = P + jQ$$

$$\underline{S}^* = \underline{U}^*\underline{I} = UIe^{-j\varphi} = UI\cos\varphi - jUI\sin\varphi = P - jQ$$



Historical



George Westinghouse. Photo

Nikola Tesla (1856–1943) and **George Westinghouse** (1846–1914) helped establish alternating current as the primary mode of electricity transmission and distribution.

Today it is obvious that ac generation is well established as the form of electric power that makes widespread distribution of electric power efficient and economical. However, at the end of the 19th century, which was the better—ac or dc—was hotly debated and had extremely outspoken supporters on both sides. The dc side was led by Thomas Edison, who had earned a lot of respect for his many contributions. Power generation using ac really began to build after the successful contributions of Tesla. The real commercial success in ac came from George Westinghouse and the outstanding team, including Tesla, he assembled. In addition, two other big names were C. F. Scott and B. G. Lamme.

The most significant contribution to the early success of ac was the patenting of the polyphase ac motor by Tesla in 1888. The induction motor and polyphase generation and distribution systems doomed the use of dc as the prime energy source.

Chapter 2: AC Circuits

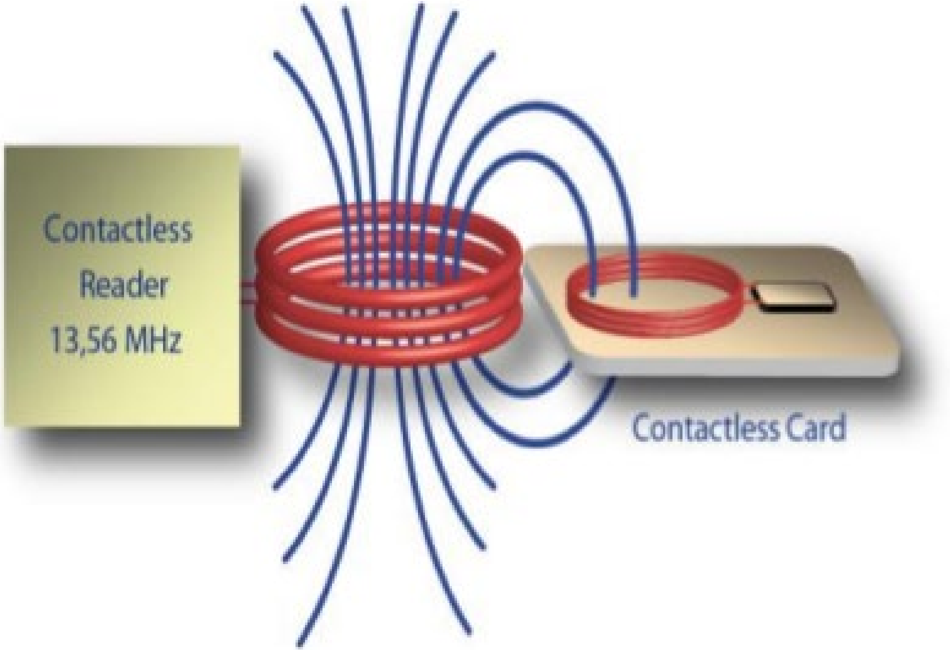
Magnetically coupled circuits

BASES OF ELECTROTECHNICS I.
Faculty of Electronics, Telecommunications and Information Technology
Specialization: IETTI
Academic year: 2023-2024

When two coils with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be *magnetically coupled.*

Inductive coupling is widely used throughout electrical technology; examples include:

- ✓ Electric motors and generators
- ✓ Inductive charging products
- ✓ Induction cookers and induction heating systems
- ✓ Induction loop communication systems
- ✓ Metal detectors
- ✓ Radio-frequency identification
- ✓ Transformers
- ✓ Wireless power transfer



Working principle of inductive coupling in NFC devices

NFC (*Near Field Communication*) works based on the principle of *inductive coupling*, where loosely coupled inductive circuits share power and data over a distance of a few centimeters. All NFC devices operate at 13.56MHz. NFC devices share the basic technology with proximity (13.56MHz) RFID (*Radio-Frequency Identification)* tags and contactless smartcards, but have a number of key additional features.

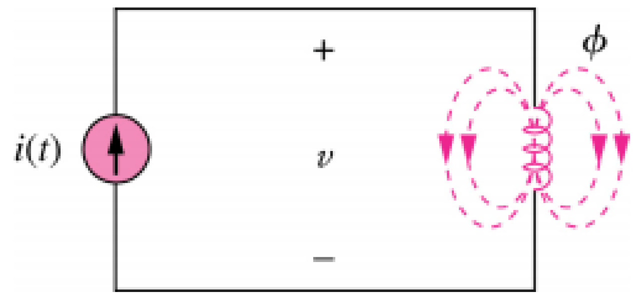
Source: [www.wireless.intgckts.com]

2. MUTUAL INDUCTANCE

When two coils are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil and inducing voltage in the latter. This phenomenon is known as mutual inductance.

- Consider a coil with N turns, when current *i* flows through the coil, a magnetic flux φ is produced around it. According to Faraday's law, the voltage *v* induced in the coil is proportional to the number of turns N and the time rate of change of the magnetic flux Φ; that is,

An inductor :
$\begin{cases} \text{inductance } L \\ N \text{ turns} \end{cases}$



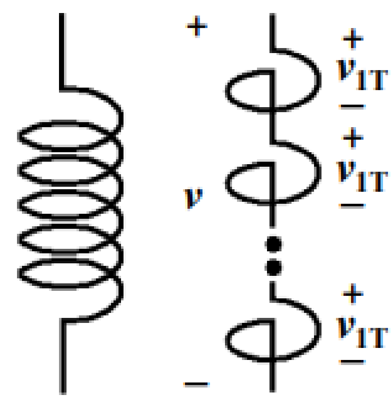
- For each turn, the induced volatge is

$$v_{1T} = \frac{d\phi}{dt} \quad \text{(Faradays's Law)}$$

- For N turns, the induced volatge is

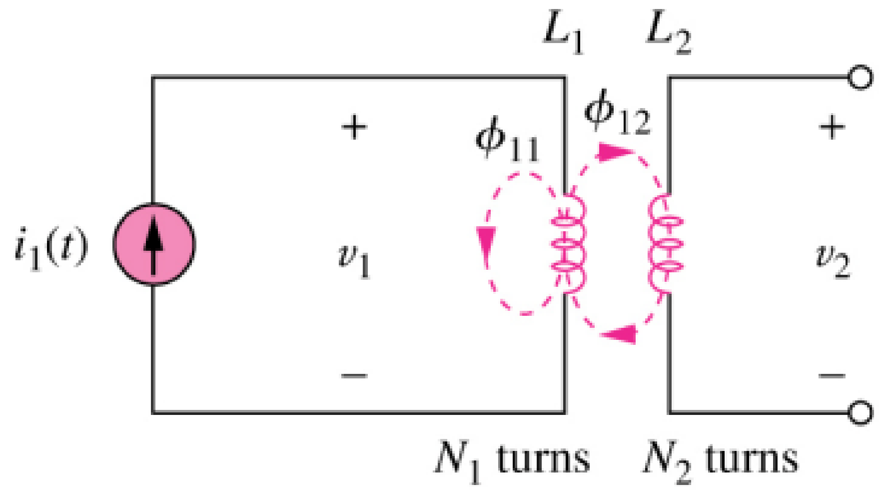
$$v = N\frac{d\phi}{dt} = N\frac{d\phi}{di}\frac{di}{dt} = L\frac{di}{dt}$$

$$\Rightarrow L = N\frac{d\phi}{di} \quad \text{(self - inductance)}$$



Coil 1: $\begin{cases} \text{self-inductances } L_1 \\ N_1 \text{ turns} \end{cases}$

Coil 2: $\begin{cases} \text{self-inductances } L_2 \\ N_2 \text{ turns} \end{cases}$



- Assuming no current in coil 2, the flux generated by coil 1 is

$$\phi_1 = \phi_{11}(\text{only coil 1}) + \phi_{12}(\text{both coils})$$

$$\Rightarrow v_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

where $L_1 = N_1 \dfrac{d\phi_1}{di_1}$

$$\Rightarrow v_2 = N_2 \frac{d\phi_{12}}{dt} = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt}$$

- The mutual-inductance of coil 2 with respect to coil 1 is

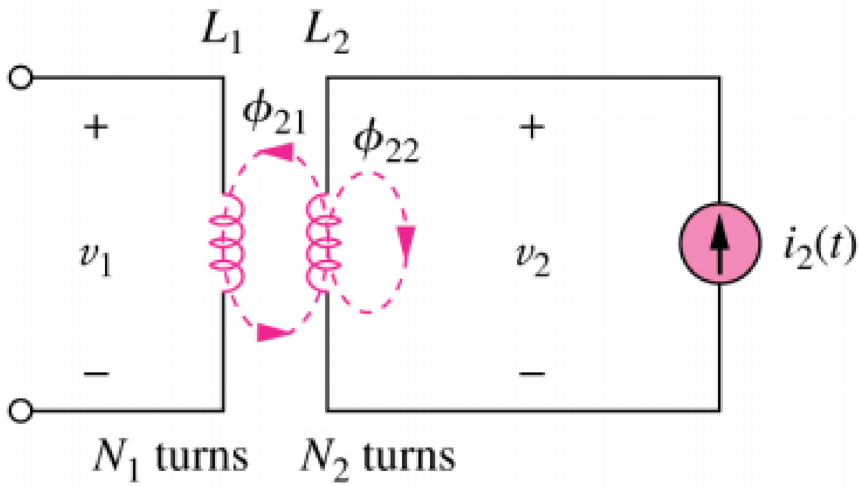
$$\boxed{M_{21} = N_2 \frac{d\phi_{12}}{di_1}}$$

- The open-circuit mutual voltage is

$$\boxed{v_2 = M_{21}\frac{di_1}{dt}}$$

Coil 1 : $\begin{cases} \text{self-inductances } L_1 \\ N_1 \text{ turns} \end{cases}$

Coil 2 : $\begin{cases} \text{self-inductances } L_2 \\ N_2 \text{ turns} \end{cases}$



- Assuming no current in coil 1, the flux generated by coil 2 is

$$\phi_2 = \phi_{22}(\text{only coil 2}) + \phi_{21}(\text{both coils})$$

$$\Rightarrow v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_2}{di_2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

where $L_2 = N_2 \dfrac{d\phi_2}{di_2}$

$$\Rightarrow v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2}\frac{di_2}{dt} = M_{12}\frac{di_2}{dt}$$

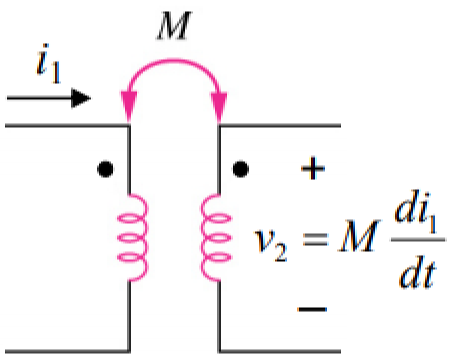
- The mutual-inductance of coil 1 with respect to coil 2 is

$$\boxed{M_{12} = N_1 \frac{d\phi_{21}}{di_2}} \quad (= M_{21})$$

- The open-circuit mutual voltage is

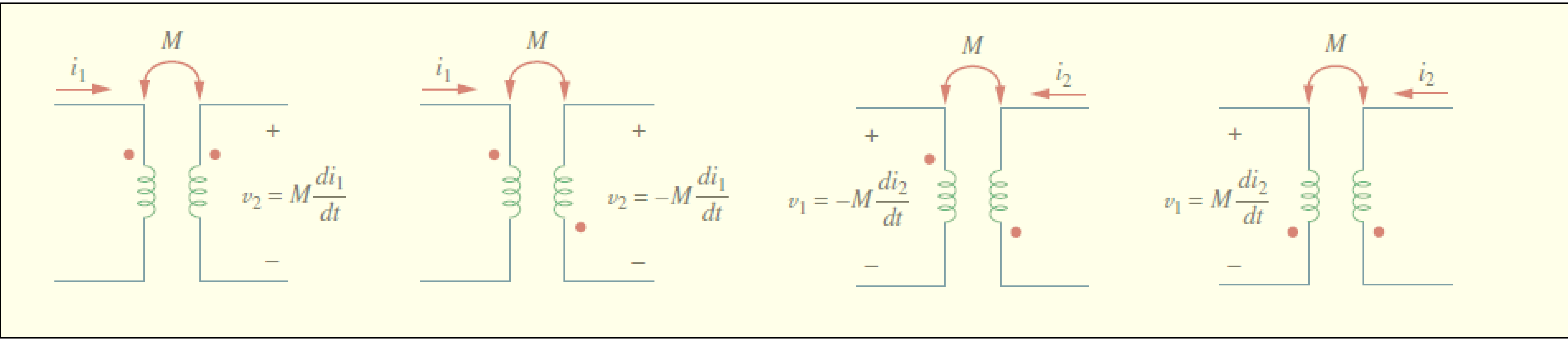
$$\boxed{v_1 = M_{12}\frac{di_2}{dt}}$$

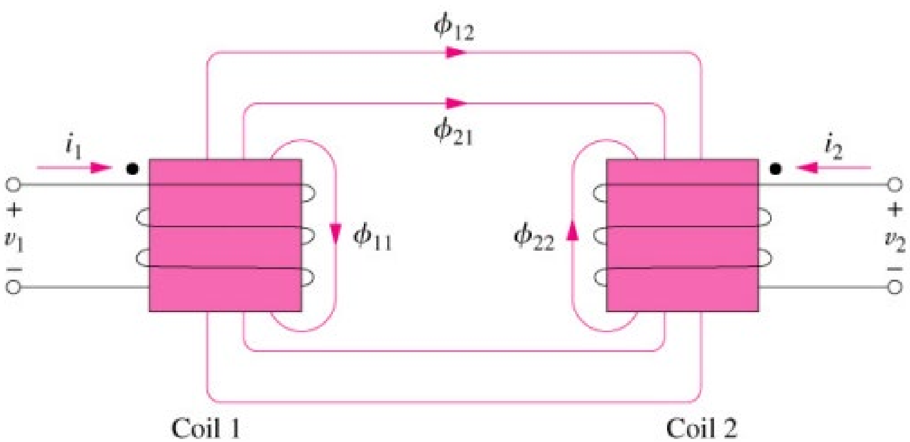
- We will see that $M_{12} = M_{21} = M$.
- ***Mutual coupling*** only exists when the inductors or coils are ***in close proximity***, and the circuits are driven by ***time-varying sources***.
- ***Mutual inductance*** is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).
- The ***dot convention*** states that a current ***entering*** the dotted terminal induces a ***positive*** polarity of the mutual voltage at the dotted terminal of the second coil.



- Mutual inductance is always a positive quantity, the mutual voltage *M di/dt* may be negative or positive, just like self-induced voltage *L di/dt*.

Examples illustrating how to apply the dot convention:





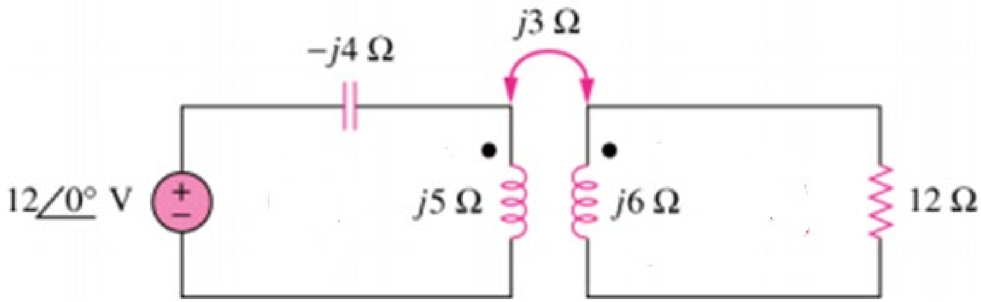
i_1 induces ϕ_{11} and ϕ_{12},
i_2 induces ϕ_{21} and ϕ_{22}.

$$\phi_1 = (\phi_{11} + \phi_{12}) + \phi_{21}$$
$$\phi_2 = \phi_{12} + (\phi_{21} + \phi_{22})$$

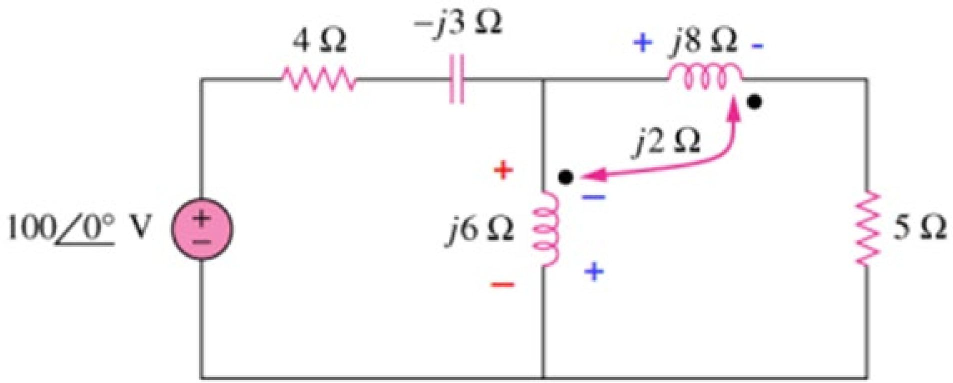
$$v_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d(\phi_{11} + \phi_{12})}{dt} + N_1 \frac{d\phi_{21}}{dt} = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt}$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d(\phi_{21} + \phi_{22})}{dt} + N_2 \frac{d\phi_{12}}{dt} = L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt}$$

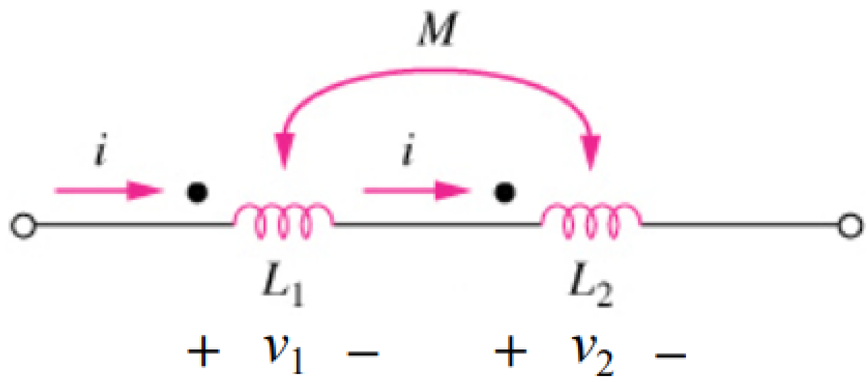
Example:

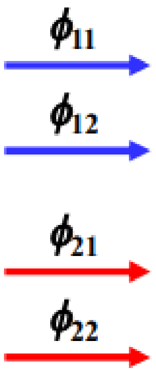


Example:



2.1. Series-Aiding Connection





$$v_1 = L_1 \frac{di}{dt} + M_{12} \frac{di}{dt}$$

$$v_2 = L_2 \frac{di}{dt} + M_{21} \frac{di}{dt}$$

$$v = v_1 + v_2$$

$$= L_1 \frac{di}{dt} + M_{12} \frac{di}{dt} + L_2 \frac{di}{dt} + M_{21} \frac{di}{dt}$$

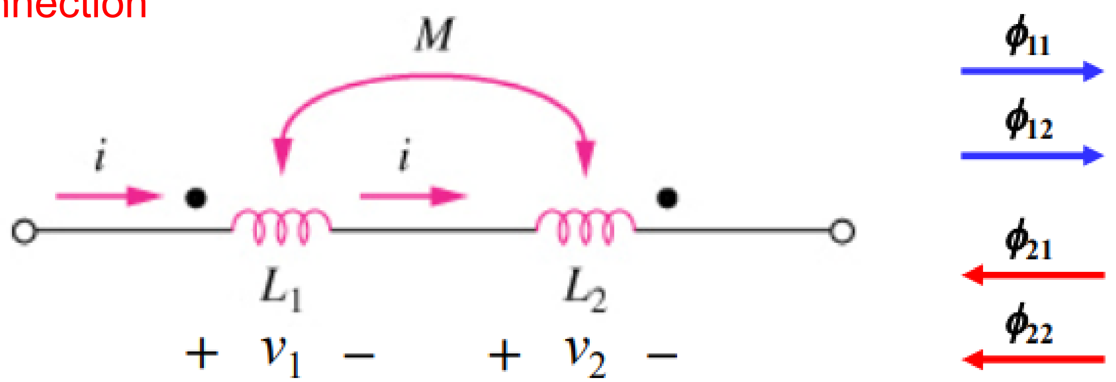
$$= (L_1 + L_2 + M_{12} + M_{21}) \frac{di}{dt}$$

But $M_{12} = M_{21} = M$,

$$\Rightarrow v = (L_1 + L_2 + 2M) \frac{di}{dt}$$

$$\Rightarrow \boxed{L_{eq} = L_1 + L_2 + 2M}$$

2.2. Series-Opposing Connection



$$v_1 = L_1\frac{di}{dt} - M_{12}\frac{di}{dt}$$

$$v_2 = L_2\frac{di}{dt} - M_{21}\frac{di}{dt}$$

$$v = v_1 + v_2$$

$$= L_1\frac{di}{dt} - M_{12}\frac{di}{dt} + L_2\frac{di}{dt} - M_{21}\frac{di}{dt}$$

$$= (L_1 + L_2 - M_{12} - M_{21})\frac{di}{dt}$$

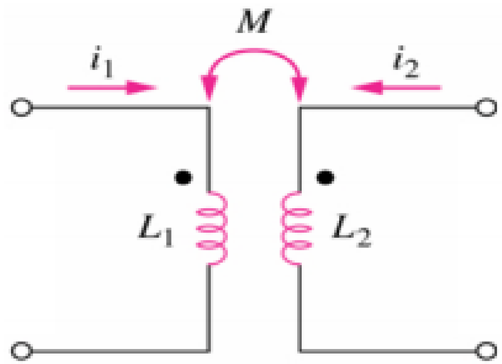
But $M_{12} = M_{21} = M$,

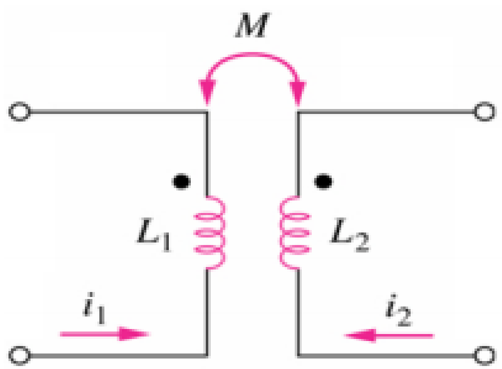
$$\Rightarrow v = (L_1 + L_2 - 2M)\frac{di}{dt}$$

$$\Rightarrow \boxed{L_{eq} = L_1 + L_2 - 2M}$$

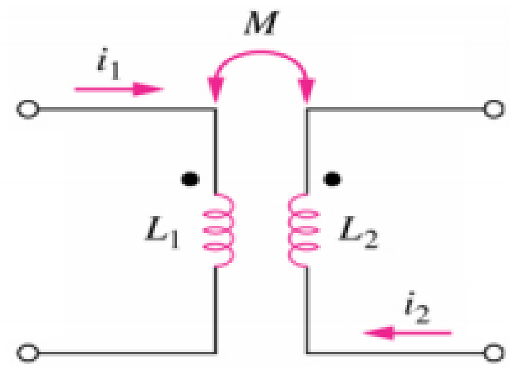
2.3. Energy in a Coupled Circuit

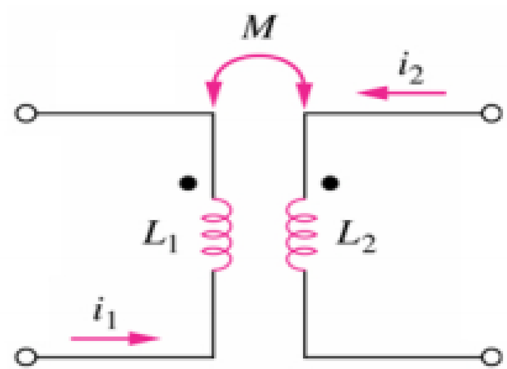
$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$





$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2$$



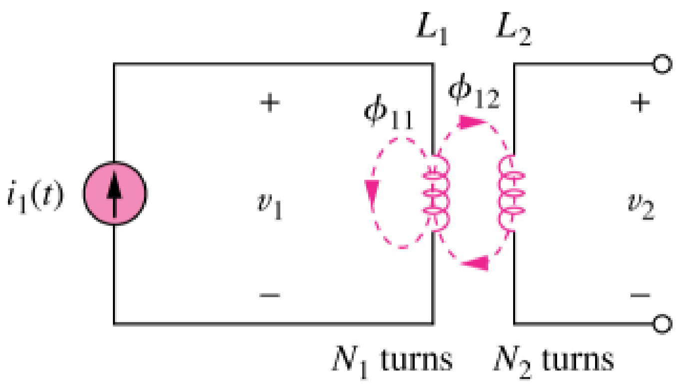


2.4. Coupling Coefficient

- The *coupling coefficient k* is defined as

$$k = \frac{M}{\sqrt{L_1 L_2}} \qquad (0 \le k \le 1)$$

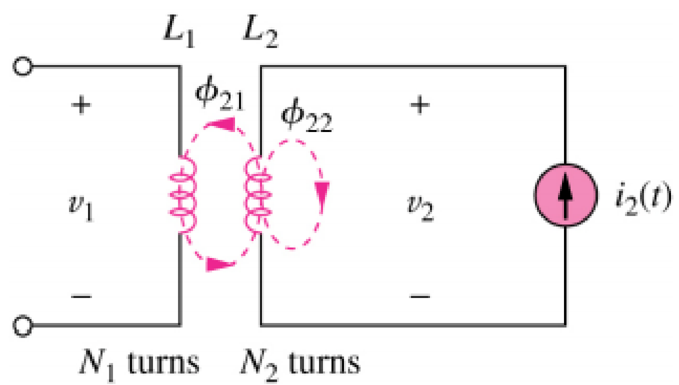
or $\quad M = k\sqrt{L_1 L_2}$



- $k = \dfrac{\phi_{12}}{\phi_{11} + \phi_{12}} = \dfrac{\phi_{21}}{\phi_{21} + \phi_{22}}$

- $k = 1$ means perfect coupling.

$$\phi_{11} = \phi_{22} = 0$$



The coupling coefficient *k* is a measure of magnetic coupling between two coils; $0 \le k \le 1$.

- *perfect coupling:* If all flux produced by one coil links another coil, then $k = 1$ and we have 100 percent coupling.
- *No coupling:* If the flux produced by one coil doesn't link the other coil, then $k = 0$ and we have 0 percent coupling.
- *loose coupling:* if $k < 0.5$, as shown in Fig 1
- *tight coupling:* if $k >= 0.5$, as shown in Fig

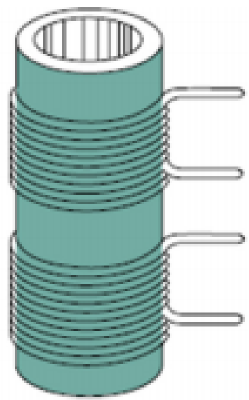


Figure 1- Loose coupling

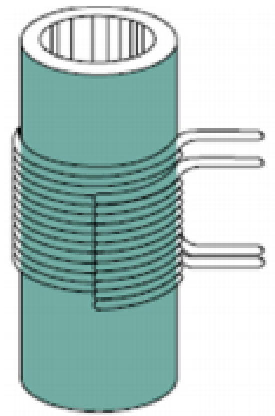


Figure 2 – Tight coupling

Example:

Q: Find k and the energy stored in the coupled inductors at $t = 1$ s.

$$Sol: \quad k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$

- For mesh 1,

$$(10 + j20)\mathbf{I}_1 + \underline{j10\mathbf{I}_2} = 60\angle 30^\circ \quad (1a)$$

- For mesh 2,

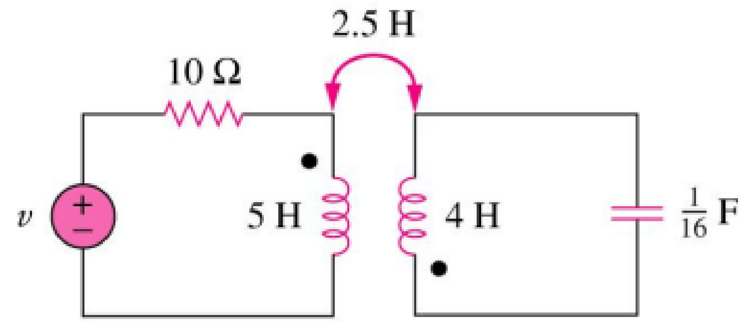
$$\underline{j10\mathbf{I}_1} + (j16 - j4)\mathbf{I}_2 = 0 \quad (1b)$$

$$\Rightarrow \begin{cases} \mathbf{I}_1 = 3.905\angle -19.4^\circ \\ \mathbf{I}_2 = 3.254\angle 160.6^\circ \end{cases}$$

$$\begin{cases} i_1 = 3.905\cos(4t - 19.4^\circ) \\ i_2 = 3.254\cos(4t + 160.6^\circ) \end{cases}$$

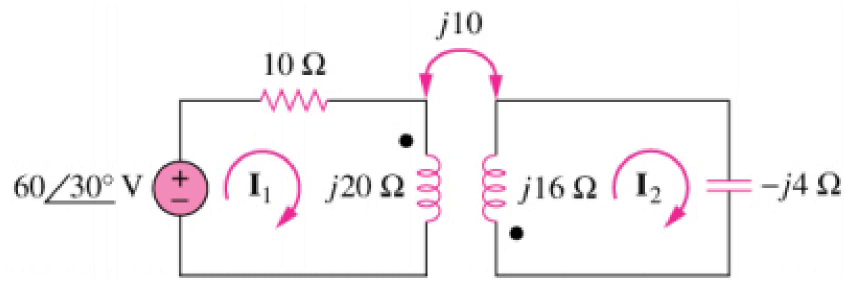
$$\Rightarrow i_1(1) = -3.389, \quad i_2(1) = 2.824$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = 20.73 \text{ J}$$



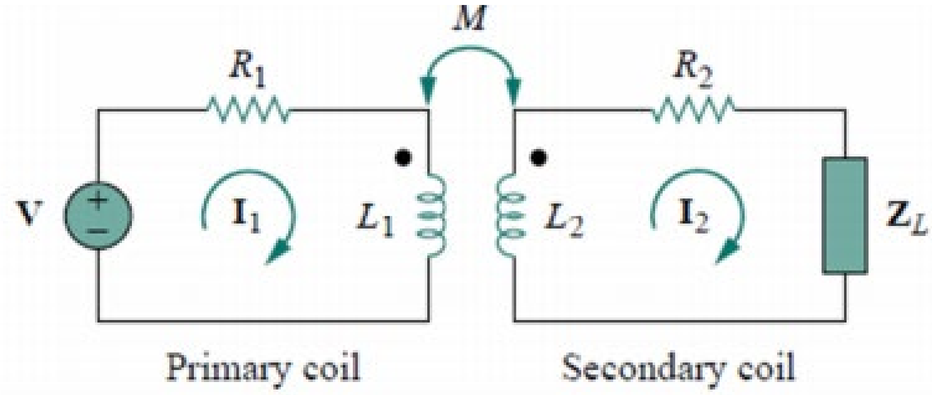
$$v = 60\cos(4t + 30^\circ) \text{ V}$$

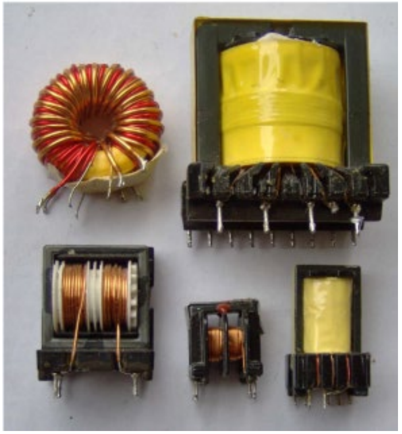
$$\omega = 4 \text{ rad/s}$$

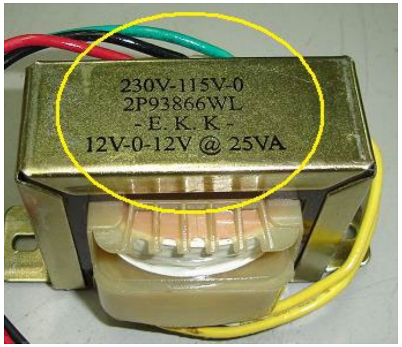


A transformer is a magnetic device that takes advantage of the phenomenon of mutual inductance.

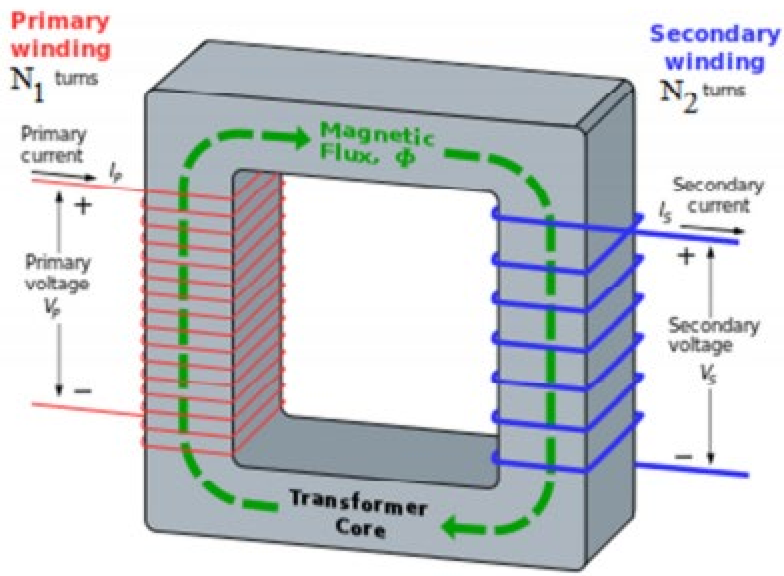
- ✓ The coil that is directly connected to the voltage source is called the primary winding.
- ✓ The coil connected to the load is called the secondary winding.
- ✓ The resistances R1 and R2 are included to account for the losses (power dissipation) in the coils.
- ✓ The transformer is said to be linear if the coils are wound on a magnetically linear material.

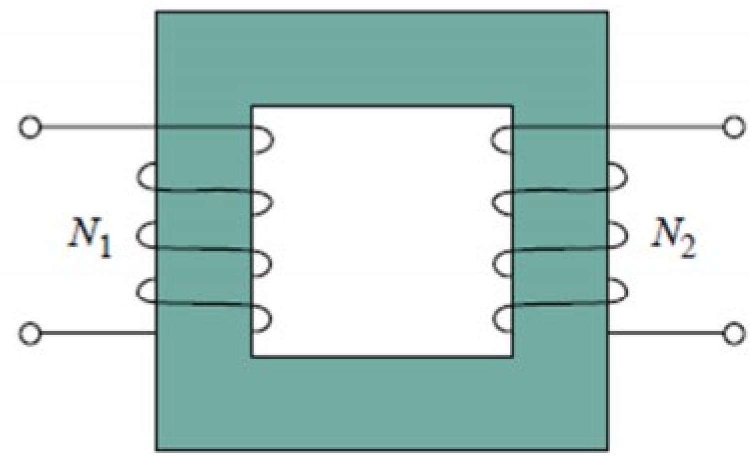






- An ideal transformer is one with perfect coupling (k = 1). It consists of two (or more) coils with a large number of turns wound on a common core of high permeability. Because of this high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling.





1. Coils have very large reactance. **(L_1, L_2, $M \sim \infty$)**
2. Coupling coefficient is equal to unity. **($k = 1$)**
3. Primary and secondary are lossless.

   (series resistances $R_1 = R_2 = 0$)

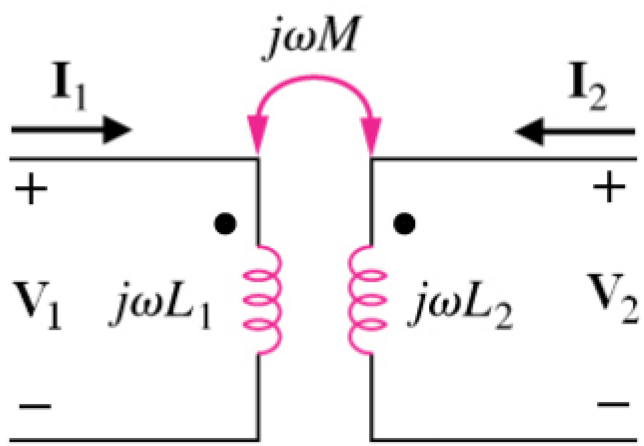
$$\begin{cases} \mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2 & \text{(1a)} \\ \mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2 & \text{(1b)} \end{cases}$$

From (1a),

$$\mathbf{I}_1 = \left(\mathbf{V}_1 - j\omega M \mathbf{I}_2\right) / j\omega L_1 \quad \text{(1c)}$$

Substituting 1(c) into (1b) gives

$$\mathbf{V}_2 = \frac{M}{L_1}\mathbf{V}_1 + \left(L_2 - \frac{M^2}{L_1}\right) j\omega \mathbf{I}_2$$

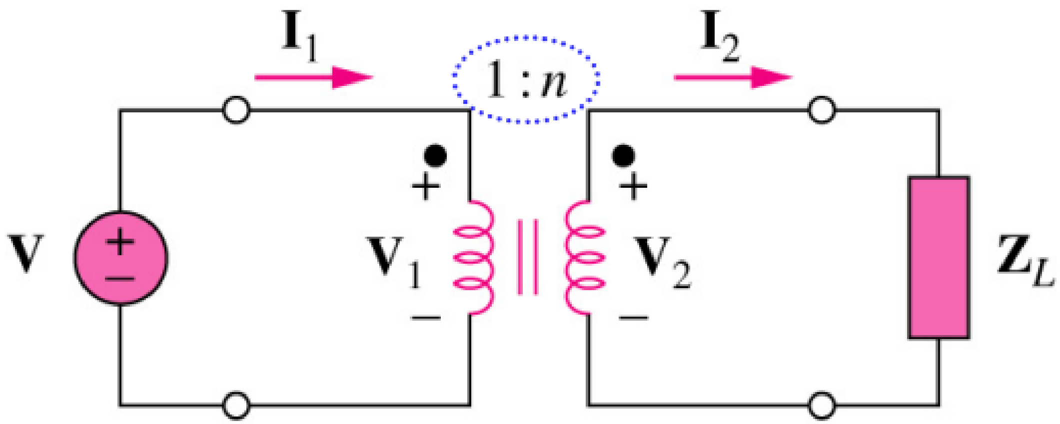


- For perfect coupling,

$$k = 1 \quad \text{or} \quad M = \sqrt{L_1 L_2}$$

$$\Rightarrow \mathbf{V}_2 = \frac{\sqrt{L_1 L_2}}{L_1}\mathbf{V}_1 = \sqrt{\frac{L_2}{L_1}}\mathbf{V}_1 = n\mathbf{V}_1$$

where n is called the *turns ratio*.



- $v_1 = N_1 \dfrac{d\phi}{dt}$, $v_2 = N_2 \dfrac{d\phi}{dt}$

$$\Rightarrow \frac{v_2}{v_1} = \frac{N_2}{N_1} = n \quad \text{or} \quad \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n$$

- The transformer is lossless $\Rightarrow v_1 i_1 = v_2 i_2$

$$\frac{i_2}{i_1} = \frac{v_1}{v_2} = \frac{1}{n} \Rightarrow \boxed{\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{n}}$$

A **step-down transformer** is one whose secondary voltage is less than its primary voltage.

A **step-up transformer** is one whose secondary voltage is greater than its primary voltage.

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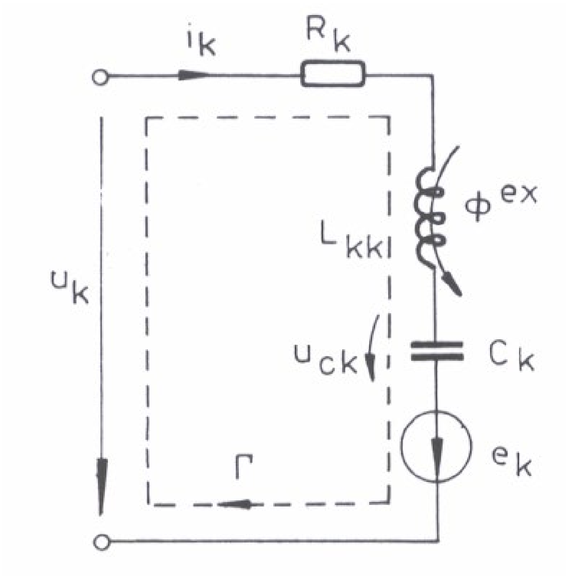
DIN CLUJ-NAPOCA

Chapter 2: AC Circuits

Sinusoidal Steady State Analysis

Content of this Subchapter:

1. The Ohm's law in complex notation
2. Kirchhoff's law
3. The superposition theorem
4. Thévenin - Norton equivalent network theorem
5. The equivalence theorem between a voltage source and a current source
6. The reciprocity theorem
7. Mesh (or loop) analysis of linear networks
8. Node analysis of linear networks



1. The Ohm's law in complex notation

$$e_k + u_k = R_k i_k + L_{kk}\frac{di_k}{dt} + \frac{1}{C_k}\int i_k dt + \sum_{\substack{j=1\\ j\neq k}}^{L} L_{kj}\frac{di_j}{dt}$$

In complex notation:

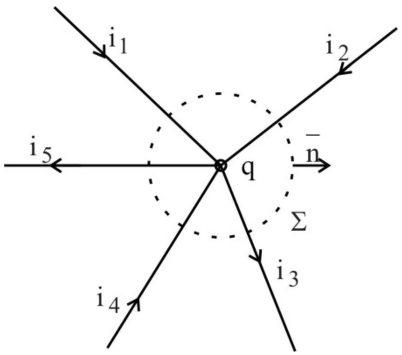
$$\underline{E}_k + \underline{U}_k = \underline{I}_k\left[R_k + j\left(\omega L_{kk} - \frac{1}{\omega C_k}\right)\right] + \sum_{\substack{j=1\\ j\neq k}}^{L} j\omega L_{kj}\underline{I}_j \Rightarrow \underline{E}_k + \underline{U}_k = \underline{Z}_{kk}\underline{I}_k + \sum_{\substack{j=1\\ j\neq k}}^{L} \underline{Z}_{kj}\underline{I}_j$$

$$\underline{Z}_{kk} = R_k + j\left(\omega L_{kk} - \frac{1}{\omega C_k}\right)$$ - the impedance if the branch *k*

$$\underline{Z}_{kj} = j\omega L_{kj}$$ - the mutual impedance between the branches *k* and *j*

2.1 Kirchhoff's current law (KCL)

For any lumped electric circuit, for any nodes, and at any time, the algebraic sum of all branch currents leaving the node is zero.



