Pulse Coded Modulation

PCM (Pulse Coded Modulation) is a voice coding technique defined by the ITU-T G.711 standard and it is used in digital telephony to encode the voice signal.

The first step in the analog to digital conversion of the voice signal is the **filtering** of the analog signal, meaning the limitation of the frequency band to [300 Hz, 3400 Hz]. The next step is the **sampling**, using a frequency which fulfills the sampling theorem, $f_s > 2*f_m$, and having the value $f_s = 8$ kHz. Have to be mentioned that the filtering has the purpose of avoiding the aliasing phenomenon. After the sampling process the next step is the **compression of the signal,** process which realizes the **non-uniform quantization**.

Companding

Companding consists in compression of the signal to be transmitted at transmission side and expansion at reception (companding = compressing + expanding). This operation is performed according to the μ (1) and A (2) **compression laws**.

μ law	A law
$f(x) = \mathrm{sgn}(x) \cdot \dfrac{\ln(1+\mu \cdot \|x\|)}{\ln(1+\mu)}, 0 \le x < 1$ (1) - employed in USA and in Japan; - μ=255;	$f(x) = \begin{cases} \dfrac{A \cdot x}{1+\ln(A)}, 0 \le \|x\| < \dfrac{1}{A} \\ \mathrm{sgn}(x) \cdot \dfrac{1+\ln(A \cdot \|x\|)}{1+\ln(A)}, \dfrac{1}{A} \le \|x\| < 1 \end{cases}$ (2) - employed in Europe; - A = 87.6, value which provides continuity of the function;

Problem: deduce the mathematical expressions of the expansion laws corresponding to the presented compression laws (variable x *function* of variable y).

In Figure 1 it is presented the processing involved by the compression and the expansion process:

- the samples of the voice signal are quantized on 16 bits using uniform quantization;
- it is employed one of the compression laws (1) and (2)
- the least significant 8 bits are suppressed (see Figure 2)

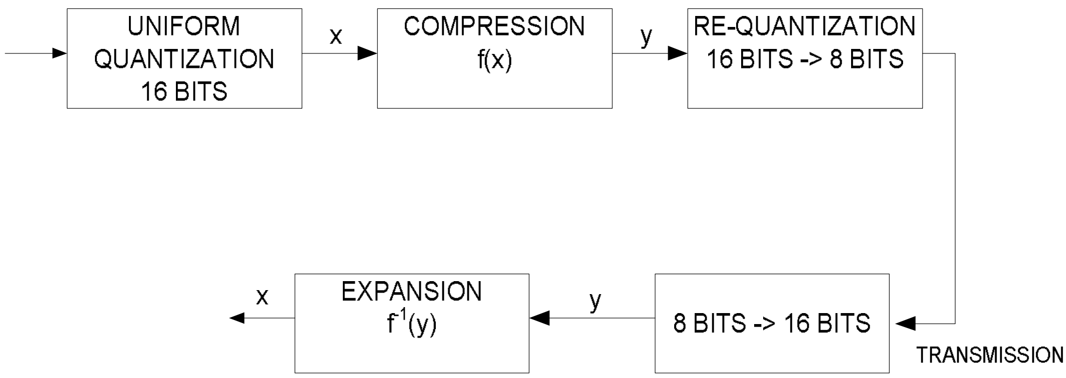

Figure 1 – Block schematic of the transmission process using companding

At the receiver the 8 bit PCM word is converted back into a 16 bit word by appending 8 LSBs having the value 10000000_2 (128_{10}) – see Fig. 2, reducing in this way to half the quantization error. In the next step the expansion function is applied to the obtained 16bit PCM word.

