Parameters of the radio channels that affect digital signal transmissions

1.Free space attenuation

- the signal undergoes an average attenuation that depends on the length of the path R and signal's frequency f (1). The exponent γ in relation (1) is called attenuation index.

$$L_{pm} = [(4\pi Rf)/c]^{\gamma} ; \quad (1)$$

- if G_t and G_r represent the gains of the antennae at the transmitter and receiver, respectively, and P_t is the transmitted power, then the power at the input of the receiver is given by the Friis's transmission equation [mil]:

$$P_r = P_t\, G_t\, G_r[c/(4\,\pi Rf)]^2; \quad (2)$$

by including the gains of the two antennae in the free space attenuation, relation ((2) expressed in dB, becomes:

$$P_{rm}(R)[dB] = P_t[dB] - L_{pm}(R)[dB]; \quad (3)$$

- the attenuation in (3) is also influenced by the geographic parameters of the environment. According to the environment where the propagation takes place, the attenuation index varies. Table 1 show the attenuation indexes for several propagation environments, [rap]:

Propagation Environment	Attenuation Index, γ
Free space	2
Urban area	2,7 to 3,5
„Shadowed" urban area	3 to 5
Direct propagation within buildings	1,6 to 1,8

Table 1 Values of the attenuation index for several propagation environments

- the attenuation of radio transmissions also increases significantly in the presence of rain, for frequencies larger than 10 GHz. The method of evaluation of the attenuation due to rain in described in [mil].

2. Log-normal fading

- expressions (1) and (3) don't take into account the fact that the attenuation can be different for two positions situated at the same distance R from the transmitter, due to environment factors (buildings, forests etc.). Measurements have shown that the attenuation, expressed in dB, in different positions at the same distance, has a normal (Gaussian) distribution around the mean value given by (1); the resulted attenuation, in dB, is expressed by (4):

$$L_p(R)\,[dB] = L_{pm}(R)\,[dB] + \Delta L_{p\sigma}\,[dB]; \quad (4)$$

- this variation of the attenuation is called log-normal fading, because its value expressed in dB (after a logarithm operation), is distributed according to a Gaussian law.
- the probability of the received signal's level to be higher than an imposed value can be expressed using the Q(t) function and relation (5):

$$P(P_r(R) > 1) = Q(\frac{1 - P_{rm}(R)}{\sigma}); \quad Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} e^{-\frac{u^2}{2}} du; \quad (5)$$

3. Doppler frequency spread

- the radio communications serving mobile stations are affected by frequency spreading of the received signal, due by the Doppler effect caused by the movement of the mobile station.
- consider a mobile station that moves with speed v and a signal transmitted with the frequency f_c, (6.a), that reaches the antenna of the receiver under an angle θ vs. its normal plane, then the frequency of the signal entering the receiver undergoes a frequency deviation f_d, due to the Doppler effect, (7.a).
- due to the change of the incidence angle, and the change in position of the mobile station, the frequency deviation f_d, modifies its value between 0 and a maximum value f_m (7.b), being spread within the range ($f_p - f_m$, $f_p + f_m$) . This shows that the received signal undergoes spreading (dispersie) in frequency, called Doppler spread. The expression of the received signal is given by equation (6.b)

$$s_t = A\cos(2\pi f_p t);\ \text{a.} \quad s_r(t) = A\cos[2\pi(f_p - f_d)t];\ \text{b.} \quad (6)$$

$$f_d = \frac{v}{\lambda}\cos\theta = \frac{vf_p}{c}\cos\theta;\ \text{a.} \quad f_m = \frac{vf_p}{c}\ \text{for}\ \theta = 0°;\ \text{b.} \quad (7)$$