, $\sum_{k\in q} i_k = 0$,

$$\sum_{k\in q} \underline{I}_k = 0$$

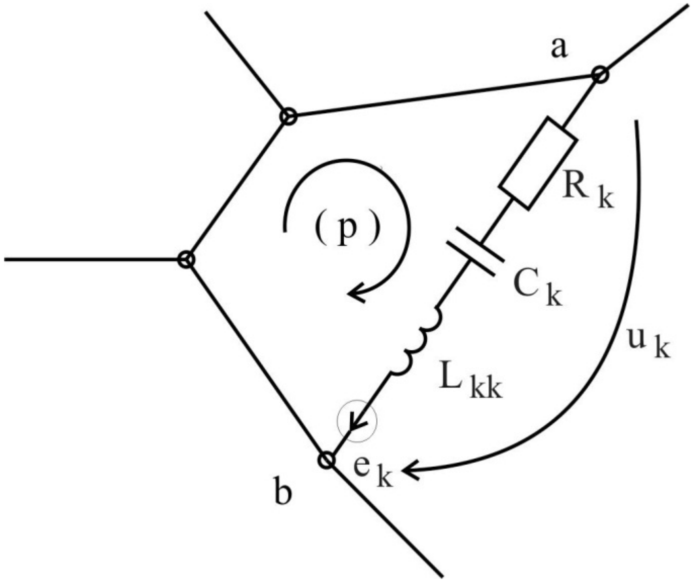
$$\sum_{k\in q} I_k \neq 0$$

2.2 Kirchhoff's voltage law (KVL)

For any lumped electric circuit, for any of its loops, and at any time, the algebraic sum of the branch voltages around the loop is zero.

$$\sum_{k\in p} \underline{U}_k = 0$$ or

$$\sum_{k\in p} \underline{Z}_k \underline{I}_k = \sum_{k\in p} \underline{E}_k$$



$$\underline{E}_k + \underline{U}_k = \underline{Z}_{kk}\underline{I}_k + j\omega\underline{\Phi}_k^{(ex)}$$

$$\underline{Z}_{kk} = R_k + j(\omega L_{kk} - \frac{1}{C_k\omega})$$

$$\Phi_k^{(ex)} = \sum_{k\neq j} L_{kj}\cdot i_j \qquad \underline{\Phi}_k^{(ex)} = \sum_{k\neq j} L_{kj}\cdot \underline{I}_j$$

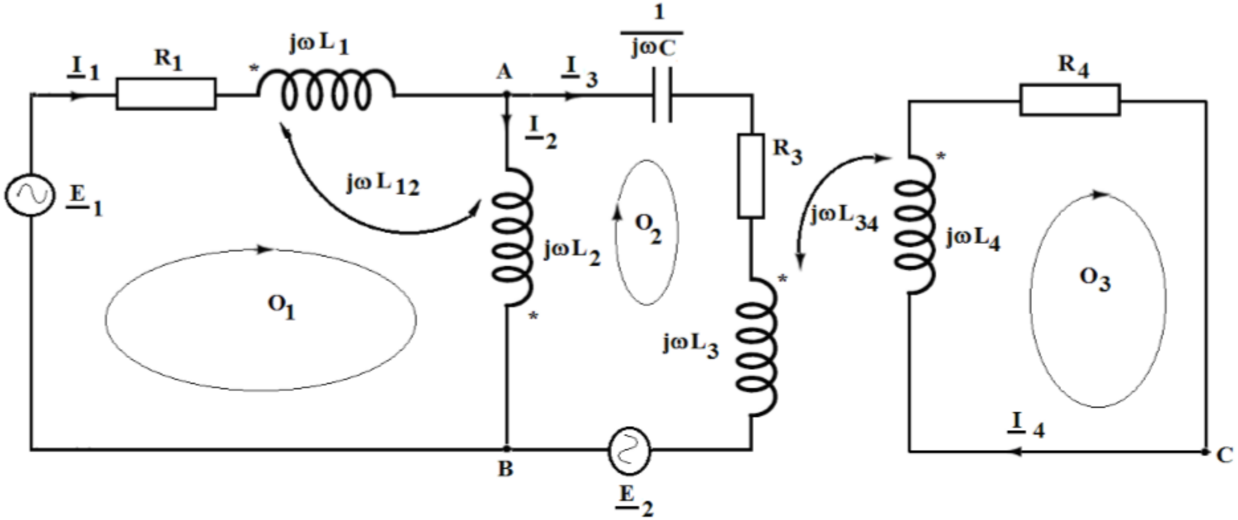
$$\underline{E}_k + \underline{U}_k = \underline{Z}_{kk}\underline{I}_k + j\omega\sum_{k\neq j} L_{kj}\underline{I}_j$$

$$\boxed{\sum_{k\in p}\underline{E}_k = \sum_{k\in p}(\underline{Z}_{kk}\underline{I}_k + \sum_{k\neq j}\underline{Z}_{kj}\underline{I}_j)}$$

where $\underline{Z}_{kj} = j\omega L_{kj}$

$$\sum_{k\in p}\underline{E}_k = \sum_{k\in p}\underline{Z}_{kk}\underline{I}_k$$

(if there is no magnetic coupling $L_{kj} = 0$)



I 1
R1
jωL 1
1
jωC
A
I 3
R4
I 2
R3
jωL 12
E 1
O2
jωL34
jωL4
jωL 2
O3
O1
jωL3
I 4
B
C
E 2

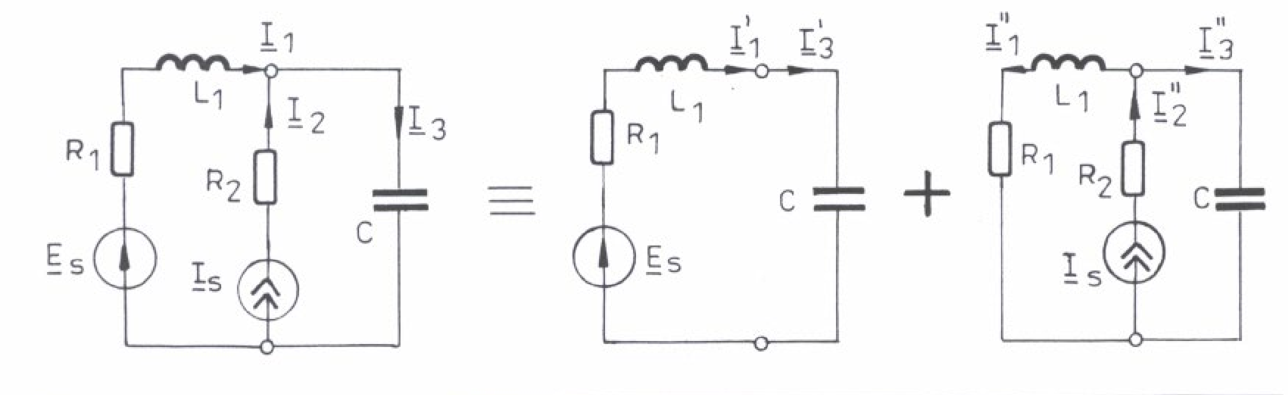
3. THE SUPERPOSITION THEOREM

The superposition theorem: *for a linear network, the zero-state response caused by several independent sources is the sum of the zero-state response due to each independent source acting alone.*

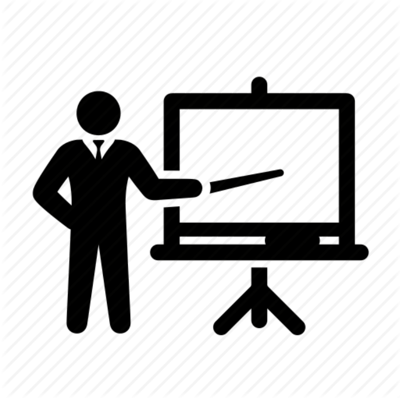
$$\underline{I}_k = \sum_{m=1}^{L} \underline{Y}_{km}\underline{E}_m$$

or

$$\underline{I}_k = \sum_{m=1}^{L} \underline{I}_{km}$$



According to the superposition theorem : $\underline{I}_3 = \underline{I}_3{}' + \underline{I}_3{}''$



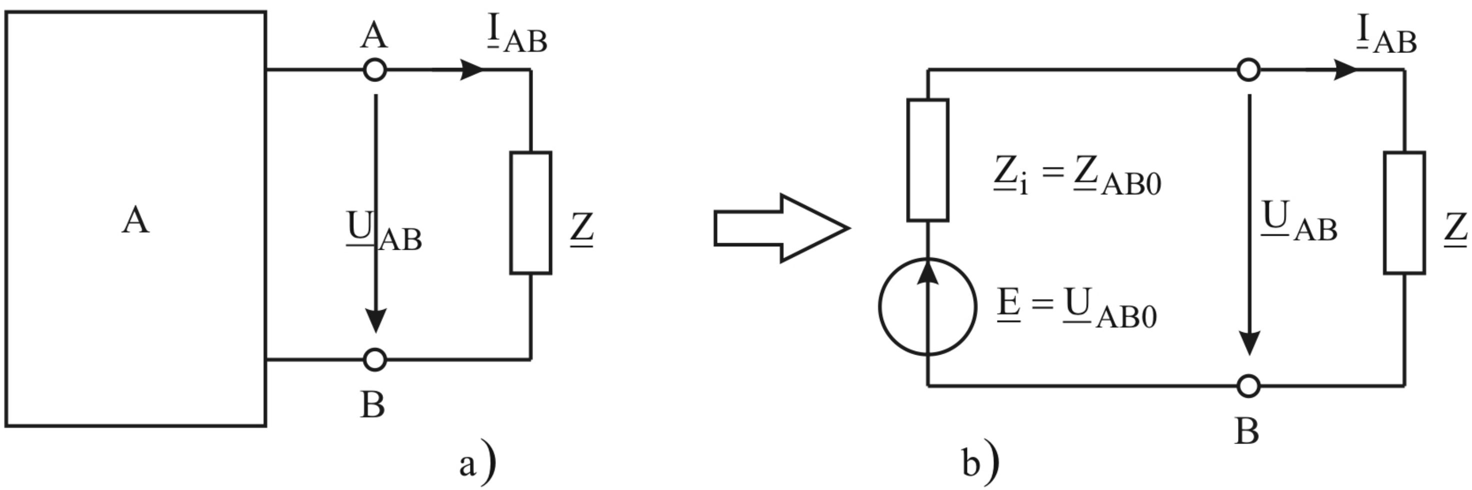
For Examples with Superposition Theorem

See the SEMINAR

4. THÉVENIN - NORTON EQUIVALENT NETWORK THEOREM

- *Powerful tool,*
- *A very general theorem*

A) The Thevenin theorem.



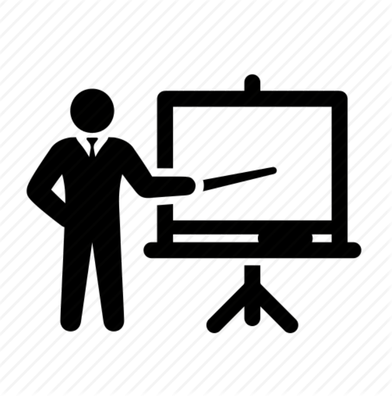
$$\underline{E} = \underline{U}_{ABo} \qquad \underline{Z}_i = \underline{Z}_{ABo} \qquad \underline{I}_{AB} = \frac{\underline{E}}{\underline{Z} + \underline{Z}_i}$$

$$\boxed{\underline{I}_{AB} = \frac{\underline{U}_{ABo}}{\underline{Z} + \underline{Z}_{ABo}}}$$

$$\underline{I}_{AB} = \frac{\underline{U}_{ABo}}{\underline{Z} + \underline{Z}_{ABo}}$$

- $\underline{U}_{AB0}$ the open-circuit voltage of the network (the voltage across the terminals A and B when the load is disconnected).

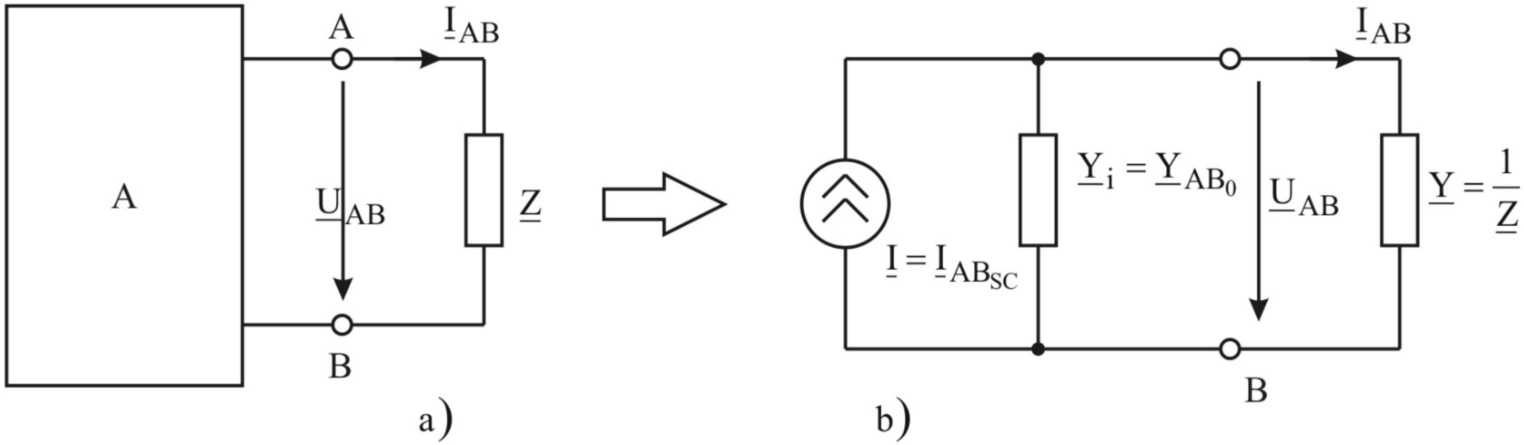
- $\underline{Z}_{AB_0}$ is obtained from the network by setting all *independent* sources to zero (i.e., by replacing every independent voltage source by a short circuit and every independent current source by an open circuit), without the load impedance $\underline{Z}$.



For Examples with

Thevenin's Theorem see the SEMINAR

B) The Norton theorem.



$$\underline{Y}_i = \underline{Y}_{ABo} \qquad \underline{I} = \underline{I}_{AB_{sc}} \qquad \underline{U}_{AB} = \frac{\underline{I}}{\underline{Y} + \underline{Y}_i}$$

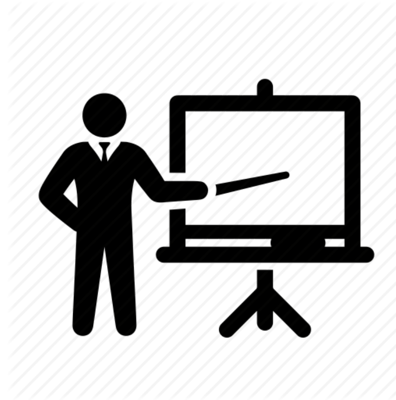
$$\boxed{\underline{U}_{AB} = \frac{\underline{I}_{ABsc}}{\underline{Y} + \underline{Y}_{ABo}}}$$

$$\underline{U}_{AB} = \frac{\underline{I}_{ABsc}}{\underline{Y} + \underline{Y}_{ABo}}$$

$$\underline{I}_{ABsc} = \underline{I}_{AB}\Big|_{\underline{Z}=0} = \frac{\underline{U}_{ABo}}{\underline{Z}_{ABo}} = \underline{U}_{ABo}\underline{Y}_{ABo}$$

$\underline{I}_{ABsc}$ is the current delivered by the circuit when the terminals A and B are short-circuited ($\underline{Z} = 0$).

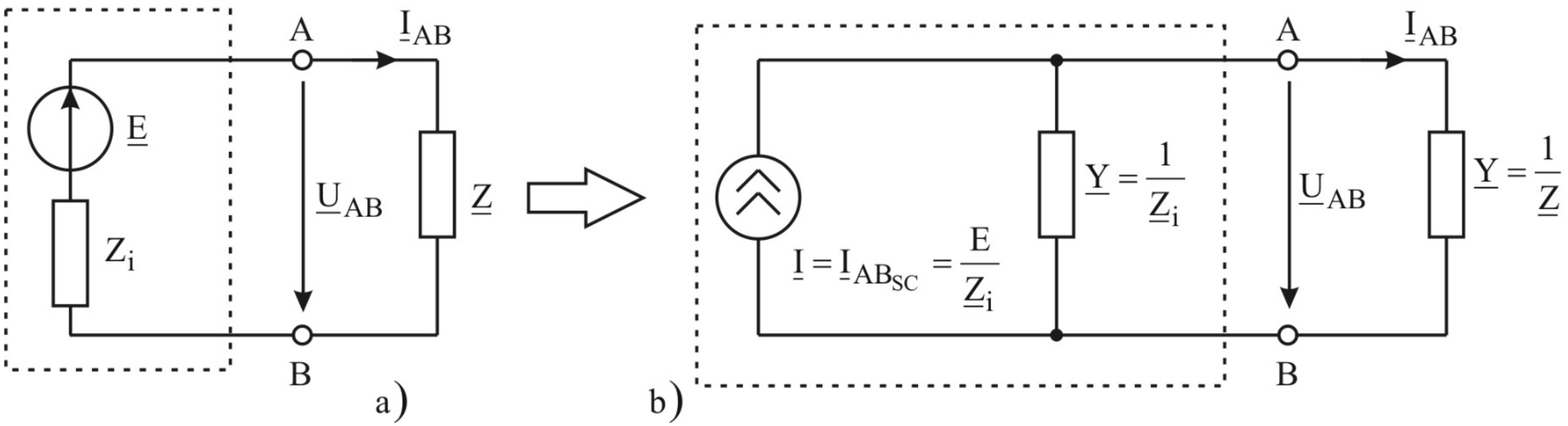
$$\underline{Y}_{ABo} = \frac{1}{\underline{Z}_{ABo}}$$



For Examples with

Norton’s Theorem see the SEMINAR

C) The equivalence theorem between a voltage source and a current source.

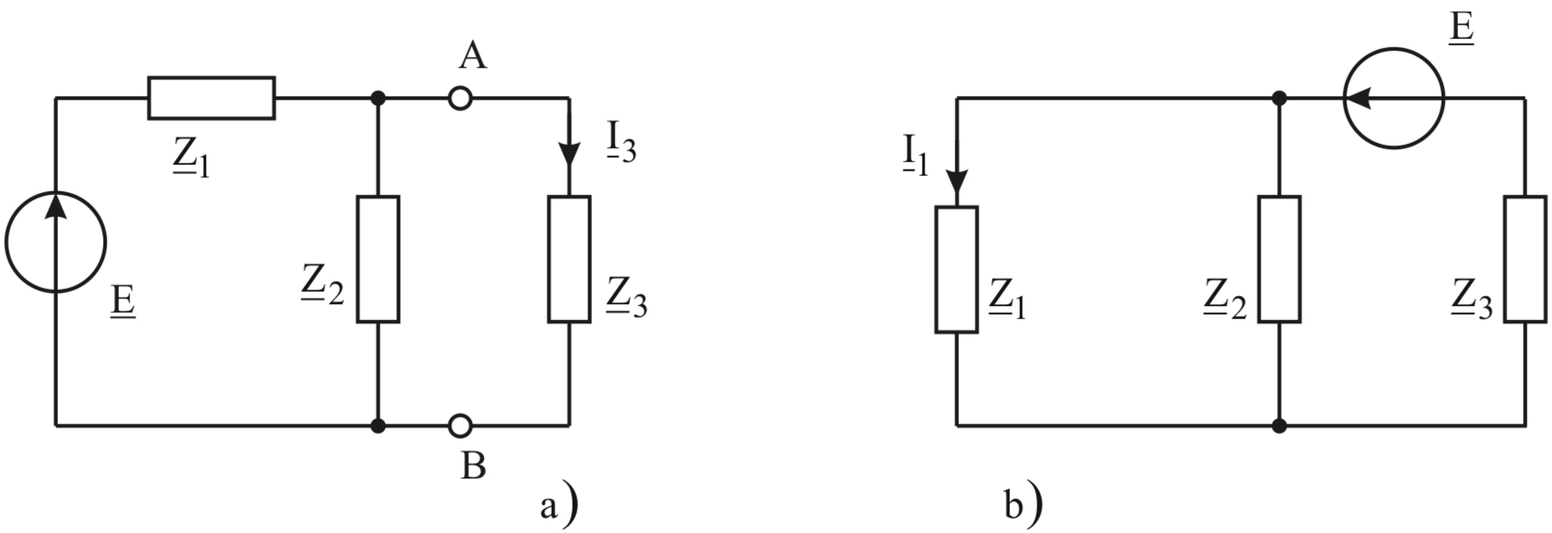


$$\underline{I}_{ABsc} = \underline{I}_{AB}\Big|_{\underline{U}_{AB}=0} = \frac{\underline{E}}{\underline{Z}_i}$$

$$\underline{Y}_i = \underline{Y}_{ABo} = \frac{1}{\underline{Z}_{ABo}} = \frac{1}{\underline{Z}_i}$$

5. THE RECIPROCITY THEOREM

The input and the output can be interchanged without altering the response of the system to a given input waveform.



$$\underline{I}_3 = \underline{I}_1$$

- In electric circuits, reciprocity applies to a subset of all linear time-invariant networks.

6. MESH (OR LOOP) ANALYSIS OF LINEAR NETWORKS

- New network variables are used: *the mesh currents (or loop currents)*

$$\underline{J}_1, \underline{J}_2, \ldots, \underline{J}_q, \ldots, \underline{J}_B$$

The branch currents: $\underline{I}_K = \sum_{k \in (p)} \underline{J}_P$

$$\begin{cases} \underline{Z}_{11}\underline{J}_1 + \underline{Z}_{12}\underline{J}_2 + \ldots + \underline{Z}_{1B}\underline{J}_B = \underline{E}_1 \\ \underline{Z}_{21}\underline{J}_1 + \underline{Z}_{22}\underline{J}_2 + \ldots + \underline{Z}_{2B}\underline{J}_B = \underline{E}_2 \\ ------------------- \\ \underline{Z}_{B1}\underline{J}_1 + \underline{Z}_{B2}\underline{J}_2 + \ldots + \underline{Z}_{BB}\underline{J}_B = \underline{E}_B \end{cases}$$

The network has N nodes and L branches; consequently it has

$B = L - N + 1$ meshes

a) $\underline{Z}_{pp}$ is called the *self impedance of mesh p*; it can be calculated as follows :

$$\underline{Z}_{pp} = \sum_{\substack{k\in(p)\\ m\in(p)}} Z_{km} = \sum_{k\in(p)} \underline{Z}_k + \sum_{\substack{k\in(p)\\ m\in(p)\\ k\neq m}} j\omega L_{km}$$

$\underline{Z}_{pp}$ is the sum of all the impedances of branches in mesh (*p*), plus the algebraic sum of the mutual impedance between branches *k* and *m*, both of them belonging to the mesh (*p*).

Remarks :

-in $\sum_{k\in(p)} \underline{Z}_k$ all terms are positive;

-because $L_{km} = L_{mk}$, in the second term of $\underline{Z}_{pp}$, each mutual inductance has to be taken twice (i.e., $\pm 2j\omega L_{km}$). The sign depends on the association of the mesh current $\underline{J}_p$ to the marked terminals of the mutual inductance.

b) $\underline{Z}_{pq}$ is called the *mutual impedance between mesh p and mesh q*; it can be calculated as follows :

$$\underline{Z}_{pq} = \sum_{\substack{k\in(p)\\ m\in(q)}} \underline{Z}_{km} = \sum_{\substack{k\in(p)\\ k\in(q)}} \underline{Z}_{k} + \sum_{\substack{k\in(p)\\ m\in(q)\\ k\neq m}} j\omega L_{km}$$

$\underline{Z}_{pq}$ is the sum of all impedances of the branches which are in common with meshes (p) and (q) plus a sum of the mutual impedances between the branch $k\in(p)$ and the branch $m\in(q)$

Remarks :

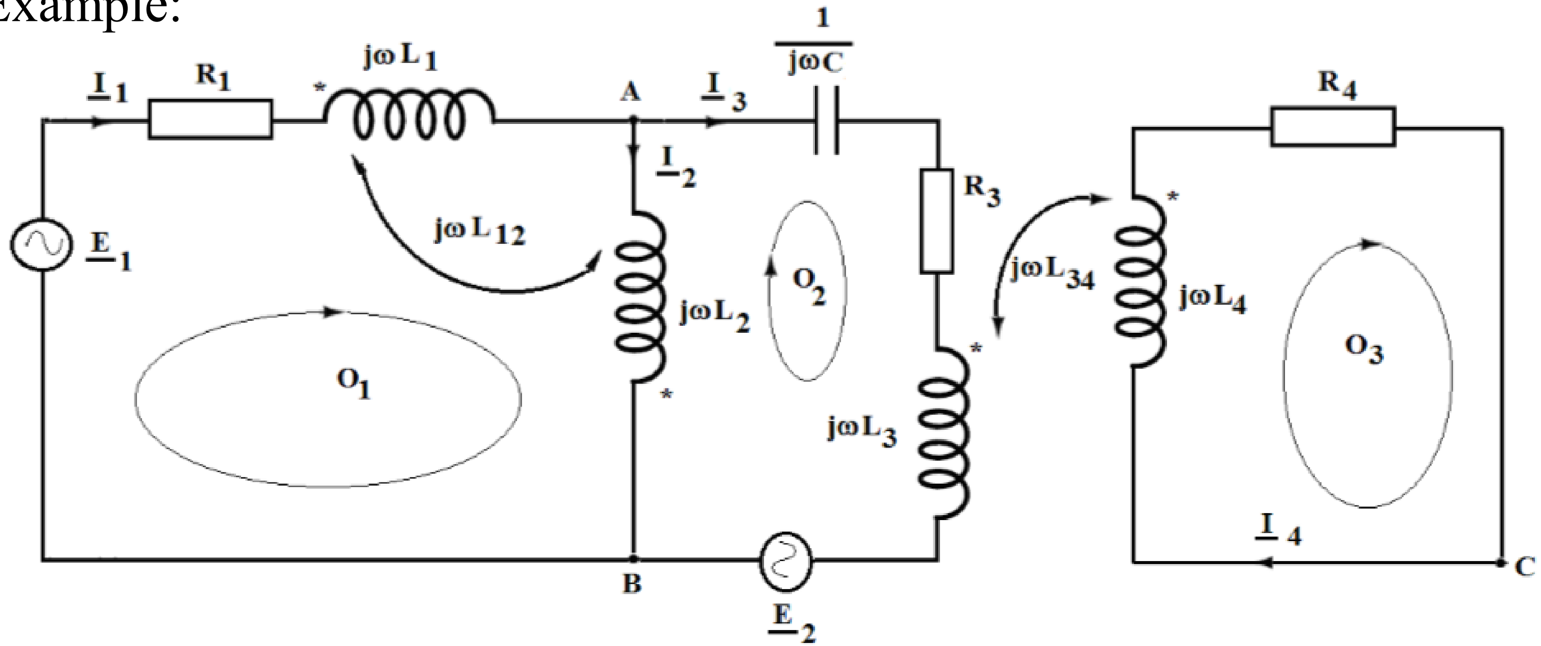
- in the first sum of the right member of $\underline{Z}_{pq}$ the impedance is positive if the mesh currents $\underline{J}_p$ and $\underline{J}_q$ have the same direction through the common impedance of the (p) mesh and (q) mesh, otherwise the sign is negative;

- in the second sum of the right member of $\underline{Z}_{pq}$ the sign depends on the association of the mesh currents $\underline{J}_p$ and $\underline{J}_q$ to the marked terminals of the two inductances situated in the branch $k\in(p)$ and branch $m\in(q)$, respectively.

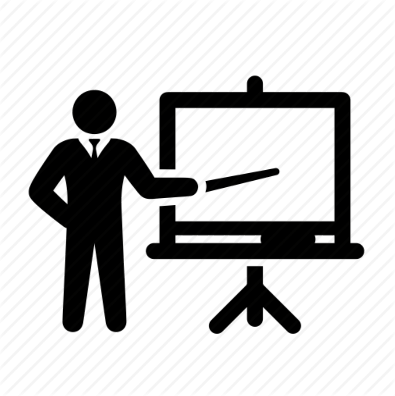
c) $\underline{E}_p$ is the algebraic sum of all the source voltages in mesh (p)

$$\underline{E}_p = \sum_{K\in(p)} \underline{E}_K$$

Example:



I 1
R1
jω L 1
A
I 3
1
jωC
R4
I 2
R3
jω L 12
E 1
O2
jωL34
jωL4
jωL2
O1
O3
jωL3
I 4
B
C
E 2



For Examples with MESH (OR LOOP) ANALYSIS
see SEMINAR

7. NODE ANALYSIS OF LINEAR NETWORKS

- New network variables are used: *the node voltages.*

- Reference node (ground) : $N - 1$ independent nodes

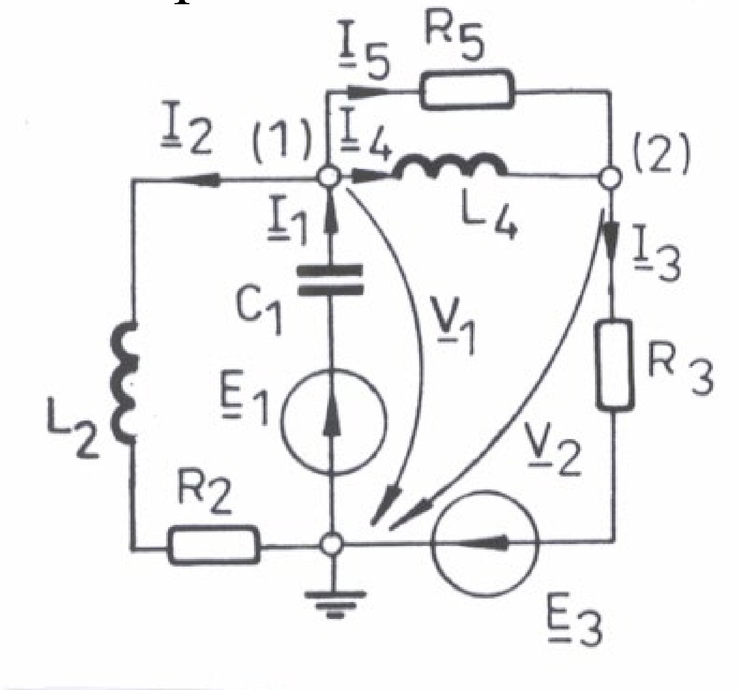
$$\begin{cases} \underline{Y}_{11}\underline{V}_1 + \underline{Y}_{12}\underline{V}_2 + \cdots + \underline{Y}_{1N-1}\underline{V}_{N-1} = \underline{I}_{SC1} \\ \underline{Y}_{21}\underline{V}_1 + \underline{Y}_{22}\underline{V}_2 + \cdots + \underline{Y}_{2N-1}\underline{V}_{N-1} = \underline{I}_{SC2} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \underline{Y}_{N-1,1}\underline{V}_1 + \underline{Y}_{N-1,2}\underline{V}_2 + \cdots + \underline{Y}_{N-1,N-1}\underline{V}_{N-1} = \underline{I}_{SCN-1} \end{cases}$$

a) $\underline{Y}_{aa}$ - the self-admittance of node a

b) $\underline{Y}_{ab}$ - the mutual admittance between node a and node b

c) $\underline{I}_{sca}$ - the short-circuit current entering the node a.

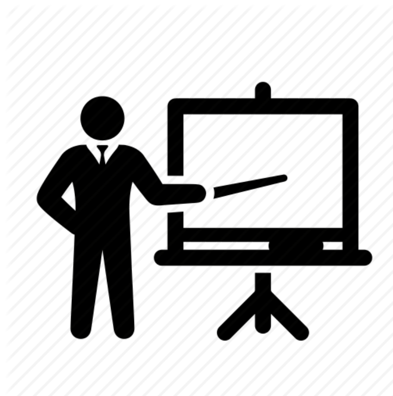
Example:



Kirchhoff's theorems: 5 eqs.

Mesh currents: 3 eqs.

Node voltages: 2 eqs.



For Examples with NODE ANALYSIS
See the SEMINAR

Chapter 2: AC Circuits

Maximum power transfer and resonance in AC circuits

BASES OF ELECTROTECHNICS I.
Faculty of Electronics, Telecommunications and Information Technology
Specialization: IETTI
Academic year: 2023-2024

1. Maximum Power Transfer Theorem

Proof of theorem at whiteboard ☺

For maximum average power transfer, the load impedance must be equal to the complex conjugate of Thevenin impedance.

$$\underline{Z}_L = R_L + X_L = R_{Th} - X_{Th} = \underline{Z}^*_{Th}$$

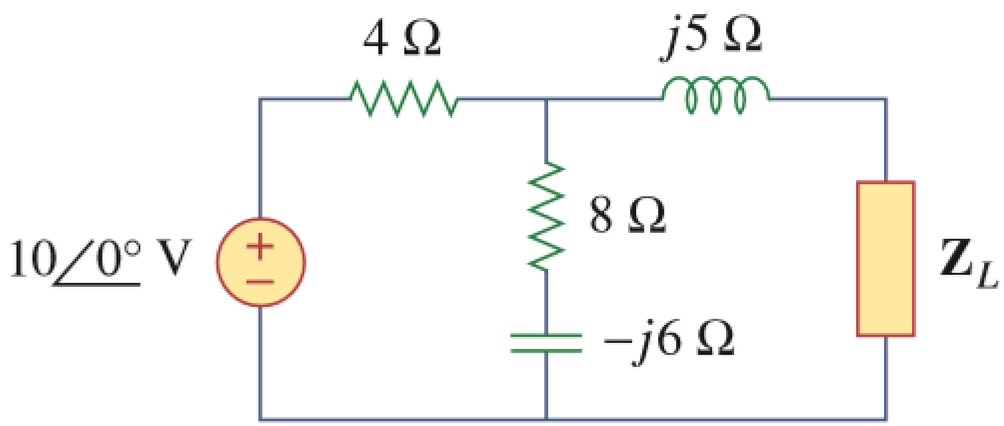
- This result is knowm as the maximum average power transfer theorem for the sinusoidal steady state.

$$P_{\max} = \frac{\left|\underline{U}_{Th}\right|^2}{8R_{Th}}$$

- For purely real load ($X_L = 0$):

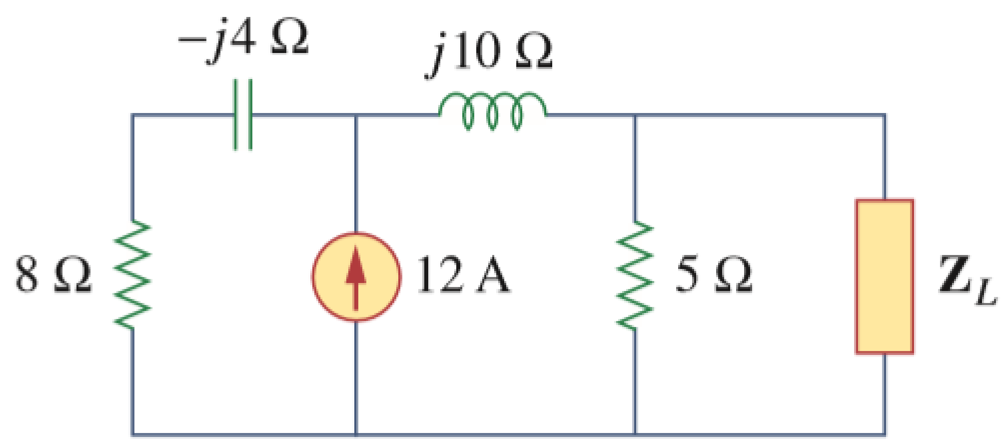
$$R_L = \sqrt{R^2_{Th} - X^2_{Th}} = \left|\underline{Z}_{Th}\right|$$

Determine the load impedance $\mathbf{Z}_L$ that maximizes the average power drawn from the circuit What is the maximum average power?



❑ Practice problem

For the circuit find the load impedance $\mathbf{Z}_L$ that absorbs the maximum average power. Calculate that maximum aver age power.



Answer: $3.415 - j0.7317\ \Omega$, 51.47 W.

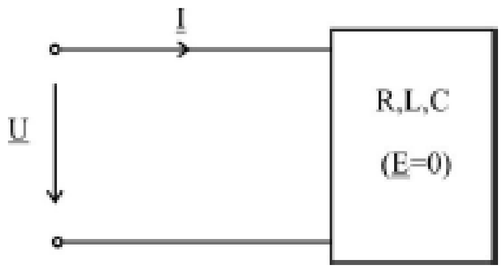
.2. Resonance in AC circuits

- ✓ Resonance in AC circuit occurs because energy is stored in two different ways: in an electric field as the capacitor is charged and in a magnetic field as current flows through the inductor. Energy can be transferred from one to the other within the circuit and this can be oscillatory.
- ✓ An important property of this circuit is its ability to resonate at a specific frequency, the resonant frequency.

- ✓ Resonance is when all the circuit parameters are all balanced and working in equal harmony like a tuning fork. ☺

- ✓ In that state, the efficiency of the circuit is very high and very little power is needed to get it to sustain oscillation at its given frequency.
- ✓ In an ideal resonant LC circuit L=C. In a perfect ideal circuit R can equal zero or infinity depending on the parallel or series nature of L & C. In the real world R does not meet those extremes, hence the Q (Quality) factor is very much less than ideal. Only at RESONANCE does this apply.

- ✓ At resonance inductive and capacitive reactances balance and the circuit behaves more like a linear resistor because such components cancel out.



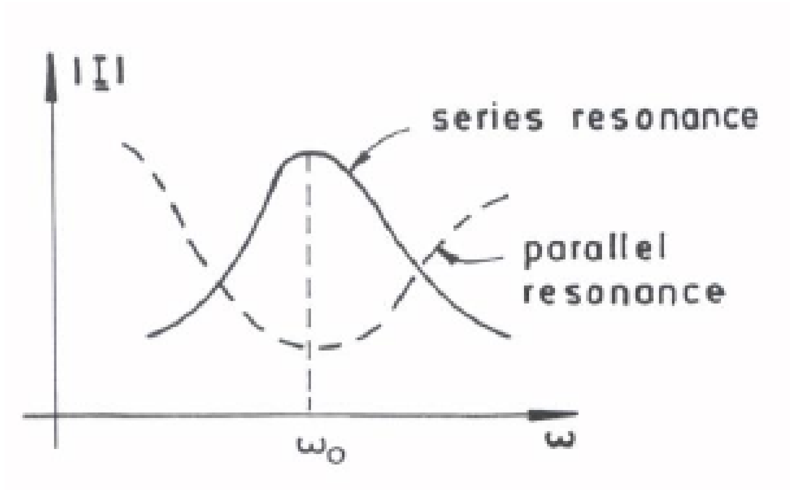
- at resonance: $Q = 0$ (meaning $X = 0$ or $B = 0$)
- the phase shift between I and U is 0 ($\sin\varphi = 0$)

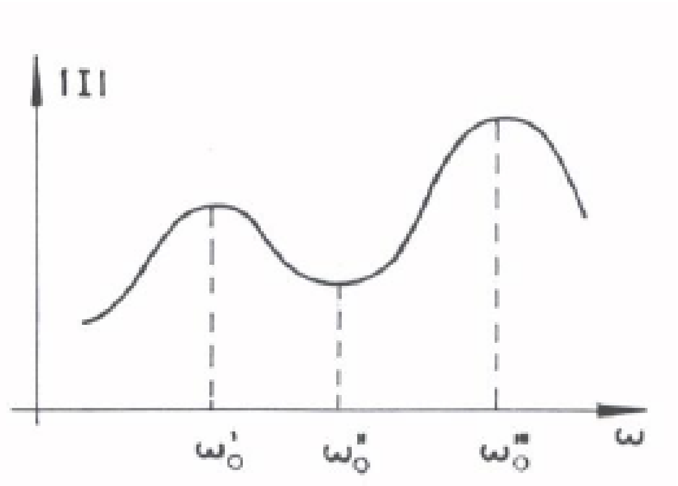
Remarks:

a) $X = 0$ corresponds to the series resonance

b) $B = 0$ corresponds to the parallel resonance

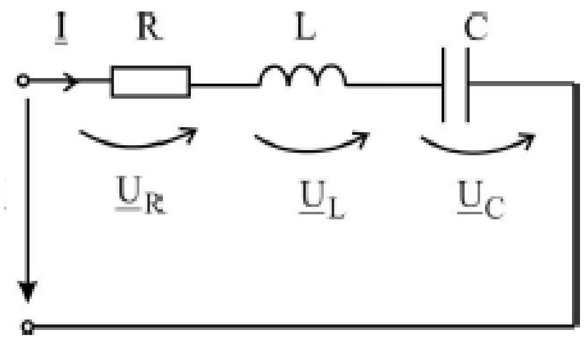
c) at resonance the current has an extreme





.2.1 Series Resonance

***Series Resonance* circuits** are one of the most important circuits used electrical and electronic circuits.
- They can be found in various forms such as in AC mains filters, noise filters and also in radio and television tuning circuits producing a very selective tuning circuit for the receiving of the different frequency channels.



$$Z = R + j(\omega L - \frac{1}{\omega C})$$

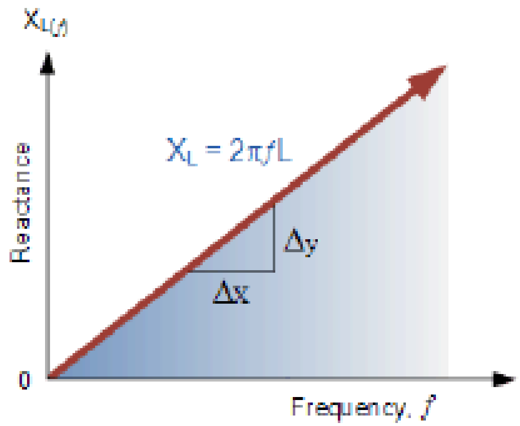
$$I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

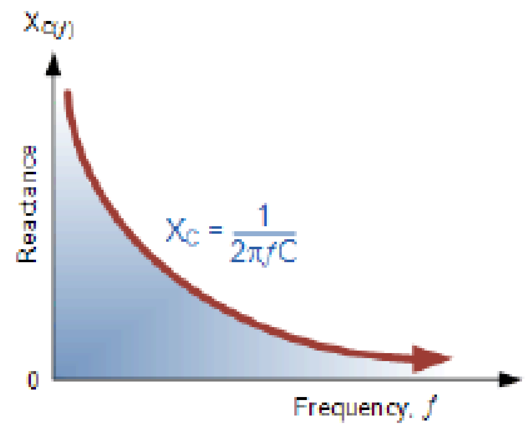
$$\varphi = arctg\frac{X}{R} = arctg\frac{\omega L - \frac{1}{\omega C}}{R}$$

-The resonance condition: $Q = 0 \Rightarrow X = 0$

$$X = \omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega L = \frac{1}{\omega C}$$

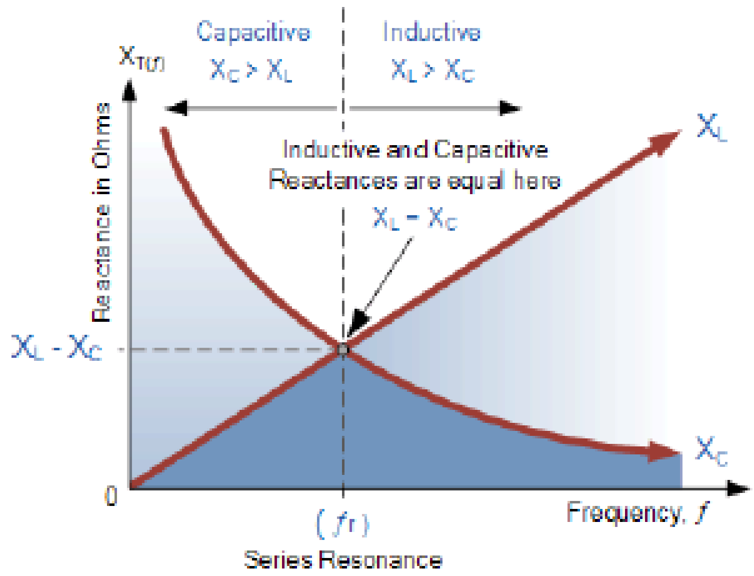
- *the angular resonant frequency:* $\omega_0 = \frac{1}{\sqrt{LC}}$

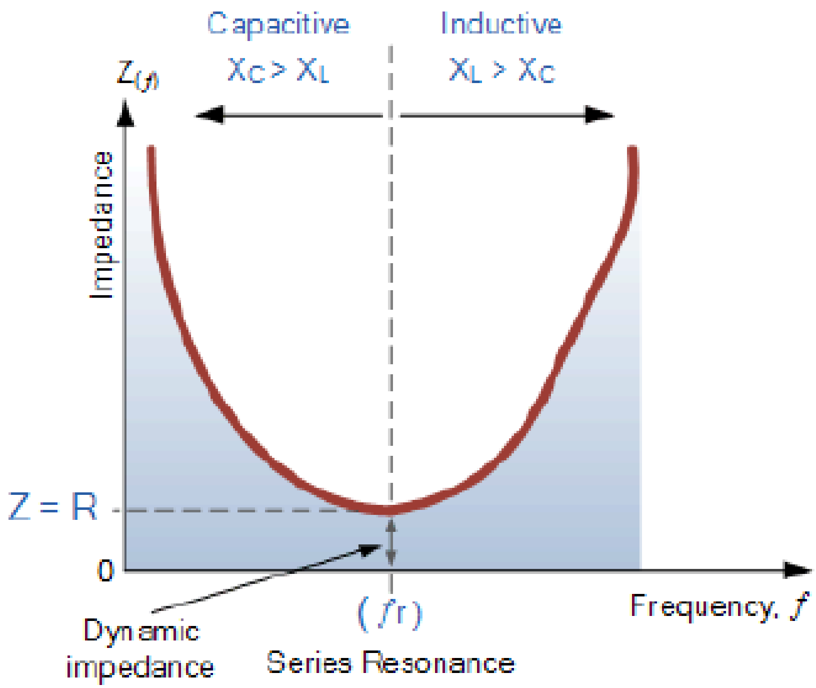




- The graph of inductive reactance against frequency is a straight line linear curve.
- The inductive reactance value of an inductor increases linearly as the frequency across it increases.
- Therefore, inductive reactance is positive and is directly proportional to frequency.