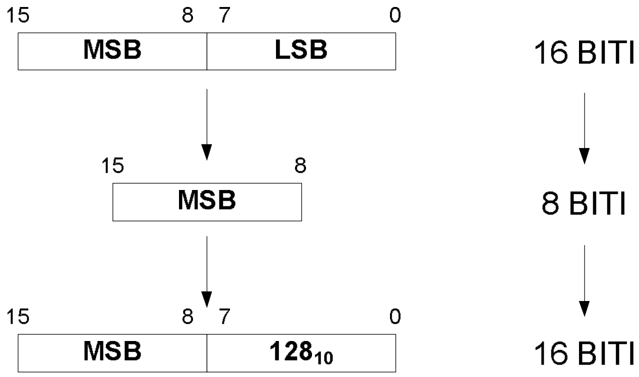

Figure 2 – The re-quantization process

The μ compression law

In Figure 3 it is presented the normalized μ law compression characteristic for different values of the μ parameter, and in Figure 4 it is presented the approximated characteristic of the μ compression law.

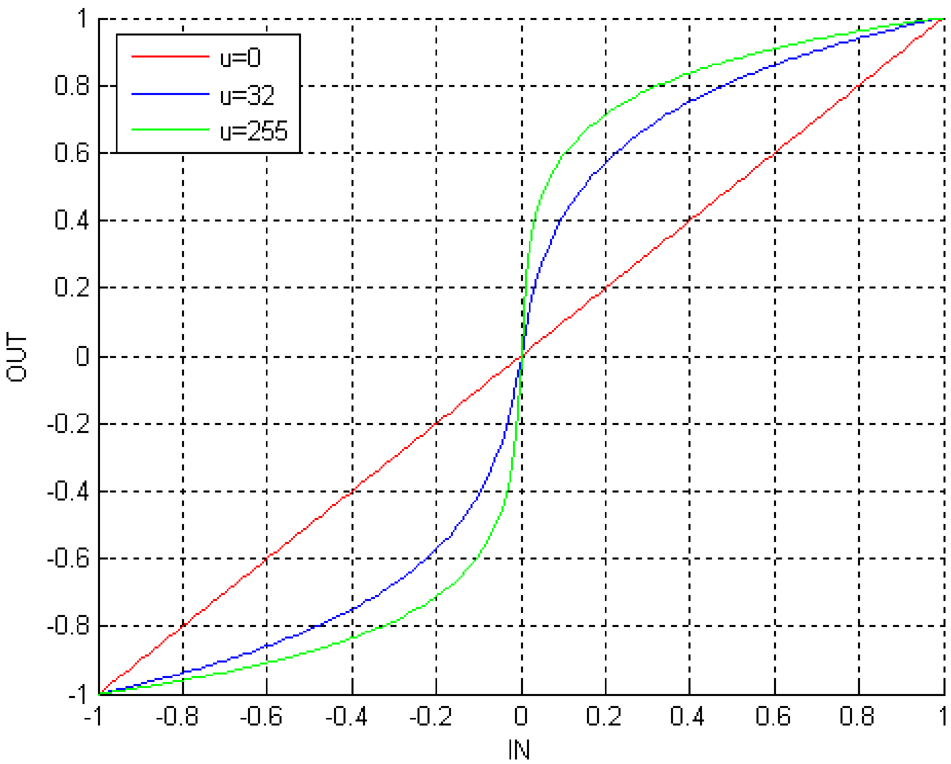

Figure 3 – Normalized μ law compression characteristic

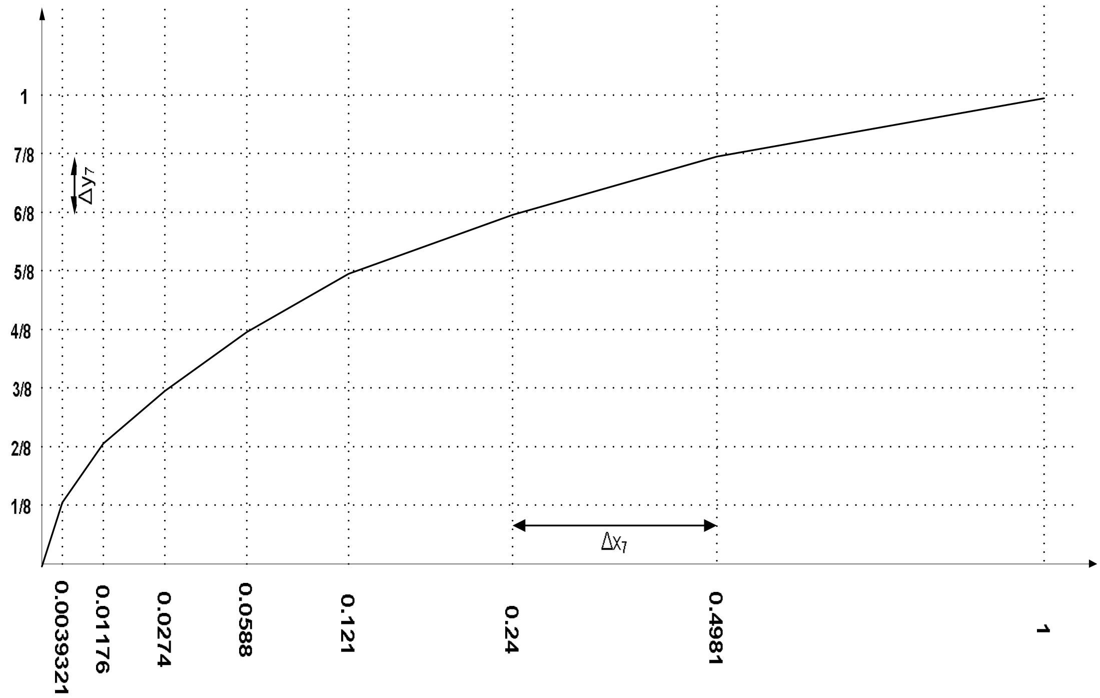

Figure 4 – Approximated/segmented μ law compression characteristic – positive values only

In practice the characteristic presented in Figure 3 (μ=255) is approximated in 16 segments, 8 for the positive values and 8 for the negative values (Figure 4), and each segment is split in 16 sub segments (Figure 5). Inside each segment on the X axis it is performed a uniform quantization on 4 bits.