4. Fading. Classification.

- *multipath propagation* affects significantly the received signal over a radio channel. The transmitted signal reaches the receiver's antenna over several propagation paths, determined by the existence of obstacles that reflect the transmitted wave. Thus, the receiver is reached by the direct wave (if any), that had no obstacles in its path, and one or several reflected waves. Because the propagation paths do not have the same length, the signals that reach the receiver can have different delays, generating the phenomena of *temporal spread.*
- *temporal spread (delay spread)* depends on the length of the secondary (reflected) paths and produces inter-symbol interference, which becomes significant for large delays (of order of one or several symbol periods). Because temporal spread is a random variable, it is characterised by its variance σ_τ.
- in order to characterize a wide band channel, the *coherence band of the channel*, B_c, is defined. This is the frequency range within which the channel can be considered to be (approximately) uniform, i.e. it has approximately the same attenuation and a linear phase variation. Within this range, two signals having different frequencies have their amplitudes strongly correlated. The coherence band can be approximated by relation (8) for correlation factors equalling 0.9 and, respectively, 0.5:

$$B_c \approx 1/(50\sigma_\tau) \text{ or } B_c \approx 1/(5\sigma_\tau); \tag{8}$$

- for radio channels with a mobile station, due to the Doppler spread, a coherence time T_c, is defined. The T_c is the time interval within which the channel's attenuation and phase characteristics remain (approximately) constant. It depends on the maximum frequency deviation (7) and its expression is:

$$T_c = 0{,}423/f_m; \tag{9}$$

- the two disturbing effects produce different types of fading. A classification of the types of fading produced by these effects is given in [rap].

- the multipath delay spread produces **flat fading** or **frequency selective fading**. Denoting the signal bandwidth by B_S and the symbol period of the transmission by T_S, the conditions of occurrence of the two types of fading are approximately:

- for flat fading: 1. $B_S << B_c$ or 2. $\sigma_\tau << T_S$; (10.a)
- for frequency selective fading: 1. $B_S >> B_c$ or 2. $\sigma_\tau >> T_S$ (10.b)

- for channel parameters having close values to transmission's ones, the two types of fading can occur simultaneously, in different proportions.

- the Doppler spread, together with the multipath propagation causes, in the case of mobile channels, **fast fading (fading rapid)** and **slow fading (fading lent)**. The conditions of occurrence for these fading types are:

- for fast fading:
  f_d large or 2. $T_c < T_S$ or 3. the channel variations are faster than those of the BB signal (11.a)
- for slow fading:
  f_d small or 2. $T_c > T_S$ or 3. channel variations are slower than those of the base-band signal (11.b)

5. Flat fading and frequency selective fading

- the effects of the multipath propagation depend on the frequency → this is a frequency selective phenomenon
- *example:* consider a QAM transmission within which s(t) (12) is the transmitted signal, and r(t) (13) is the received signal, and for which the reflected component undergoes attenuation b and the delay τ, with respect to the direct wave; indexes *i* and *q* denote the in-phase and quadrature signal components of the involved signals

$$s(t) = v_i \cos 2\pi f_p t + v_q \sin 2\pi f_p t; \quad (12); \qquad r(t) = s(t) + bs(t-\tau); \quad (13)$$

- the received signal will be:

$$r(t) = v_i \cos 2\pi f_p t + v_q \sin 2\pi f_p t + bv_i(t-\tau)\cos 2\pi f_p(t-\tau) + bv_q(t-\tau)\sin 2\pi f_p(t-\tau); \tag{14}$$

- using the synchronised carrier for the QAM demodulation for the in-phase branch:

$$p_i(t) = 2\cos \pi f_p t\,; \tag{15}$$

after the multiplication of the carrier with the received signal and after LP filtering, operations denoted by *, we get:

$$r(t) * p_i(t) = v_i(t) + bv_i(t-\tau)\cos 2\pi f_p \tau + bv_q(t-\tau)\sin 2\pi f_p \tau \tag{16}$$

- (16) shows two disturbing effects:
- the occurrence of interference between the two modulating signals (third term) and
- the superposition of the delayed version over the use full signal (second term).