- The graph of capacitive reactance against frequency is a hyperbolic curve.
- The Reactance value of a capacitor has a very high value at low frequencies but quickly decreases as the frequency across it increases.
- Therefore, capacitive reactance is negative and is inversely proportional to frequency

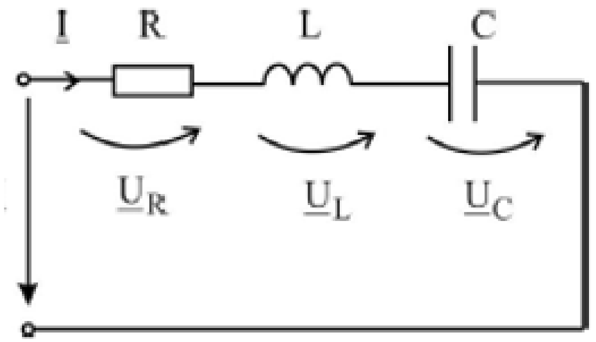




- When the capacitive reactance dominates the circuit the impedance curve has a hyperbolic shape to itself, but when the inductive reactance dominates the circuit the curve is non-symmetrical due to the linear response of X_L.

- If the circuits impedance is at its minimum at resonance then consequently, the circuits **admittance** must be at its maximum and one of the characteristics of a series resonance circuit is that admittance is very high. But this can be a bad thing because a very low value of resistance at resonance means that the resulting current flowing through the circuit may be dangerously high.

The vector diagram (phasorial representation):



Remarks:

a) *R* does **not** influence the resonance.

b) $U = U_R = RI$

$Z = \sqrt{R^2 + X^2} = R$ is minimum ($X = 0$),

the current is maximum: $I = \frac{U}{R} = I_{max}$

c) $U_L = U_C$: *voltage resonance!*

d) if $U_L = U_C >> U$ → *overvoltages.*

$$\frac{U_L}{U} = \frac{\omega_0 LI}{RI} = \frac{\omega_0 L}{R} > 1 \text{ or } \omega_0 L > R$$, because $\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow \frac{1}{\sqrt{LC}} L = \sqrt{\frac{L}{C}} > R$

$$\omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}} = \rho$$

the characteristic impedance of a series resonant circuit

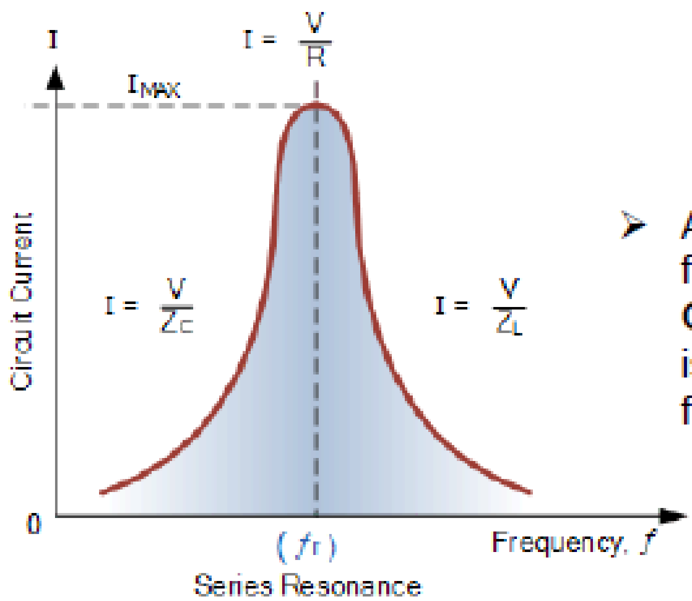
$$\frac{U_L}{U} = \frac{U_C}{U} = \frac{\rho\, I}{RI} = \boxed{\frac{\rho}{R} = Q}$$

- the ***quality factor*** **(or *Q-factor*)**

It shows how many times the voltage across the inductor or across the capacitance of a series resonant circuit (at resonance) is greater than the applied voltage.

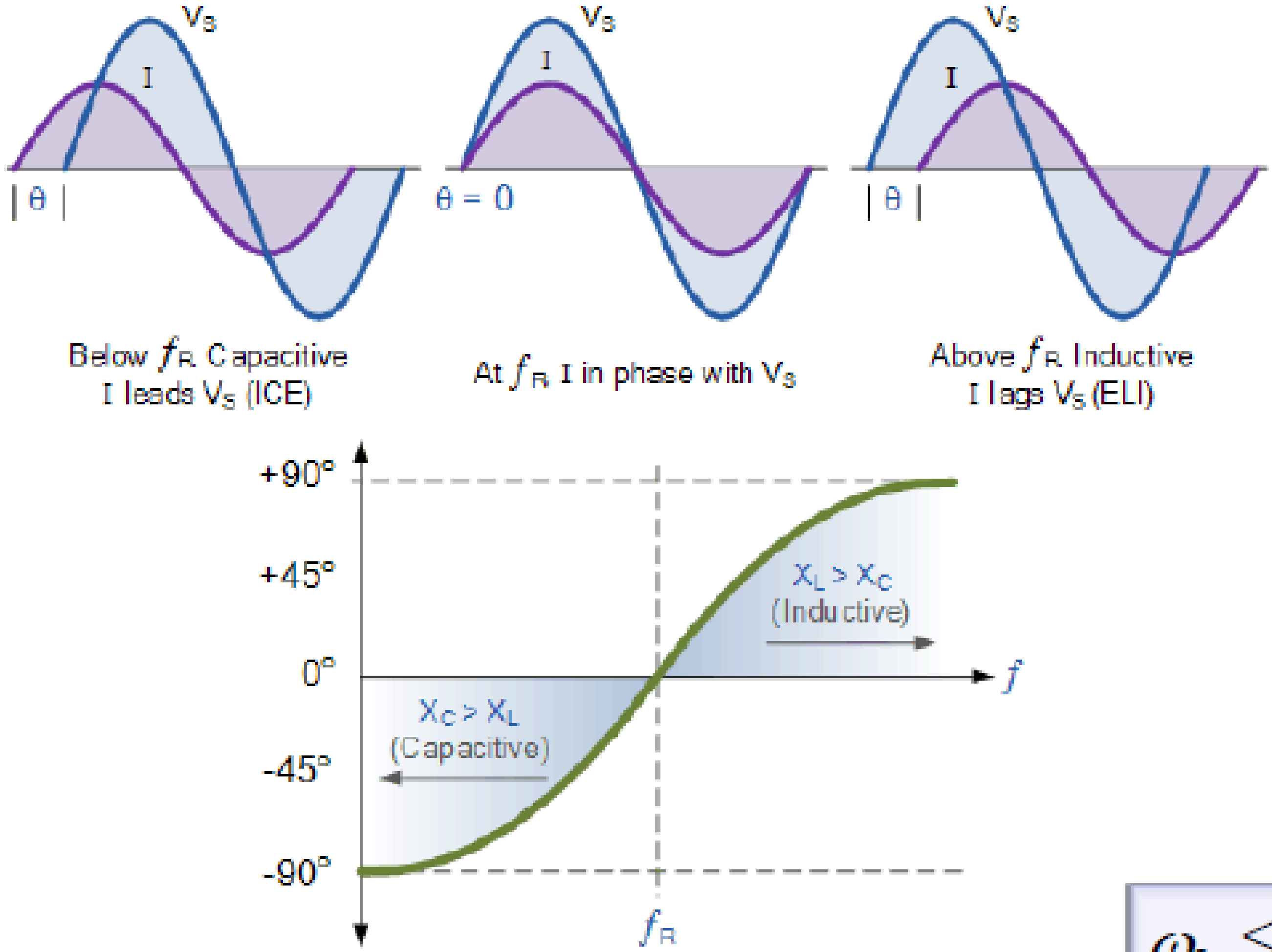
$$\boxed{d = \frac{1}{Q} = \frac{R}{\rho}}$$

- the *damping factor*



- As a series resonance circuit only functions on resonant frequency, this type of circuit is also known as an **Acceptor Circuit** because at resonance, the impedance of the circuit is at its minimum so easily accepts the current whose frequency is equal to its resonant frequency.

Phase Angle of a Series Resonance Circuit



$$\omega_C \leq \omega_0 \leq \omega_L$$

2.2 Parallel Resonance

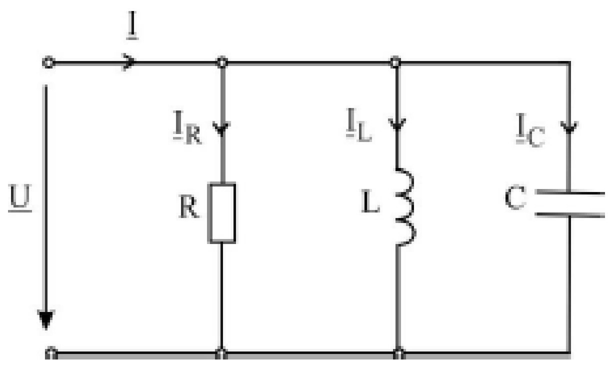
In many ways a **parallel resonance** circuit is exactly the same as the series resonance circuit.

- Both are 3-element networks that contain two reactive components making them a second-order circuit, both are influenced by variations in the supply frequency and both have a frequency point where their two reactive components cancel each other out influencing the characteristics of the circuit. Both circuits have a resonant frequency point.
- The difference this time however, is that a parallel resonance circuit is influenced by the currents flowing through each parallel branch within the parallel LC tank circuit.

A **tank circuit** is a parallel combination of L and C that is used in filter networks to either select or reject AC frequencies.

A ***parallel resonant circuit*** stores the circuit energy in the magnetic field of the inductor and the electric field of the capacitor.

- This energy is constantly being transferred back and forth between the inductor and the capacitor which results in zero current and energy being drawn from the supply. This is because the corresponding instantaneous values of I_L and I_C will always be equal and opposite and therefore the current drawn from the supply is the vector addition of these two currents and the current flowing in I_R.



$$\underline{I} = \underline{I}_R + \underline{I}_L + \underline{I}_C$$

$$\underline{I} = \frac{\underline{U}}{R} + \frac{\underline{U}}{j\omega L} + \underline{U} j\omega C$$

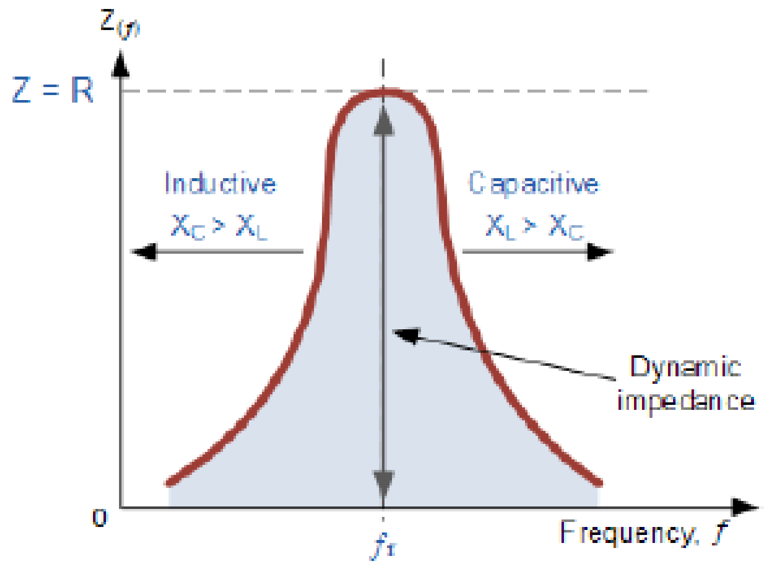
$$\underline{I} = \underline{U}\left[\frac{1}{R} - j\left(\frac{1}{\omega L} - \omega C\right)\right] \qquad \underline{I} = \underline{U}(G - jB)$$

-The resonance condition:

$$\mathbf{Q = 0} \quad \Rightarrow \quad B = 0 \qquad B = \frac{1}{\omega L} - \omega C = 0 \qquad \boxed{\omega L = \frac{1}{\omega C}}$$

- the angular resonant frequency: $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

__Remark__: In practice (that is having real *L* and *C*), the resonant frequencies are different for the series and parallel connections.

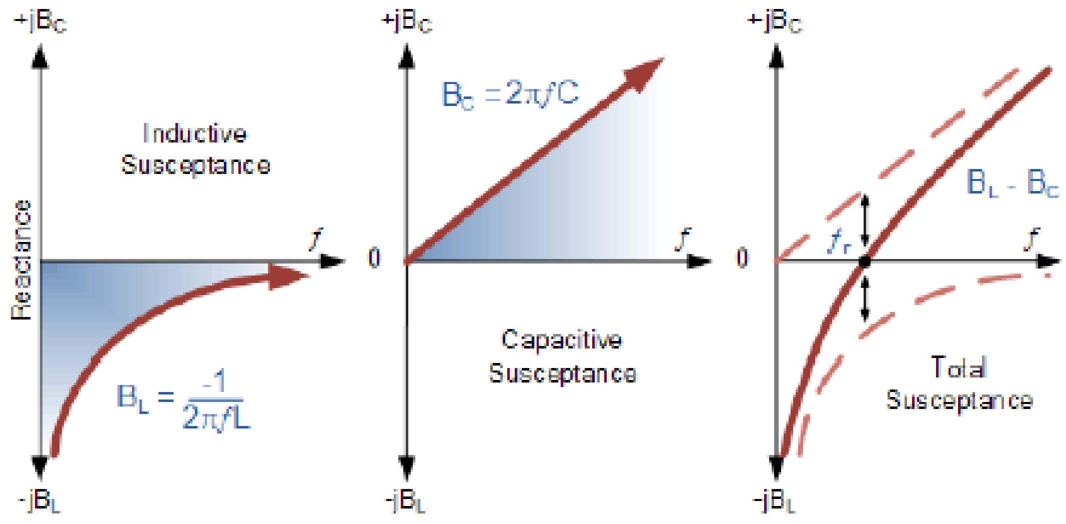


Parallel Resonance

- If the parallel circuits impedance is at its maximum at resonance then consequently, the circuits **admittance** must be at its minimum and one of the characteristics of a parallel resonance circuit is that admittance is very low limiting the circuits current.
- Unlike the series resonance circuit, the resistor in a parallel resonance circuit has a damping effect on the circuits bandwidth making the circuit less selective.

- Since the circuit current is constant for any value of impedance, Z, the voltage across a parallel resonance circuit will have the same shape as the total impedance and for a parallel circuit the voltage waveform is generally taken from across the capacitor.

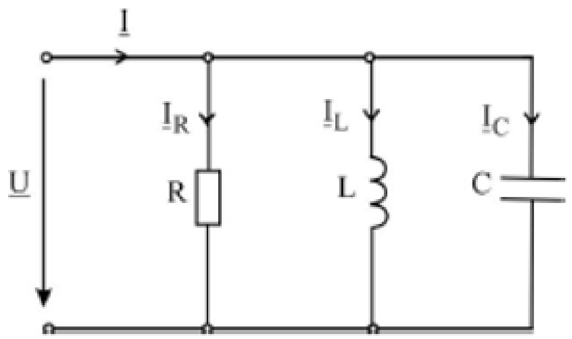
Susceptance at Resonance



- The *inductive susceptance*, B_L is inversely proportional to the frequency as represented by the hyperbolic curve.
- The *capacitive susceptance*, B_C is directly proportional to the frequency and is therefore represented by a straight line.
- The final curve shows the plot of total susceptance of the parallel resonance circuit versus the frequency and is the difference between the two susceptance's.

We can see that at the resonant frequency point were it crosses the horizontal axis the total circuit susceptance is zero.
Below the resonant frequency point, the inductive susceptance dominates the circuit producing a "lagging" power factor, whereas above the resonant frequency point the capacitive susceptance dominates producing a "leading" power factor.

The vector diagram (phasorial representation):



Remarks:

a) $I = I_R = GU = U/R$

$Y = \sqrt{G^2 + B^2} = G$ **is minimum (B = 0),**

the current is minimum: $I = GU = I_{min}$

b) $I_L = I_C$ **:** ***current resonance!***

c) **if** $I_L = I_C >> I$ $\rightarrow$ ***overcurrents.***

$\omega_0 C = \frac{1}{\omega_0 L} > \frac{1}{R}$ or $\omega_0 L = \frac{1}{\omega_0 C} < R$, **because** $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\omega_0 C = \frac{1}{\omega_0 L} = \sqrt{\frac{C}{L}} = \gamma$$

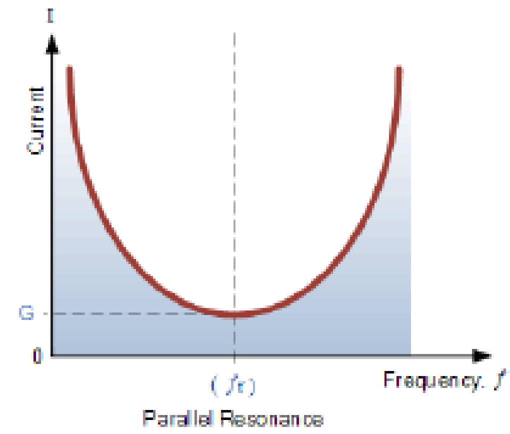
the characteristic admittance **of parallel resonant circuit**

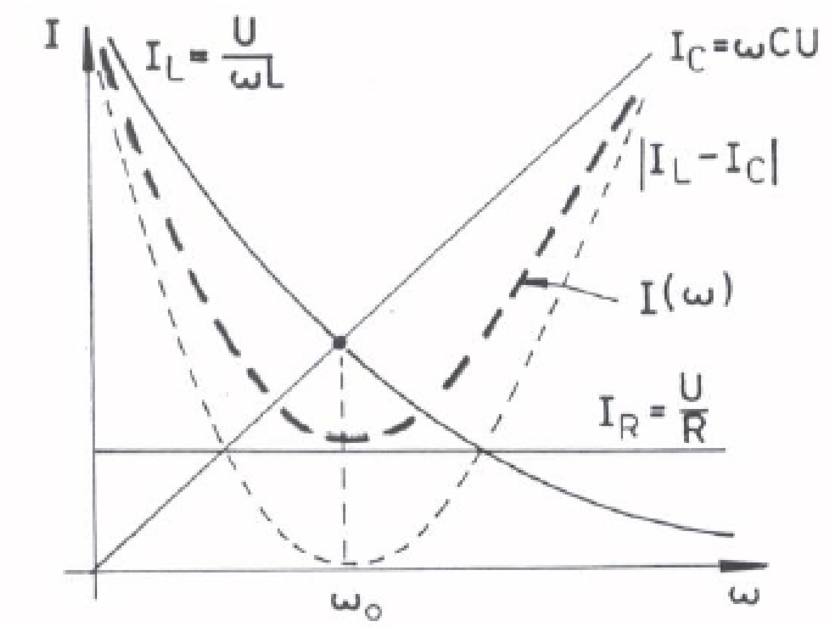
$$Q = \frac{\gamma}{G} = \gamma R$$

- the ***quality factor***

$$d = \frac{1}{Q} = \frac{G}{\gamma}$$

- the ***damping factor***





$$I_R = \frac{U}{R}$$

$$I_L = \frac{U}{\omega L}$$

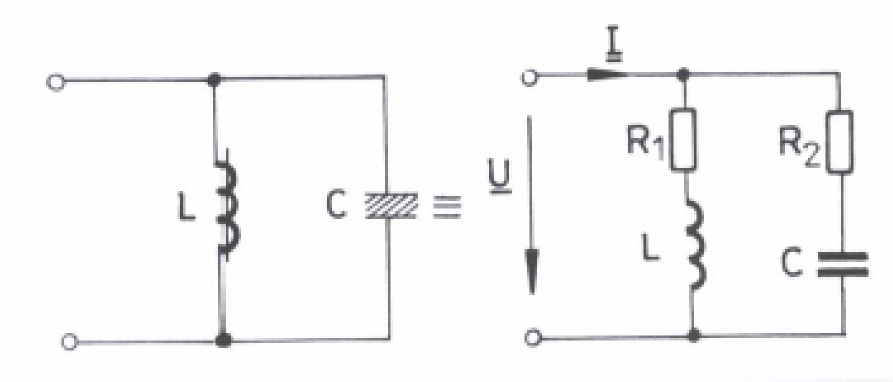
$$I_C = \omega C U$$

$$\underline{I} = \underline{I}_R + \underline{I}_L + \underline{I}_C$$

$$\underline{I} = \underline{U}\left[\frac{1}{R} - j\left(\frac{1}{\omega L} - \omega C\right)\right]$$

$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

2.3 Resonance in real circuits



$$\underline{Z}_1 = R_1 + j\omega L \quad \text{and} \quad \underline{Z}_2 = R_2 + \frac{1}{j\omega C}$$

$$\underline{Y}_1 = \frac{1}{R_1 + j\omega L} \quad \text{and} \quad \underline{Y}_2 = \frac{1}{R_2 - \dfrac{j}{\omega C}}$$

$$\underline{Y}_e = \underline{Y}_1 + \underline{Y}_2 = \frac{R_1}{R_1^2 + \omega^2 L^2} + \frac{R_2}{R_2^2 + \dfrac{1}{\omega^2 C^2}} - j\left(\frac{\omega L}{R_1^2 + \omega^2 L^2} - \frac{\dfrac{1}{\omega C}}{R_2^2 + \dfrac{1}{\omega^2 C^2}} \right)$$

In a parallel circuit, the resonance occurs when $B_e = 0$

$$B_e = \frac{\omega L}{R_1^2 + \omega^2 L^2} - \frac{\dfrac{1}{\omega C}}{R_2^2 + \dfrac{1}{\omega^2 C^2}} = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{\frac{L}{C} - R_1^2}{\frac{L}{C} - R_2^2}} = \frac{1}{\sqrt{LC}} \sqrt{\frac{\rho^2 - R_1^2}{\rho^2 - R_2^2}}$$

Remarks:

a) **if** $R_1 > \rho$ and $R_2 > \rho$ or $R_1 < \rho$ and $R_2 < \rho$ **there is resonance;**

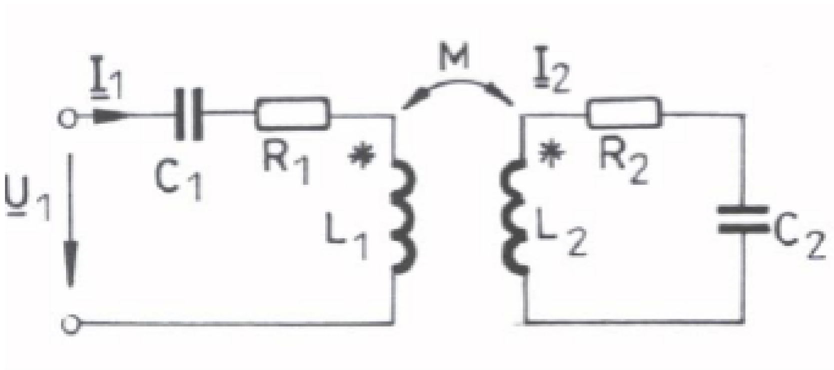
b) **if** $R_1 < \rho < R_2$ or $R_1 > \rho > R_2$ **there is no resonance;**

c) **if** $R_1 = R_2 \neq \rho$, $\omega_0 = \frac{1}{\sqrt{LC}}$ $\rightarrow$ **the same as for a series resonant circuit**

d) **if** $R_1 = R_2 = \rho$, $\omega_0 = \frac{0}{0}$ $\rightarrow$ **the resonance can occur at any frequency**

in this case: $\underline{Z}_e = \frac{\underline{Z}_1 \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} = \frac{(R + j\omega L)\left(R - \frac{j}{\omega C}\right)}{2R + j\left(\omega L - \frac{1}{\omega c}\right)} = \rho$

2.4 Resonance in inductively coupled circuits



$$\begin{cases} \underline{U}_1 = \left[R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1} \right) \right] \underline{I}_1 + j\omega M \underline{I}_2 \\ 0 = \left[R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2} \right) \right] \underline{I}_2 + j\omega M \underline{I}_1 \end{cases}$$

It is possible to obtain resonance in the primary circuit, in the secondary circuit or simultaneously in both circuits.

$$\begin{cases} \frac{\underline{U}_1}{\underline{I}_1} = \underline{Z}_{e1} = R_1 + jX_1 + j\omega\ M \frac{\underline{I}_2}{\underline{I}_1} \\ 0 = (R_2 + jX_2)\frac{\underline{I}_2}{\underline{I}_1} + j\omega\ M \end{cases} \longrightarrow \frac{\underline{I}_2}{\underline{I}_1} = -\frac{j\omega M}{R_2 + jX_2}$$

where $X_1 = \omega L_1 - \frac{1}{\omega C_1},\ X_2 = \omega L_2 - \frac{1}{\omega C_2}$

It results:

$$\underline{Z}_{e1} = R_1 + jX_1 + \frac{\omega^2 M^2}{R_2 + jX_2} = R_1 + \frac{R_2 \omega^2 M^2}{R_2^2 + X_2^2} + j\left(X_1 - \frac{\omega^2 M^2 X_2}{R_2^2 + X_2^2} \right)$$

The resonance occur when $X_e = 0$:

$$X_1 = \frac{\omega^2 M^2 X_2}{R_2^2 + X_2^2}$$

Remarks:

a) the resistance R_1 does not influence the resonance, while R_2 influences the resonance;

b) the realization of the resonance of inductively coupled circuits is important especially in *radio frequencies circuits*, we can approximate

$$R_2 \langle\langle X_2$$

$$X_1 X_2 = \omega^2 M^2$$

$$\left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_2}\right) = \omega^2 M^2$$

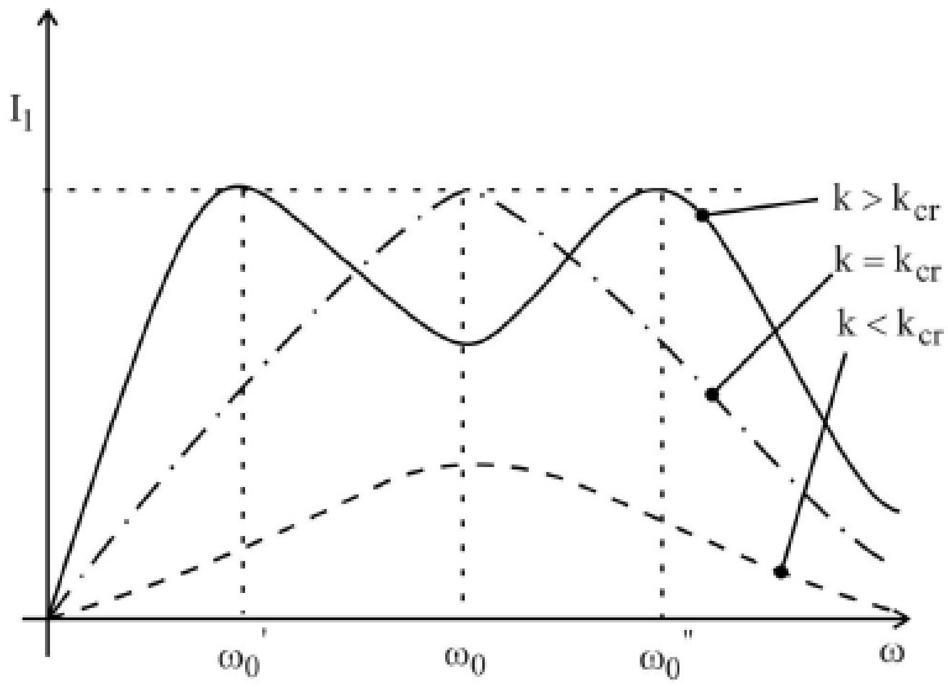
$$\omega^4 (1 - k^2) - \omega^2 (\omega_{01}^2 + \omega_{02}^2) + \omega_{01}\omega_{02} = 0$$

where: $k = \frac{M}{\sqrt{L_1 L_2}}$ **is the coefficient of coupling,**

$\omega_{01} = \frac{1}{\sqrt{L_1 C_1}}$ **is the resonance frequency of the primary circuit**

$\omega_{02} = \frac{1}{\sqrt{L_2 C_2}}$ **is the resonance frequency of the secondary circuit**

$$\omega_0', \omega_0'' = \sqrt{\frac{\omega_{01}^2 + \omega_{02}^2 \pm \sqrt{(\omega_{01}^2 + \omega_{02}^2)^2 - 4(1-k^2)\,\omega_{01}^2\omega_{02}^2}}{2(1-k^2)}}$$



$$k_{cr} = \sqrt{d^2 - \frac{d^4}{4}} \cong d$$ **(critical coupling)**

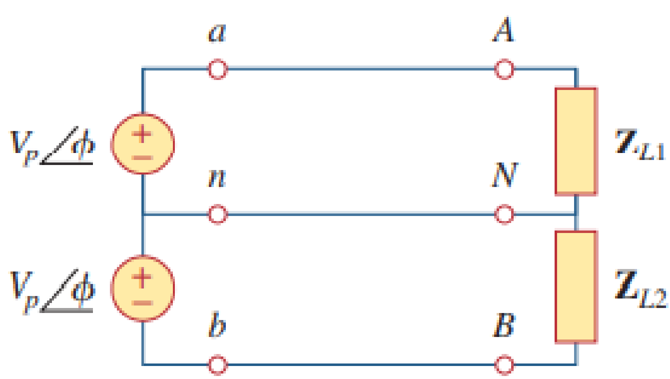
Chapter 2: AC Circuits

THREE–PHASE CIRCUITS

BASES OF ELECTROTECHNICS I.
Faculty of Electronics, Telecommunications and Information Technology
Specialization: IETTI
Academic year: 2022-2023

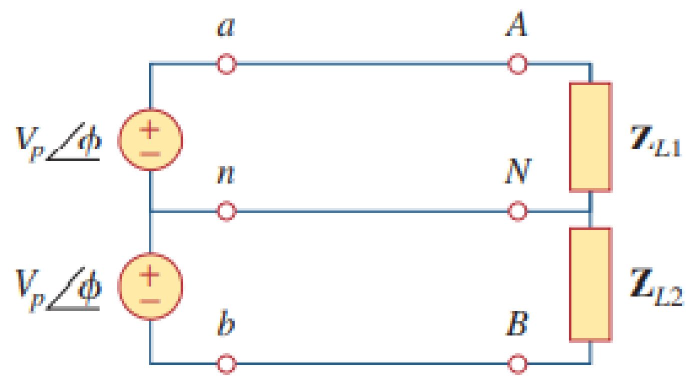
Single phase systems:

Two-wire type

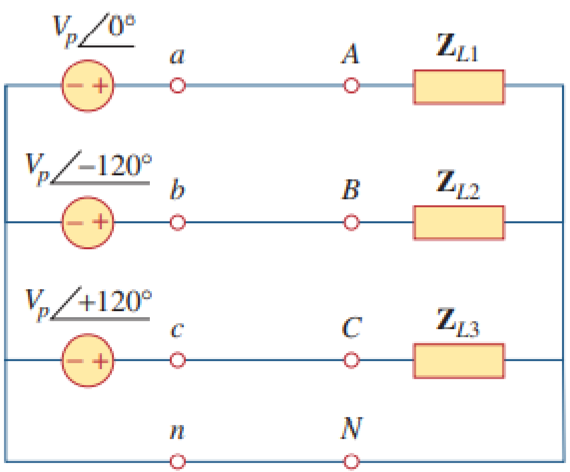


Three-wire type

Polyphase systems:



Two-phase three-wire system



Three-phase four-wire system

Three-phase circuits.

- Three-phase systems are the most common, although, for certain special jobs, greater number of phases is also used.
- For example, almost all mercury-arc rectifiers for power purposes are either six-phase or twelve-phase and most of the rotary converters in use are six-phase. All modern generators are practically three-phase. For transmitting large amounts of power, three-phase is invariably used.

- The reasons for the immense popularity of three-phase apparatus are that:
  - ✓ it is more efficient
  - ✓ the instantaneous power can be constant (not pulsating)
  - ✓ it uses less material for a given capacity
  - ✓ it costs less than single-phase apparatus etc.

"Phase" – two meanings (in electrical engineering):

1) a stage of a periodic process,
2) a portion of a polyphase system of electric circuits.

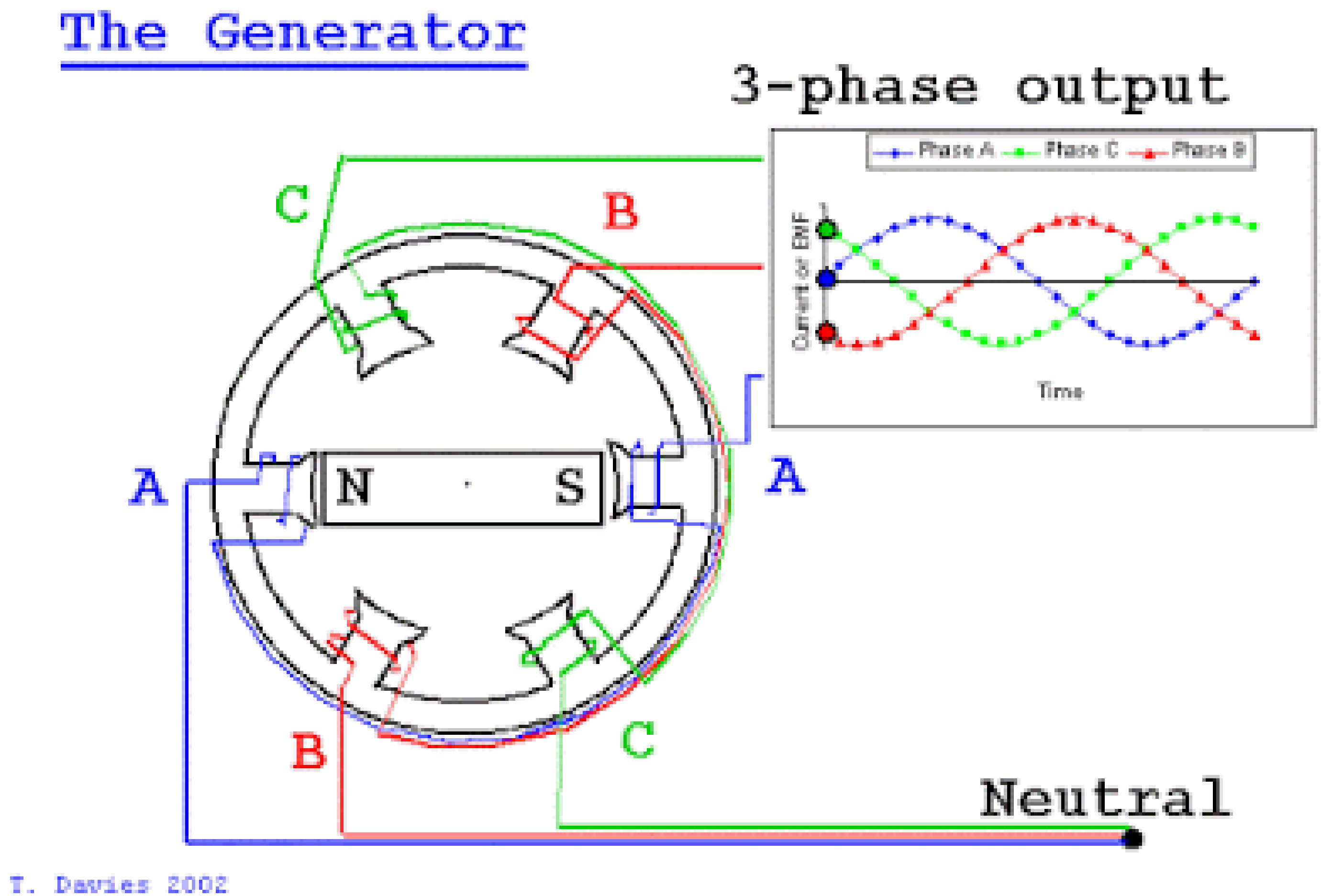
Historical

Nikola Tesla (1856–1943) was a Croatian-American engineer whose inventions—among them the induction motor and the first polyphase ac power system—greatly influenced the settlement of the ac versus dc debate in favor of ac. He was also responsible for the adoption of 60 Hz as the standard for ac power systems in the United States.

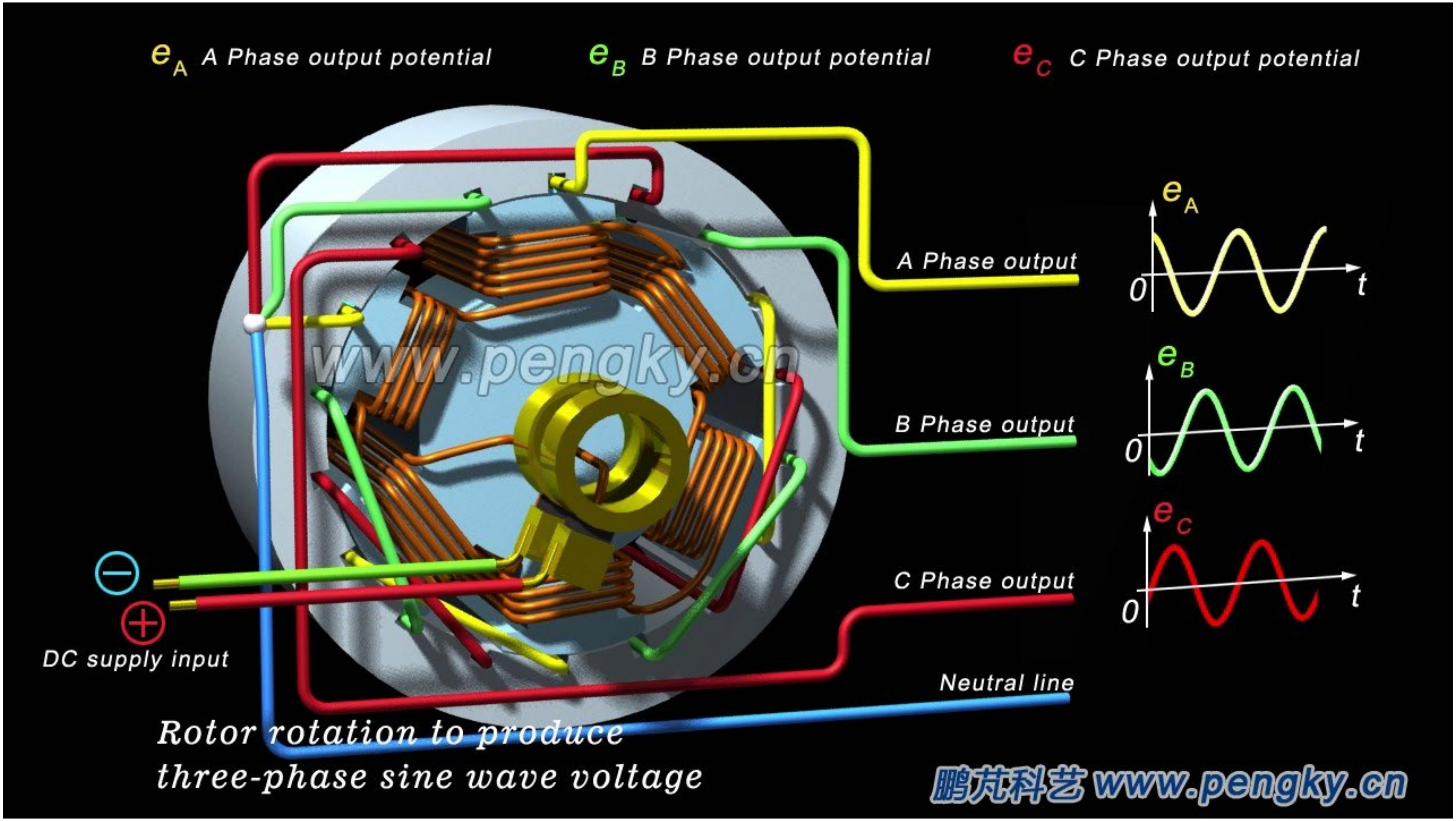
Born in Austria-Hungary (now Croatia), to a clergyman, Tesla had an incredible memory and a keen affinity for mathematics. He moved to the United States in 1884 and first worked for Thomas Edison. At that time, the country was in the "battle of the currents" with George Westinghouse (1846–1914) promoting ac and Thomas Edison rigidly leading the dc forces. Tesla left Edison and joined Westinghouse because of his interest in ac. Through Westinghouse, Tesla gained the reputation and acceptance of his polyphase ac generation, transmission, and distribution system. He held 700 patents in his lifetime. His other inventions include high-voltage apparatus (the tesla coil) and a wireless transmission system. The unit of magnetic flux density, the tesla, was named in honor of him.

