Theoretical estimation for 802.11a bit rates and their adaptive employment

1.1. Main parameters

The 802.11a amendment to the original standard was ratified in 1999. The 802.11a standard uses the same core protocol as the original standard, operates in 5 GHz band, 9 channels of 20MHz bandwidth. Their central frequencies are given by the following rule:

$$f_c = 5000 + 20 \cdot k_{ch} [MHz]$$

The transmission has a preamble of 4-12 OFDM symbols. The length of the transmitted packets at MAC level varies between 1 and 4095 bytes.

The parameters that impose the data rate are the modulation and code employed. For modulation, the standard recommends BPSK (2-PSK with 1 bit/symbol), QPSK (2 bits/symbol), 16-QAM (4 bits/symbol) and 64-QAM(6 bits/symbol) modulations. The codes employed are convolutional codes of constraint K=7 and rates R_c=1/2, 2/3 and 3/4.

802.11a uses 52 subcarriers, modulated by means of an Inverse Fast Fourier transform (IFFT). Out of these 52 (from -26 to 26), subcarrier 0 is not modulated (it will coincide with the channel carrier after the frequency translation). On the subcarriers -21, -7, 7 and 21 pilot signals are transmitted, therefore only 48 subcarriers are left for data.

The configurations of the coded modulations imposed by the standard are presented below :

Modulation	Rate of the convolutional code
BPSK	R_c=1/2; R_c=3/4
QPSK	R_c=1/2; R_c=3/4
16-QAM	R_c=1/2; R_c=3/4
64-QAM	R_c=2/3; R_c=3/4

Table 3.1. Configurations of the coded modulations

1.2. Nominal bit rate

In order to compute the actual data rate, one must take into account that only 48 subcarriers are used for data transmission and that at the beginning 4 OFDM symbols are used for preamble.

$$N_{subcarriers}=48; \quad T_s=3{,}2\mu s; \; T_{preamble}=16\ \mu s \text{ (4 OFDM symbols)}$$
$$T'_s=T_s+T_G=3.2+0.8=4\mu s$$

and the effective symbol frequency : $f'_s=250kHz$

The frequency separation between the subcarriers is 312,5kHz. The bandwidth of the signal, centered on the carrier frequency of the radio channel, can be computed as follows:

$$BW=2(26 \cdot 0.3125)MHz+312.5kHz=16.5625MHz$$

The 312,5 kHz is added in order to ensure a guard interval in both sides.

In order to emit the same level of power all the time, the constellations should be scaled. The scaling factors for the employed modulations are presented in the table below.

BPSK	QPSK	16-QAM	64-QAM
1	$(1/2)^{1/2}$	$(1/10)^{1/2}$	$(1/42)^{1/2}$

Table 3.2. Scaling factors that ensure a constant level of emitted power for QAM constellation

The computation of the nominal bit rate takes into account the effective symbol frequency (f'_s), number of subcarriers (N), number of bits/symbol given by the constellation (n) and the coding rate (R_c) of the convolutional code. Therefore, the Nominal bit rate can be expressed as :

$$D_n= f'_s \cdot N \cdot n \cdot R_c=250kHz \cdot 48symols/bin \cdot n\ bits/symbol \cdot R_c=12000ksymbQAM/s \cdot n \cdot R_c=$$
$$=12Msymb/s \cdot n \cdot R_c$$

This formula allows the computation of the nominal bit rates for coded modulations in the 802.11a standard. The values are presented in the table below:

R_c |Constelation	BPSK	QPSK	16-QAM	64-QAM
1/2	6 Mbps	12 Mbps	24 Mbps	36 Mbps
2/3	-	-	-	48 Mbps
3/4	9 Mbps	18 Mbps	36 Mbps	54 Mbps

Table 3.3. Nominal bit rates ensured by the coded modulations

1.3. SNR Thresholds

This section will present a theoretical computation of the values for the SNR thresholds, for which the coded modulations in the 802.11a ensure an imposed bit error rate (BER). The symbol error probability will be calculated, and from this BER will be computed for each modulation.

