Seminar 5 (homework) Discrete probabilities

1. In the poker game there are 52 cards and a player receives 5 cards. Find the probabilities of each of the following hands:

a. royal flush (ace, king, queen, jack, 10 in same suit);

b. straight flush (5 cards of 5 consecutive ranks, all in 1 suit, but not a royal flush);

c. four of a kind (four cards in 1 rank and a fifth card);

d. full house (3 cards of 1 rank, 2 of another rank);

e. flush (5 cards in 1 suit, but neither royal nor straight flush);

f. straight (5 cards in 5 consecutive ranks, but not all of the same suit);

g. three of a kind (3 cards of 1 rank and 2 cards of 2 different ranks);

h. two pairs (2 cards in each of 2 different ranks and a fifth card of a third rank);

i. one pair (2 cards in 1 rank, plus 3 cards from 3 other ranks).

2. (from counting) If P(n,k) denotes the number of integer partitions of n into k parts, then

$$P(n,k) = P(n-k,k) + P(n-k,k-1) + \ldots + P(n-k,1).$$

Hint: Given a partition of n in k parts, we can obtain a partition of n - k in k (or fewer) parts by subtracting 1 from each part (and conversely, also).