Courtesy Smithsonian Institution

❑ Three-phase voltages are often produced with a three-phase AC generator (or alternator).



Three-phase circuits.



Watch:
https://www.youtube.com/watch?v=RycspJC4OKM

1. SYMMETRICAL (BALANCED) THREE-PHASE SYSTEMS.

The ends of the phase windings: "starts" or "beginnings", respectively "finishes" or "ends".

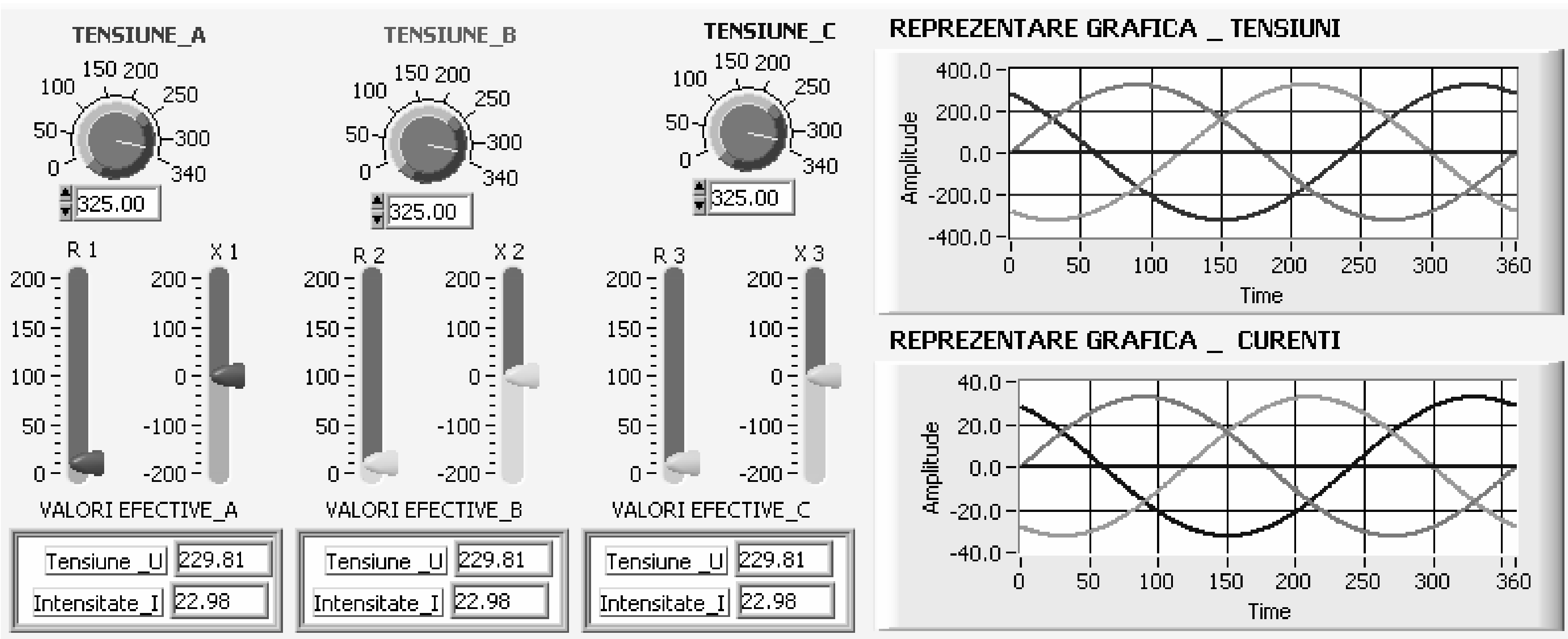
If
$$\begin{cases} i_1(t) = I_1\sqrt{2}\sin(\omega t + \gamma_1) \\ i_2(t) = I_2\sqrt{2}\sin(\omega t + \gamma_2) \\ i_3(t) = I_3\sqrt{2}\sin(\omega t + \gamma_3) \end{cases} \qquad \begin{cases} \underline{I}_1 = I_1 e^{j\gamma_1} \\ \underline{I}_2 = I_2 e^{j\gamma_2} \\ \underline{I}_3 = I_3 e^{j\gamma_3} \end{cases}$$

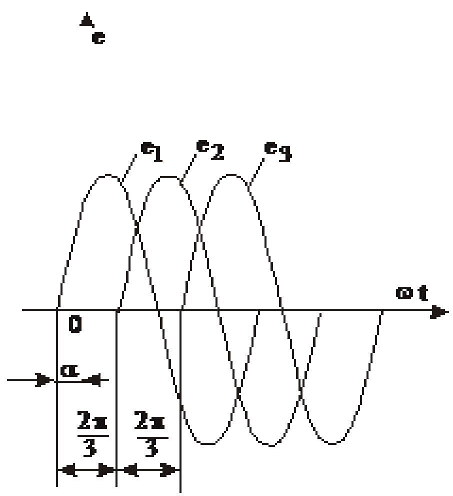
it results a **symmetrical (balanced) three-phase system**.

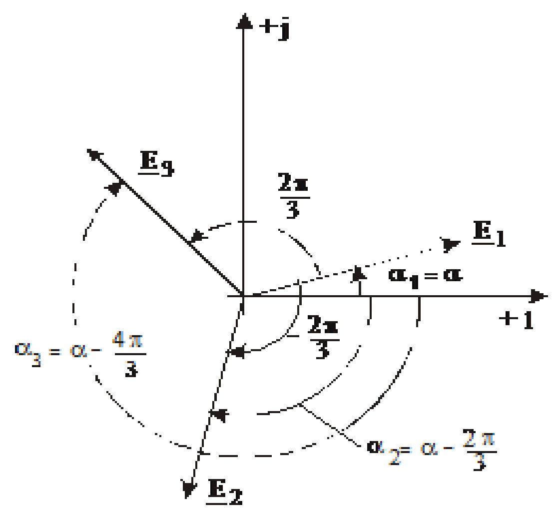
$$I_1 = I_2 = I_3 = I \quad \text{and} \quad \gamma_1 - \gamma_2 = \gamma_2 - \gamma_3 = \gamma_3 - \gamma_1 = 2\pi/3$$

Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.

Three-phase circuits.



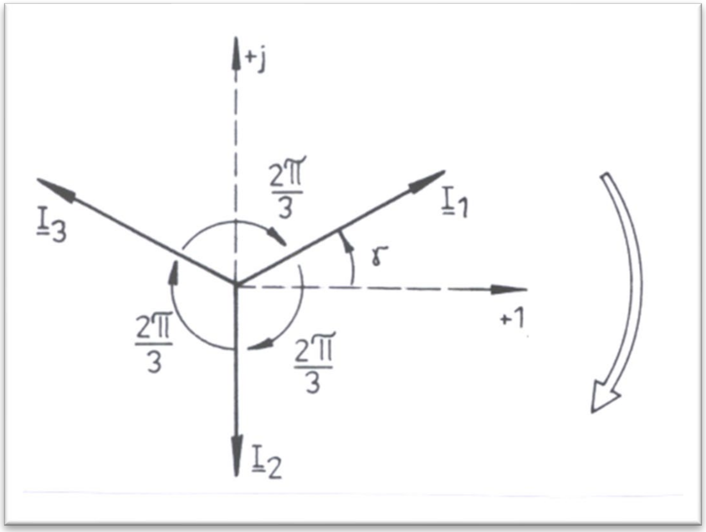




The **phase sequence** is the time order in which the voltages pass through their respective maximum values.

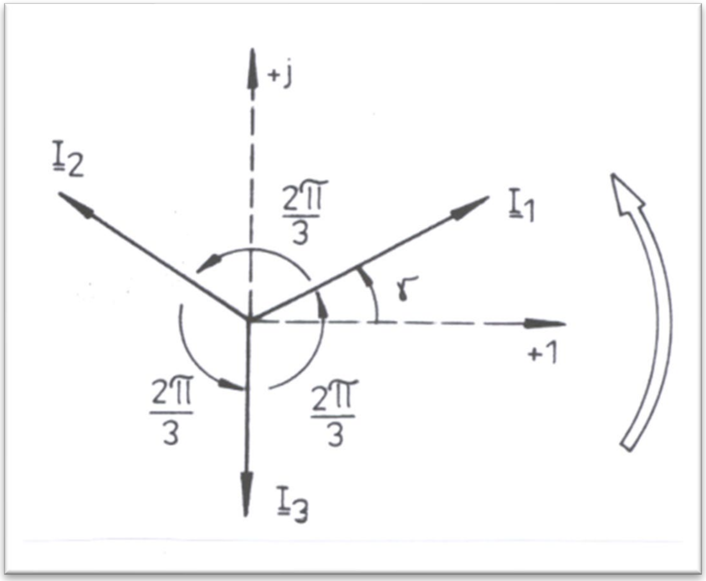
1) Positive phase-sequence (direct system)

$$\begin{cases} i_1(t) = I\sqrt{2}\sin(\omega t + \gamma) \\ i_2(t) = I\sqrt{2}\sin\left(\omega t + \gamma - \frac{2\pi}{3}\right) \\ i_3(t) = I\sqrt{2}\sin\left(\omega t + \gamma - \frac{4\pi}{3}\right) \end{cases} \qquad \begin{cases} \underline{I}_1 = Ie^{j\gamma} \\ \underline{I}_2 = Ie^{j(\gamma - \frac{2\pi}{3})} \\ \underline{I}_3 = Ie^{j(\gamma - \frac{4\pi}{3})} \end{cases}$$



2) Negative phase-sequence (inverse system)

$$\begin{cases} i_1(t) = I\sqrt{2}\sin(\omega t + \gamma) \\ i_2(t) = I\sqrt{2}\sin\left(\omega t + \gamma + \frac{2\pi}{3}\right) \\ i_3(t) = I\sqrt{2}\sin\left(\omega t + \gamma + \frac{4\pi}{3}\right) \end{cases} \qquad \begin{cases} \underline{I}_1 = Ie^{j\gamma} \\ \underline{I}_2 = Ie^{j(\gamma + \frac{2\pi}{3})} \\ \underline{I}_3 = Ie^{j(\gamma + \frac{4\pi}{3})} \end{cases}$$



Determine the phase sequence of the set of voltages

$$v_{an} = 200\cos(\omega t + 10^\circ)$$

$$v_{bn} = 200\cos(\omega t - 230^\circ), \qquad v_{cn} = 200\cos(\omega t - 110^\circ)$$

Solution:

The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200\underline{/10^\circ}\text{ V}, \qquad \mathbf{V}_{bn} = 200\underline{/-230^\circ}\text{ V}, \qquad \mathbf{V}_{cn} = 200\underline{/-110^\circ}\text{ V}$$

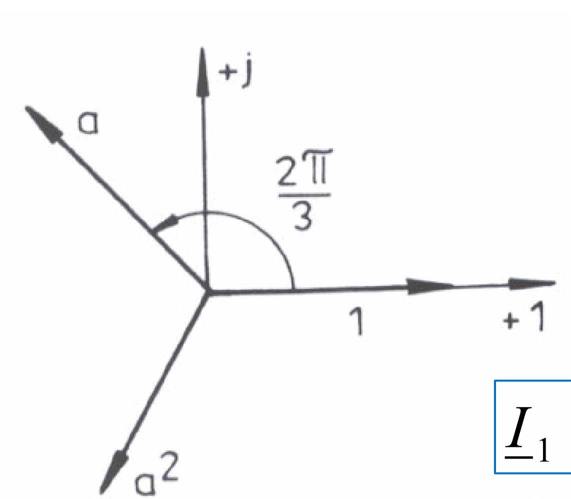
We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120° and $\mathbf{V}_{cn}$ in turn leads $\mathbf{V}_{bn}$ by 120°. Hence, we have an ………………… **PHASE-SEQUENCE**

The ***phase operator***:

$$a = e^{j\frac{2\pi}{3}} = e^{-j\frac{4\pi}{3}} = \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

Multiplication by a advances the position of a vector in a counter-clockwise (or forward) direction by 2π/3 or in a clockwise (or reverse) direction by 4 π/3.

$$a^2 = e^{j\frac{4\pi}{3}} = e^{-j\frac{2\pi}{3}} = -\frac{1}{2} - j\frac{\sqrt{3}}{2} \quad \Rightarrow \quad \boxed{1 + a + a^2 = 0.}$$



$$(1)\quad \begin{cases} \underline{I}_1 = Ie^{j\gamma} = \underline{I}_1 \cdot 1 \\ \underline{I}_2 = \underline{I}_1 a^2 \\ \underline{I}_3 = \underline{I}_1 a \end{cases} \qquad (2)\quad \begin{cases} \underline{I}_1 = \underline{I}_1 \cdot 1 \\ \underline{I}_2 = \underline{I}_1 a \\ \underline{I}_3 = \underline{I}_1 a^2 \end{cases}$$

$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = \underline{I}(1 + a + a^2) = 0,$$

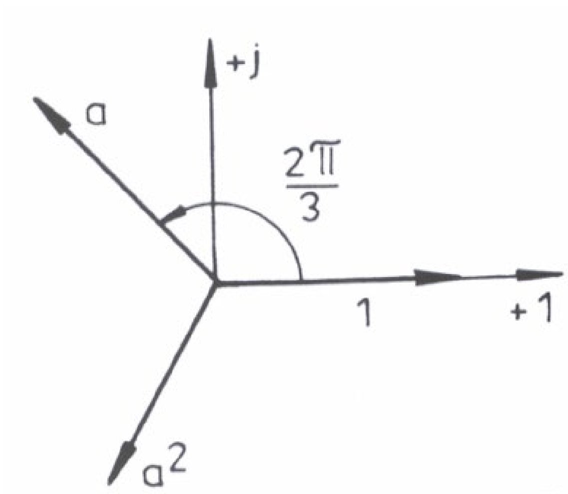
$$I_1 + I_2 + I_3 \neq 0$$

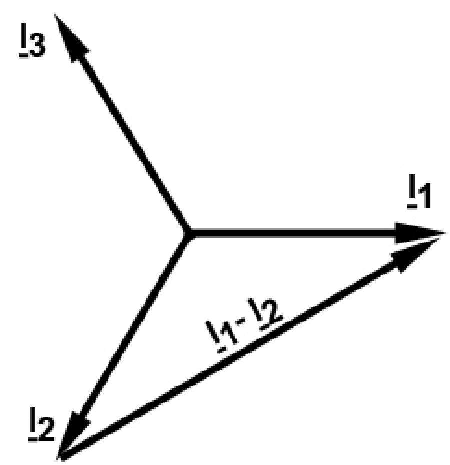
Operations – three-phase systems:

a) addition: $\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0 \quad \Rightarrow \quad \boxed{\underline{I}_1 + \underline{I}_2 = -\underline{I}_3}$

b) difference:

$$\underline{I}_1 - \underline{I}_2 = \underline{I} - a^2 \underline{I} = \underline{I}(1 - a^2) = \underline{I}\sqrt{3}e^{j\frac{\pi}{6}}. \qquad \boxed{\underline{I}_1 - \underline{I}_2 = \underline{I}\sqrt{3}e^{j\frac{\pi}{6}}.}$$

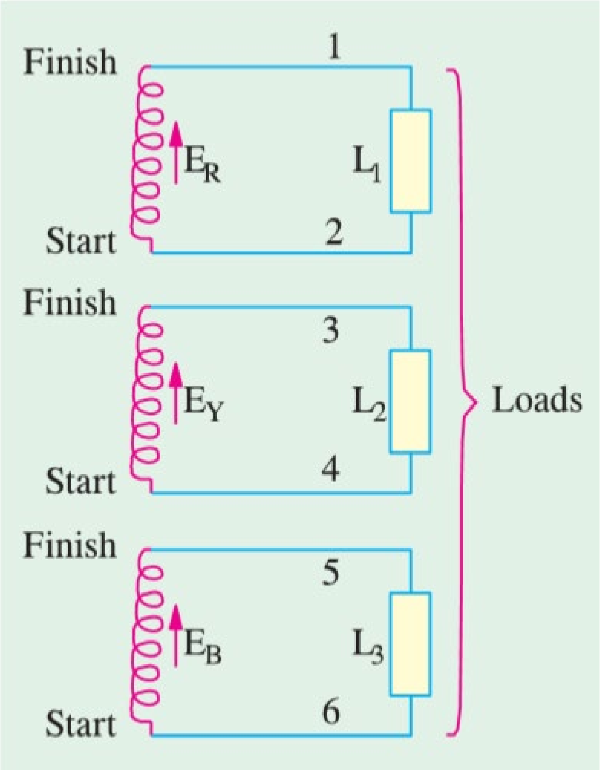




Three-phase circuits.

- ✓ If the three armature coils of the 3-phase alternator are not interconnected but are kept separate, then each phase or circuit would need two conductors, the total number of conductors, in that case, being six. It means that each transmission cable would contain six conductors which will make the whole system complicated and expensive.
- ✓ Hence, the three phases are generally interconnected which results in substantial saving of copper.

3-phase alternator



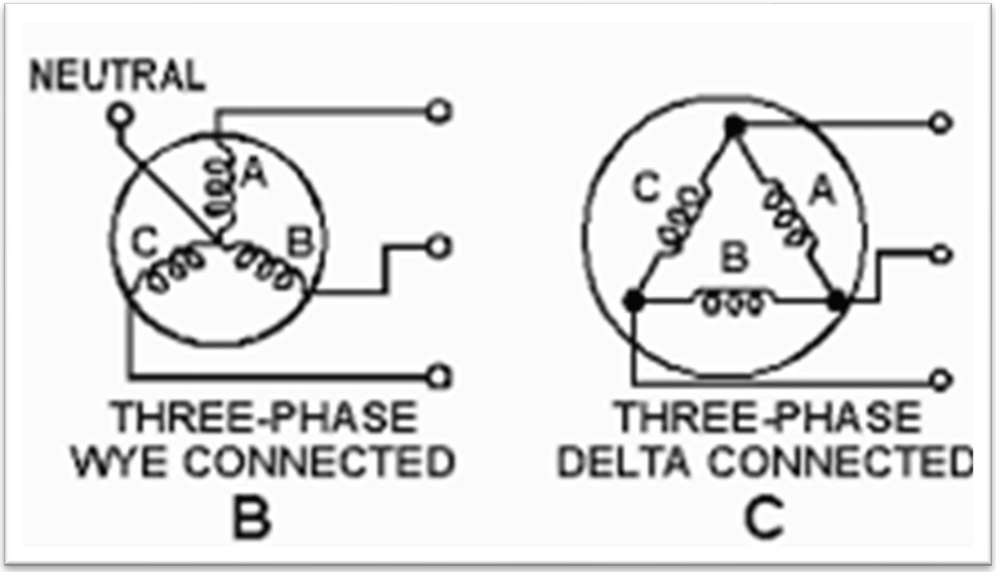
3-phase alternator are not interconnected

2. PHASE INTERLINKAGE

- voltages, currents: *symmetrical* or *unsymmetrical,*
- load impedances: *balanced* or *unbalanced*

The general methods of interconnection are:

(a) Star or Wye (Y) connection
(b) Mesh or Delta (Δ) connection.

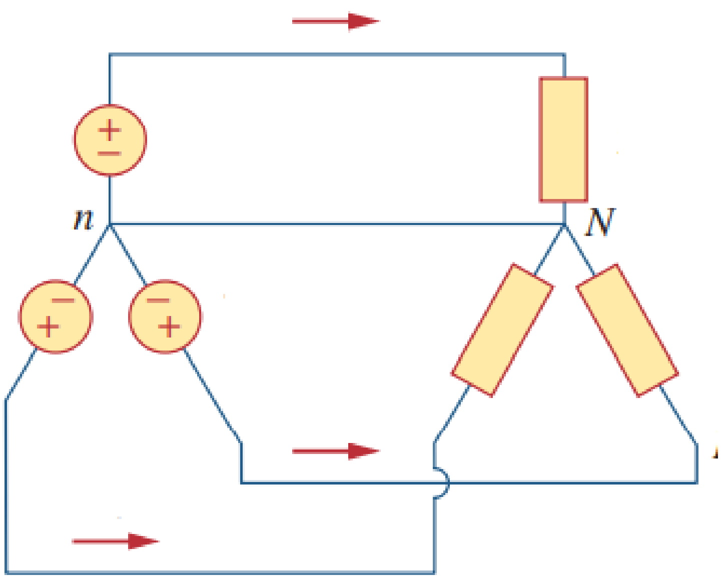


Three-phase source and three-phase load possible conections:

- Y-Y connection
- Y-Δ connection.
- Δ-Δ connection.
- Δ-Y connection.

Three-phase circuits.

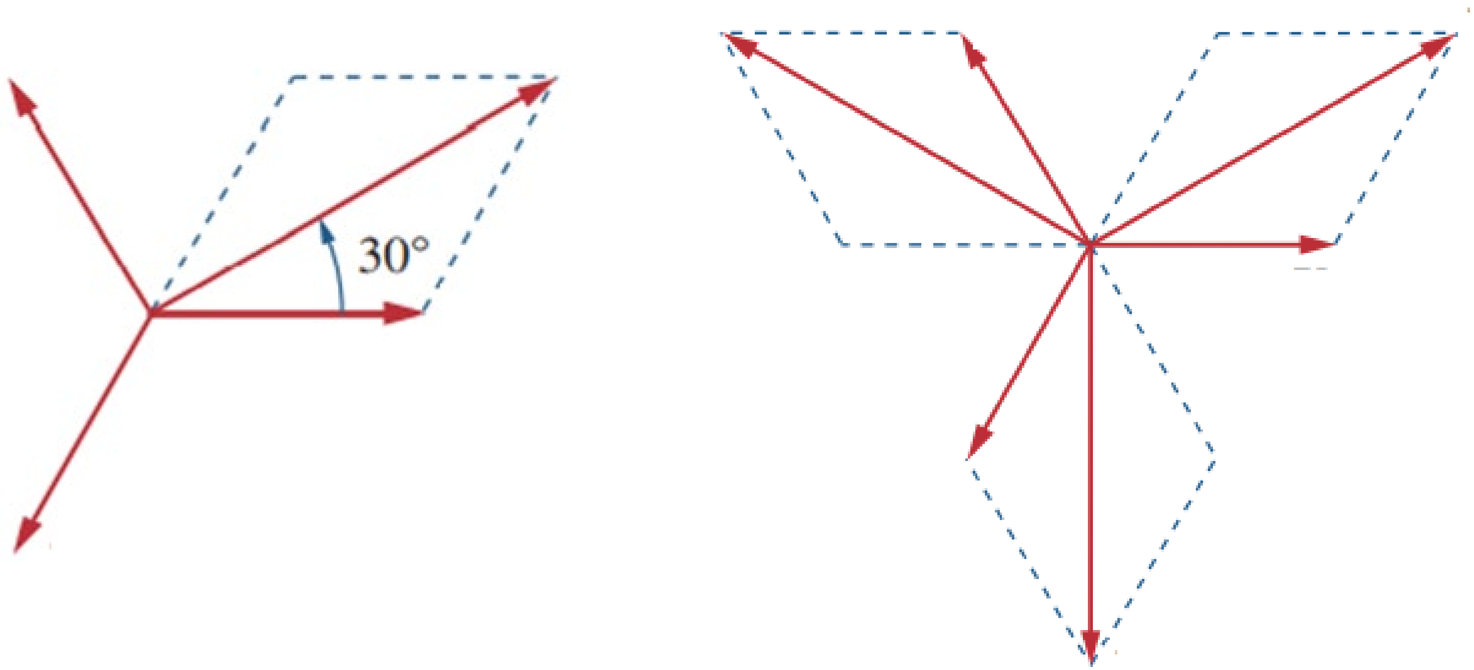
A **balanced Y-Y system** is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.



n, N – neutral points
n–N – neutral wire
1-1′, 2-2′, 3-3′ – line wires
$\underline{Z}_1$, $\underline{Z}_2$, $\underline{Z}_3$ – phase impedances

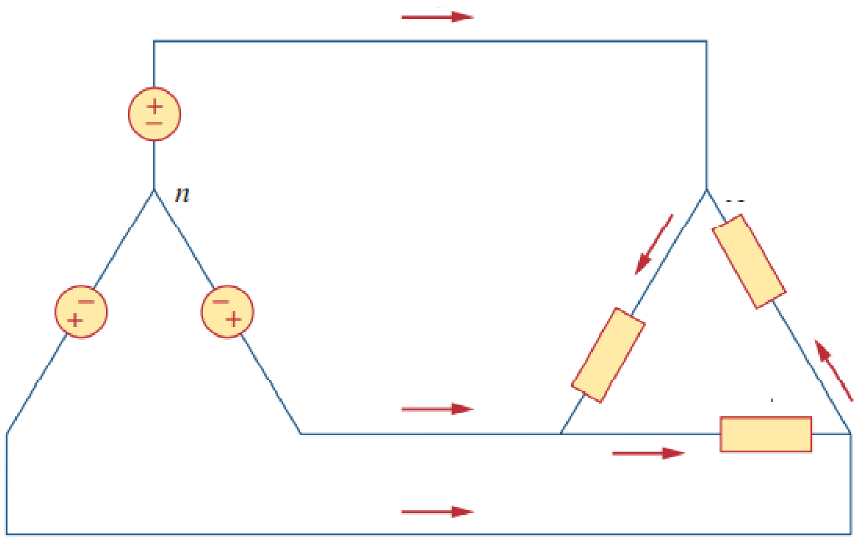
$$U_l = \sqrt{3}U_{ph}$$

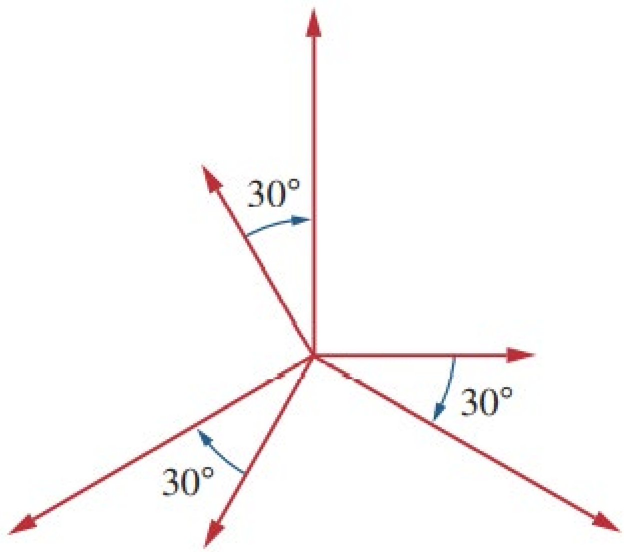
$$I_l = I_{ph}$$



Phasor diagrams illustrating the relation- ship between line voltages and phase voltages.

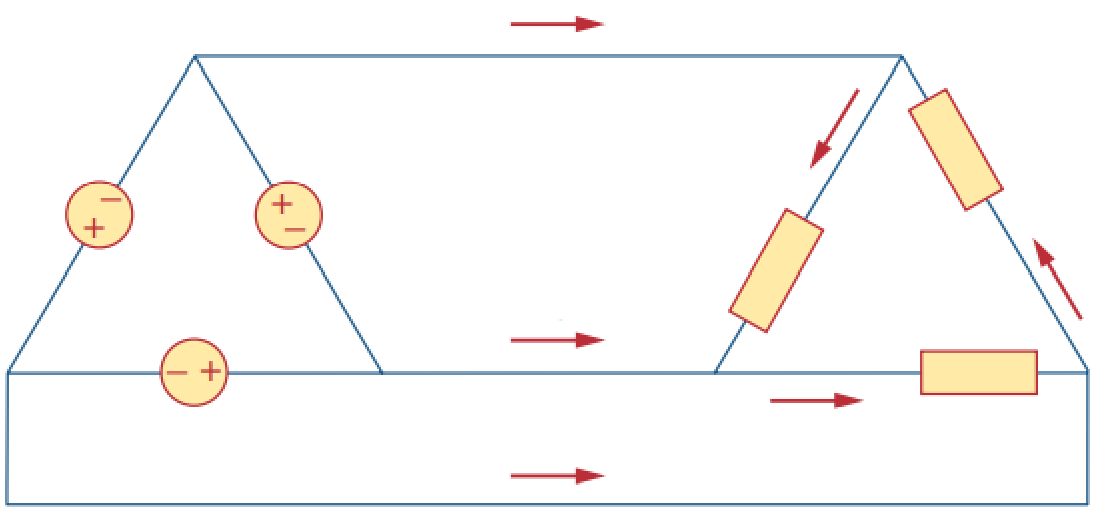
A **balanced Y-Δ system** consists of a balanced Y-connected source feeding a balanced Δ-connected load.





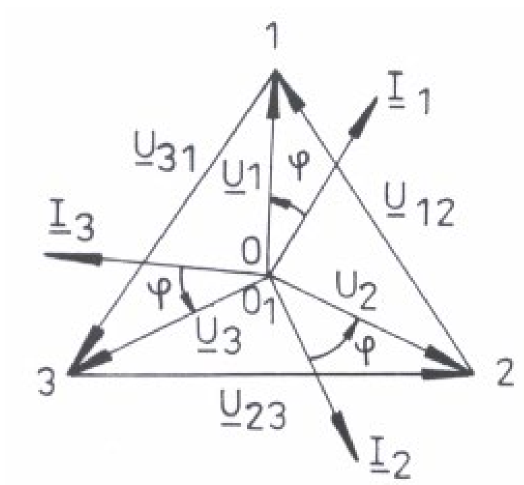
Phasor diagram illustrating the relationship between phase and line currents.

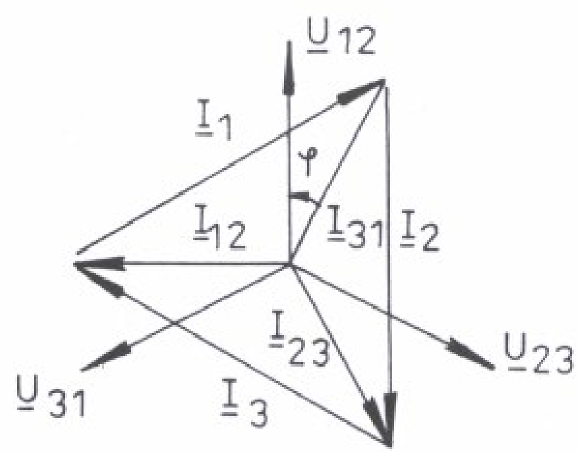
A **balanced Δ-Δ system** is one in which both the balanced source and balanced load are Δ-connected.



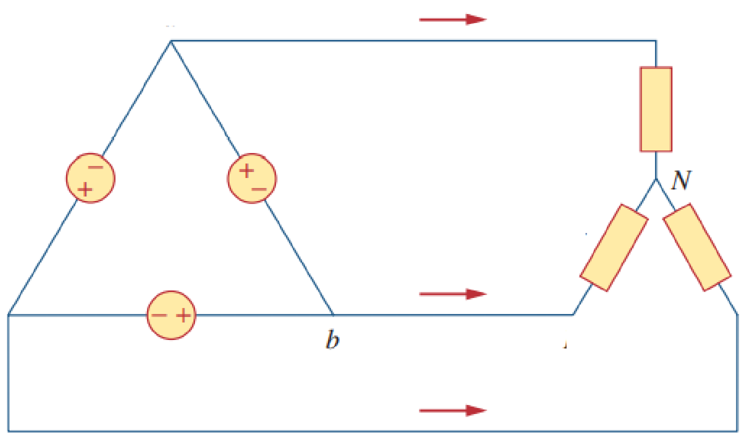
$$U_l = U_{ph}$$

$$I_l = \sqrt{3} I_{ph}$$





A **balanced Δ-Y system** consists of a balanced Δ-connected source feeding a balanced Y-connected load.

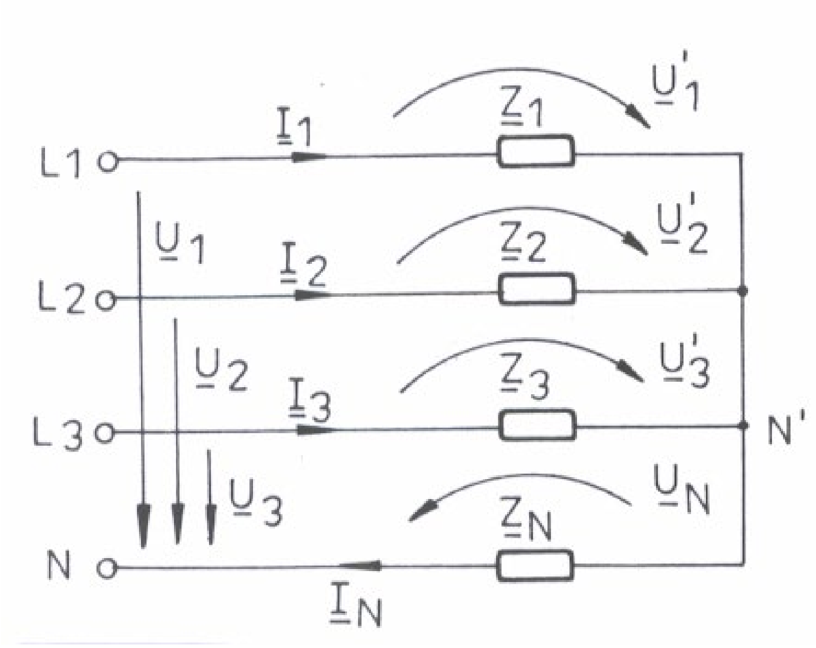


$$P = 3U_{ph}I_{ph}\cos\varphi = \sqrt{3}U_l I_l \cos\varphi$$

$$Q = 3U_{ph}I_{ph}\sin\varphi = \sqrt{3}U_l I_l \sin\varphi$$

$$S = 3U_{ph}I_{ph} = \sqrt{3}U_l I_l$$

4. CALCULATION OF UNBALANCED THREE-PHASE CIRCUITS.



$$\begin{cases} \underline{I}_1 = \dfrac{\underline{U}_1'}{\underline{Z}_1} = (\underline{U}_1 - \underline{U}_N)\underline{Y}_1 \\ \underline{I}_2 = \dfrac{\underline{U}_2'}{\underline{Z}_2} = (\underline{U}_2 - \underline{U}_N)\underline{Y}_2 \\ \underline{I}_3 = \dfrac{\underline{U}_3'}{\underline{Z}_3} = (\underline{U}_3 - \underline{U}_N)\underline{Y}_3 \\ \underline{I}_N = \dfrac{\underline{U}_N}{\underline{Z}_N} = \underline{U}_N \underline{Y}_N \end{cases}$$

$$\underline{I}_N = \underline{I}_1 + \underline{I}_2 + \underline{I}_3$$

$$\underline{U}_N \underline{Y}_N = (\underline{U}_1 - \underline{U}_N)\underline{Y}_1 + (\underline{U}_2 - \underline{U}_N)\underline{Y}_2 + (\underline{U}_3 - \underline{U}_N)\underline{Y}_3$$

$$\underline{U}_N (\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3 + \underline{Y}_N) = \underline{U}_1\underline{Y}_1 + \underline{U}_2\underline{Y}_2 + \underline{U}_3\underline{Y}_3$$

$$\underline{U}_N = \frac{\underline{U}_1\underline{Y}_1 + \underline{U}_2\underline{Y}_2 + \underline{U}_3\underline{Y}_3}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3 + \underline{Y}_N}$$

the Millman Theorem

a) Symmetrical voltages (i.e. $\underline{U}_1 = \underline{U}, \quad \underline{U}_2 = a^2\underline{U}, \quad \underline{U}_3 = a\underline{U}$) :

$$\underline{U}_N = \underline{U}\frac{\underline{Y}_1 + a^2\underline{Y}_2 + a\underline{Y}_3}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3 + \underline{Y}_N}$$

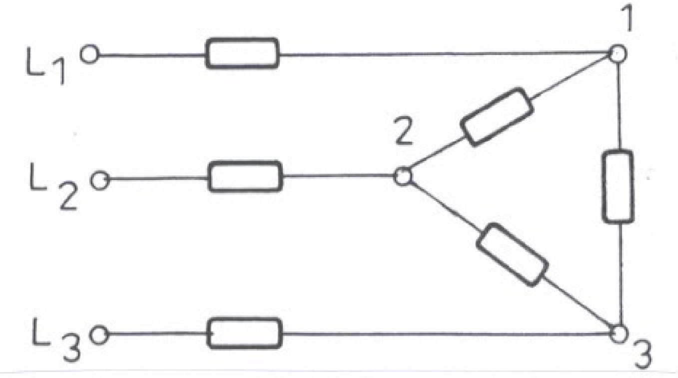
b) Symmetrical (balanced) load (i.e. $\underline{Y}_1 = \underline{Y}_2 = \underline{Y}_3 = \underline{Y}$) :

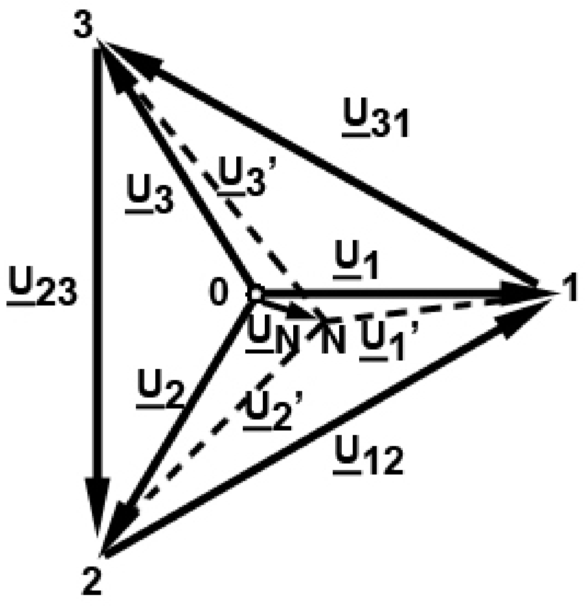
$$\underline{U}_N = \frac{\underline{Y}(\underline{U}_1 + \underline{U}_2 + \underline{U}_3)}{3\underline{Y} + \underline{Y}_N}$$

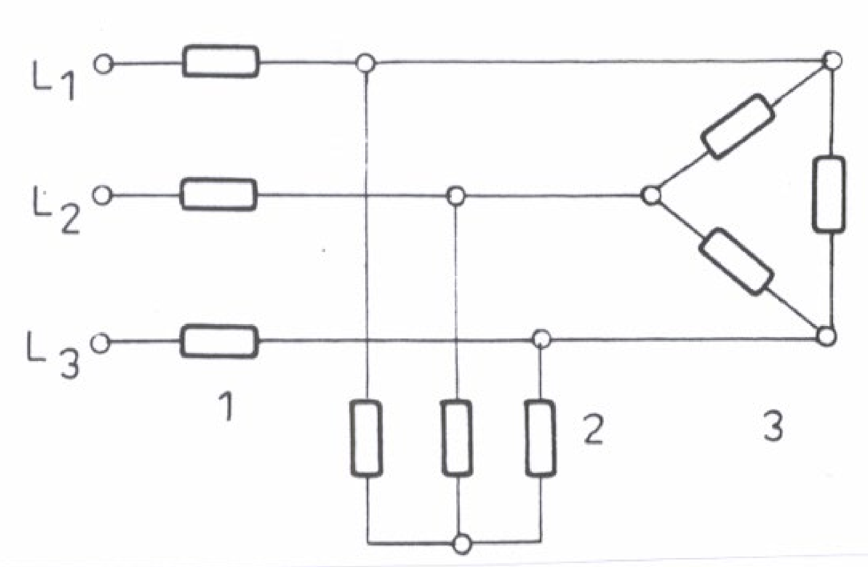
c) Symmetrical circuit : $\underline{U}_N = 0$

d) In the absence of the neutral wire: $\underline{Z}_N = \infty \quad \rightarrow \quad \underline{Y}_N = 0$

$$\underline{U}_N = \frac{\underline{U}_1\underline{Y}_1 + \underline{U}_2\underline{Y}_2 + \underline{U}_3\underline{Y}_3}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3}$$

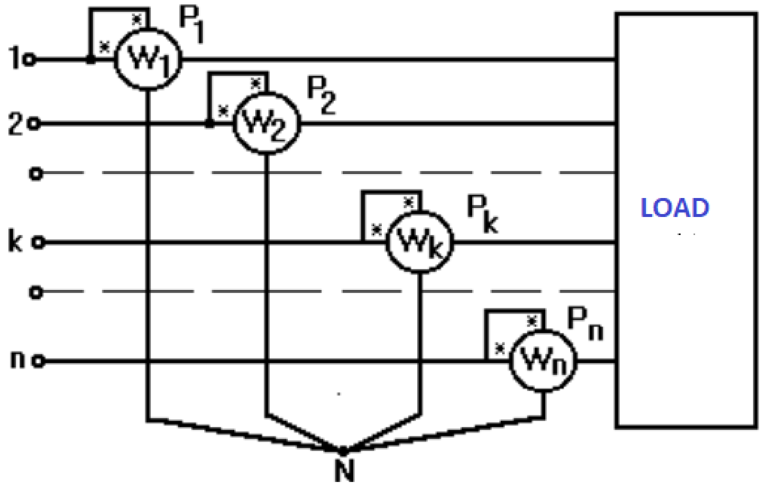




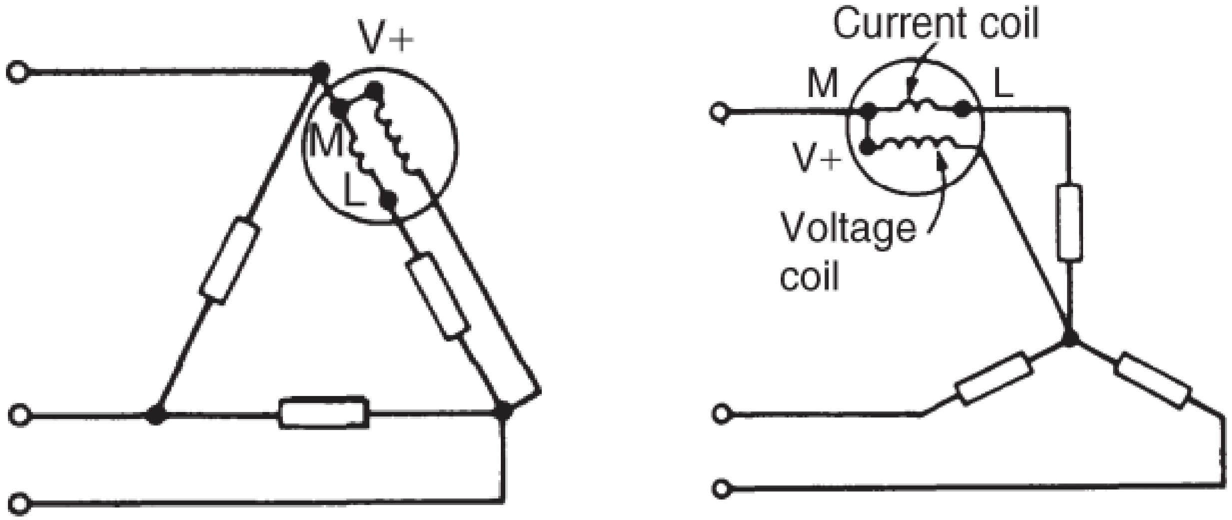


5. MEASUREMENT OF POWER IN THREE-PHASE SYSTEMS

- The total active power in a polyphase circuit with *n* conductors is equal to the sum of single-phase powers and, consequently, can be measured with the help of wattmeters, connected so that the current coil is traversed by the line current and the voltage coil is or connected between the conductor of the respective phase and a common *N* point taken as a reference.