Note that for certain values of Tp/τ, the second term is added or subtracted from the value of the useful data signal $v_i(t)$, according to the relation between T_p and τ. The extreme situations are given by the equations below:

$$\cos 2\pi f_p \tau = \pm 1 \Rightarrow 2\pi f_p \tau = 2n\pi; \quad \text{or} \Rightarrow 2\pi f_p \tau = (2n+1)\pi; \tag{17}$$

from (17) =>

$$r(t) * p_i(t) = v_i(t) + b v_i(t-\tau) \text{ for } f_p = \frac{2n}{\tau} \Leftrightarrow \tau = 2nT_p \tag{18}$$

$$r(t) * p_i(t) = v_i(t) - b v_i(t-\tau) \text{ for } f_p = \frac{2n+1}{\tau} \Leftrightarrow \tau = (2n+1)T_p \tag{19}$$

- if the signal frequency takes the values given by (18), the amplitude of the signal increases, and if it has the values given by (19), the signal is attenuated; the values of the signal's increases or decreases depend on the attenuation coefficient of the signal on the delayed path and on the modulated signal. There should also be noted that, since the symbol period usually is much greater than de delay τ, the demodulated baseband signals of the two paths could be considered as synchronous.

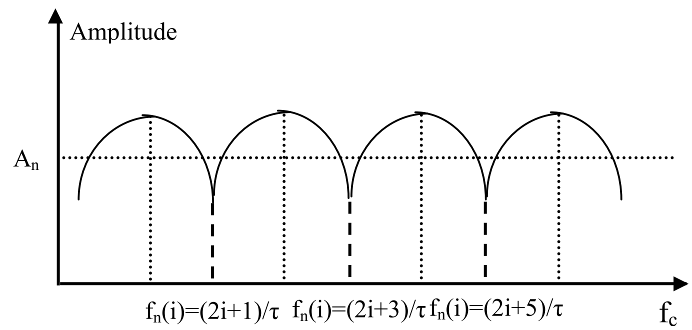

Figure 1 Schematic representation of the frequency selectiv fading: f_n represent the notch frequencies

- the frequency values given by (19) are called „notch frequencies". For fixed radio channels, these frequencies have fixed values, but for mobile channels, due to the change of the position of mobile stations, the values of these frequencies change. For QAM signals, such frequencies also occur for the term introduced by the quadrature component of the modulated signal. Figure 1 presents schematically the frequency selective fading phenomenon.
- if the frequency bandwidth of the modulated signal < 1/τ, i.e., the frequency spacing between a minima and a maxima, then the attenuation is relatively constant and we have a flat fading.
- if the frequency bandwidth > 1/τ, then the channel exhibits a frequency selective fading phenomenon.
- the same phenomenon can occur if two reflected waves, delayed in different ways and having a delay difference Δτ, arrive at the receiver
- if the receiver is reached by several reflected waves with distinct delays, then several groups of notch frequencies occur, determined by each delay time τ_i, and the attenuation of each group is different, according to the attenuation factor b_i of each reflected wave.

6. Fast fading and slow fading.

- fast fading and slow fading refer to the speed of variation of the envelope of the received signal, i.e. the evolution of the received signal's amplitude in time.
- for fixed radio channels, they are caused by the variations of the channel parameters, but these variations are slow, thus, allowing large symbol period values of the transmission.
- for mobile radio channels, they are caused by the frequency shift produced by the Doppler effect, the symbol period decreasing with the increase of the relative speed between transmitter and receiver.
- *example:* let us analyse the case of a mobile station that receives two waves, for which the arriving angles differ by *Θ*; the speed of the mobile station is *v*.
- considering the transmitted signal to be:

$$s(t) = R_0 \cos 2\pi f_p t \tag{20}$$

the received signal will have the following form:

$$r(t) = R_0 \cos[2\pi t(f_p - \frac{v f_p}{c})] + R_0 \cos[2\pi t(f_p - \frac{v f_p}{c}\cos\Theta)] \tag{21}$$