$$p_e \approx \frac{4(\sqrt{N}-1)}{\sqrt{N}} \cdot \frac{\sqrt{N-1}}{\sqrt{3}} \cdot \frac{e^{-\frac{3}{N-1} \cdot \frac{\rho_{ech}}{2}}}{\sqrt{\pi \cdot \rho_{ech}}};$$ for QPSK, 16-QAM, 64-QAM

$$p_e \approx \frac{e^{-\rho_{ech}}}{2\sqrt{\pi \cdot \rho_{ech}}};$$ for BPSK

$$\rho_{ech} = \rho \cdot K_{C_G} \Rightarrow K_G = 10^{\frac{C_G}{10}};$$

K_{Cg} represents the coding gain expressed in linear, C_G, the coding gain expressed in dB

The standard imposes a packet error rate under 10% (PER<0.1) for packets of 1000 bytes. Therefore, the probability of receiving o correct packet should exceed 90% (P_{Pc}>0.9) and can be expressed as $P_{Pc} \geq (1\text{-BER})^{8000}$. From this, the value for the BER will be: $\text{BER}<1-10^{\lg 0.9} <1.3\cdot10^{-5}$. A more detailed analysis gave the approximate values given in table 3.4.1, considering also the impact of the approximation involved by the "classical" approximate relation between BER and symbol-error probability:

BER≈p_e/n , where n is the number of bits/symbol employed by each constellation.

For packets of 4195 bytes the required BER would be BER $<3\cdot10^{-6}$.
The accepted BER of transmissions using BPSK and QPSK are increased by to approx. 10^{-4} to allow their employment at smaller SNR values.

From these relations, the values of p_e can be computed for each constellation. The results are presented in the table below, along with the values of the SNT thresholds for non-coded modulations:

Constellation	Symbol error rate(p_e)	BER	SNR[dB]
BPSK	$1\cdot10^{-4}$	$1\cdot10^{-4}$	8
QPSK	$2.\cdot10^{-4}$	$1\cdot10^{-4}$	11
16-QAM	$5.2\cdot10^{-5}$	$1.3\cdot10^{-5}$	18.0
64-QAM	$7.8\cdot10^{-5}$	$1.3\cdot10^{-5}$	24

Table 3.4.1. SNR thresholds and symbol error rates for non-coded constellations

Note: the SNR values of the BPSK and QPSK are smaller with approx. 1.5 dB than the values corresponding to $p_e = 10^{-5}$, due to the greater BER accepted.
Considering the coding gain introduced by the convolutional codes with R_c=1/2, 2/3, 3/4 and constraint K=7, the SNR thresholds change. The equivalent SNR can be expressed as: $SNR_{equiv}=SNR-C_G$. The approximate values for the equivalent SNR are presented in the table below, along with the coding gain for each code.

	Coding Gain [dB]	BPSK	QPSK	16-QAM	64-QAM
1/2	7-5	3	6	11	-
2/3	5.	-	-	-	19.
3/4	3-2.5	5	8	15	21

Table 3.4.2. SNR thresholds and symbol error rates for coded constellations
Note: the coding gain of the R_c=1/2 decreases significantly due the different p_e and effects of the Gray mapping

The equivalent SNR for 64-QAM with a code of rate 1/2 would be approximately 17 dB and the number of data bits/symbol would be 6·1/2=3. For 16-QAM with a code of rate 3/4, the equivalent SNR is 15 dB and the number of data bits/symbol is 4·3/4=3. Therefore the same number of data bits/symbol is ensured, but for an equivalent SNR of 15 and not 17. The same performances (3 data bits/symbol) can be ensured on a channel with an SNR with 2 dB smaller when using a 16-QAM with a code having R_c=3/4, than when using a 64-QAM with a code of rate R_c=1/2.

From the table, the ranges of use for every coded modulation can be extracted. For example, between 3 and 5 dB –BPSK with code of rate 1/2,from 5 to 6 dB, BPSK with code of rate 3/4, from 6 to 8 dB –QPSK with code of rate 1/2, from 8 to 11dB – QPSK with code 3/4, from 11 to 15dB – 16-QAM with code 1/2, from 15 to 19 dB 16-QAM with code ¾, from 19 to 21 64-QAM with code 2/3 and from 21 dB on – 64-QAM with code 3/4.
The method used to compute these thresholds is an approximate one.
Note: In a real Rayleigh-faded channel a margin of 5-7 dB should be included, i.e. the thresholds should be increased by 5-7 dB according to the following correspondence rule.:3 → 8; 5 → 10; 6 → 11; 8 → 14; 11 → 18; 19 → 26; 21 → 28.