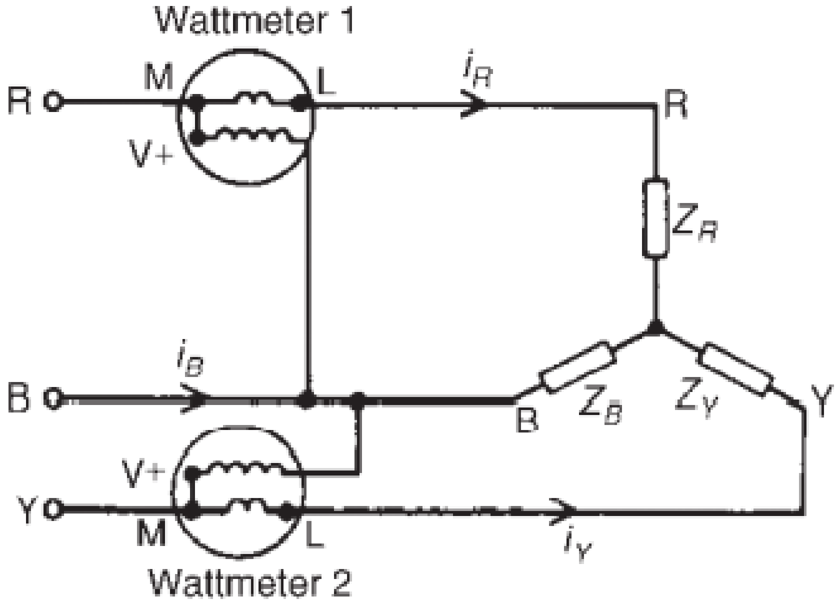


ONE-WATTMETER METHOD FOR A BALANCED LOAD



Total power = 3 × wattmeter reading

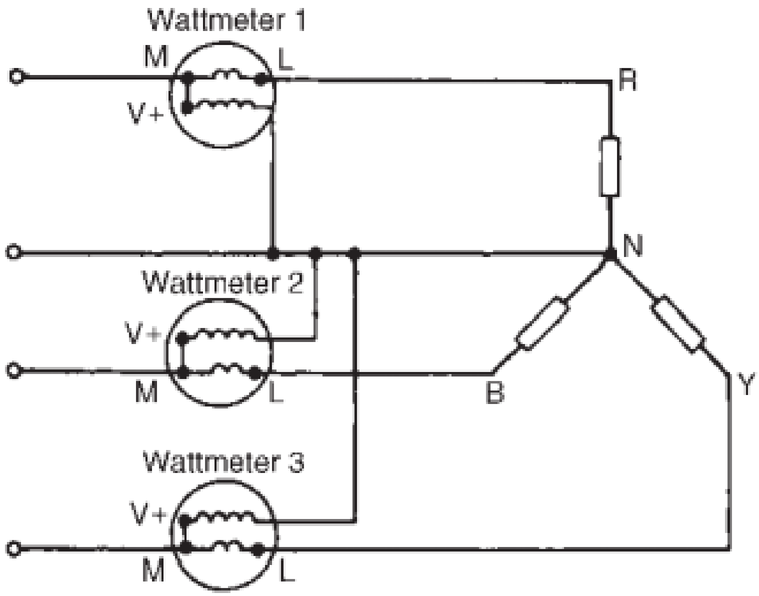
TWO-WATTMETER METHOD FOR A BALANCED OR



$$\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)$$

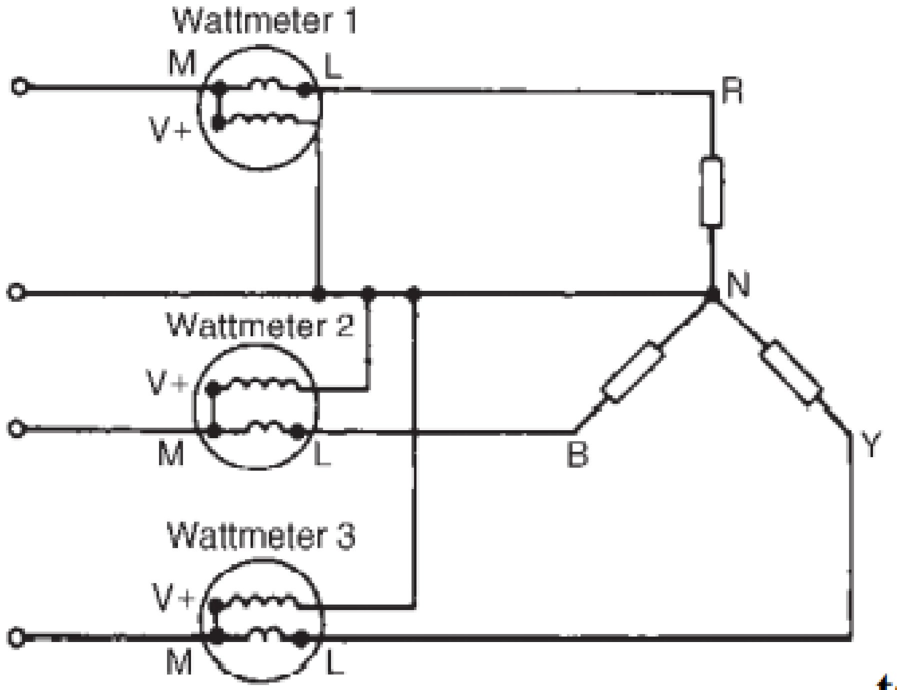
$$\text{Total power} = \text{sum of wattmeter readings} = P_1 + P_2$$

THREE-WATTMETER METHOD FOR A THREE-PHASE, 4-WIRE SYSTEM FOR BALANCED AND UNBALANCED LOADS.



$$\text{Total power} = P_1 + P_2 + P_3$$

THREE-WATTMETER METHOD FOR A THREE-PHASE, 4-WIRE SYSTEM FOR BALANCED AND UNBALANCED LOADS.



$$\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)$$

$$\boxed{\text{Total power} = P_1 + P_2 + P_3}$$

Comparison of star and delta connections

- Loads connected in delta dissipate three times more power than when connected in star to the same supply.

- For the same power, the phase currents must be the same for both delta and star connections (since power = 3Ip 2Rp), hence the line current in the delta-connected system is greater than the line current in the corresponding star-connected system. To achieve the same phase current in a star-connected system as in a delta-connected system, the line voltage in the star system is p3 times the line voltage in the delta system.

Thus for a given power transfer, a delta system is associated with larger line currents (and thus larger conductor cross-sectional area) and a star system is associated with a larger line voltage (and thus greater insulation).

ADVANTAGES OF THREE-PHASE SYSTEMS

Advantages of three-phase systems over single-phase supplies include:

- For a given amount of power transmitted through a system, the three-phase system requires conductors with a smaller cross-sectional area. This means a saving of copper (or aluminium) and thus the original installation costs are less.
- Two voltages are available.
- Three-phase motors are very robust, relatively cheap, generally smaller, have self-starting properties, provide a steadier output and require little maintenance compared with single-phase motors.

Review Questions

1 What is the phase sequence of a three-phase motor for which $\mathbf{V}_{AN} = 220\underline{/-100^\circ}$ V and $\mathbf{V}_{BN} = 220\underline{/140^\circ}$ V?

(a) *abc* (b) *acb*

2 If in an *acb* phase sequence, $V_{an} = 100\underline{/-20^\circ}$, then $\mathbf{V}_{cn}$ is:

(a) $100\underline{/-140^\circ}$ (b) $100\underline{/100^\circ}$
(c) $100\underline{/-50^\circ}$ (d) $100\underline{/10^\circ}$

3 Which of these is not a required condition for a balanced system:

(a) $|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$
(b) $\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$
(c) $V_{an} + V_{bn} + V_{cn} = 0$
(d) Source voltages are 120° out of phase with each other.
(e) Load impedances for the three phases are equal.

4 In a Y-connected load, the line current and phase current are equal.

(a) True (b) False

5 In a Δ-connected load, the line current and phase current are equal.

(a) True (b) False

6 In a Y-Y system, a line voltage of 220 V produces a phase voltage of:

(a) 381 V (b) 311 V (c) 220 V
(d) 156 V (e) 127 V

7 In a Δ-Δ system, a phase voltage of 100 V produces a line voltage of:

(a) 58 V (b) 71 V (c) 100 V
(d) 173 V (e) 141 V

8 When a Y-connected load is supplied by voltages in *abc* phase sequence, the line voltages lag the corresponding phase voltages by 30°.

(a) True (b) False

9 In a balanced three-phase circuit, the total instantaneous power is equal to the average power.

(a) True (b) False

10 The total power supplied to a balanced Δ-load is found in the same way as for a balanced Y-load.

(a) True (b) False

Answers: 12.1a, 12.2a, 12.3c, 12.4a, 12.5b, 12.6e, 12.7c, 12.8b, 12.9a, 12.10a.

Problems[1]

Practice on Problems proposed in:

Charlews K. Alexander, Matthew N.O.Sadiku, Fundamentals of Electric Circuits (Fifth Edition), published by McGraw-Hill, 2013

Pages 544 - 553

References

[1] Charlews K. Alexander, Matthew N.O.Sadiku, *Fundamentals of Electric Circuits (Fifth Edition),* published by McGraw-Hill, 2013

[2] John Bird, Electrical Circuit Theory and Technology, published by Newnes, 2003

[3] Radu V. Ciupa, Vasile Topa, *The Theory of Electric Circuits,* published by Casa Cartii de Stiinta, 1998

[4] Dan. D Micu, Laura Darabant, Denisa Stet et al., *Teoria circuitelor electrice. Probleme,* published by UTPress, 2027

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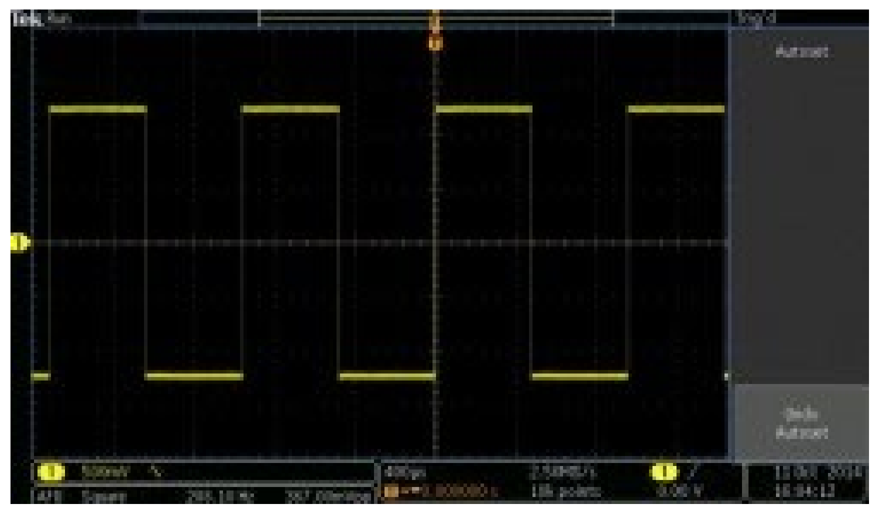
Chapter 3:

Steady-state periodic non-sinusoidal regime

BASES OF ELECTROTECHNICS I.
Faculty of Electronics, Telecommunications and Information Technology
Specialization: IETTI
Academic year: 2022-2023

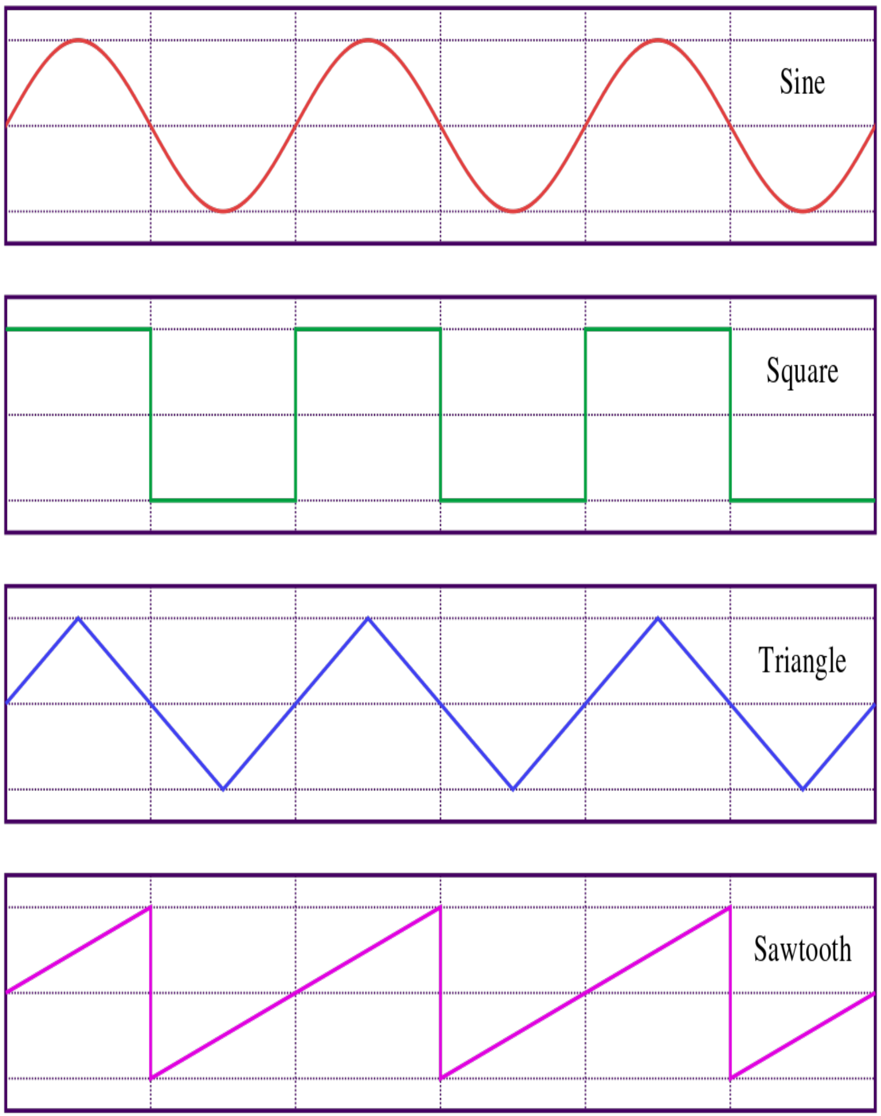
Not all signals in electrical and computer engineering are sinusoidal.

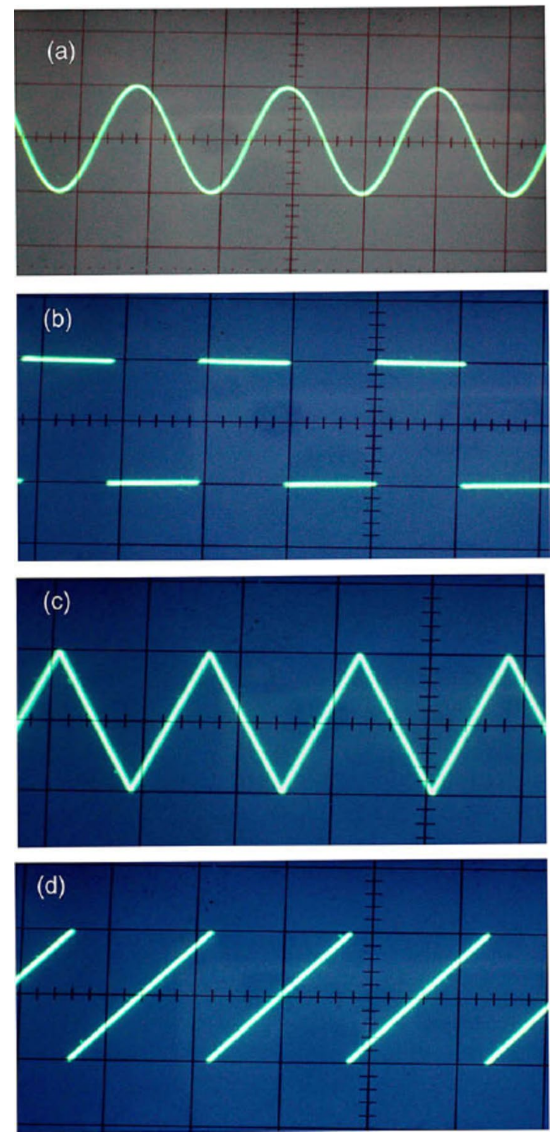
- ✓ **Most digital systems use square waveforms. Although at high switching speeds, these waveforms are starting to look trapezoidal.**
- ✓ **Most bioelectric signals are non-sinusoidal, many are composite ramp functions.**
- ✓ **When a switch is opened or closed, the time required for the signals to return to steady state is accompanied by sinusoidal and non-sinusoidal transients.**



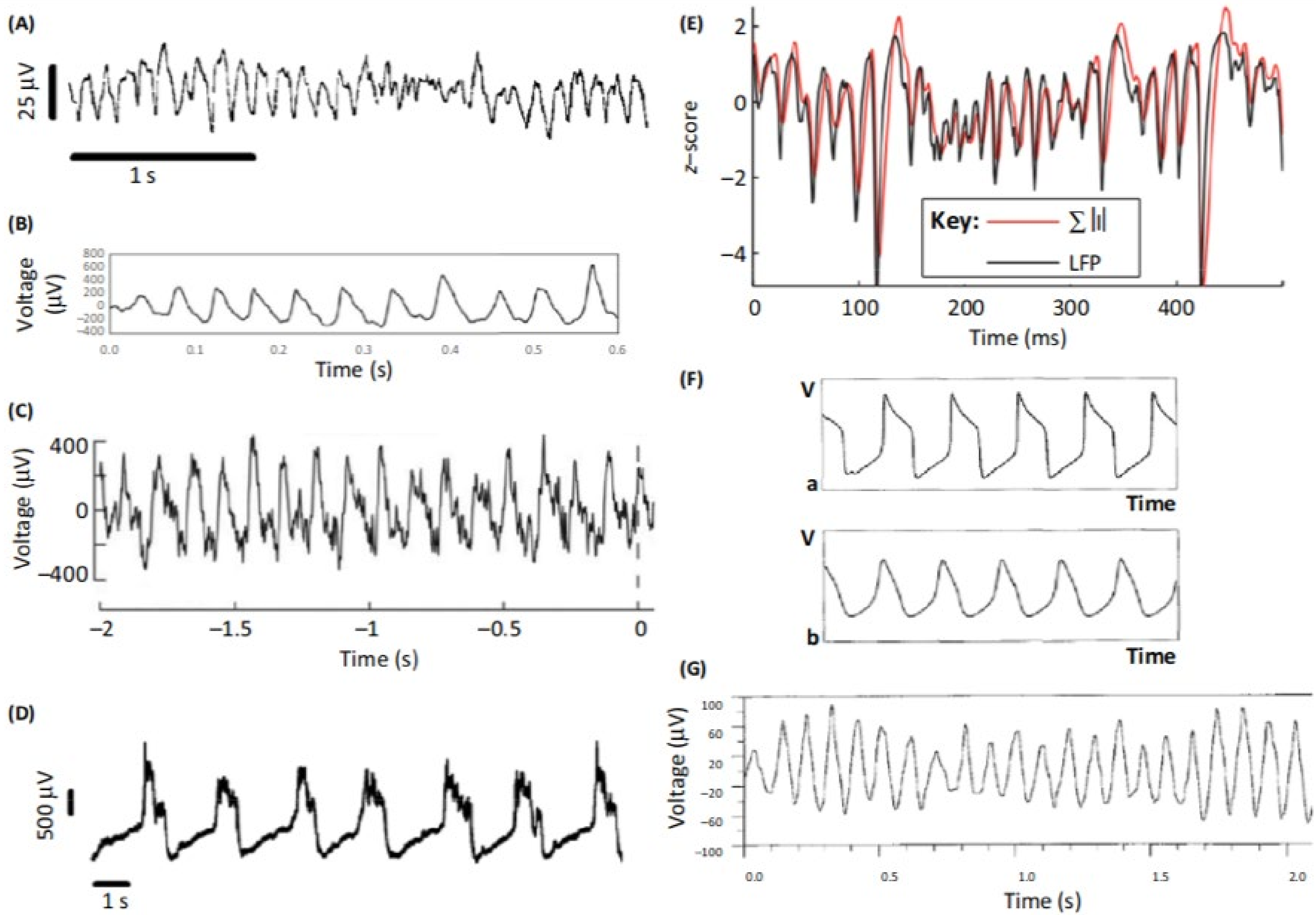
Chapter 3. Non-sinusoidal regime.

Samples of non-sinusoidal signals:

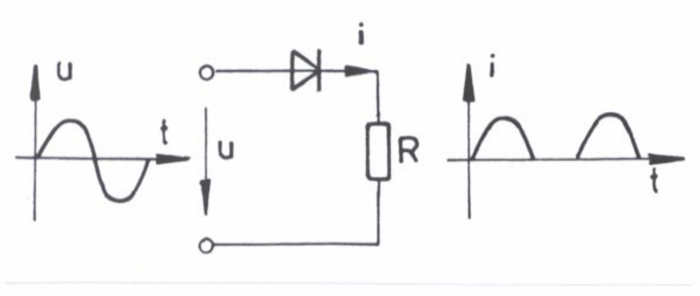




Samples of non-sinusoidal signals:



3.1 FOURIER EXPANSION.



$$f(t) = A_0 + \sum_{k=1}^{\infty} \sqrt{2} A_k \sin(k\omega t + \varphi_k)$$

A_0 – a constant (the average value),

$\sqrt{2} A_k \sin(k\omega t + \varphi_k)$ - the k-th order harmonic (for k=1: fundamental),

$\sqrt{2} A_k = A_m$ - the amplitude of the k-th harmonic,

φ_k - the phase of the k-th harmonic

$$f(t) = A_0 + \sum_{k=1}^{\infty} \sqrt{2} A_k \cos\varphi_k \sin k\omega t + \sum_{k=1}^{\infty} \sqrt{2} A_k \sin\varphi_k \cos k\omega t$$

$$f(t) = A_0 + \sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\omega t + \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\omega t$$

$$B_k = A_k \cos\varphi_k$$
$$C_k = A_k \sin\varphi_k$$

Finding the Coefficients of the Fourier Series.

$$f(t) = A_0 + \sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\omega t + \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\omega t$$

1) $\boldsymbol{A_0}$

$$\int_0^T f(t)dt = \int_0^T A_0 dt + \int_0^T \sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\omega t dt + \int_0^T \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\omega t dt$$

$$\int_0^T f(t)dt = \int_0^T A_0 dt + \sum_{k=1}^{\infty} \left[\int_0^T \sqrt{2} B_k \sin k\omega t dt + \int_0^T \sqrt{2} C_k \cos k\omega t dt \right]$$

$$\int_0^T f(t)dt = \int_0^T A_0 dt + \sum_{k=1}^{\infty} [0 + 0]$$

$$\boxed{A_0 = \frac{1}{T} \int_0^T f(t)dt}$$

- the average value of the function over a period

$$f(t) = A_0 + \sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\omega t + \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\omega t$$

2) $\sqrt{2} B_k$

$$\int_{)}^{T} f(t) \sin m\omega t dt = \int_0^T A_0 \sin m\omega t dt + \int_0^T \sin m\omega t \left[\sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\omega t + \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\omega t \right] dt$$

$$\int_0^T \sin m\omega t \sin k\omega t dt = \begin{cases} 0, & if \ m \neq k \\ \frac{T}{2}, & if \ m = k \end{cases} \quad , \quad \int_0^T \sin m\omega t dt = 0$$

$$\int_0^T f(t) \sin k\omega t dt = \sqrt{2} B_k \int_0^T \sin k\omega t \sin k\omega t dt = \frac{\sqrt{2} B_k}{2} T$$

$$\boxed{\sqrt{2} B_k = \frac{2}{T} \int_0^T f(t) \sin k\omega t dt}$$

$$f(t) = A_0 + \sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\omega t + \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\omega t$$

3) $\sqrt{2} C_k$

$$\int_0^T f(t) \cos n\omega t dt = \int_0^T A_0 \cos n\omega t dt + \int_0^T \cos n\omega t \left[\sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\omega t + \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\omega t \right] dt$$

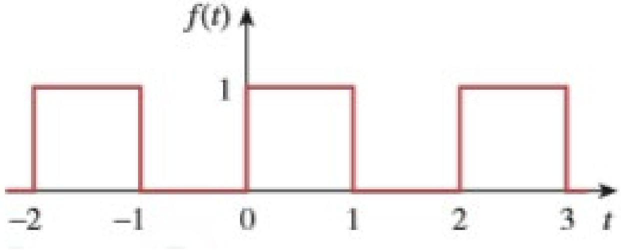
$$\int_0^T \cos n\omega t \cos k\omega t dt = \begin{cases} 0, & if \ n \neq k \\ \frac{T}{2}, & if \ n = k \end{cases} , \qquad \int_0^T \cos n\omega t dt = 0$$

$$\int_0^T f(t) \cos k\omega t dt = \sqrt{2} C_k \int_0^T \cos k\omega t \cdot \cos k\omega t dt = \frac{\sqrt{2} C_k}{2} T$$

$$\boxed{\sqrt{2} C_k = \frac{2}{T} \int_0^T f(t) \cos k\omega t dt}$$

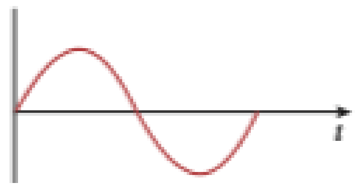
The **Fourier series** of a periodic function $f(t)$ is a representation that resolves $f(t)$ into a dc component and an ac component comprising an infinite series of harmonic sinusoids.

Function	Value
$\cos 2n\pi$	1
$\sin 2n\pi$	0
$\cos n\pi$	$(-1)^n$
$\sin n\pi$	0
$\cos\dfrac{n\pi}{2}$	$\begin{cases}(-1)^{n/2}, & n = \text{even}\\ 0, & n = \text{odd}\end{cases}$
$\sin\dfrac{n\pi}{2}$	$\begin{cases}(-1)^{(n-1)/2}, & n = \text{odd}\\ 0, & n = \text{even}\end{cases}$
$e^{j2n\pi}$	1
$e^{jn\pi}$	$(-1)^n$
$e^{jn\pi/2}$	$\begin{cases}(-1)^{n/2}, & n = \text{even}\\ j(-1)^{(n-1)/2}, & n = \text{odd}\end{cases}$



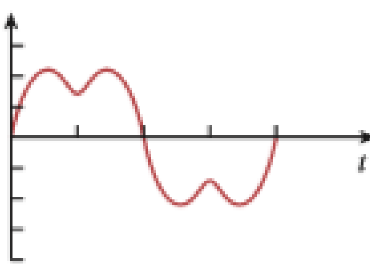
$$f(t) = A_0 + \sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\omega t + \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\omega t$$

dc component

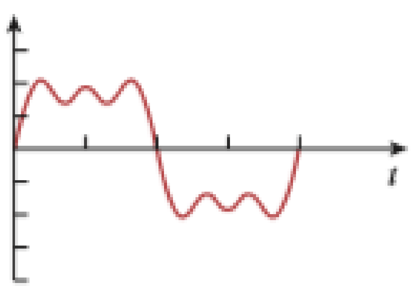


Fundamental ac component

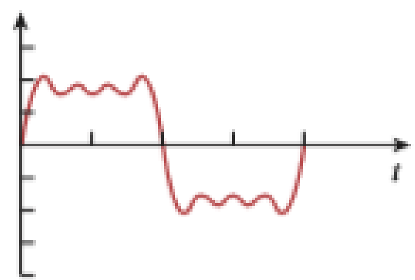
$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sin \pi t + \frac{2}{3\pi}\sin 3\pi t + \frac{2}{5\pi}\sin 5\pi t + \cdots$$



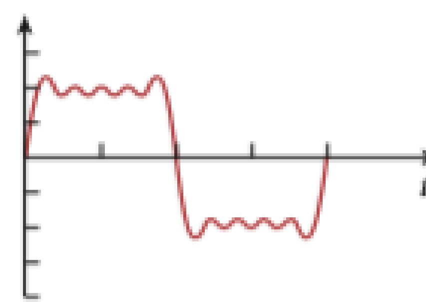
Sum of first two ac components



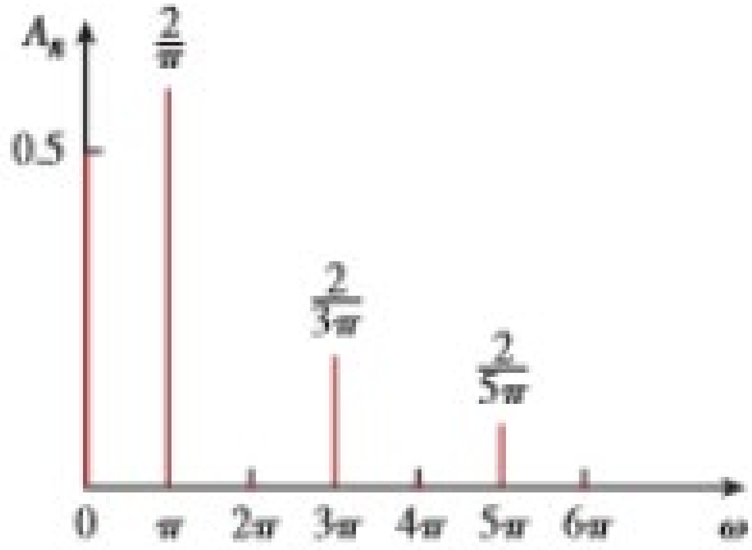
Sum of first three ac components



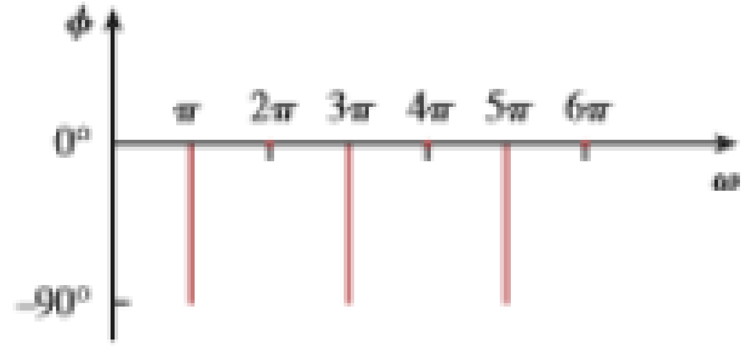
Sum of first four ac components



Sum of first five ac components



Amplitude spectrum of the function



Phase spectrum of the function

Historical

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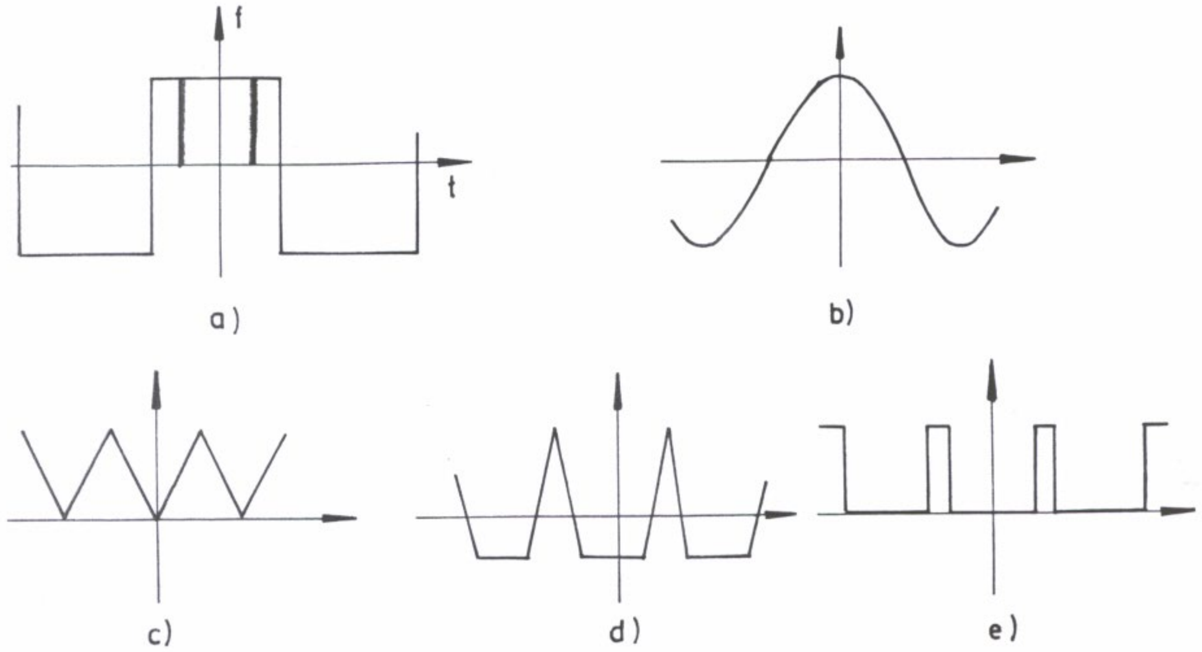
Jean Baptiste Joseph Fourier (1768–1830), a French mathematician, first presented the series and transform that bear his name. Fourier's results were not enthusiastically received by the scientific world. He could not even get his work published as a paper.

Born in Auxerre, France, Fourier was orphaned at age 8. He attended a local military college run by Benedictine monks, where he demonstrated great proficiency in mathematics. Like most of his contemporaries, Fourier was swept into the politics of the French Revolution. He played an important role in Napoleon's expeditions to Egypt in the later 1790s. Due to his political involvement, he narrowly escaped death twice.

3.2 EVEN AND ODD SYMMETRY.

1) ***Even** function*: ***f(t) = f(-t)*** or ***f(α) = f(-α)***



$$\sin(-k\alpha) = -\sin k\alpha$$

$$\cos(-k\alpha) = \cos k\alpha$$

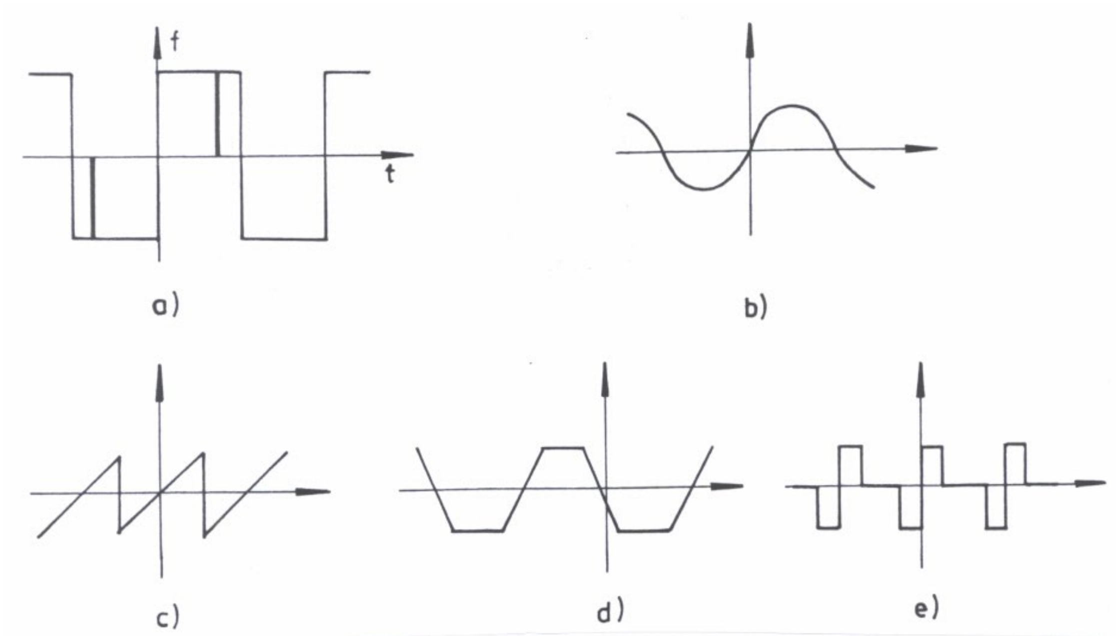
$$f(\alpha) = A_0 + \sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\alpha + \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\alpha$$

$$f(-\alpha) = A_0 - \sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\alpha + \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\alpha$$

$$B_k = 0$$

Even function: ***the Fourier series does not contain sin harmonics***

2) ***Odd*** *function*: ***f(t) = - f(-t)*** or ***f(α) = - f(-α)***



$$\sin(-k\alpha) = -\sin k\alpha$$

$$\cos(-k\alpha) = \cos k\alpha$$

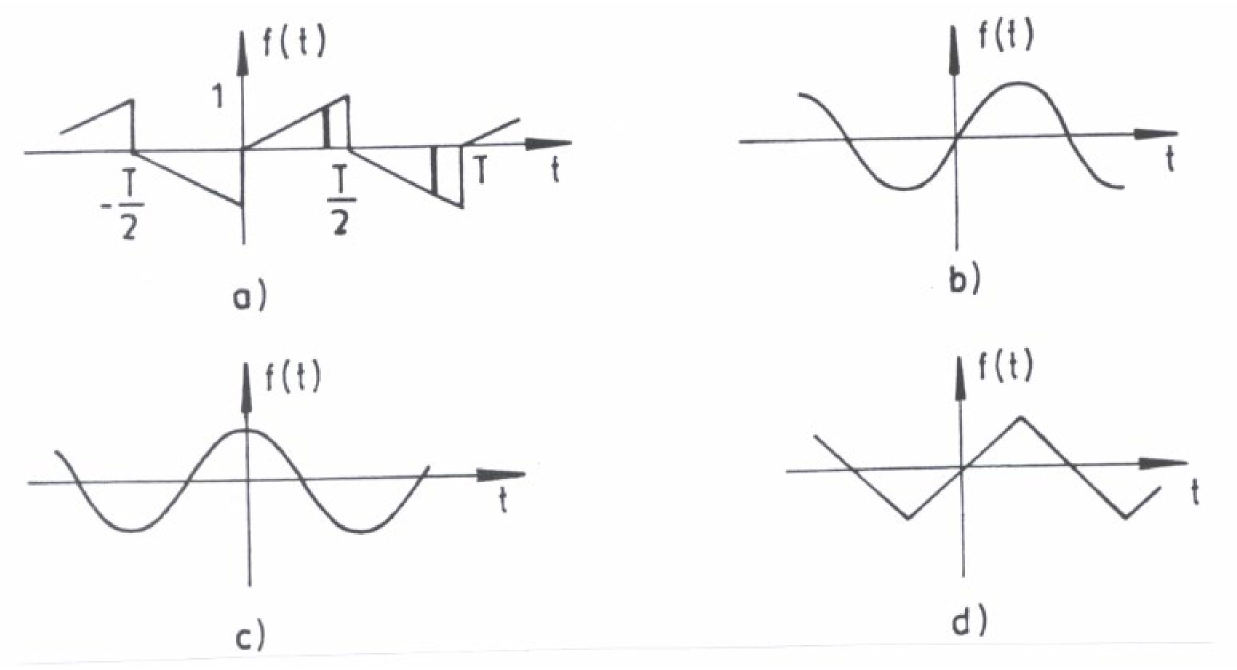
$$f(-\alpha) = A_0 + \sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\alpha + \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\alpha$$

$$-f(-\alpha) = -A_0 + \sum_{k=1}^{\infty} \sqrt{2} B_k \sin k\alpha - \sum_{k=1}^{\infty} \sqrt{2} C_k \cos k\alpha$$

$$A_0 = 0 \qquad C_k = 0$$

Odd function: ***the Fourier series contains only sin harmonics***

3) ***Half-wave*** *symmetry*: $f(t) = -f\left(t + \frac{T}{2}\right)$ or $f(\alpha) = -f(\alpha + \pi)$



$A_0 = 0$

$B_{2p} = 0$

$C_{2p} = 0$

$$f(\alpha) = \sum_{p=1}^{\infty} \sqrt{2} B_{2p+1} \sin(2p+1)\alpha + \sum_{p=1}^{\infty} \sqrt{2} C_{2p+1} \cos(2p+1)\alpha$$

Half-wave symmetry: ***the Fourier series contains only odd sin and cos harmonics***

3.4 CHARACTERISTIC VALUES FOR PERIODIC NON-SINUSOIDAL FUNCTIONS.

a) *The maximum value (or the peak value)*: the biggest value of periodic non-sinusoidal function over a period.

b) The average mean value :

$$A_0 = \frac{1}{T}\int_0^T f(t)dt$$

c) The effective value (or r.m.s. value) :

$$A = A_{r.m.s.} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}$$

$$f(t) = A_0 + \sqrt{2}A_1\sin(\omega t + \gamma_1) + \sqrt{2}A_2\sin(2\omega t + \gamma_2) + \ldots$$

$$A = \sqrt{\frac{1}{T}\int_0^T \left[A_0 + \sqrt{2}A_1\sin(\omega t + \gamma_1) + \sqrt{2}A_2\sin(2\omega t + \gamma_2) + \ldots\right]^2 dt}$$

$$A = \sqrt{\frac{1}{T}\int_0^T \left[A_0^2 + \sum_{k=1}^{\infty} A_k^2 2\sin^2(k\omega t + \gamma_k) \right] dt}$$

$$A = \sqrt{\frac{1}{T}\int_0^T A_0^2 dt + \sum_{k=1}^{\infty} \frac{1}{T}\int_0^T A_k^2 2\sin^2(k\omega t + \gamma_k) dt}$$

$$\frac{1}{T}\int_0^T A_k^2 2\sin^2(k\omega t + \gamma_k) dt = \frac{A_k^2}{T}\int_0^T [1 - \cos 2(k\omega t + \gamma_k)] dt =$$

$$= \frac{A_k^2}{T}\left[\int_0^T dt - \int_0^T \cos 2(k\omega t + \gamma_k) dt \right] = A_k^2$$

$$A = \sqrt{A_0^2 + A_1^2 + A_2^2 + \ldots} = \sqrt{A_0^2 + \sum_{k=1}^{\infty} A_k^2}$$

$$U = \sqrt{U_0^2 + U_1^2 + U_2^2 + \ldots}$$

$$I = \sqrt{I_0^2 + I_1^2 + I_2^2 + \ldots}$$

More information about non-sinusoidal periodic functions:

a) The ***form factor* - k_f** : the ratio of the r.m.s. value of a quantity to its half-period average value.

$$k_f = \frac{A}{A_{ha}}$$, for sinusoids $k_f = \pi / 2\sqrt{2} = 1.11$

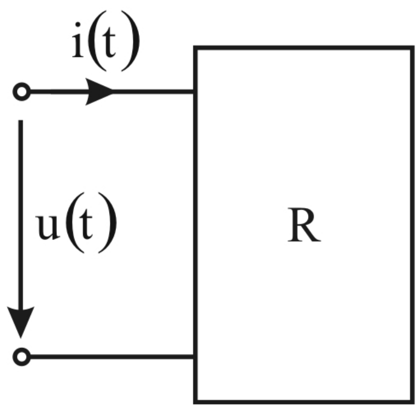
b) The ***peak factor* - k_p** : the ratio of the peak value of a quantity to its r.m.s. value.

$$k_p = \frac{A_m}{A}$$, for sinusoids $k_p = \sqrt{2} = 1.41$

c) The ***distorsion factor* - k_d** : the ratio of the r.m.s. value of the harmonics to the r.m.s. value of the function as a whole, neglecting the constant component

$$k_d = \frac{\sqrt{\sum_{k=2}^{\infty} A_k^2}}{\sqrt{A^2 - A_0^2}} = \frac{\sqrt{A_2^2 + A_3^2 + \ldots}}{\sqrt{A_1^2 + A_2^2 + A_3^2 + \ldots}}$$

3.6 POWER FOR NON-SINUSOIDAL PERIODIC VARIABLES.



$$u(t) = U_0 + \sum_{k=1}^{\infty} \sqrt{2} U_k \sin(k\omega t + \gamma_{uk}) = U_0 + \sum_{k=1}^{\infty} u_k(k\omega t)$$

$$i(t) = I_0 + \sum_{k=1}^{\infty} \sqrt{2} I_k \sin(k\omega t + \gamma_{ik}) = I_0 + \sum_{k=1}^{\infty} i_k(k\omega t)$$

a) The instantaneous power p(t):

$$p(t) = u(t) \cdot i(t) = \left(U_0 + \sum_{k=1}^{\infty} u_k \right)\left(I_0 + \sum_{k=1}^{\infty} i_k \right)$$

b) The active power P:

$$P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T u(t) \cdot i(t)dt$$

$$P=\frac{1}{T}\int_0^T\left(U_0+\sum_{k=1}^{\infty}u_k\right)\left(I_0+\sum_{k=1}^{\infty}i_k\right)dt=\frac{1}{T}\int_0^T\left(U_0I_0+U_0\sum_{k=1}^{\infty}i_k+I_0\sum_{k=1}^{\infty}u_k+\sum_{k=1}^{\infty}\sum_{k=1}^{\infty}u_ki_k\right)dt=$$

$$=\frac{1}{T}\int_0^T U_0I_0dt+\frac{1}{T}U_0\sum_{k=1}^{\infty}\int_0^T\sqrt{2}I_k\sin(k\omega t+\gamma_{ik})dt+\frac{1}{T}I_0\sum_{k=1}^{\infty}\int_0^T\sqrt{2}U_k\sin(k\omega t+\gamma_{uk})dt+$$

$$+\frac{1}{T}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\int_0^T 2U_kI_k\sin(k\omega t+\gamma_{uk})\sin(n\omega t+\gamma_{ik})dt$$

$$\frac{1}{T}\int_0^T 2U_kI_n\sin(k\omega t+\gamma_{uk})\sin(n\omega t+\gamma_{ik})dt=\begin{cases}0 & n\neq k\\ U_kI_k\cos\varphi_k & n=k\end{cases}$$

where $\varphi_k=\gamma_{uk}-\gamma_{ik}$

$$\boxed{P=U_0I_0+\sum_{k=1}^{\infty}U_kI_k\cos\varphi_k}\qquad [W]$$

c) *The reactive power Q* :

$$Q = \sum_{k=1}^{\infty} U_k I_k \sin\varphi_k$$

[VAR]

d) *The apparent power S* :

$$S = UI = \sqrt{U_0^2 + U_1^2 + \dots} \cdot \sqrt{I_0^2 + I_1^2 + \dots}$$

[VA]

e) *The distorsion power D* : $S^2 = P^2 + Q^2 + D^2$, $D = \sqrt{S^2 - P^2 - Q^2}$

$$D = \sqrt{\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\left[U_k^2 I_n^2 + U_n^2 I_k^2 - 2U_k U_n I_k I_n \cos(\varphi_k - \varphi_n)\right]}$$

[VAD]

$$k_p = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2 + D^2}}$$

3.7. NETWORK ANALYSIS IN NON-SINUSOIDAL REGIME

Steps for Applying Fourier Series:

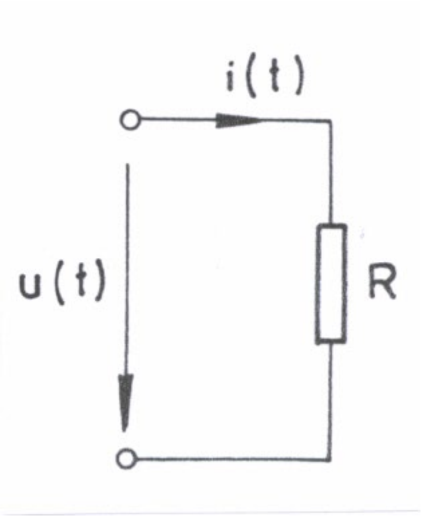
1. Express the excitation as a Fourier series.
2. Transform the circuit from the time domain to the frequency domain.
3. Find the response of the dc and ac components in the Fourier series.
4. Add the individual dc and ac responses using the superposition principle.