- the received signal decomposed according to its in-phase and quadrature components, $R_i(t)$ and $R_q(t)$, is:

$$r(t) = R_i(t)\cos 2\pi f_p t + R_q(t)\sin 2\pi f_p t; \quad \text{a.}$$

$$R_i(t) = R_0[\cos(2\pi\frac{v f_p}{c}t) + \cos(2\pi\frac{v f_p}{c}t\cos\Theta); \quad \text{b.} \quad R_q(t) = R_0[\sin(2\pi\frac{v f_p}{c}t) + \sin(2\pi\frac{v f_p}{c}t\cos\Theta); \quad \text{c.} \tag{22}$$

- the envelope of the received signal is: $R(t)=\sqrt{R_i^2(t)+R_q^2(t)}=2R_0\cos[2\pi\frac{vf_p}{2c}(1-\cos\Theta)]$ (23)

- from (23) → the envelope of the received signal varies with the frequency:

$$f_D=\frac{f_m}{2}(1-\cos\Theta) \quad (24)$$

- the maximum speed of variation is reached for Θ = 180° and has the value f_m, induced by the Doppler effect.
- if the frequency of the envelope's variation has its period larger than the symbol period of the transmission, we have fast fading; if the frequency of variation is smaller than the symbol frequency, we have slow fading.

7. Envelope Distribution Laws of the Faded Signal

7.1. N reflected waves – the Rayleigh distribution

- if the received signal is composed of *N* reflected waves, each having different but constant amplitude R_n, an incidence angle Θ_n, and a Doppler shift f_{dn}, then the received signal would be expressed by:

$$r(t)=\sum_{n=1}^{N}R_n\cos(2\pi f_c t-2\pi f_{dn}t);\quad f_{dn}=\frac{vf_c}{c}\cos\Theta_n \quad (25)$$

- by a similar reasoning as in (20,…,24), the expression of the received signal decomposed into its I and Q components is given by the second line of (26):

$$\begin{aligned}&r(t)=R_i(t)\cos 2\pi f_c t+R_q(t)\sin 2\pi f_c t;\\&R_i(t)=\sum_{n=1}^{N}R_n\cos 2\pi f_{dn}t;\quad R_q(t)=\sum_{n=1}^{N}R_n\sin 2\pi f_{dn}t;\end{aligned} \quad (26)$$

- If *N* has great values, then acc. to the Central Limit Theorem, the $R_i(t)$ and $R_q(t)$ are random variables with Gaussian distributions and null mean values, and the received signal's envelope $R(t)$, (27) will be a random variable with a Rayleigh distribution [mil].

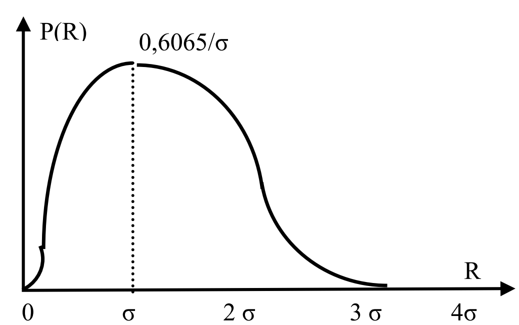

Figura 2 Reprezentarea aproximativă a distribuției Rayleigh

$$R(t)=\sqrt{R_i^2(t)+R_q^2(t)}; \quad (27)$$

- The probability density of this distribution, i.e. the probability of the envelope to equal *R*, if the average power of the received signal equals σ_R^2, is given by (28) and is schematically represented in figure 2.

$$p(R)=\frac{R}{\sigma_R}e^{\frac{-R^2}{2\sigma_R^2}};\quad 0\le R<\infty; \quad (28)$$

- the frequency band of the received signal equals $[f_c - f_m; f_c + f_m]$ and is determined by the Doppler spread.