1.4. Correlation with the standard's levels for the received signal

The standard specifies the minimum levels of the received signal for every coded modulation. The purpose of this section is to check the values obtained for the SNR thresholds. Knowing the bandwidth of the signal and considering that the power spectral distribution of the noise (N_0) is constant, the obtained values should approximately match the ones in the standard using the formula:

$$SNR[dB] = 10\lg\left(\frac{P_{sgn}}{P_{noise}}\right) = 10\lg\left(\frac{P_{sgn}}{N_0 \cdot BW}\right) = P_{sgn}[dBm] - N_0[dBm/kHz] - 10\lg BW$$

Correlating these thresholds with the received signal levels from the standard, for a constant bandwidth, the power spectral distribution of the noise should turn out to be approximately constant. Considering the bandwidth BW= 52·0.3125=16,25MHz and using the formula from the beginning of this section with a power spectral distribution of the noise $N_0 \approx$-127dBm/kHz we get the values of the SNR as:

$$SNR[dB] = P_{sgn} + 127 - 42.1 = P_{sgn} + 85\ (84.9)$$

The levels of the received signals given in the standard, and the corresponding SNR thresholds are presented in the table below for each coded modulation.

Code rate	BPSK	QPSK	16-QAM	64-QAM
1/2	-82dBm (3)	-79dBm (6)	-74dBm (11)	-
2/3	-	-	-	-66dBm (19)
3/4	-80dBm (5)	-77dBm (8)	-70dBm (15)	-64dBm (21)

Table 3.5. Minimum received levels for the coded constellations

In order to check the validity of the theoretical results, simulations concerning the use of the presented modulations, simulations were performed with the DTL radio channel simulator. The results presented in figure 3.5. match the theoretical estimations made for the coded modulations, with their thresholds and nominal bit rates. The 64-QAM with code of rate 1/2, is also represented, in order to illustrate what was explained in section 1.3.
For packets of 4195 byte-long the threshold should be increased with 0.8 -1 dB.

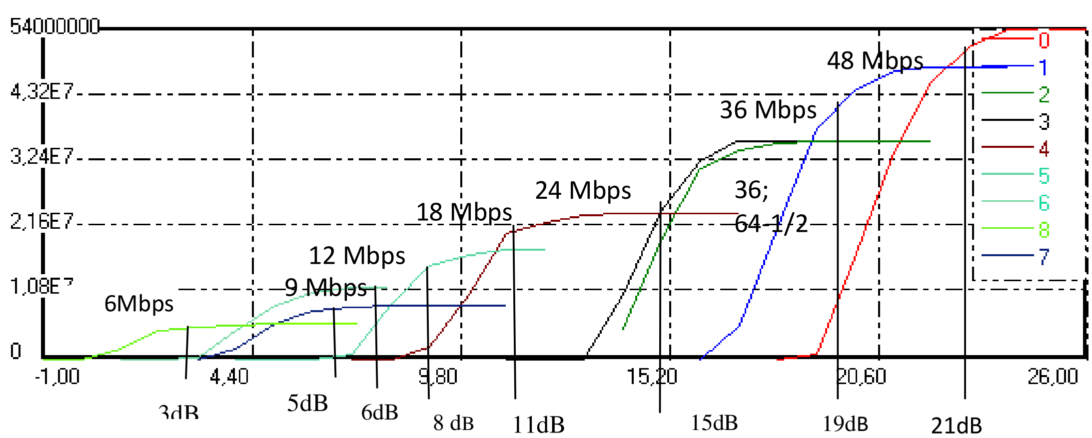

Fig. 3.5. SNR thresholds and bit rates for BPSK with R_c=1/2; BPSK with R_c=3/4; QPSK with R_c=1/2; QPSK with R_c=3/4; 16-QAM with R_c=1/2; 16-QAM with R_c=3/4; 64-QAM with R_c=1/2; 64-QAM with R_c=2/3; 64-QAM with R_c=3/4;