3.7. NETWORK ANALYSIS IN NON-SINUSOIDAL REGIME.

$$u(t) = U_0 + \sum_{k=1}^{\infty} \sqrt{2} U_k \sin(k\omega t + \gamma_{uk})$$

$$i(t) = I_0 + \sum_{k=1}^{\infty} \sqrt{2} I_k \sin(k\omega t + \gamma_{ik})$$

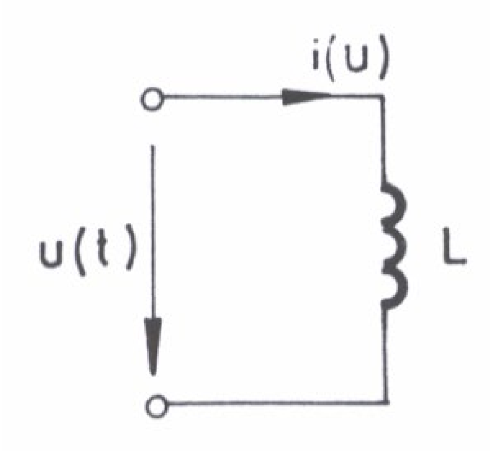
a) Ideal resistor .



$$i(t) = \frac{u(t)}{R} = \frac{U_0}{R} + \sum_{k=1}^{\infty} \frac{U_k}{R} \sin(k\omega t + \gamma_{uk})$$

The voltage and the current are <u>in phase</u>.

b) Ideal inductor.



$$u(t) = U_0 + \sum_{k=1}^{\infty} \sqrt{2} U_k \sin(k\omega t + \gamma_{uk})$$

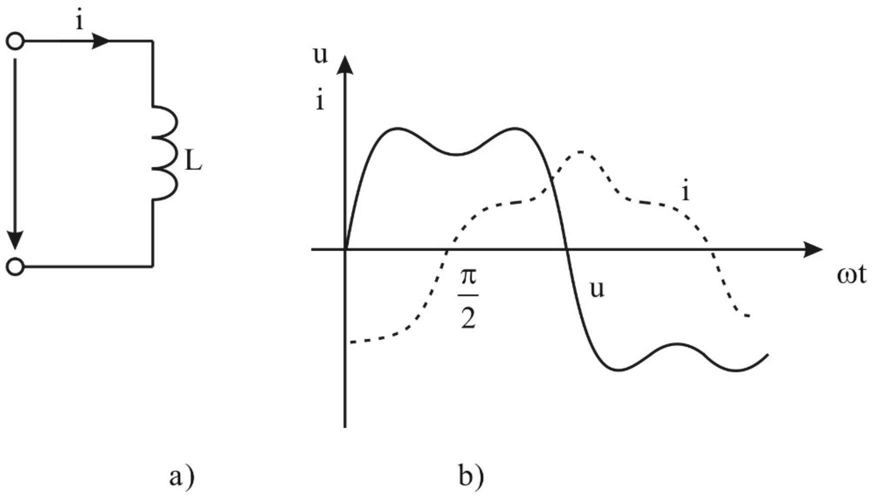
$$u_L = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_0^T u(t)dt = \frac{1}{L}U_0 t + \sqrt{2}\frac{U_1}{\omega L}\sin(\omega t + \gamma_{u1} - \frac{\pi}{2}) + \ldots + \sqrt{2}\frac{U_k}{k\omega L}\sin(k\omega t + \gamma_{uk} - \frac{\pi}{2}) + \ldots$$

Important remark: Assuming $t \to \infty$, the first current component $\frac{U_0}{L}t \to \infty$:

It is forbidden to apply d.c. voltage across an inductance (having negligible resistance, an ideal inductance represents a short circuit).

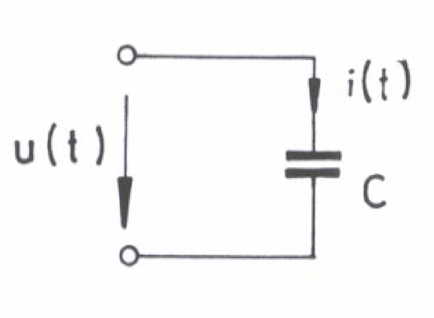
$$i(t)=\sum_{k=1}^{\infty}\sqrt{2}\,\frac{U_k}{k\omega L}\sin\left(k\omega t+\gamma_{uk}-\frac{\pi}{2}\right)$$



$$I_k=\frac{U_k}{k\omega L}$$

$$\gamma_{ik}=\gamma_{uk}-\frac{\pi}{2}\quad or\quad \varphi_k=\gamma_{uk}-\gamma_{ik}=\frac{\pi}{2}$$

c) Ideal capacitor.



$$u(t) = U_0 + \sum_{k=1}^{\infty} \sqrt{2} U_k \sin(k\omega t + \gamma_{uk})$$

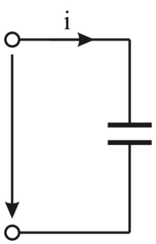
$$u_c = \frac{1}{C}\int i dt$$

$$i = C\frac{du}{dt} = \sqrt{2}\omega C U_1 \sin\left(\omega t + \gamma_{u1} + \pi/2\right) + \ldots + \sqrt{2}k\omega C U_k \sin\left(k\omega t + \gamma_{uk} + \pi/2\right) + \ldots$$

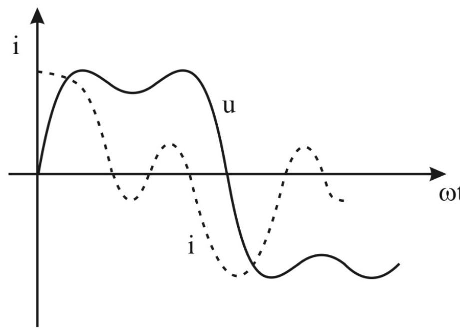
$$i(t) = \sum_{k=1}^{\infty} \sqrt{2}k\omega C U_k \sin\left(k\omega t + \gamma_{uk} + \pi/2\right)$$

$$I_k = k\omega C \cdot U_k$$

$$\varphi = \gamma_{uk} - \gamma_{ik} = -\frac{\pi}{2}$$

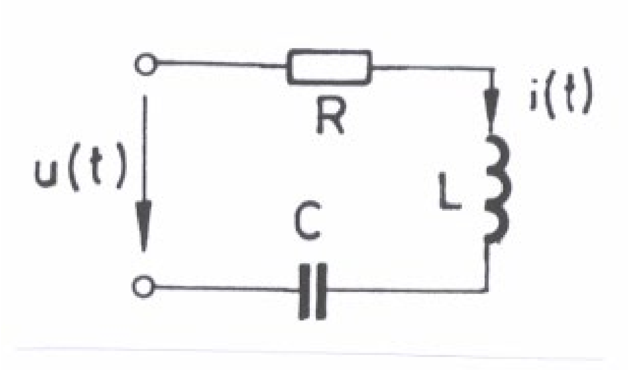


a)



b)

d) R, L, C circuit.



if $$u(t) = U_0 + \sum_{k=1}^{\infty} \sqrt{2} U_k \sin(k\omega t + \gamma_{uk})$$

then: $$i(t) = \sum_{k=1}^{\infty} \sqrt{2} I_k \sin(k\omega t + \gamma_{ik})$$

where: $$I_k = \frac{U_k}{\sqrt{R^2 + \left(k\omega L - \dfrac{1}{k\omega C}\right)^2}}, \quad \gamma_{ik} = \gamma_{uk} - \varphi_k$$

$$\varphi_k = \arctan \frac{k\omega L - \dfrac{1}{k\omega C}}{R}$$

The **resonance** condition:

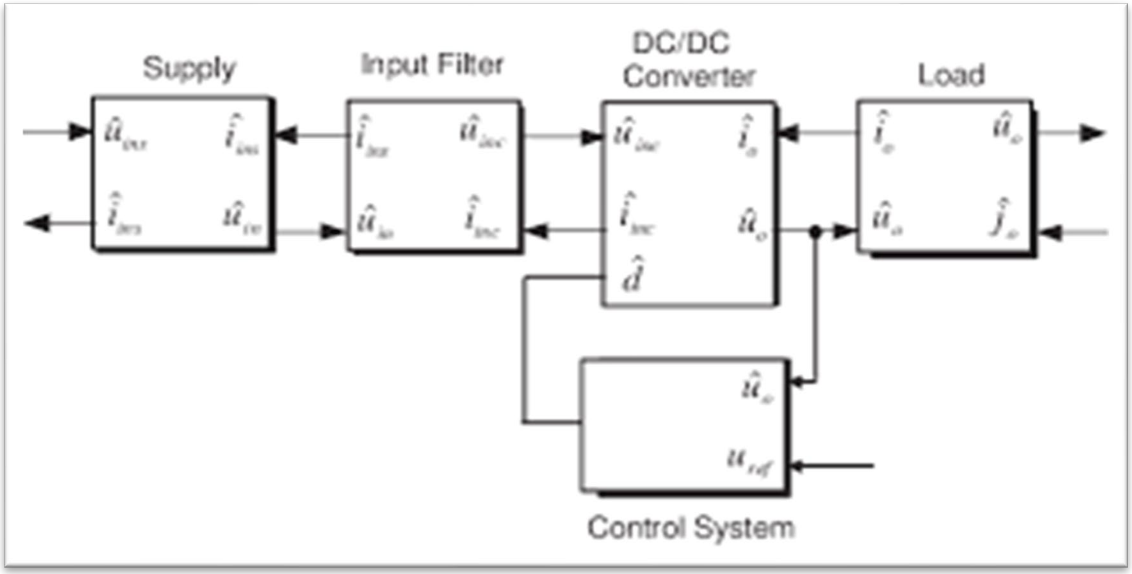
$$X_k = k\omega L - \frac{1}{k\omega C} = 0$$

Chapter 4: TWO-PORT NETWORKS

BASES OF ELECTROTECHNICS I.
Faculty of Electronics, Telecommunications and Information Technology
Specialization: IETTI
Academic year: 2023-2024

Contents

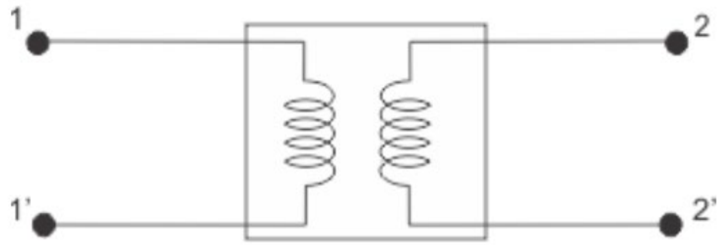
- One of the most frequent tasks to which networks are applied is the process of shaping (or filtering) some electrical signal information.
- Another examples are: coupling and matching networks, alternators, phase-shifter, amplifiers etc.
- Such networks are useful in communications, control system, power systems and electronics and to facilitate cascades design.
- Knowing the parameters of a two-port network enables us to treat it as a "black box" which is embedded within a larger network.



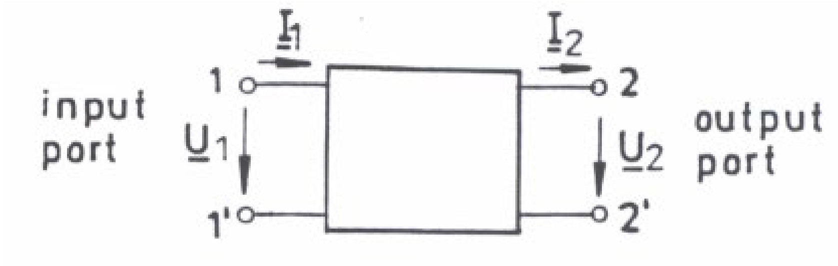
A **two-port network** is an electrical network with two separate ports for input and output.

Two Part Network

- ✓ A single phase transformer is an ideal example of two port network.



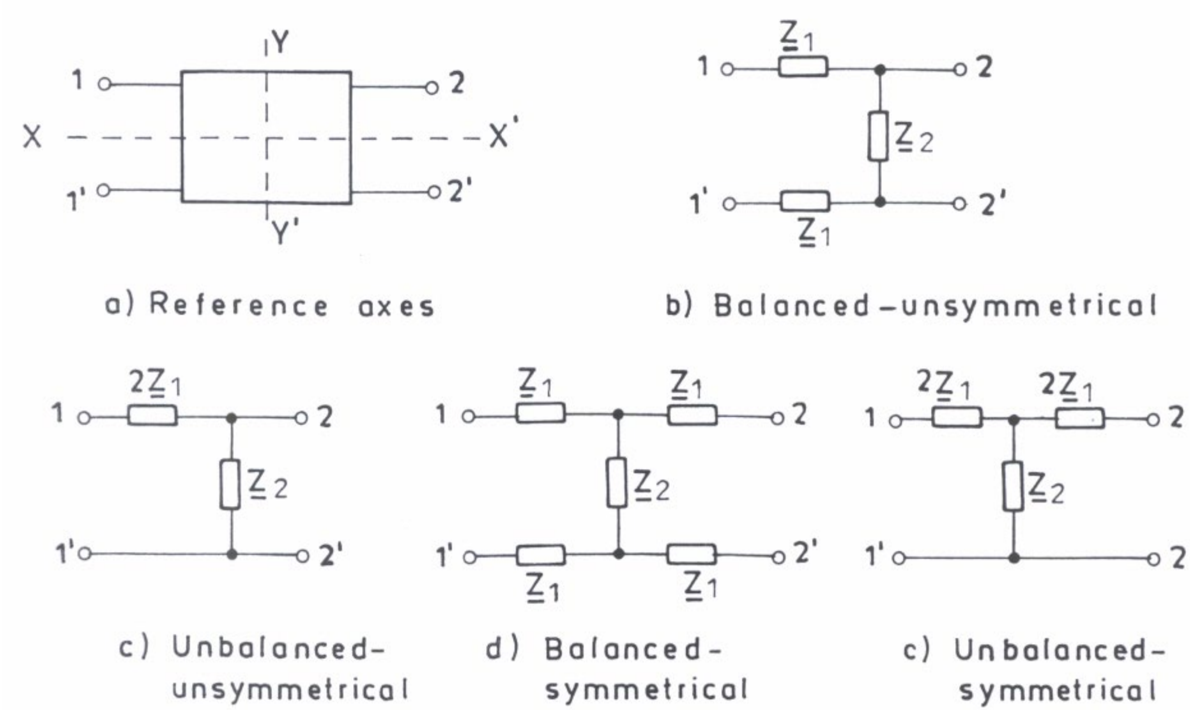
- ✓ The relation between input and output signals of the network can be determined by transferring various network parameters, such as, impedance, admittance, voltage ratio and current ratio .



- **port**: a pair of terminals through which a current may enter or leave a network *(an access to the network and consists of a pair of terminals; the current entering one terminal leaves thtough the other terminal so that the net current entering the port equals zero)*

Definitions:
- input port
- output port
- linear
- passive
- reciprocity

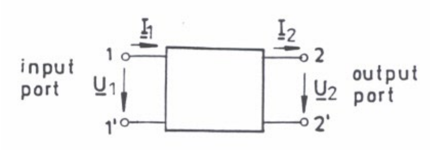
- symmetrical:
- balanced:



1. TWO-PORT NETWORK. EQUATIONS AND PARAMETERS.

1.1. The fundamental equations. ABCD parameters (Transmission parameters).

$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{cases}$$



where $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$ are called ***fundamental*** (or ***transmission***) ***parameters***

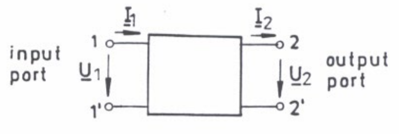
($\underline{A}$ and $\underline{D}$ – dimensionless, $\underline{B}$ is impedance, $\underline{C}$ is admittance)

$$\underline{A} = \frac{1}{\left(\frac{\underline{U}_2}{\underline{U}_1}\right)_{\underline{I}_2=0}}$$

- is the reciprocal of the *open-circuit voltage transfer ratio* from port 1 to port 2

$$\underline{C} = \frac{1}{\left(\frac{\underline{U}_2}{\underline{I}_1}\right)_{\underline{I}_2=0}}$$

- is the reciprocal of the *open-circuit transfer impedance* from port 1 to port 2



$$\underline{B} = \frac{1}{\left(\frac{\underline{I}_2}{\underline{U}_1}\right)_{\underline{U}_2=0}}$$

- is the reciprocal of the *short-circuit transfer admittance* from port 1 to port 2.

$$\underline{D} = \frac{1}{\left(\frac{\underline{I}_1}{\underline{I}_2}\right)_{\underline{U}_2=0}}$$

- is the reciprocal of the *short-circuit current transfer ratio* from port 1 to port 2.

$$\underline{A}\underline{D} - \underline{B}\underline{C} = 1$$

- the ***condition of reciprocity***
(if the network is symmetrical : $\underline{A} = \underline{D}$)

1.2.Impedance parameters.

$$\begin{cases} \underline{U}_1 = \underline{Z}_{11}\underline{I}_1 + \underline{Z}_{12}\underline{I}_2 \\ \underline{U}_2 = \underline{Z}_{21}\underline{I}_1 + \underline{Z}_{22}\underline{I}_2 \end{cases}$$

$$\underline{Z}_{11} = \left(\frac{\underline{U}_1}{\underline{I}_1}\right)_{\underline{I}_2=0} \qquad \underline{Z}_{12} = \left(\frac{\underline{U}_1}{\underline{I}_2}\right)_{\underline{I}_1=0}$$

$$\underline{Z}_{21} = \left(\frac{\underline{U}_2}{\underline{I}_1}\right)_{\underline{I}_2=0} \qquad \underline{Z}_{22} = \left(\frac{\underline{U}_2}{\underline{I}_2}\right)_{\underline{I}_1=0}$$

where $\underline{Z}_{ij}$ are called the *Z parameters*.

$$\begin{cases} \underline{A} = \dfrac{\underline{Z}_{11}}{\underline{Z}_{21}} \\ \underline{B} = -\dfrac{\underline{Z}_{11}\cdot\underline{Z}_{22} - \underline{Z}_{12}\cdot\underline{Z}_{21}}{\underline{Z}_{21}} \\ \underline{C} = \dfrac{1}{\underline{Z}_{21}} \\ \underline{D} = -\dfrac{\underline{Z}_{22}}{\underline{Z}_{21}} \end{cases} \qquad \begin{cases} \underline{Z}_{11} = \dfrac{\underline{A}}{\underline{C}} \\ \underline{Z}_{12} = -\dfrac{\underline{A}\cdot\underline{D} + \underline{B}\cdot\underline{C}}{\underline{C}} \\ \underline{Z}_{21} = \dfrac{1}{\underline{C}} \\ \underline{Z}_{22} = -\dfrac{\underline{D}}{\underline{C}} \end{cases}$$

- for a reciprocal network:

$$\underline{Z}_{12} = -\underline{Z}_{21}$$

- for symmetrical network:

$$\underline{Z}_{11} = -\underline{Z}_{22}$$

$$\underline{Z}_{11} = \left(\frac{\underline{U}_1}{\underline{I}_1} \right)_{\underline{I}_2 = 0}$$

$$\underline{Z}_{12} = \left(\frac{\underline{U}_1}{\underline{I}_2} \right)_{\underline{I}_1 = 0}$$

$$\underline{Z}_{21} = \left(\frac{\underline{U}_2}{\underline{I}_1} \right)_{\underline{I}_2 = 0}$$

$$\underline{Z}_{22} = \left(\frac{\underline{U}_2}{\underline{I}_2} \right)_{\underline{I}_1 = 0}$$

$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{cases}$$

$$\begin{cases} \underline{U}_1 = \underline{Z}_{11}\underline{I}_1 + \underline{Z}_{12}\underline{I}_2 \\ \underline{U}_2 = \underline{Z}_{21}\underline{I}_1 + \underline{Z}_{22}\underline{I}_2 \end{cases}$$

1.3. Admittance parameters.

$$\begin{cases} \underline{I}_1 = \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2 \\ \underline{I}_2 = \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 \end{cases}$$

$$\underline{Y}_{11} = \left(\frac{\underline{I}_1}{\underline{U}_1}\right)_{\underline{U}_2=0} \qquad \underline{Y}_{12} = \left(\frac{\underline{I}_1}{\underline{U}_2}\right)_{\underline{U}_1=0}$$

$$\underline{Y}_{21} = \left(\frac{\underline{I}_2}{\underline{U}_1}\right)_{\underline{U}_2=0} \qquad \underline{Y}_{22} = \left(\frac{\underline{I}_2}{\underline{U}_2}\right)_{\underline{U}_1=0}$$

where $\underline{Y}_{ij}$ are called the *Y parameters*.

$$\begin{cases} \underline{A} = -\dfrac{\underline{Y}_{22}}{\underline{Y}_{21}} \\ \underline{B} = \dfrac{1}{\underline{Y}_{21}} \\ \underline{C} = -\dfrac{\underline{Y}_{12}\cdot\underline{Y}_{21}-\underline{Y}_{11}\cdot\underline{Y}_{22}}{\underline{Y}_{21}} \\ \underline{D} = \dfrac{\underline{Y}_{11}}{\underline{Y}_{21}} \end{cases} \qquad \begin{cases} \underline{Y}_{11} = \dfrac{\underline{D}}{\underline{B}} \\ \underline{Y}_{12} = -\dfrac{\underline{A}\cdot\underline{D}-\underline{B}\cdot\underline{C}}{\underline{B}} \\ \underline{Y}_{21} = \dfrac{1}{\underline{B}} \\ \underline{Y}_{22} = -\dfrac{\underline{A}}{\underline{B}} \end{cases}$$

- for a reciprocal network:

$$\underline{Y}_{12} = -\underline{Y}_{21}$$

- for symmetrical network:

$$\underline{Y}_{11} = -\underline{Y}_{22}$$

$$\underline{Y}_{11} = \left(\frac{\underline{I}_1}{\underline{U}_1}\right)_{\underline{U}_2=0}$$

$$\underline{Y}_{12} = \left(\frac{\underline{I}_1}{\underline{U}_2}\right)_{\underline{U}_1=0}$$

$$\underline{Y}_{21} = \left(\frac{\underline{I}_2}{\underline{U}_1}\right)_{\underline{U}_2=0}$$

$$\underline{Y}_{22} = \left(\frac{\underline{I}_2}{\underline{U}_2}\right)_{\underline{U}_1=0}$$

$$\begin{cases} \underline{I}_1 = \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2 \\ \underline{I}_2 = \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 \end{cases}$$

$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{cases}$$

$$\underline{Y}_{11} = \left(\frac{\underline{I}_1}{\underline{U}_1}\right)_{\underline{U}_2=0}$$

$$\underline{Y}_{12} = \left(\frac{\underline{I}_1}{\underline{U}_2}\right)_{\underline{U}_1=0}$$

$$\underline{Y}_{21} = \left(\frac{\underline{I}_2}{\underline{U}_1}\right)_{\underline{U}_2=0}$$

$$\underline{Y}_{22} = \left(\frac{\underline{I}_2}{\underline{U}_2}\right)_{\underline{U}_1=0}$$

$$\begin{cases} \underline{I}_1 = \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2 \\ \underline{I}_2 = \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 \end{cases}$$

$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{cases}$$

1.4. Hybrid parameters.

The hybrid parameters result when I_1 and U_2 are chosen as independent variables.

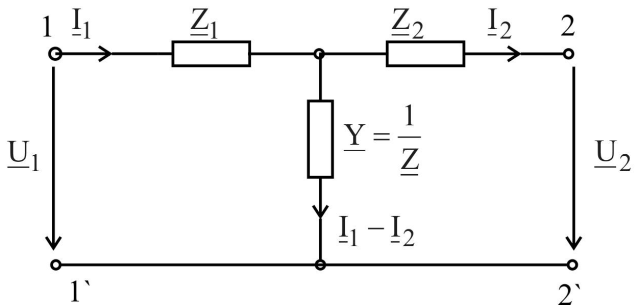
$$\begin{cases} \underline{U}_1 = h_{11} \cdot \underline{I}_1 + h_{12} \cdot \underline{U}_2 \\ \underline{I}_2 = h_{21} \cdot \underline{I}_1 + h_{22} \cdot \underline{U}_2 \end{cases}$$

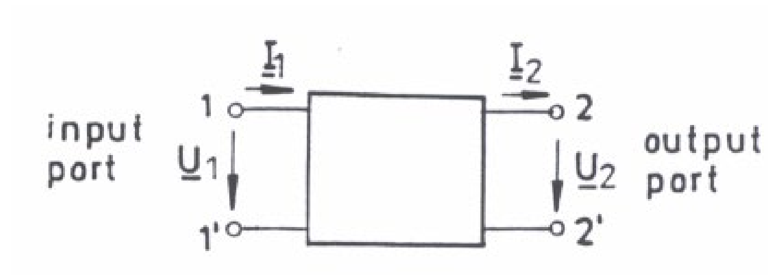
$$h_{11} = \left(\frac{\underline{U}_1}{\underline{I}_1} \right)_{\underline{U}_2 = 0} \qquad h_{12} = \left(\frac{\underline{U}_1}{\underline{U}_2} \right)_{\underline{I}_1 = 0} \qquad h_{21} = \left(\frac{\underline{I}_2}{\underline{I}_1} \right)_{\underline{U}_2 = 0} \qquad h_{22} = \left(\frac{\underline{I}_2}{\underline{U}_2} \right)_{\underline{I}_1 = 0}$$

- ✓ The **h** terms are known as the *hybrid parameters* (or, simply, *h parameters*) because they are a hybrid combination of ratios.
- ✓ They are very useful for describing electronic devices such as transistors; it is much easier to measure experimentally the *h* parameters of such devices than to measure their *z* or *y* parameters.
- ✓ The ideal transformer can be also described by the hybrid parameters

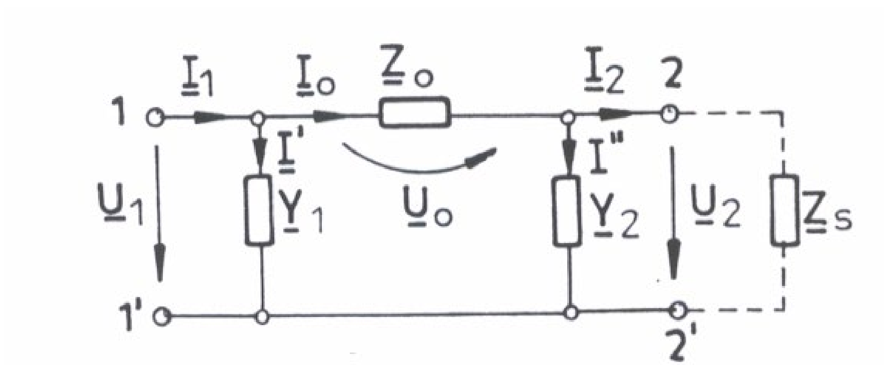
2. EQUIVALENT “T” AND “Π” NETWORKS.

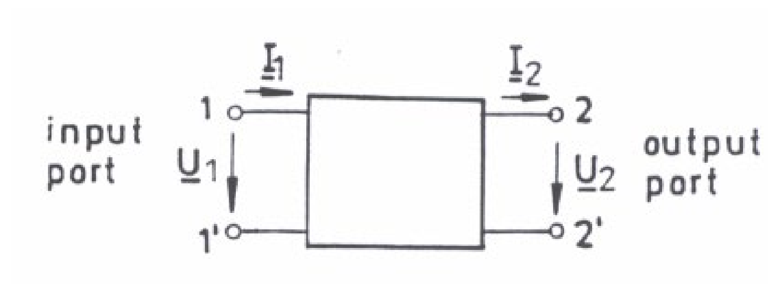
2.1 The T - network.





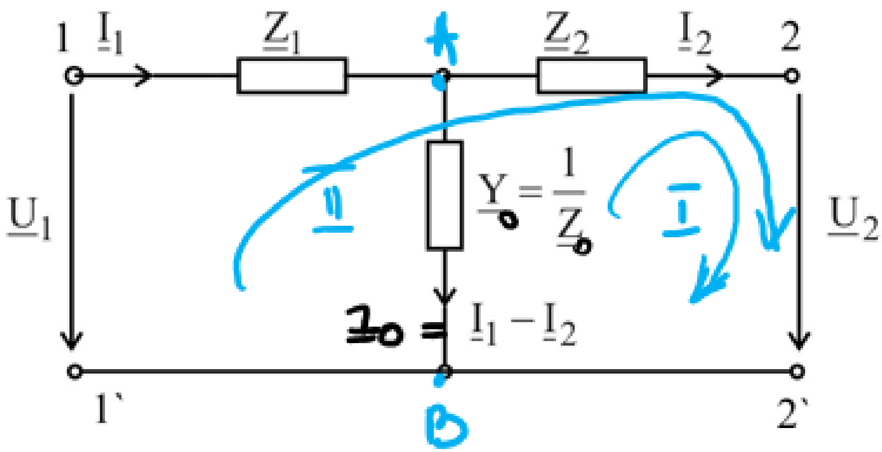
2.2 The Π - network





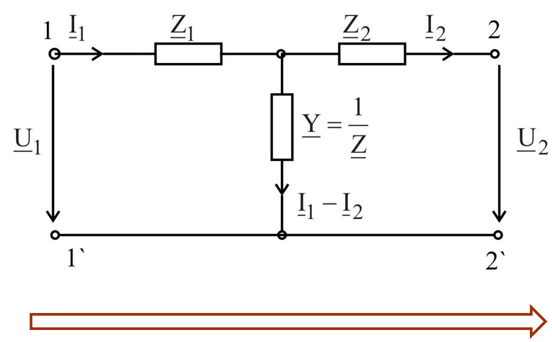
$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{cases}$$

2.1 The T - network.



$$\begin{cases} \underline{U}_1 = (1+\underline{Z}_1\cdot\underline{Y}_0)\cdot\underline{U}_2 + (\underline{Z}_1+\underline{Z}_2+\underline{Z}_1\cdot\underline{Z}_2\cdot\underline{Y}_0)\cdot\underline{I}_2 \\ \underline{I}_1 = \underline{Y}_0\cdot\underline{U}_2 + (1+\underline{Z}_2\cdot\underline{Y}_0)\cdot\underline{I}_2 \end{cases}$$

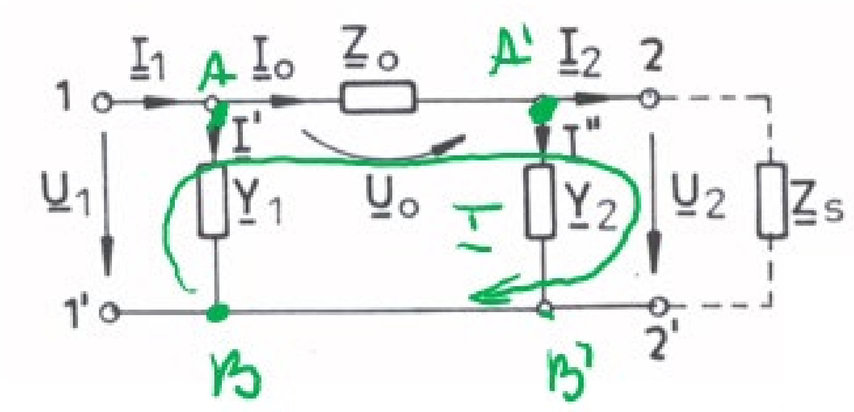
$$\begin{cases} \underline{A} = 1+\underline{Z}_1\underline{Y}_0 \\ \underline{B} = \underline{Z}_1+\underline{Z}_2+\underline{Z}_1\underline{Z}_2\underline{Y}_0 \\ \underline{C} = \underline{Y}_0 \\ \underline{D} = 1+\underline{Z}_2\underline{Y}_0 \end{cases}$$



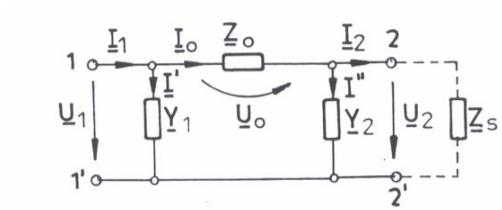
$$\begin{cases} \underline{Y}_0 = \underline{C} \\ \underline{Z}_1 = \dfrac{\underline{A}-1}{\underline{C}} \\ \underline{Z}_2 = \dfrac{\underline{D}-1}{\underline{C}} \end{cases}$$

$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{cases}$$

2.2 The Π - network.



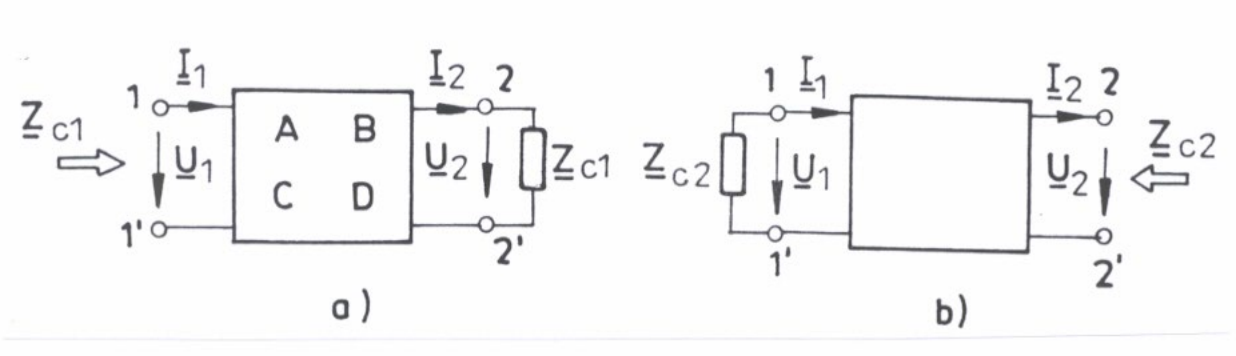
$$\begin{cases} \underline{U}_1 = (1+\underline{Z}_0\cdot\underline{Y}_2)\cdot\underline{U}_2 + \underline{Z}_0\cdot\underline{I}_2 \\ \underline{I}_1 = (\underline{Y}_1+\underline{Y}_2+\underline{Y}_1\cdot\underline{Y}_2\cdot\underline{Z}_0)\cdot\underline{U}_2 + (1+\underline{Z}_0\cdot\underline{Y}_2)\cdot\underline{I}_2 \end{cases}$$



$$\begin{cases} \underline{A} = 1+\underline{Z}_0\cdot\underline{Y}_2 \\ \underline{B} = \underline{Z}_0 \\ \underline{C} = \underline{Y}_1+\underline{Y}_2+\underline{Y}_1\cdot\underline{Y}_2\cdot\underline{Z}_0 \\ \underline{D} = 1+\underline{Z}_0\cdot\underline{Y}_1 \end{cases} \Rightarrow \begin{cases} \underline{Z}_0 = \underline{B} \\ \underline{Y}_1 = \dfrac{\underline{D}-1}{\underline{B}} \\ \underline{Y}_2 = \dfrac{\underline{A}-1}{\underline{B}} \end{cases}$$

3. ITERATIVE/IMAGE IMPEDANCES and PROPAGATION CONSTANT

3.1 Iterative impedances.



$$\frac{\underline{U}_1}{\underline{I}_1} = \frac{\underline{U}_2}{\underline{I}_2} = \underline{Z}_C$$

in general: two iterative impedances, one for each direction

$$\begin{cases} \underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{cases}$$

$$\underline{U}_2 = \underline{Z}_{C1}\underline{I}_2$$

$$\underline{Z}_{C1} = \frac{\underline{U}_1}{\underline{I}_1} = \frac{\underline{A}\underline{Z}_{C1} + \underline{B}}{\underline{C}\underline{Z}_{C1} + \underline{D}} \quad \text{or} \quad \underline{C}\underline{Z}_{C1}^2 + (\underline{D} - \underline{A})\underline{Z}_{C1} - \underline{B} = 0$$

$$\underline{Z}_{C1} = \frac{1}{2 \cdot \underline{C}}\left[\underline{A} - \underline{D} \pm \sqrt{(\underline{D} - \underline{A})^2 + 4\underline{B} \cdot \underline{C}}\right]$$

Similarly:

$$\underline{Z}_{C2} = \frac{1}{2\underline{C}}\left[\underline{D} - \underline{A} \pm \sqrt{(\underline{D} - \underline{A})^2 + 4\underline{B} \cdot \underline{C}}\right]$$

If the network is **symmetrical** $\underline{A} = \underline{D}$ and:

$$\underline{Z}_{C1} = \underline{Z}_{C2} = \underline{Z}_C = \pm\sqrt{\frac{\underline{B}}{\underline{C}}}$$

- the ***characteristic impedance***

Particular case: $\underline{B} = \underline{C} = 0$

- what does this mean?