7.2. Direct wave and N reflected waves – the Rice distribution

- the received signal is composed of a direct wave and *N* reflected wave, each having different but constant amplitude R_n, an incidence angle Θ_n, and a Doppler shift f_{dn}, → the received signal can be expressed as a sum between the direct signal *d(t)* and a signal *r(t)* composed of the reflected waves, whose envelope would take values acc. to the Rayleigh distribution.
- the received signal *s(t)* would be, after its decomposition in the I and Q components, expressed by, see also (26):

$$s(t) = d(t) + r(t) = D_i(t)\cos 2\pi f_c t + D_q(t)\sin 2\pi f_c t + R_i(t)\cos 2\pi f_c t + R_q(t)\sin 2\pi f_c t; \quad (29)$$

- Literature shows that the envelope *S(t)* of the received signal:

$$S(t)=\sqrt{[D_i(t)+R_i(t)]^2+[D_q(t)+R_q(t)]^2} \quad (30)$$

would take values acc. to the Rice distribution, which has the probability density function:

$$p(S)=\frac{S}{\sigma^2}e^{-\frac{(S+A)^2}{2\sigma^2}}\cdot I_0(\frac{A^2}{2\sigma^2});\quad \text{for } A\ge 0 \text{ şi } S\ge 0; \quad (31)$$

- expression (31) is also expressed in literature in terms of ratio K between the power of the direct signal, of amplitude A, and the sum of the powers of the reflected waves, which is proportional to σ^2, see (32.a). $I_0(t)$ denotes the order-zero modified Bessel function, (32.b):

$$K = \frac{A^2}{2\sigma^2};a. \quad I_0(t) = \sum_{n=0}^{\infty} \frac{t^{2n}}{2^{2n}\, n!};b. \tag{32}$$

- If the power of the direct signal decreases significantly ($A \to 0$), then the received signal is composed predominantly of reflected signals, and its envelope's values will be distributed according to the Rayleigh distribution. This means that for $K \to 0$, the Rice distribution degenerates into a Rayleigh distribution.

8. Effects of the final radio frequency amplifiers

- in order to ensure power efficiency, final radio amplifiers are used close to the saturation region of their transfer characteristic $P_o = f(P_i)$.
- under these circumstances, the more the output power gets closer to the maximum allowed value, the more non-linear the power transfer characteristic becomes, and the amplifier enters the saturation region of its transfer characteristic.
- for these cases, the transfer characteristic can no longer be approximated with a linear characteristic, i.e., $U_o = a \cdot U_i$, but can be approximated with a polynomial transfer function of type (35) that includes the relative delays between the components of the output signal:
- to point out the effects of this non-linearity upon the output signal, we approximate the transfer (amplification) characteristic by a polynomial function (33), which would include the relative delays of the output signal's components :

$$U_0(t) = a \cdot U_i(t) + b \cdot U_i^2(t-\tau_2) + c \cdot U_i^3(t-\tau_3) + \ldots; \quad a,b,c - \text{constants}; \tag{33}$$

- the modulated signal that has to be amplified is:

$$s_i(t) = R(t)\cos(\omega_p t + \varphi(t)); \tag{34}$$

- if the modulated signal (34) has a constant envelope $R(t) = R_0$, its phase is denoted by $\Phi(t)$, and the phase shifts inserted by the delays t_1 and t_2 are denoted by $\varphi_1(t)$ and $\varphi_2(t)$, then the output signal of such an amplifier (hard-limiting) will be:

$$\begin{aligned} s_o(t) &= a \cdot R_0 \cdot \cos\Phi(t) + \frac{b \cdot R_0}{2} + \frac{b \cdot R_0}{2} \cdot \cos[2(\Phi(t) - \phi_1)] + \\ &+ 4c \cdot R_0 \cdot \cos[3(\Phi(t) - \phi_2)] - 3c \cdot R_0 \cdot \cos(\Phi(t) - \phi_2); \quad \Phi(t) = w_p t + \varphi(t) \end{aligned} \tag{35}$$