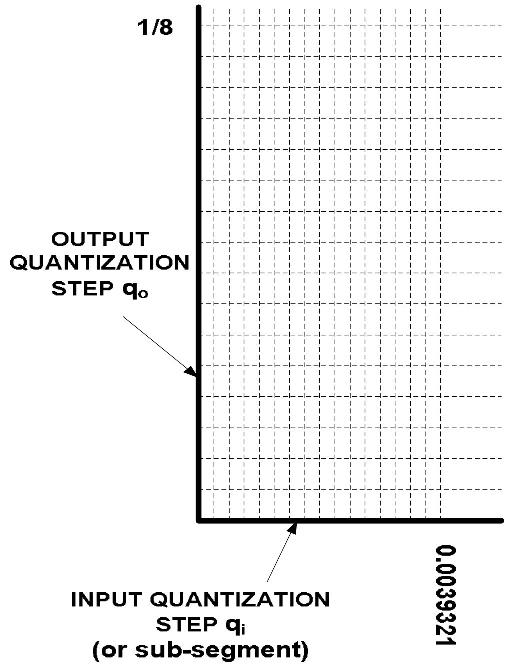

Figure 5 - Segment 1 (splitting in sub-segments)

Encoding of the input signal is performed in the following way (Figure 6):

- the most significant bit, b_7, indicates the signal polarity;
- the next 3 bits, b_6 b_5 b_4 indicate the segment;
- the last 4 bits, b_3 b_2 b_1 b_0, indicate the sub segment;

b_7	b_6	b_5	b_4	b_3	b_2	b_1	b_0
SIGN	SEGMENT			SUB-SEGMENT			

Figure 6 – PCM encoding with compression

Questions:

1) It is given a compression characteristic approximated in 4 segments, and each segment is split into 8 sub-segments. How many bits are required for PCM encoding?
2) A compression characteristic is described by the sequence of coordinates (0,0) – (0.1,0.25) - (0.3,0.5) - (0.6,0.75) – (1,1). Give the sequence of coordinates for the segmented expansion characteristic.