For the T – network:

$$\begin{cases} \underline{B} = \underline{Z}_1 + \underline{Z}_2 + \underline{Z}_1\underline{Z}_2\underline{Y}_0 = 2\underline{Z}_1 + \underline{Z}_1{}^2\underline{Y}_0 = 0 \\ \underline{C} = \underline{Y}_0 = 0 \end{cases}$$

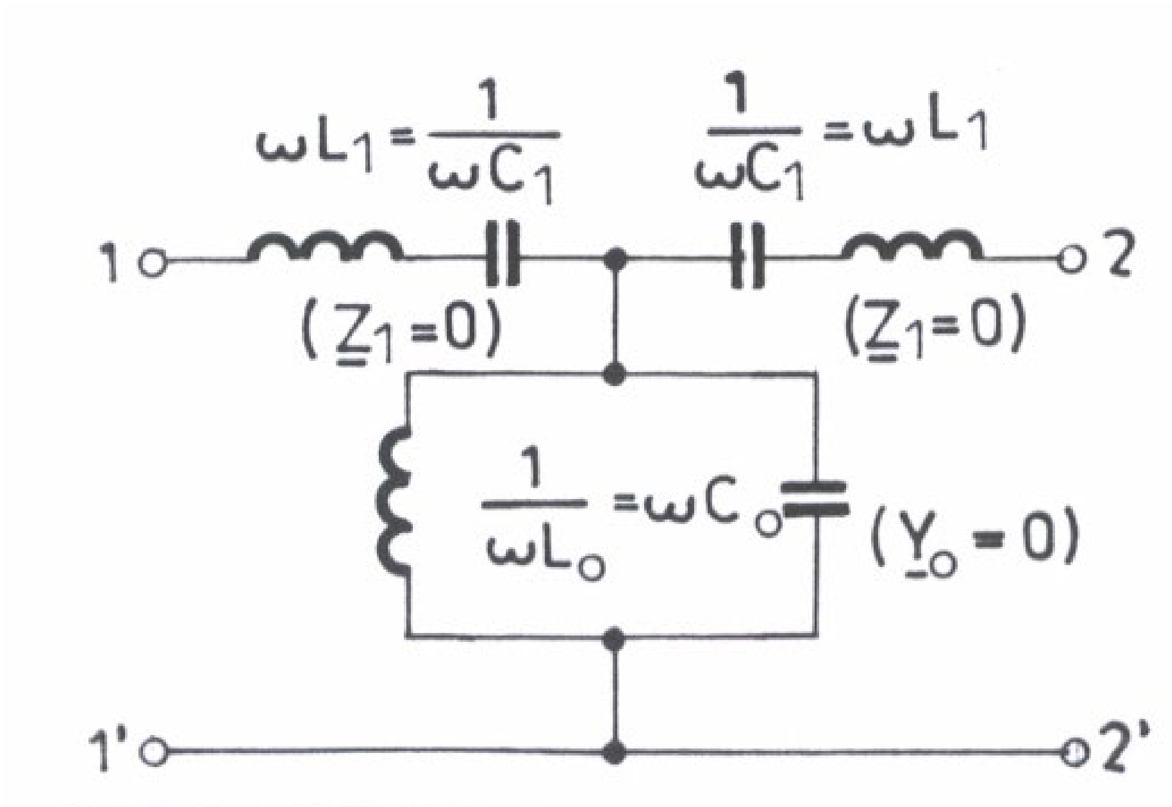
it results $\underline{Z}_1 = 0 \text{ and } \underline{Y}_0 = 0$

For the Π – network:

$$\begin{cases} \underline{C} = \underline{Y}_1 + \underline{Y}_2 + \underline{Y}_1 \cdot \underline{Y}_2 \cdot \underline{Z}_0 = 2\underline{Y}_1 + \underline{Y}_1{}^2 \cdot \underline{Z}_0 = 0 \\ \underline{B} = \underline{Z}_0 = 0 \end{cases}$$

it results $\underline{Y}_1 = 0 \text{ and } \underline{Z}_0 = 0$

➢ these conditions are satisfied only for *two-port networks build from reactive elements (inductors and capacitors) tuned at resonance*.



Reactive two-pot network tuned at resonance

3.2 The propagation constant.

If the two-port network is symmetrical ($\underline{A} = \underline{D}$) and the load impedance is an iterative impedance ($\underline{Z}_c$) we get:

$$\underline{U}_1 = \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 = \underline{U}_2(\underline{A} + \underline{B}\frac{\underline{I}_2}{\underline{U}_2}) = \underline{U}_2(\underline{A} + \underline{B}\sqrt{\frac{\underline{C}}{\underline{B}}}) = \underline{U}_2(\underline{A} + \sqrt{\underline{B}\underline{C}})$$

$$\underline{I}_1 = \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 = \underline{I}_2(\underline{A} + \underline{C}\frac{\underline{U}_2}{\underline{I}_2}) = \underline{I}_2(\underline{A} + \underline{C}\sqrt{\frac{\underline{B}}{\underline{C}}}) = \underline{I}_2(\underline{A} + \sqrt{\underline{B}\underline{C}})$$

$$\frac{\underline{U}_1}{\underline{U}_2} = \frac{\underline{I}_1}{\underline{I}_2} = \underline{A} + \sqrt{\underline{B} \cdot \underline{C}}$$

$$\ln\frac{\underline{U}_1}{\underline{U}_2} = \ln\frac{\underline{I}_1}{\underline{I}_2} = \ln(\underline{A} + \sqrt{\underline{B} \cdot \underline{C}}) = \underline{\gamma}$$

- the propagation constant

$\underline{\gamma} = \alpha + j\beta$ α - the *damping constant*, β - the *phase constant*

$$e^{\gamma} = \underline{A} + \sqrt{\underline{B} \cdot \underline{C}} \quad \text{or} \quad \gamma = \ln \frac{\underline{U}_1}{\underline{U}_2} = \ln\left[\frac{U_1}{U_2} e^{j\varphi}\right] = \ln \frac{U_1}{U_2} + j\varphi$$

$$\begin{cases} \alpha = \ln \dfrac{U_1}{U_2} \\ \beta = \varphi \end{cases} \qquad \begin{cases} \dfrac{U_1}{U_2} = e^{\alpha} \\ \varphi = \beta \end{cases}$$

Remarks:

- if $\alpha = 0$, the signal pass through the network without damping (unchanged);

- if $\alpha > 0$, the signal is damped at the output port ($U_2 < U_1$);

- if $\alpha < 0$, the signal is amplified ($U_2 > U_1$);

The reciprocity condition: $\underline{A}\underline{D} - \underline{B}\underline{C} = 1$

If the network is symmetrical: $\underline{A} = \underline{D}$ and $\underline{A}^2 - \underline{B}\underline{C} = 1$

$$(\underline{A} - \sqrt{\underline{B} \cdot \underline{C}})(\underline{A} + \sqrt{\underline{B} \cdot \underline{C}}) = 1$$ it results that :

$$\begin{cases} \underline{A} + \sqrt{\underline{B} \cdot \underline{C}} = e^{\underline{\gamma}} \\ \underline{A} - \sqrt{\underline{B} \cdot \underline{C}} = e^{-\underline{\gamma}} \end{cases}$$

$$\underline{Z}_C = \sqrt{\frac{\underline{B}}{\underline{C}}}$$

$$\begin{cases} \underline{A} = \frac{1}{2}\left(e^{\underline{\gamma}} + e^{-\underline{\gamma}}\right) = ch\,\underline{\gamma} \\ \underline{B} = \underline{Z}_C sh\,\underline{\gamma} \\ \underline{C} = \frac{1}{\underline{Z}_C} sh\,\underline{\gamma} \\ \underline{D} = \underline{A} \end{cases}$$

The two-port network equations are:

$$\begin{cases} \underline{U}_1 = \underline{U}_2\, ch\underline{\gamma} + \underline{I}_2 \underline{Z}_C sh\underline{\gamma} \\ \underline{I}_1 = \frac{\underline{U}_2}{\underline{Z}_C} sh\underline{\gamma} + \underline{I}_2 ch\underline{\gamma} \end{cases}$$

If the output port is *open-circuited* and, respectively, *short-circuited*:

$$\begin{cases} \underline{U}_{10} = \underline{U}_2 \, ch \, \underline{\gamma} \\ \underline{I}_{10} = \dfrac{\underline{U}_2}{\underline{Z}_C} sh \, \underline{\gamma} \end{cases}$$

$$\begin{cases} \underline{U}_{1SC} = \underline{I}_2 \underline{Z}_C sh \, \underline{\gamma} \\ \underline{I}_{1SC} = \underline{I}_2 ch \, \underline{\gamma} \end{cases}$$

$$\underline{Z}_{10} = \frac{\underline{U}_{10}}{\underline{I}_{10}} = \frac{\underline{Z}_C}{th\underline{\gamma}}$$

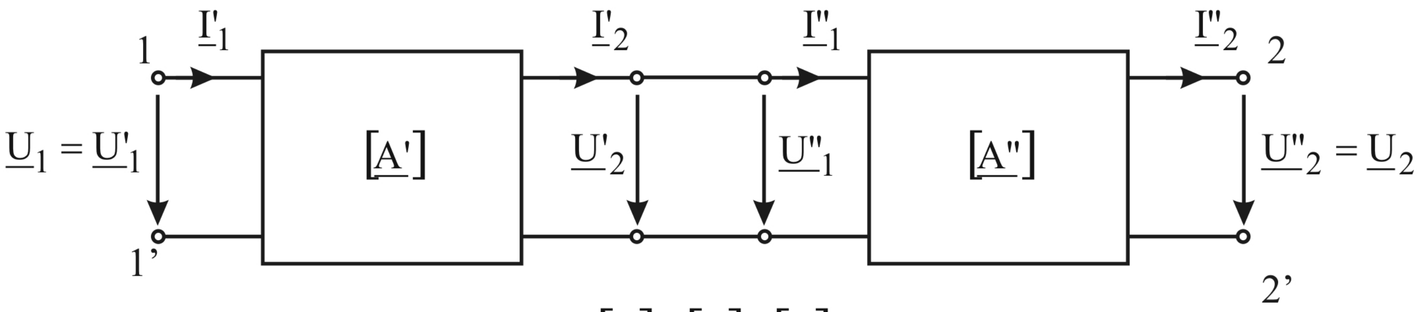
$$\underline{Z}_{1SC} = \frac{\underline{U}_{1SC}}{\underline{I}_{1SC}} = \underline{Z}_C th\underline{\gamma}$$

It results:

$$\begin{cases} \underline{Z}_C = \sqrt{\underline{Z}_{10} \underline{Z}_{1SC}} \\ th\underline{\gamma} = \sqrt{\dfrac{\underline{Z}_{1SC}}{\underline{Z}_{10}}} \end{cases}$$

5. THE INTERCONNECTION OF TWO-PORT NETWORKS.

5.1 Cascade connection.



$$[\underline{A}] = [\underline{A}] + [\underline{A}]$$

$$\begin{cases} \underline{U}_1 = \underline{U'}_1; & \underline{I}_1 = \underline{I'}_1 \\ \underline{U'}_2 = \underline{U''}_1; & \underline{I'}_2 = \underline{I''}_1 \\ \underline{U}_2 = \underline{U''}_2; & \underline{I}_2 = \underline{I''}_2 \end{cases}$$

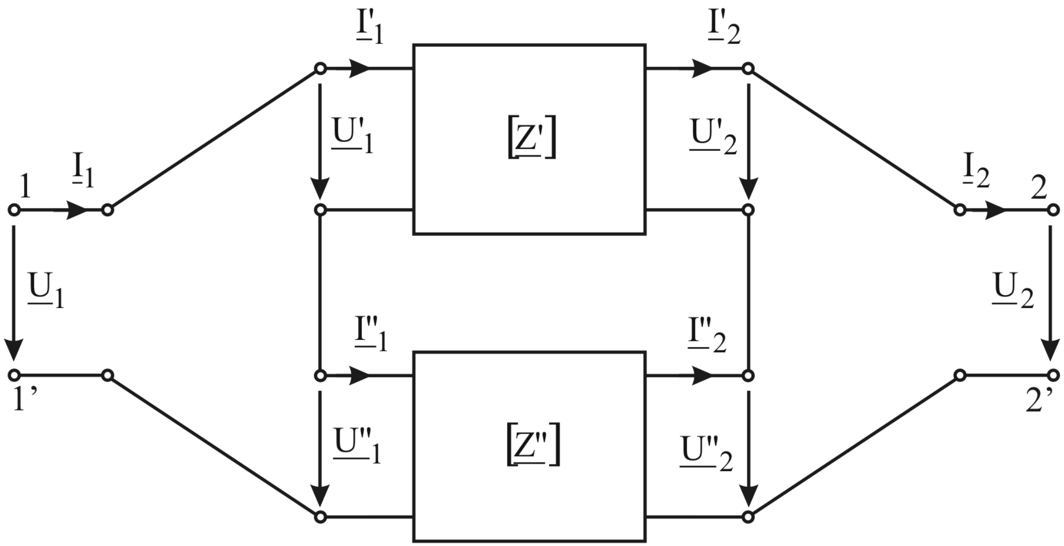
$$\begin{bmatrix} \underline{U}'_1 \\ \underline{I}'_1 \end{bmatrix} = [\underline{A}'] \begin{bmatrix} \underline{U}'_2 \\ \underline{I}'_2 \end{bmatrix}, \quad \begin{bmatrix} \underline{U}''_1 \\ \underline{I}''_1 \end{bmatrix} = [\underline{A}''] \begin{bmatrix} \underline{U}''_2 \\ \underline{I}''_2 \end{bmatrix}, \quad \begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} = [\underline{A}] \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} \underline{U}'_1 \\ \underline{I}'_1 \end{bmatrix} = [\underline{A}'] \begin{bmatrix} \underline{U}'_2 \\ \underline{I}'_2 \end{bmatrix} = [\underline{A}'] [\underline{A}''] \begin{bmatrix} \underline{U}''_2 \\ \underline{I}''_2 \end{bmatrix} = [\underline{A}'] [\underline{A}''] \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \end{bmatrix}$$

$$[\underline{A}] = [\underline{A}'][\underline{A}'']$$

$$\begin{cases} \underline{U}_1 = \underline{U}'_1; & \underline{I}_1 = \underline{I}'_1 \\ \underline{U}'_2 = \underline{U}''_1; & \underline{I}'_2 = \underline{I}''_1 \\ \underline{U}_2 = \underline{U}''_2; & \underline{I}_2 = \underline{I}''_2 \end{cases}$$

5.2 Series connection



$$[\underline{Z}] = [\underline{Z}'] + [\underline{Z}'']$$

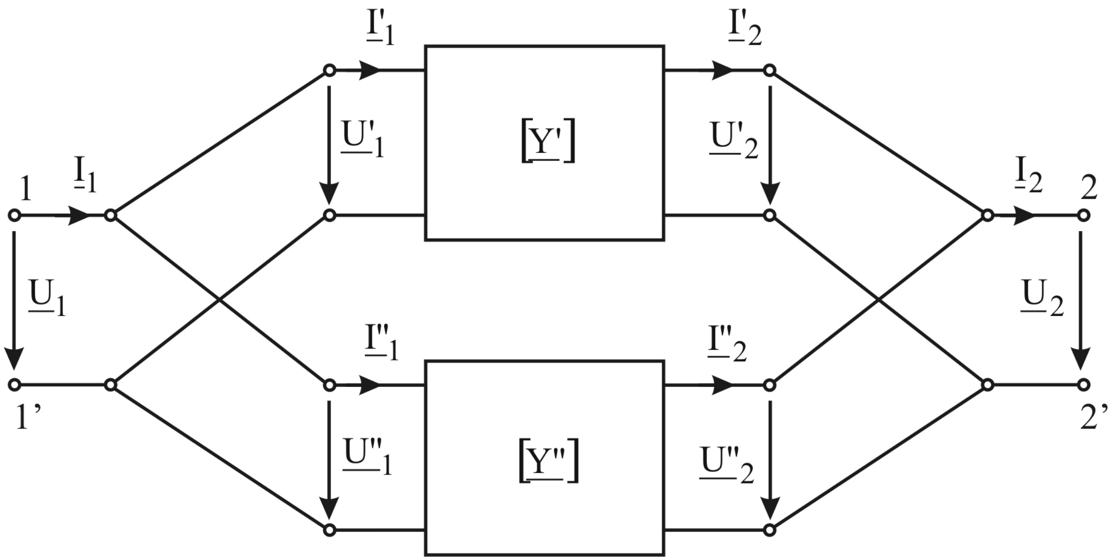
$$\begin{cases} \underline{U}_1 = \underline{U}'_1 + \underline{U}''_1 \\ \underline{I}_1 = \underline{I}'_1 = \underline{I}''_1 \\ \underline{I}_2 = \underline{I}'_2 = \underline{I}''_2 \\ \underline{U}_2 = \underline{U}'_2 + \underline{U}''_2 \end{cases}$$

$$\begin{bmatrix} \underline{U}_1' \\ \underline{U}_2' \end{bmatrix} = \left[\underline{Z}'\right] \begin{bmatrix} \underline{I}_1' \\ \underline{I}_2' \end{bmatrix} \quad ; \quad \begin{bmatrix} \underline{U}_1'' \\ \underline{U}_2'' \end{bmatrix} = \left[\underline{Z}''\right] \begin{bmatrix} \underline{I}_1'' \\ \underline{I}_2'' \end{bmatrix} \quad , \quad \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} = \left[\underline{Z}\right] \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} = \begin{bmatrix} \underline{U}_1' \\ \underline{U}_2' \end{bmatrix} + \begin{bmatrix} \underline{U}_1'' \\ \underline{U}_2'' \end{bmatrix} = \left[\underline{Z}'\right] \begin{bmatrix} \underline{I}_1' \\ \underline{I}_2' \end{bmatrix} + \left[\underline{Z}''\right] \begin{bmatrix} \underline{I}_1'' \\ \underline{I}_2'' \end{bmatrix} = \left(\left[\underline{Z}'\right] + \left[\underline{Z}''\right]\right) \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}$$

$$\left[\underline{Z}\right] = \left[\underline{Z}'\right] + \left[\underline{Z}''\right]$$

5.3 Parallel connection



$$[\underline{Y}] = [\underline{Y}'] + [\underline{Y}'']$$

$$\begin{cases} \underline{U}_1 = \underline{U}'_1 = \underline{U}''_1 \\ \underline{I}_1 = \underline{I}'_1 + \underline{I}''_1 \\ \underline{I}_2 = \underline{I}'_2 + \underline{I}''_2 \\ \underline{U}_2 = \underline{U}'_2 = \underline{U}''_2 \end{cases}$$

$$\begin{bmatrix} \underline{I}_1^{'} \\ \underline{I}_2^{'} \end{bmatrix} = \left[\underline{Y}^{'}\right] \begin{bmatrix} \underline{U}_1^{'} \\ \underline{U}_2^{'} \end{bmatrix} \quad ; \quad \begin{bmatrix} \underline{I}_1^{''} \\ \underline{I}_2^{''} \end{bmatrix} = \left[\underline{Y}^{''}\right] \begin{bmatrix} \underline{U}_1^{''} \\ \underline{U}_2^{''} \end{bmatrix} \quad , \quad \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \left[\underline{Y}\right] \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix}$$

$$\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} \underline{I}_1^{'} \\ \underline{I}_2^{'} \end{bmatrix} + \begin{bmatrix} \underline{I}_1^{''} \\ \underline{I}_2^{''} \end{bmatrix} = \left[\underline{Y}^{'}\right] \begin{bmatrix} \underline{U}_1^{'} \\ \underline{U}_2^{'} \end{bmatrix} + \left[\underline{Y}^{''}\right] \begin{bmatrix} \underline{U}_1^{''} \\ \underline{U}_2^{''} \end{bmatrix} = \left(\left[\underline{Y}^{'}\right] + \left[\underline{Y}^{''}\right]\right) \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix}$$

$$\left[\underline{Y}\right] = \left[\underline{Y}'\right] + \left[\underline{Y}''\right]$$

Advanced Circuit Analysis

Chapter 5. Transient regime of linear circuits

BASES OF ELECTROTECHNICS I.
Faculty of Electronics, Telecommunications and Information Technology
Specialization: IETTI
Academic year: 2023-2024

- **Transient regime in electrical circuits are the processes of transition from one steady-state regime to another, characterized by different parameters.**

- Transient processes are caused by the *commutation* in the circuit (closing and opening of the circuit with electrical switch, the sudden variation of the circuit parameters due to special functioning conditions).

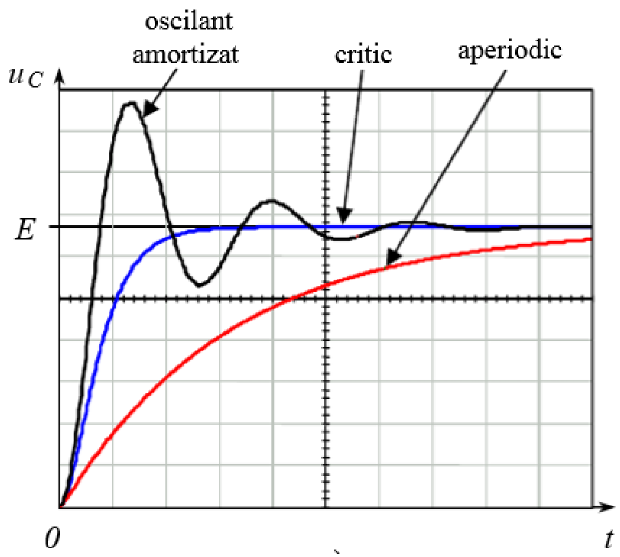
- Transient process can be define as the process of energy state transition of the circuit from prior-commutation state to after-commutation state.

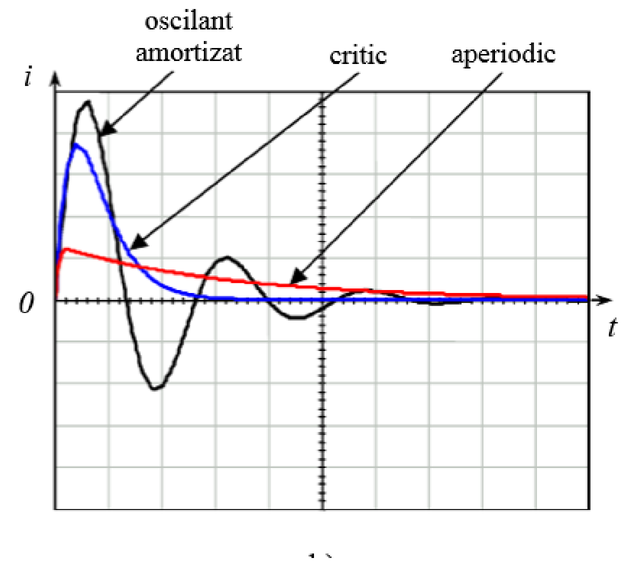
- Transient processes are very short, usually about a ten-hundredth of second. However, it is important to know the transient process length, the how the signal changes between the circuits states.

- **The study of transient regimes involves the computation of currents and voltages as time functions.**

- In some situations, the transient regimes characterize the normal operation of the circuits (operation of protection circuits, operation of static converters, etc.).

- In other cases, operation of a circuit in transient regime can result in increases of voltages or currents that cause dielectric, thermal or mechanical stresses that far exceed the stresses corresponding to a permanent regime. These can lead to total or partial destruction of some electrical appliances, which shows the importance of studying the circuits in transient mode.

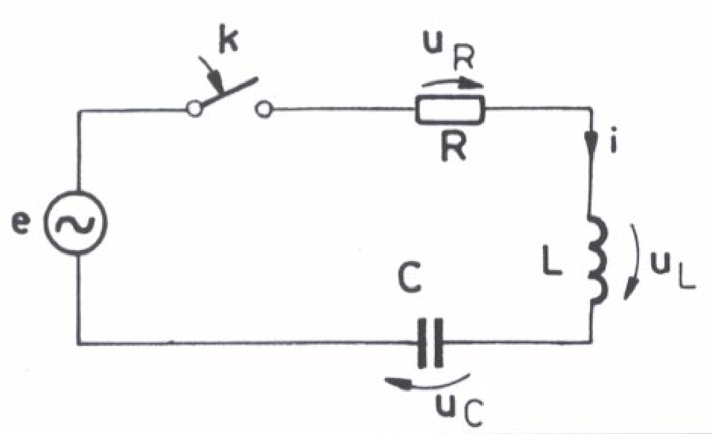




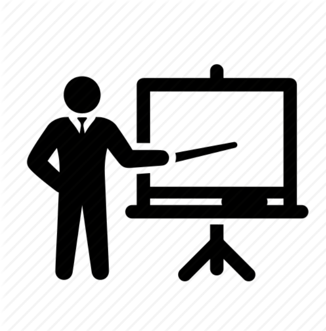
5.1 INTRODUCTION

-Transient regime:

- L, C
- moment: t = 0, or modification of circuit parameters values



$$u_R + u_L + u_C = e(t)$$



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$$i(t) = i_L(t) + i_p(t)$$

where: **i_p** is the *forced response* (or the *steady-state response*)

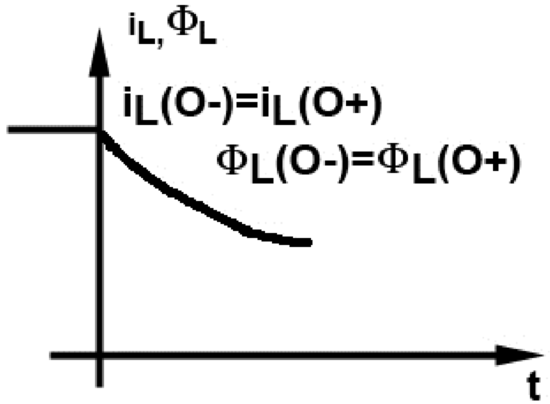
i_l is the *natural response* (or the *transient response*).

5.2 CONTINUITY CONDITIONS

There is a problem : how does the transition between initial and final states take place?

Is it possible for the capacitor voltage or for the inductor current simply to jump up to their final values immediately ?

a) **The first continuity condition** (for circuits containing inductors)



The current and the magnetic flux are continuous functions of time :

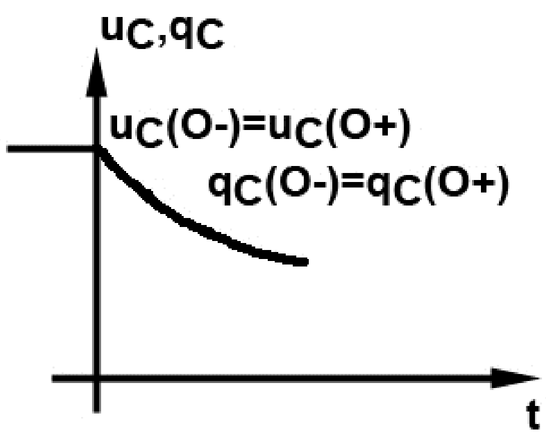
$$i_L(0-) = i_L(0+) = i(0)$$

$$\Phi_L(0-) = \Phi_L(0+) = \Phi(0)$$

If there is a sudden jump in an inductor current:

$$u_L(0+) = \left(\frac{d\Phi_L}{dt}\right)_{0+} = L\left(\frac{di_L}{dt}\right)_{0+} \to \infty$$

b) **The second continuity condition** (**for circuits containing capacitors**)



The voltage and the electric charge are continuous functions of time :

$$u_c(0-) = u_c(0+) = u_C(0)$$

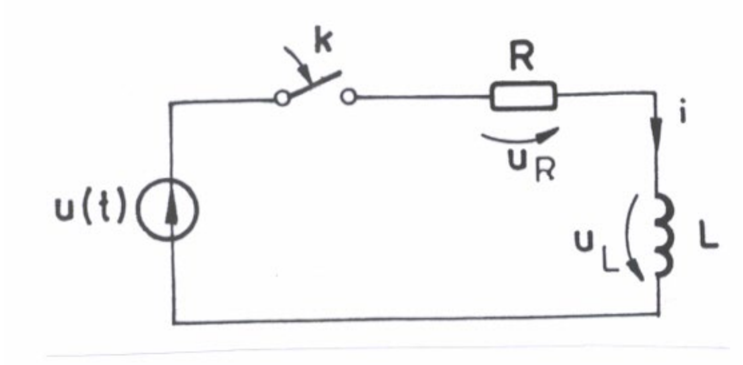
$$q_c(0-) = q_c(0+) = q_C(0)$$

If there is a sudden jump in a capacitor voltage:

$$i_c(0+) = \left(\frac{dq_c}{dt}\right)_{0+} = C\left(\frac{du_c}{dt}\right)_{0+} \to \infty$$

5.3 TRANSIENT BEHAVIOUR OF THE R, L CIRCUITS

- *first-order circuits*: circuits containing a single type of energy-storage element



$$u_R + u_L = u(t)$$

$$Ri + L\frac{di}{dt} = u(t)$$

$$i(t) = i_p + i_l$$

The natural response - when $u(t) = 0$:

$$L\frac{di_l}{dt} + Ri_l = 0\,, \qquad Lp + R = 0, \qquad \Rightarrow \qquad p = -\frac{R}{L}$$

$$i_l = Ae^{-\frac{R}{L}t}; \qquad\qquad i(t) = i_p + Ae^{-\frac{R}{L}t}$$

$$i(t) = i_p + Ae^{-\frac{R}{L}t}$$

- A is determined by the initial conditions:

when t = 0: $i(0) = i_p(0) + i_l(0)$ *or* $i_0 = i_{p0} + A$ $A = i_0 - i_{p0}$

$$i = i_p + (i_0 - i_{p0})e^{-\frac{R}{L}t}$$

❖ $i_p(t)$ has a form quite similar to the particular form of the excitation function being used; it is logically called the *particular solution.*

❖ i_{p0} represents the value of the steady-state current: $i_{p0} = i_p(t = 0)$

❖ i_0 represents the value of the current immediately <u>before</u> the switching operation: $i_0 = i(0-)$

The quantity $\boxed{\dfrac{L}{R} = \tau}$ has the dimension of time:

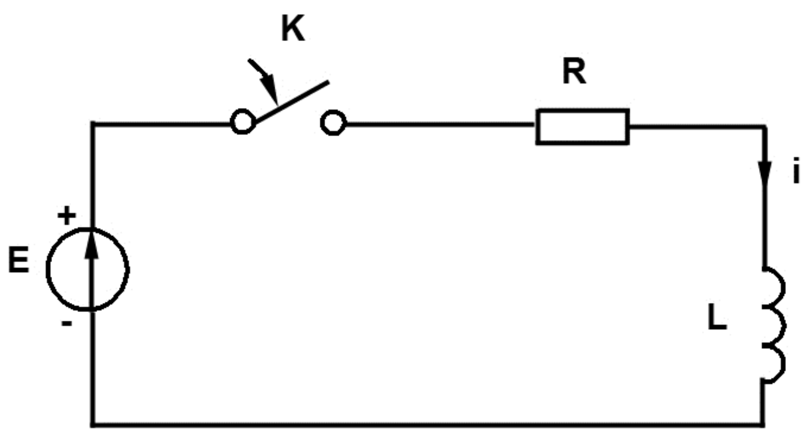
$$\left[\frac{L}{R}\right] = \frac{[\omega L]}{[R]}\frac{1}{[\omega]} = \frac{\Omega}{\Omega}\frac{1}{s^{-1}} = s \quad or \quad \left[\frac{L}{R}\right] = \frac{1H}{1\Omega} = \frac{1V \cdot 1s}{1\Omega \cdot 1A} = 1s$$

$\tau = L/R$ is called the *time constant* of the circuit.

$$i = i_p + \left(i_0 - i_{p0}\right)e^{-\frac{t}{\tau}}$$

5.3.1 Response to sources with constant excitation.

a) The switch is closed.



$$i(t) = i_p + \left(i_0 - i_{p0}\right)e^{-\frac{R}{L}t}$$

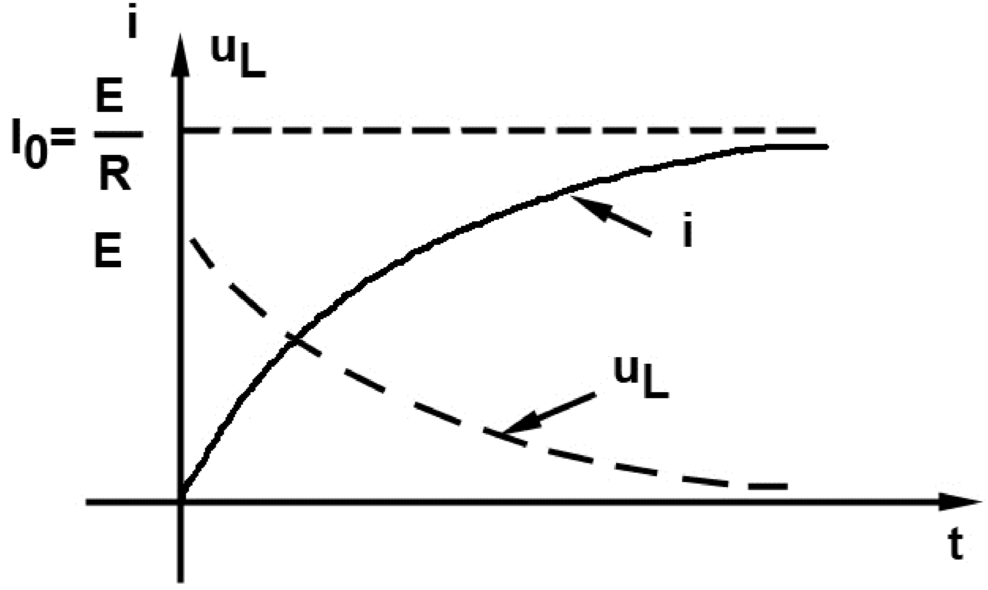
$$i_0 = 0$$

$$i_p = \frac{E}{R}, \quad i_{p0} = \frac{E}{R}$$

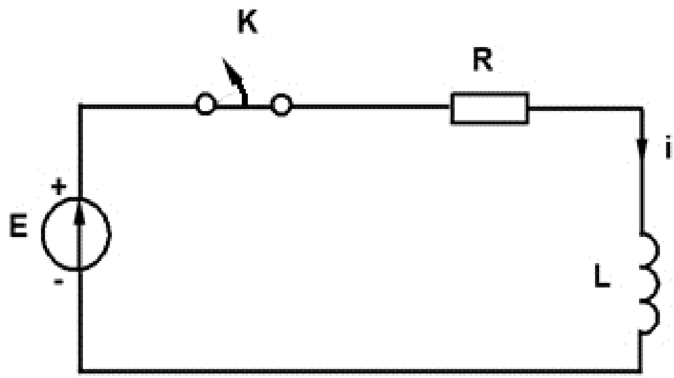
$$i(t) = \frac{E}{R} - \frac{E}{R}e^{-\frac{t}{\tau}} = \frac{E}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

The voltage across the inductance:

$$u_L = L\frac{di}{dt} = L\cdot\frac{E}{R}\cdot\frac{1}{\tau}e^{-\frac{t}{\tau}} \qquad \Rightarrow \qquad u_L = Ee^{-\frac{t}{\tau}}$$



b) The switch is opened.



$$i(t) = i_p + \left(i_0 - i_{p0}\right)e^{-\frac{R}{L}t}$$

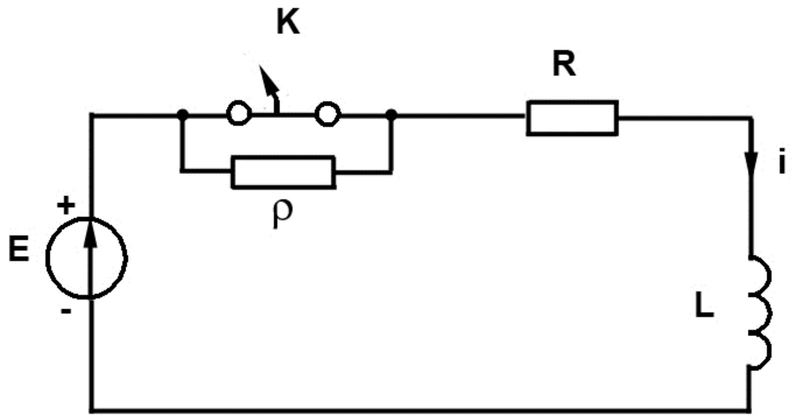
$$i_0 = \frac{E}{R}$$

$$i_p = 0\,, \qquad i_{p0} = 0$$

$$i(t) = \frac{E}{R}e^{-\frac{t}{\tau}}$$

There is a problem: how great is the value of the voltage across the inductance at t=0 ?

$$i(t) = i_p + \left(i_0 - i_{p0}\right)e^{-\frac{R}{L}t}$$

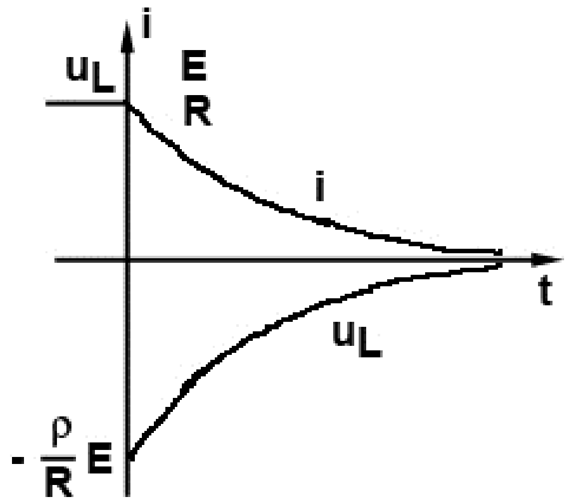


Let ρ be the resistance between the switch contacts.

$$0 \leq \rho \leq \infty$$

$$i(t) = \frac{E}{R+\rho} + \frac{\rho E}{R(R+\rho)} e^{-\frac{R+\rho}{L}t}$$

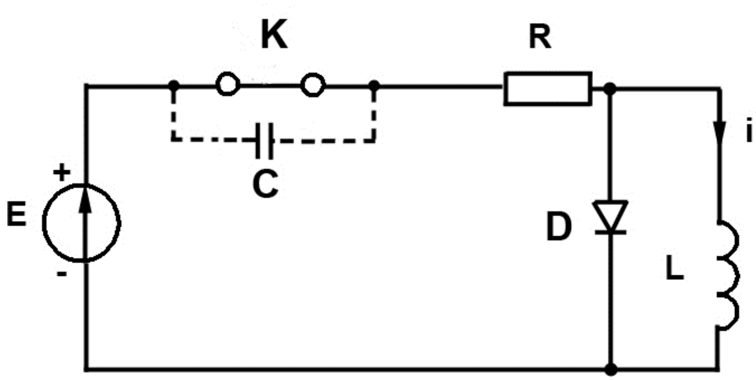
The voltage across the inductance:



$$u_L = L\frac{di}{dt} = -\frac{\rho}{R}Ee^{-\frac{R+\rho}{L}t}$$

Overvoltages, for instance if:

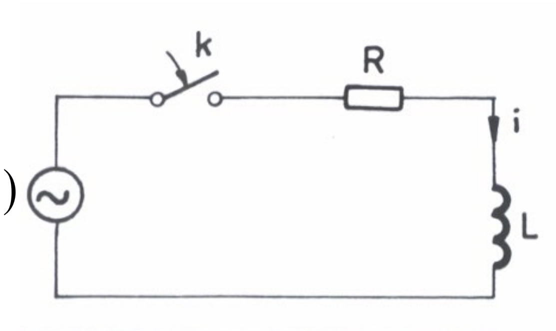
$$\left(\frac{\rho}{R} = 10,\ E = 100\ \text{V} \quad \Rightarrow \frac{\rho}{R}E = 1.000\ \text{V}\right)$$



$\Rightarrow$ Solutions for protection

5.3.2 Response to sources with sinusoidal excitation.

$$e(t) = \sqrt{2}\mathrm{E}\sin(\omega t + \beta)$$



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$$i(t) = i_p + \left(i_0 - i_{p0}\right)e^{-\frac{R}{L}t}$$

$$i = i_p + i_l = \frac{E\sqrt{2}}{Z}\sin(\omega t + \beta - \varphi) - \frac{\sqrt{2}E}{Z}\sin(\beta - \varphi)e^{-\frac{R}{L}t}$$

Two cases:

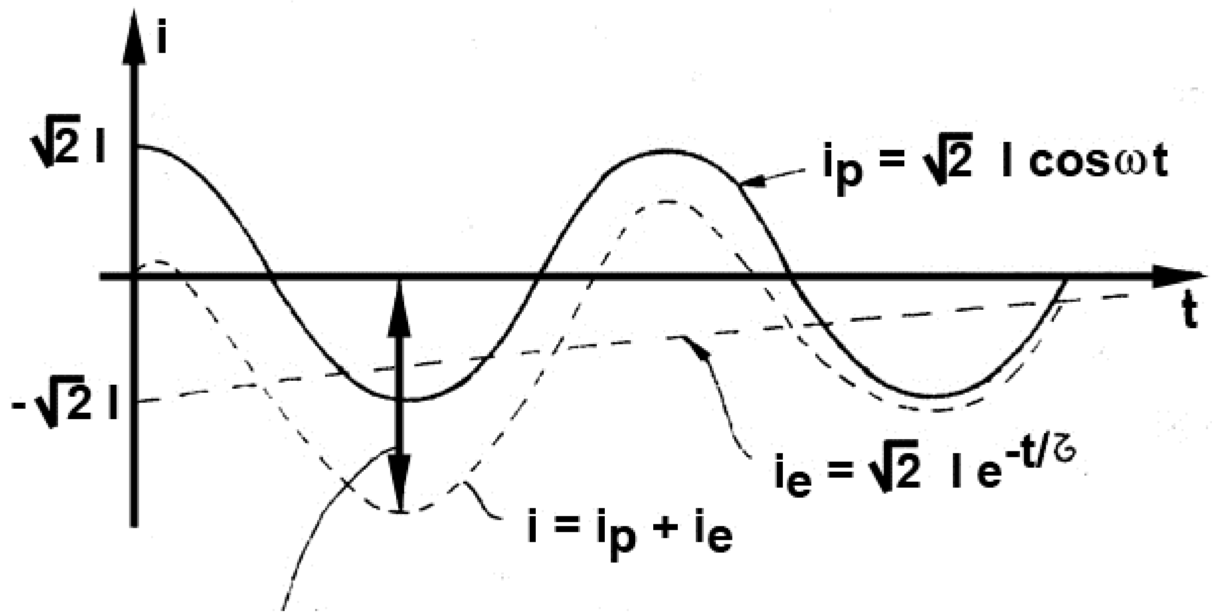
a) if $\beta - \varphi = 0 \quad or \quad \beta - \varphi = \pi$: $i(t) = i_p$ the *steady-state regime appears immediately, without a transient response !*

b) If $\beta - \varphi = \frac{\pi}{2}$ *it appears a stroke current*:

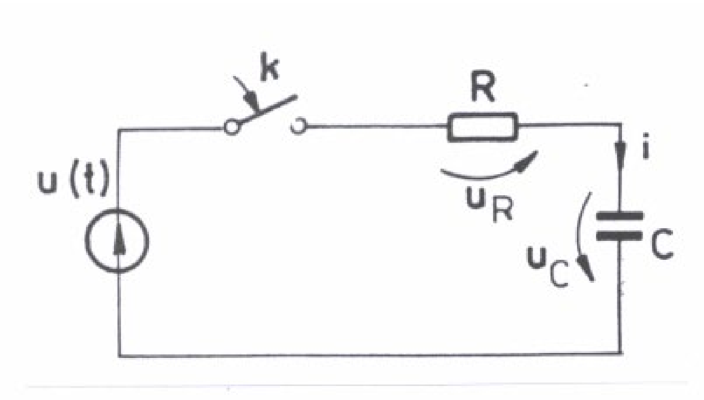
$$i(t) = \sqrt{2}I(\cos\omega t - e^{-\frac{t}{\tau}})$$

$$i(t) = \sqrt{2}I(\cos\omega t - e^{-\frac{t}{\tau}})$$

If the time constant $\tau = L/R$ has a very high value ($L/R >>$) :



5.4 TRANSIENT BEHAVIOUR OF THE RC CIRCUITS.



$$Ri + \frac{1}{C}\int i\,dt = u(t)$$

$$i = \frac{dq}{dt} \qquad q = \int i\,dt$$

$$R\frac{dq}{dt} + \frac{1}{C}q = u(t) \qquad q(t) = q_p + q_l$$

$$R\frac{dq_l}{dt} + \frac{1}{C}q_l = 0 \quad \Rightarrow q_l = Ae^{-\frac{t}{RC}}$$

$$q(t) = q_p + Ae^{-\frac{t}{RC}}$$

For t = 0: $q(0) = q_0\ ,\ q_p(0) = q_{p0}$ and $A = q_0 - q_{p0}$

thus:

$$q = q_p + \left(q_0 - q_{p0}\right) e^{-\frac{t}{\tau}}$$

where $\tau = RC$ is **the time constant of the RC circuit**

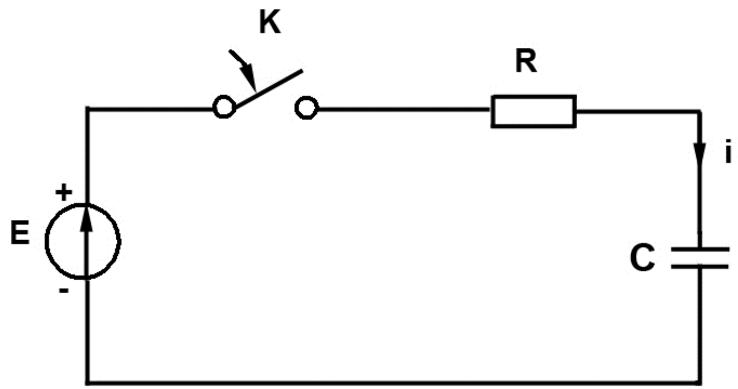
$$[\tau] = [\mathrm{RC}] = \frac{[\mathrm{R}]}{\left[\frac{1}{\omega \mathrm{C}}\right][\omega]} = \frac{\Omega}{\Omega \cdot \mathrm{s}^{-1}} = \mathrm{s}$$

q_o is the electric charge immediately before the switching time;

q_p the steady-state electric charge;

q_{p0} the value of q_p for $t=0$.