- after some trigonometric manipulations (35) can be expressed as:

$$\begin{aligned} s_o(t) &= [a - 3c \cdot \cos\phi_2(t)] \cdot \cos\Phi(t) + [3c \cdot \sin\phi_2(t)] \cdot \sin\Phi(t) + \frac{b \cdot R_0}{2} + \\ &+ \frac{b \cdot R_0}{2} \cdot \cos[2(\Phi(t) - \phi_1)] + 4c \cdot R_0 \cdot \cos[3(\Phi(t) - \phi_2)] = \\ &= R_0 \cdot \sqrt{a^2 + 9c^2 - 9ac \cdot \cos\phi_2} \cdot \cos[\Phi(t) + \operatorname{arctg} \frac{3c \cdot \sin\phi_2}{(a - 3c) \cdot \cos\phi_2}] + \frac{b \cdot R_0}{2} + \\ &+ \frac{b \cdot R_0}{2} \cdot \cos[2(\Phi(t) - \phi_1)] + 4c \cdot R_0 \cdot \cos[3(\Phi(t) - \phi_2)]; \end{aligned} \tag{36}$$

- relation (36) shows that the output signal has a baseband spectral component, $b \cdot R_0/2$, a spectral component on the second harmonic of the carrier signal, the second term, and a spectral component on the third harmonic of the carrier signal, the last term.
- the presence of these terms shows that the non-linearity of the amplifier inserts spectral components outside the useful (allowed) frequency band, that are not present in the input signal; this phenomenon is called "spectral regrowth". The levels of these components decrease for more linear amplifiers, constant b and c very small, and for small variations of the modulated signal's envelope around its average value, i.e. small PAPR values.
- coming back to the first term of (36), this is placed in the allowed frequency band, but both its amplitude and phase are modified (distorted).
- if the envelope $R(t)$ is not constant, the signal at the output of a non-linear amplifier is expressed by (37); in (37) only the spectral components in the allowed frequency band are included

$$s_e(t) = u(R(t)) \cdot \cos[\omega_p t + v(R(t)) + \varphi(t)]; \tag{37}$$

- the variation of *R(t), u(R(t))*, is denoted as the AM/AM characteristic, while the variation of the phase is denoted as the AM/PM characteristic of the amplifier, both indicating the ways in which the variation of the modulated signal that has to be amplified, affect the amplitude and phase of the output (amplified) signal.
- the degree of distortion inserted by such an amplifier decreases with the decrease of the PAPR value of the modulated signal.

Summarizing, the nonlinear characteristic of the final RF amplifier lead to the following consequences:
1. distortion of the signal's envelope - by the characteristic AM/AM
2. distortion of the signal's phase by the characteristic AM/PM
3. spectral regrowth of some components outside the allowed frequency band, which were removed by the transmitter's filters before the final amplification;
- the first two consequences affect the quality of the signal that reaches the receiver, leading to an increased bit-error rate after the demodulation.
- the third consequence generates some undesired signals that interfere with the transmissions in the neighbouring frequency bands.

- to decrease the amplitudes of the components outside the allowed frequency band, which occur due to the spectral regrowth, as well as for decreasing the distortions of the signal in the allowed frequency band, two aspects should be taken into account:

- the decrease of the non-linearity degree of the final power amplifier;
- the employment of modulations that ensure an amplitude of the modulated signal as constant as possible;

- the decrease of the degree of non-linearity of the final amplifier can be achieved by the following approaches:

- lowering the operation point in the linear region of the amplifier's characteristic $P_o = f(P_i)$. This process requires the decrease of the output power, called output back-off („repliere a ieşirii") and denoted with B_o [dB], that is obtained by decreasing the power of the input signal, called input back-off („repliere a intrării") and denoted with B_i [dB].
- employment of some methods to compensate the non-linearity, e.g., the method called "linear amplification with nonlinear components (LINC)".
- employment of amplifiers with AM/AM characteristics as linear as possible, and with AM/PM characteristics as constant as possible.