Calculation of some of the compression characteristics parameters:

- **Compression ratio for segment i**: $Rc_i = \frac{\Delta x_i}{\Delta y_i}$, where Δx_i represents the length of segment i at the "input" of the characteristic (X axis), and Δy_i is the length of segment i at the "output" of the characteristic (Y axis) (see Figure 4); the length of any segment at the "output" is 0.125. When the compression ratio is smaller than 1 means that we have expansion, and when the compression ration is larger than 1 means that we have compression.
- **Input quantization step on segment i:** represents the length of the sub-segments of the "input" segment i and can be computed as: $q_i = \frac{\Delta x_i}{No.\,sub-segments} = \frac{\Delta x_i}{16}$.
- **Output quantization step:** represents the length of the sub-segments of the "output" segments and can be computed as: $q_o = \frac{\Delta y_i}{16} = \frac{0.125}{16} = 0.0078125$.
- **Elementary quantization step:** represents the quantization step used for uniform quantization providing the same (or similar) performances as the non-uniform quantization (14 bits in the case of μ law): $q_e = \frac{2}{2^b} = \frac{2}{2^{14}} = 0.00012207$.
- **Number of elementary quantization steps of the input quantization step:** $n_i = \frac{q_i}{q_e}$
- **The power of the quantization noise on segment i:** $Pnq_i = \frac{q_i^2}{12}$.

- **Total (average) power of the quantization noise:** $Pnq = \sum_{\substack{i=-8 \\ i\neq 0}}^{8} p_i \cdot Pnq_i$ **,**

  where p_i represents the probability that the amplitude of the sample is located in segment i. If we have the same occurrence probability of each sample (uniform amplitude distribution) then the probability that a sample is located in segment i is equal with the length of this segment, if the characteristic is normalized, meaning that $p_i = \Delta x_i$.

- **Total (average) power of the quantization noise (considering only positive segments):** $Pnq' = \sum_{i=1}^{8} p_i \cdot Pnq_i$, p_i has the same definition.

Segment (i)	Δx_i	Rc_i	q_i	n_i	Pnq_i
1	0.003921	0.031368	0.000245063	2.007552	5.00464E-09
2	0.007839	0.062712	0.000489938	4.013568	2.00032E-08
3	0.01564	0.12512	0.0009775	8.00768	7.96255E-08
4	0.0314	0.2512	0.0019625	16.0768	3.20951E-07
5	0.0622	0.4976	0.0038875	31.8464	1.25939E-06
6	0.119	0.952	0.0074375	60.928	4.6097E-06
7	0.2581	2.0648	0.01613125	132.1472	2.16848E-05
8	0.5019	4.0152	0.03136875	256.9728	8.19999E-05

Table 1 – Computed parameters for the μ compression characteristic

If we compute the total power of the quantization noise, when non-uniform quantization is employed, we obtain $Pnq' = 47.39 \cdot 10^{-6}$W and if we compute the total power of the uniform quantization, on the same number of bits, we get $Pnq_{uniform}' = 5 \cdot 10^{-6}$W. Even if the noise power of the uniform quantization is smaller than the noise power of the non-uniform quantization, we can notice in Table 1 that for the first segments, meaning for low signal values where the hearing is more sensitive, the noise power is much higher in the case of uniform quantization than in the case of non-uniform quantization. The quantization error at high signal levels is less detectable by the human hearing (having less importance in this case) but influences significantly the total quantization noise power, which is more important for data transmissions.

Supplementary information at:

- http://www.educypedia.be/electronics/telephonetopics.htm
- http://telecom.tbi.net

Questions

1) Specify the advantages and disadvantages of the non-uniform quantization.
2) Calculate the parameters given in Table 1 for the A compression law.
3) A compression characteristic is approximated in 4 segments: (0,0) – (1/8,0.25) – (1/4,0.5) – (1/2,0.75) – (1,1), and each segment is divided into 4 sub-segments. Which is the code generated if the input sample's amplitude is 0.755? (The coordinates of the characteristic are specified as (x,y))