5.4.1 Response to sources with constant excitation.



$$q = q_p + \left(q_0 - q_{p0}\right) e^{-\frac{t}{\tau}}$$

$$q_0 = 0$$

$$q_p = CE \quad , \qquad q_{p0} = CE$$

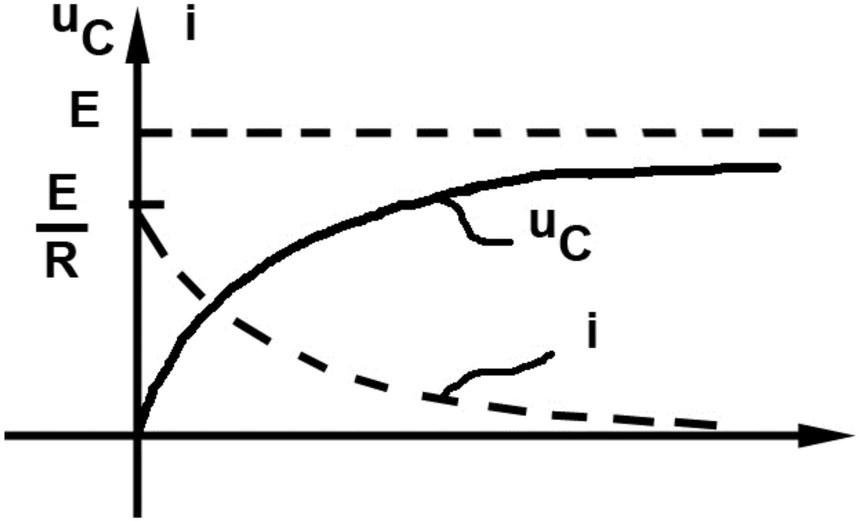
$$q = CE - CEe^{-\frac{t}{\tau}} = CE\left(1 - e^{-\frac{t}{\tau}}\right)$$

where $\tau = RC$

$$u_C = Ee^{-\frac{t}{RC}}$$

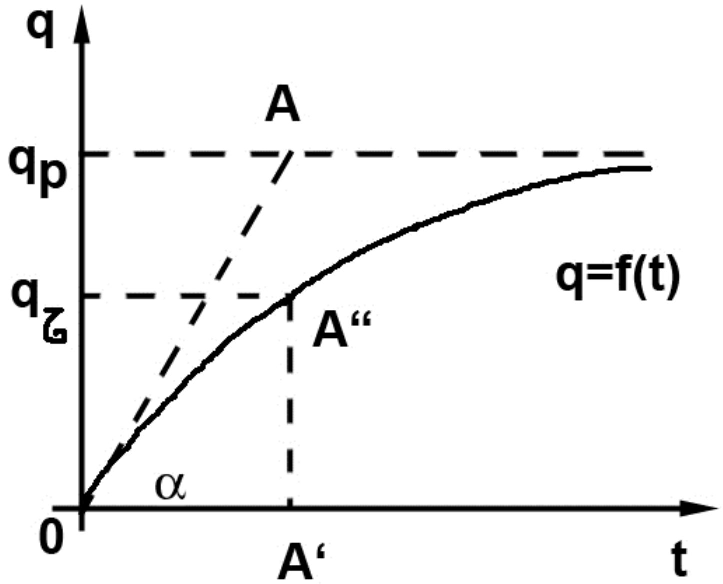
$$i = \frac{dq}{dt} = CE\frac{1}{\tau}e^{-\frac{t}{\tau}}$$

$$i = \frac{E}{R}e^{-\frac{t}{\tau}}$$



If $\boldsymbol{R}$ is very small : $i(0+) = \frac{E}{R}$ will be very high !

5.5 TIME CONSTANT



$$q = q_p\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$OA' = \frac{AA'}{tg\alpha} = \frac{q_p}{tg\alpha}$$

$$tg\alpha = \left(\frac{dq}{dt}\right)_{t=0} = \frac{q_p}{\tau} \Longrightarrow \boxed{\tau = OA'}$$

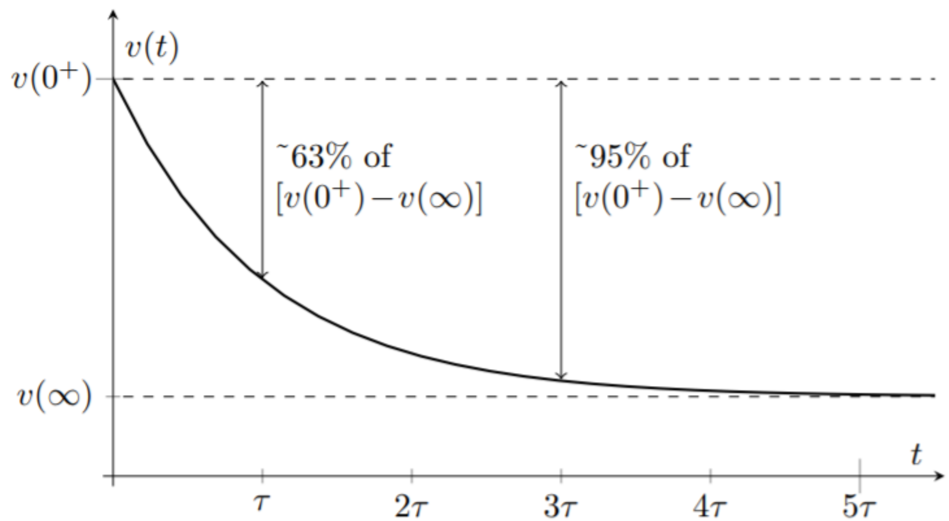
If $t = \tau$:

$$q_\tau = q_p\left(1 - e^{-1}\right) = q_p\frac{e-1}{e} = 0{,}632 q_p$$

$$or \quad u_\tau = E\left(1 - e^{-1}\right) = 0{,}632 E$$

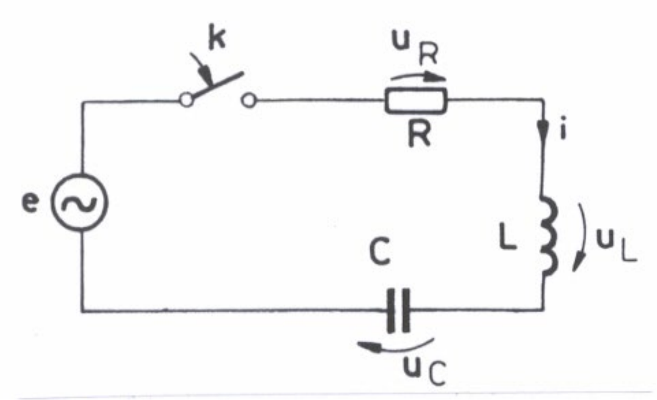
$u_\tau = E\left(1 - e^{-1}\right) = 0{,}632E$ means that the time constant represents the time required for *u* to reach 63.2% of its final value (physical significance).

After 3τ , the circuit will have gotten 1 − e −3 ≈ 95% of the way, and after 5τ, more than 99%.
So, after a few time constants, for practical purposes, the circuit has reached steady state.
Thus, the time constant is itself a good rough guide to **"how long" the transient response will take**.



Of course, mathematically, the steady state is actually an asymptote: it never truly reaches steady state.
But, unlike mathematicians, engineers don't sweat over such inconsequential details. ☺

5.6.1. The General Transient Expression of the Charge and Current of an RLC Series Circuit



$$L\frac{di}{dt}+Ri+\frac{1}{C}\int idt=u(t)$$

$$q=\int idt; \quad i=\frac{dq}{dt} \quad , \quad \frac{di}{dt}=\frac{d^2q}{dt^2}$$

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=u(t) \quad , \quad q=q_p+q_l \quad ,$$

$$i=i_p+i_l$$

Notations:

$$\delta = \frac{R}{2L}$$ -*the damping constant*

$$\omega_0 = \frac{1}{\sqrt{LC}}$$ - *the resonant frequency*

$$\omega_0' = \sqrt{\delta^2 - \omega_0^2}$$ - *the frequency of oscillation*

$$p_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -\delta \pm \omega_0' \,,$$

Because there are 2 roots (usually distinct values):

$$q_l = Ae^{p_1 t} + Be^{p_2 t}$$

$$i_l = \frac{dq_l}{dt} = Ap_1 e^{p_1 t} + Bp_2 e^{p_2 t}$$

At t = 0 : $\begin{cases} q(0) = q_p(0) + q_l(0) \\ i(0) = i_p(0) + i_l(0) \end{cases}$ or $\begin{cases} q_l(0) = q_0 - q_{p0} \\ i_l(0) = i_0 - i_{p0} \end{cases}$

Because: $\begin{cases} q_l(0) = A + B \\ i_l(0) = Ap_1 + Bp_2 \end{cases} \Rightarrow \begin{cases} A + B = q_0 - q_{p0} \\ Ap_1 + Bp_2 = i_0 - i_{p0} \end{cases}$

$$A = \frac{p_2(q_0 - q_{p0}) - (i_0 - i_{p0})}{p_2 - p_1} \qquad B = \frac{p_1(q_0 - q_{p0}) - (i_0 - i_{p0})}{p_1 - p_2}$$

$$\Rightarrow \quad q_l = Ae^{(-\delta+\omega_0')t} + Be^{(-\delta-\omega_0')t}$$

$$q_l = e^{-\delta t}\left(Ae^{\omega_0' t} + Be^{-\omega_0' t}\right)$$

Euler formulas:

$$e^{\omega_0' t} = ch\,\omega_0' t + sh\,\omega_0' t; \quad e^{-\omega_0' t} = ch\,\omega_0' t - sh\,\omega_0' t$$

$$\Rightarrow \quad q_l = e^{-\delta t}\left[(A+B)ch\,\omega_0' t + (A-B)sh\,\omega_0' t\right]$$

Replacing *A* and *B* :

$$q_l = e^{-\delta t}\left[\left(q_0 - q_{p0}\right)ch\omega_0' t + \frac{\delta}{\omega_0' t}\left(q_0 - q_{p0}\right)sh\omega_0' t + \frac{1}{\omega_0}\left(i_0 - i_{p0}\right)sh\omega_0' t\right]$$

or

$$q_l = e^{-\delta t}\left[\left(q_0 - q_{p0}\right)\left(ch\omega_0' t + \frac{\delta}{\omega_0'}sh\omega_0' t\right) + \frac{i_0 - i_{p0}}{\omega_0'}sh\omega_0' t\right]$$

If $th\alpha' = \frac{\omega_0'}{\delta}$ we can consider: $sh\alpha' = \frac{\omega_0'}{\omega_0}$ and $ch\alpha' = \frac{\delta}{\omega_0}$

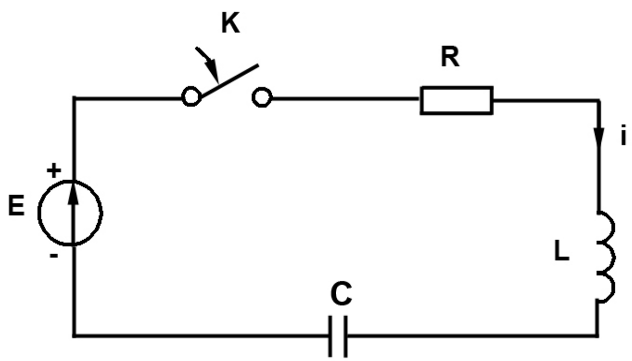
The final expressions are:

$$q_l = \frac{\omega_0}{\omega_0'} e^{-\delta t} \left[\left(q_0 - q_{p0}\right) sh\left(\omega_0' t + \alpha'\right) + \frac{i_0 - i_{p0}}{\omega_0} sh\omega_0' t \right]$$

$$i_l = \frac{dq_l}{dt} = -\frac{\omega_0}{\omega_0'} e^{-\delta t} \left[\omega_0 \left(q_0 - q_{p0}\right) sh\omega_0' + \left(i_0 - i_{p0}\right) sh\left(\omega_0' t - \alpha'\right) \right]$$

$$\begin{cases} q(t) = q_p + q_l \\ i(t) = i_p + i_l \end{cases}$$

5.6.2. The transient response of the second order circuits.



$$e(t) = E$$

$$q_0 = 0; q_p = CE; q_{p0} = CE$$

$$i_0 = 0; i_p = 0; i_{p0} = 0$$

It results:

$$q(t) = CE\left[1 - \frac{\omega_0}{\omega'} e^{-\delta t} sh\left(\omega_0' t + \alpha'\right)\right]$$

$$i(t) = CE\frac{\omega_0^2}{\omega_0'} e^{-\delta t} sh\,\omega_0' t$$

The roots of the characteristic equation assume three possible conditions:

$$p_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -\delta \pm \omega_0' \ ,$$

(where $\delta = \dfrac{R}{2L}$ and $\omega_0 = \dfrac{1}{\sqrt{LC}}$)

1) two real and distinct roots, when $\delta^2 > \omega_0^2$

→ the circuit is said to be **overdamped**

2) two real, equal roots, when $\delta^2 = \omega_0^2$

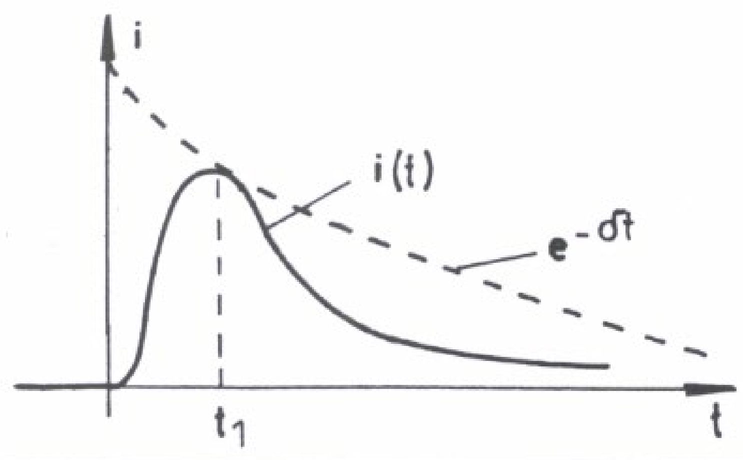
→ the circuit is said to be **critically damped**

3) two complex roots, when $\delta^2 < \omega_0^2$

→ the circuit is said to be **underdamped**

1) An **overdamped response** is the response that does not oscillate about the steady-state value but takes longer to reach than the critically damped case.

$$i(t) = CE\frac{\omega_0^2}{\omega_0'}e^{-\delta t}sh\omega_0't$$

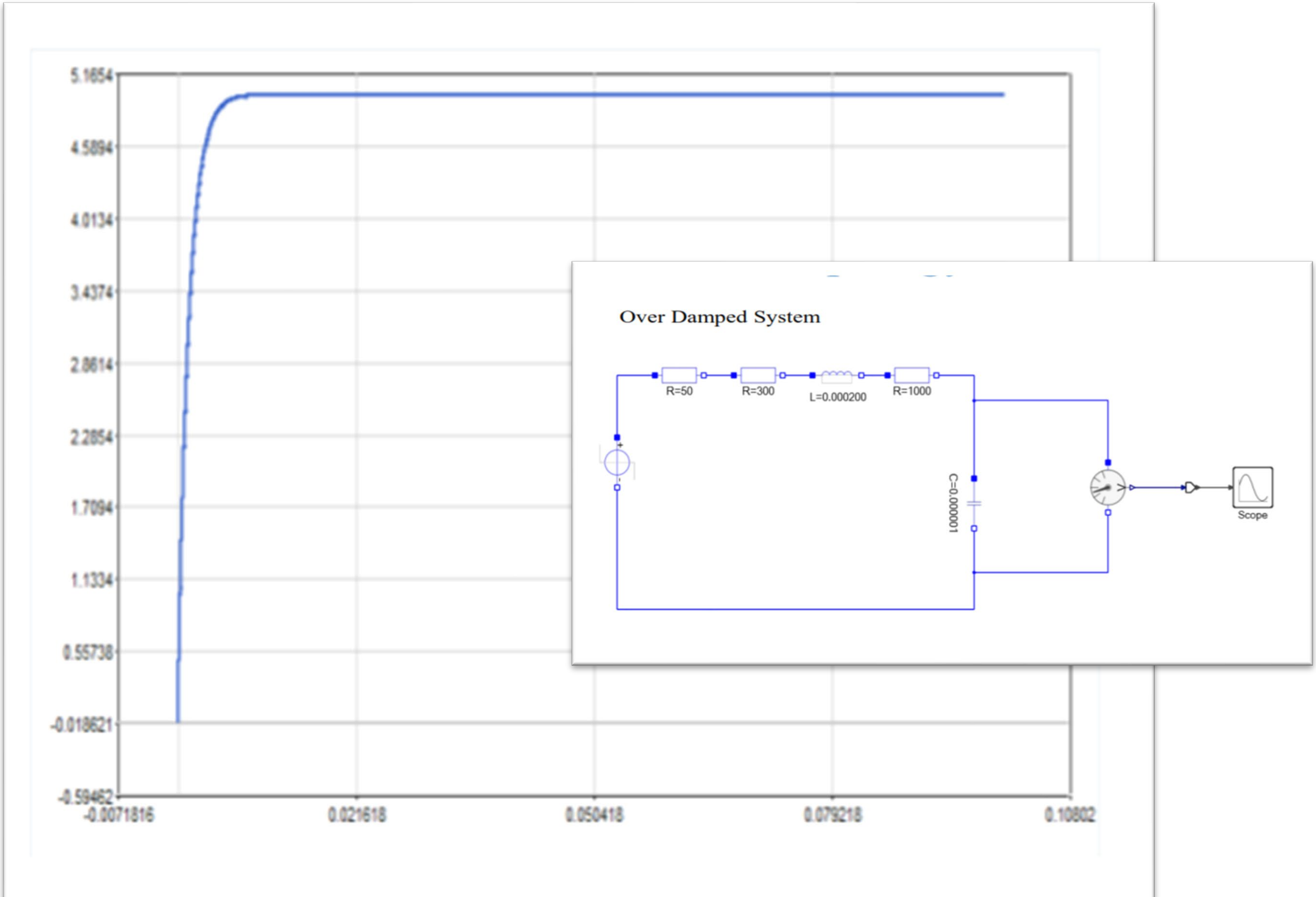


- the current is only positive

- the maximum of the current occurs at the instant t_1, when the first derivative of the current is zero:

$$\frac{\partial i}{\partial t} = -\delta\, e^{-\delta t}\, sh\, \omega_0't + \omega_0'\, e^{-\delta t}\, ch\, \omega_0't = 0$$

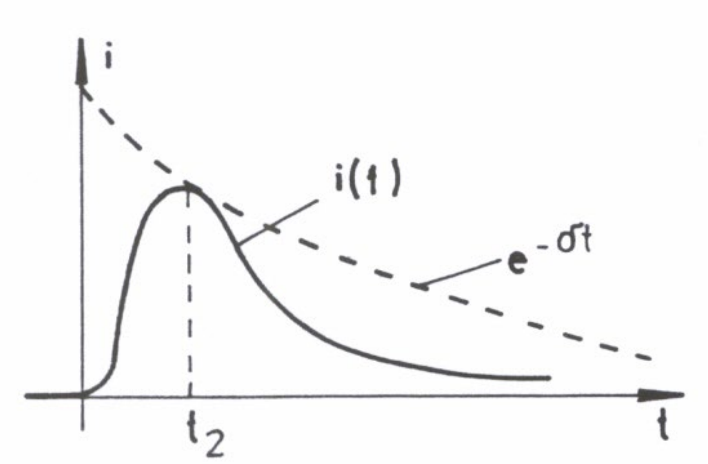
$$\Rightarrow \quad t_1 = \frac{\alpha'}{\omega_0'}$$


5.1654
4.5894
4.0134
3.4374
2.8614
2.2854
1.7094
1.1334
0.55738
-0.018621
-0.59462
-0.0071816
0.021618
0.050418
0.079218
0.10802
Over Damped System
R=50
R=300
L=0.000200
R=1000
C=0.000001
Scope

2) A **critically damped** response is that response that reaches the steady-state value the fastest without being underdamped. It is related to critical points in the sense that it straddles the boundary of underdamped and overdamped responses. Here, damping ratio is always equal to one. There should be no oscillation about the steady state value in the ideal case.

$$i(t) = CE\frac{\omega_0^2}{\omega_0'}e^{-\delta t} sh\,\omega_0' t$$

$$i = \omega_0^2 e^{-\delta t} \cdot CE \lim_{\omega_{0'} \to 0} \frac{sh\ \omega_{0'} t}{\omega_{0'}} = \omega_0^2 CEe^{-\delta t} \cdot t$$

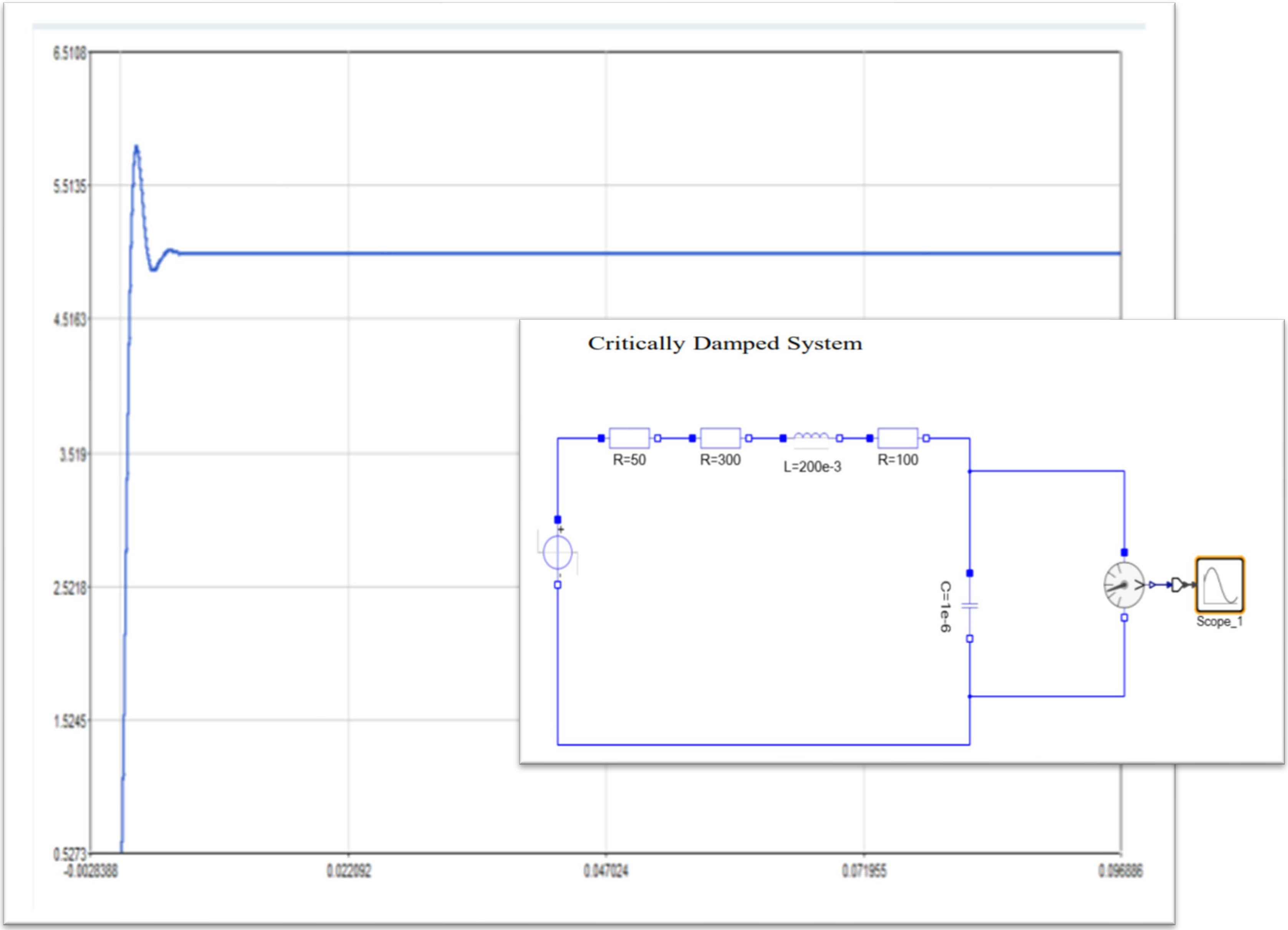


- the current is only positive

- the maximum of the current occurs at the instant t_2, when the first derivative of the current is zero:

$$\frac{\partial i}{\partial t} = -\delta\, e^{-\delta t} \cdot t + e^{-\delta t} = 0$$

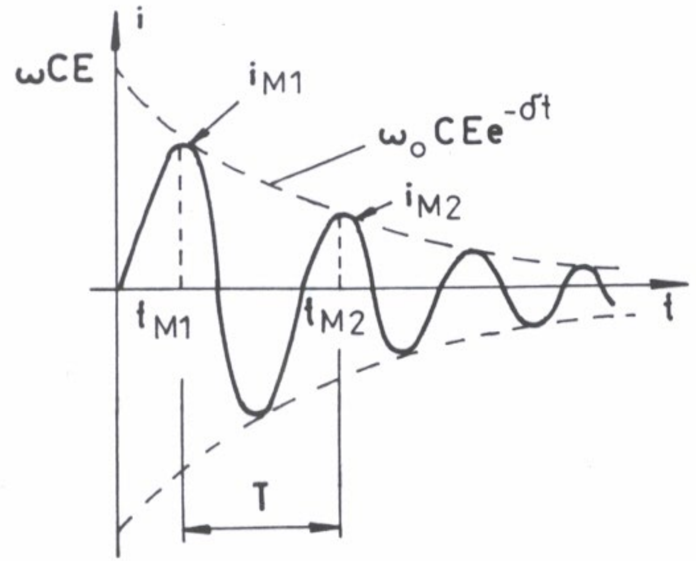
$$t_2 = \frac{1}{\delta} = \frac{1}{\omega_0}$$


6.5108
5.5135
4.5163
3.519
2.5218
1.5245
0.5273
-0.0028388
0.022092
0.047024
0.071955
0.096886
Critically Damped System
R=50
R=300
L=200e-3
R=100
C=1e-6
Scope_1

3) An **underdamped response** is one that oscillates within a decaying envelope. The more underdamped the system, the more oscillations and longer it takes to reach steady-state. Here damping ratio is always <1.

$$i(t) = CE\frac{\omega_0^2}{\omega_0'}e^{-\delta t} sh\,\omega_0' t$$

$$i(t) = CE\frac{\omega_0^2}{\omega_0''}e^{-\delta t}\sin\omega_0'' t$$



-the period of the damped oscillations:

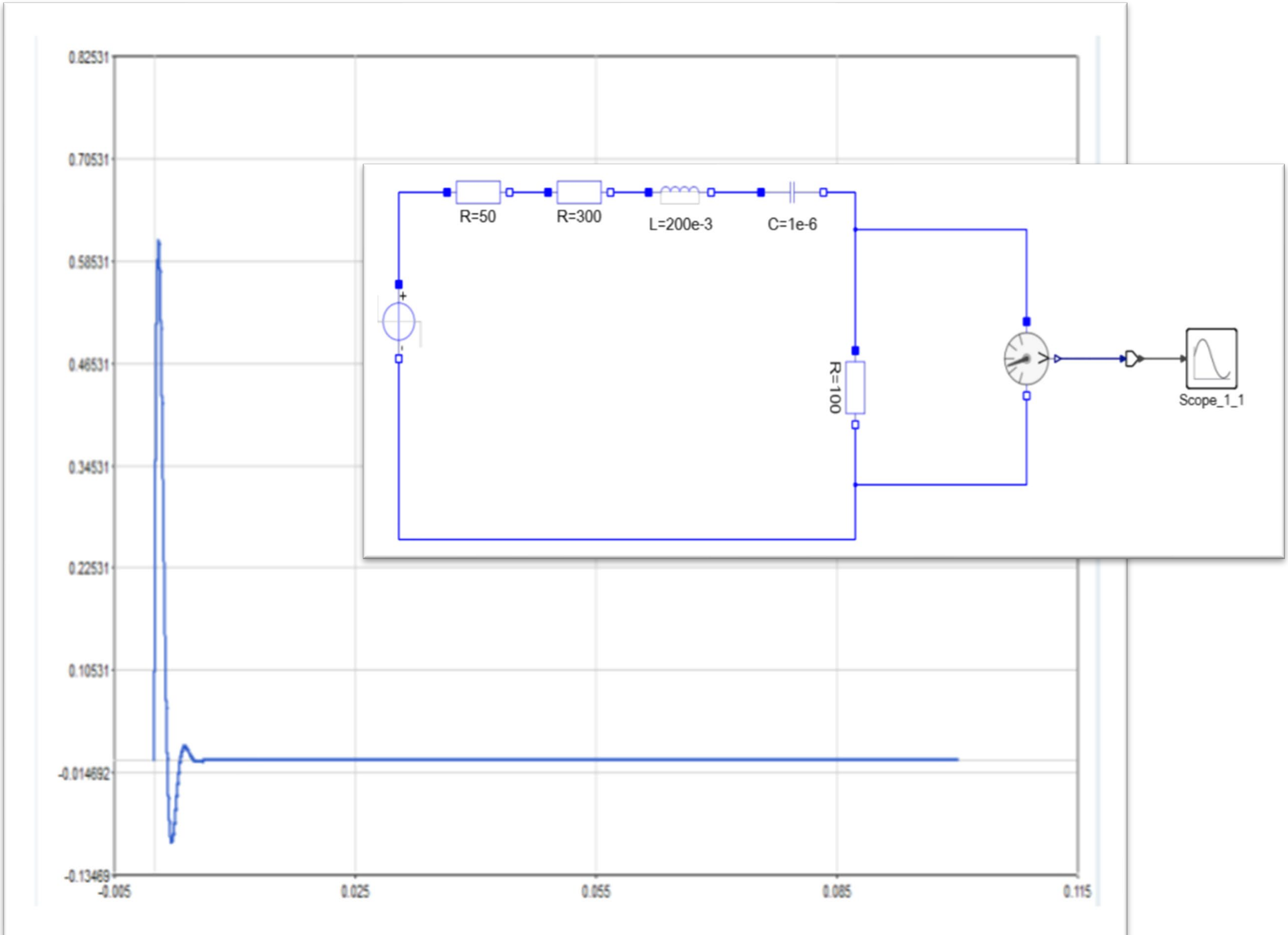
$$T = \frac{2\pi}{\omega_0''}$$

It can be shown that

$$\delta T = \ln\frac{i_{M_1}}{i_{M2}}$$

→ logarithmic decrement

where i_{M_1} and i_{M_2} are the amplitude of two succesive peaks.


0.82531
0.70531
0.58531
0.46531
0.34531
0.22531
0.10531
-0.014692
-0.13469
-0.005
0.025
0.055
0.085
0.115
R=50
R=300
L=200e-3
C=1e-6
R=100
Scope_1_1

5.7. THE LAPLACE TRANSFORM

- The Laplace transform is an integral transform perhaps second only to the Fourier transform in its utility in solving physical problems.
- The Laplace transform is particularly useful in solving linear ordinary differential equations such as those arising in the analysis of electronic circuits.
- In other words it can be said that the Laplace transform is nothing but a shortcut method of solving differential equation.

Laplace transform definition: $L[f(t)] = \int_0^{\infty} f(t)\, e^{-st}\, dt = F(s)$

where $s = \sigma + j\omega$ (complex quantity)

1) The impulse function

$$L[\delta(t)] = \int_0^{\infty} e^{-st}\,\delta(t)dt = 1$$

2)The step function

$$L[\gamma(t)] = F(s) = \int_0^{\infty} e^{-st}\,dt = \frac{1}{s}$$

3) The transform of the first derivative of $f(t)$:

$$\mathcal{L}\,[f'(t)] = sF(s) - f(0)$$

4) The transform of the integral of *f(t)* :

$$L[\int_0^{\infty} f(t)dt] = \frac{1}{s}F(s)$$

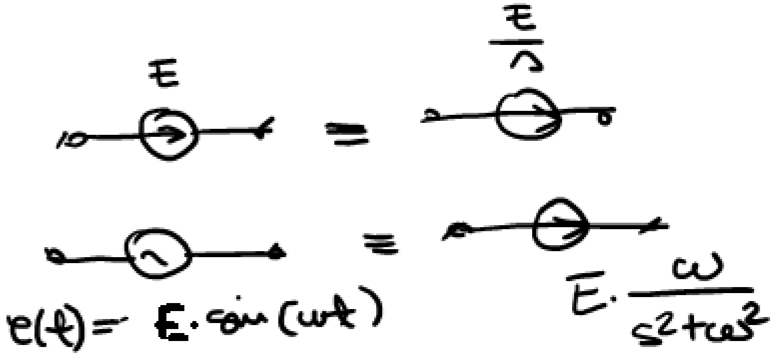
Properties of the Laplace transform:

- linearity : $\mathcal{L}[af(t)] = a\mathcal{L}[f(t)] = aF(s)$
- superposition: $\mathcal{L}[f_1(t) \pm f_2(t)] = \mathcal{L}[f_1(t)] \pm \mathcal{L}[f_2(t)] = F_1(s) \pm F_2(s)$

$f(t)$	$L[f(t)]$	$f(t)$	$L[f(t)]$
$\delta(t)$	1	$shat$	$\frac{a}{s^2 - a^2}$
$\gamma(t)$	$\frac{1}{s}$	$chat$	$\frac{s}{s^2 - a^2}$
$e^{\pm at}$	$\frac{1}{s \pm a}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
te^{-at}	$\frac{1}{(s \pm a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\frac{1}{a+b}\left(e^{at} - e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$\frac{1}{a-b}\left(ae^{at} - be^{bt}\right)$	$\frac{a}{(s-a)(s-b)}$
$\sin(\omega t + \varphi)$	$\frac{s \sin \varphi + \omega \cos \varphi}{s^2 + \omega^2}$	$f'(t)$	$sL[f(t) = f(0)]$
$\cos(\omega t + \varphi)$	$\frac{s \cos \varphi - \omega \sin \varphi}{s^2 + \omega^2}$	$\int_0^1 f(t)dt$	$\frac{1}{s} L[f(t)]$

Network analysis by Laplace transform.

- the voltage-current relationship for a resistance

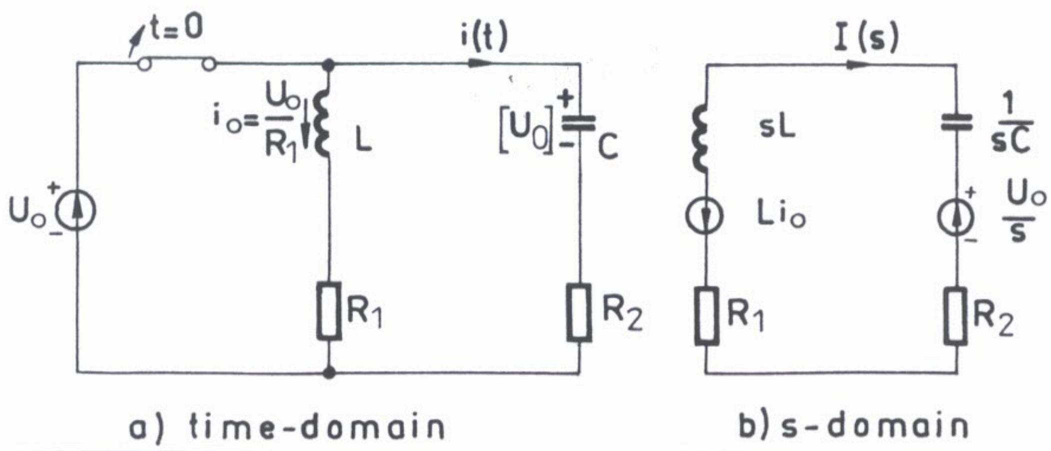


Network analysis by Laplace transform.

- the voltage-current relationship for an inductance carrying initial current i_0

Network analysis by Laplace transform.

- the transform corresponding to the voltage-current relationship for capacitance, charged to an initial voltage u_0



Example of circuit transformation.

Applying Kirchhoff's voltage law to the *s*-domain circuit:

$$\left(sL + \frac{1}{sC} + R_1 + R_2\right) I(s) = -\left(\frac{U_0}{s} + Li_0\right)$$

or

$$I(s) = \frac{-(U_0/s + Li_0)}{sL + 1/sC + R_1 + R_2}$$

Inverse Laplace transform : $f(t) = \mathcal{L}^{-1}\,[F(s)]$

$$F(s) = \frac{P(s)}{Q(s)} = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}$$

$$F(s) = \frac{P(s)}{Q(s)} = \frac{P(s)}{(s - s_1)(s - s_2)\cdots(s - s_k)\cdots(s - s_n)}$$

where $s_1, s_2 \ldots, s_n$ are the roots of $Q(s) = 0 \;\Rightarrow$ (poles !)

$$F(s) = \frac{P(s)}{Q(s)} = \frac{A_1}{s - s_1} + \frac{A_2}{s - s_2} + \cdots + \frac{A_k}{s - s_k} + \cdots + \frac{A_n}{s - s_n}$$

- the original function: $f(t) = \sum_{k=1}^{n} A_k e^{s_k t}$

Ohm's Law and Kirchhoff's Laws (using the Laplace transform):

$$u + e_g = Ri + \frac{1}{C}\int_{-\infty}^{t} i dt + L\frac{di}{dt} + \frac{d\phi^{ex}}{dt}$$

$$u + e_g = Ri + U_{C_0} + \frac{1}{C}\int_{0}^{t} i dt + \frac{d\phi}{dt}$$

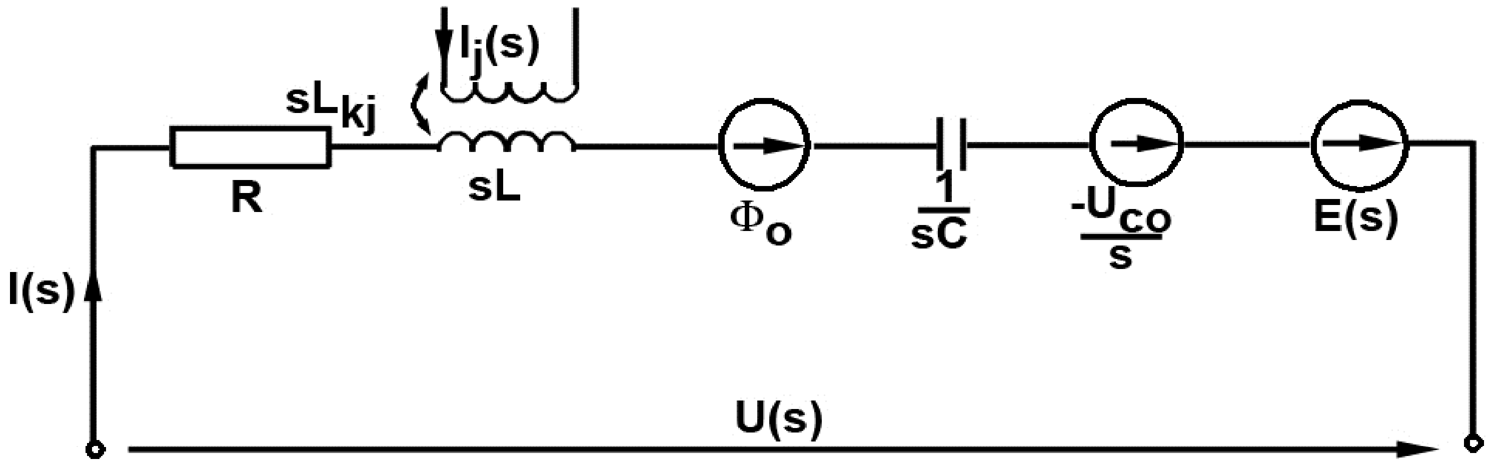
where: $U_{C_0} = \frac{1}{C}\int_{-\infty}^{0} i dt; \quad \phi = Li + \phi^{ex}$

Applying the Laplace transform it results *the Ohm's law*:

$$U(s) + E_g(s) + \phi_0 - \frac{U_{C_0}}{s} = I(s)\cdot Z(s) + \sum_{\substack{j=1 \\ j\neq k}}^{L} Z_{kj}\cdot I_j(s)$$

$$U(s) + E_g(s) + \phi_0 - \frac{U_{C_0}}{s} = I(s) \cdot Z(s) + \sum_{\substack{j=1 \\ j \neq k}}^{L} Z_{kj} \cdot I_j(s)$$

The corresponding circuit is:



Kirchhoff's Laws (using the Laplace transform):

1) *Kirchhoff's Current Law (KCL)*

$$\sum_{k\in q} i_k = 0$$

$$\sum_{k\in q} I_k(s) = 0$$

2) *Kirchhoff's Voltage Law (KVL)*

$$\sum_{k\in p}\left[E_{gk}(s)+\Phi_{k0}-\frac{U_{C_0}}{s}\right]=\sum_{k\in p}\left[Z_k(s)\cdot I_k(s)+\sum_{\substack{j=1\\ j\neq k}}^{L} Z_{kj}(s)\cdot I_j(s)\right]$$

References

[1] Charlews K. Alexander, Matthew N.O.Sadiku, *Fundamentals of Electric Circuits (Fifth Edition),* published by McGraw-Hill, 2013

[2] Radu V. Ciupa, Vasile Topa, *The Theory of Electric Circuits,* published by Casa Cartii de Stiinta, 5518

[3] Dan. D Micu, Laura Darabant, Denisa Stet et al., *Teoria circuitelor electrice. Probleme,* published by UTPress, 2016

[4] *Transient response of RC and RL circuits ENGR 40M lecture notes* — July 26, 2017, Chuan-Zheng Lee, Stanford University (https://web.stanford.edu/class/archive/engr/engr40m.1178/slides/transient.pdf)

[5] Transient Response Of R LC Network – Altair Innovation Intelligence (https://altairuniversity.com/wp-content/uploads/2017/06/Determination-of-transient-response-of-an-R-L-C-network-.pdf)

[6] John Bird, Electrical Circuit Theory and Technology, published by Newnes, 200