DELTA problems:

1. It is given the following signal: x(t)=sin(2π100t). The sampling frequency is $f_e = 10f_m$ (f_m – the maximum frequency from the spectrum of the signal).
   a. Compute the quantization step for non-adaptive Delta modulation.
   b. Employing the non-adaptive Delta modulation find the sequence of bits obtained at the output of the modulator.
   c. Represent graphically the predicted signal using the non-adaptive Delta modulation.
   d. Calculate the signal to noise ratio in the case of non-adaptive Delta modulation.
   e. Employing the Song modulation find the sequence of bits obtained at the output of the modulator. The initial quantization step is $\Delta_0 = 0.2$, and the elementary quantization step is $\Delta_e = 0.2$.
   f. Represent graphically the predicted signal using the Song modulation.
   g. Employing the Jayant modulation find the sequence of bits obtained at the output of the modulator. The initial quantization step is $\Delta_0 = 0.2$, and the adaptation constant is p = 1.2.
   h. Represent graphically the predicted signal using the Jayant modulation.

   Solution:

   a.

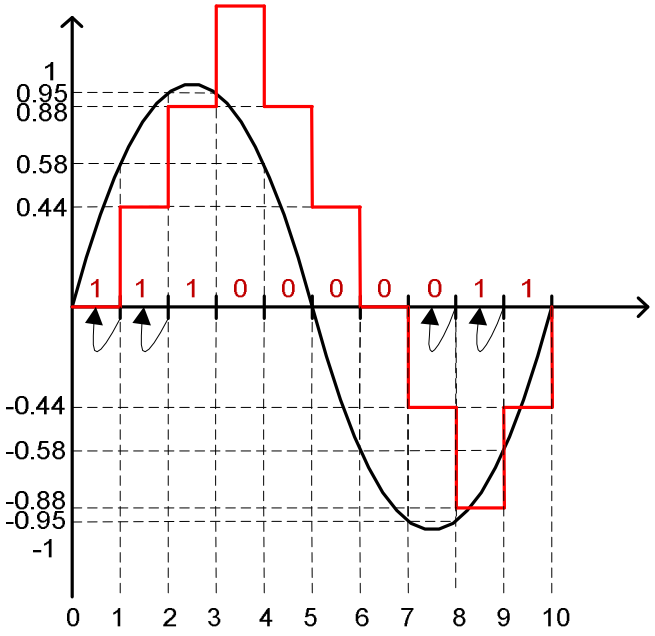
   - f_s=100Hz; $T_s=1/f_s$=0.1s (f_s – signal frequency)
   - Quantization step: $\Delta = \left|\frac{dx(t)}{dt}\right| T_e$
   - $T_e = 1/f_e = 1/10f_s = 1/10*100Hz = 1/1000Hz = 1ms$
   - The value of Δ which fulfills the conditions necessary for no slope distortion:
     $\Delta = 10^{-3}*200\pi\cos(2\pi100t)$
   - Possible values for Δ:

     $$\left|\frac{dx(t)}{dt}\right| = 2\pi 100 \cdot \cos(2\pi 100t)$$

     $$\begin{cases} x'(0) = 2\pi 100 \Rightarrow \Delta = 2\pi 100 \cdot 10^{-3} = 0.628 \\ x'(T_s/8) = 2\pi 100 \cdot 1/\sqrt{2} \Rightarrow \Delta = 2\pi \cdot 100 \cdot 1/\sqrt{2} \cdot 10^{-3} = 0.444 \\ x'(T_s/4) = 0 \end{cases}$$

   - We choose $\Delta = 10^{-3}*200\pi\cos(2\pi100\cdot T_s/8) = 0.2\pi*1/\sqrt{2} = 0.44$
     - Ensures relatively low slope distortion
     - Ensures a small quantization noise

   b. c. Find the values of the signal in the sampling moments:
   $x(10^{-3}) = \sin(2\pi0.1) = 0.58$
   $x(2*10^{-3}) = \sin(2\pi0.2) = 0.95$
   $x(3*10^{-3}) = \sin(2\pi0.3) = 0.95$
   $x(4*10^{-3}) = \sin(2\pi0.4) = 0.58$
   $x(5*10^{-3}) = \sin(2\pi0.5) = 0$
   $x(6*10^{-3}) = \sin(2\pi0.6) = -0.58$
   $x(7*10^{-3}) = \sin(2\pi0.7) = -0.95$
   $x(8*10^{-3}) = \sin(2\pi0.8) = -0.95$
   $x(9*10^{-3}) = \sin(2\pi0.9) = -0.58$
   $x(10*10^{-3}) = \sin(2\pi1) = 0$



d. $P_s = \frac{A^2}{2} = 0.5$, $P_{zq} = \frac{\Delta^2}{12} = \frac{0.44^2}{12} = 0.016133$, $SNR = \frac{P_s}{P_{zq}} = \frac{0.5}{0.016133} = 30.99 = 14.91dB$

e. f.

- **The Song algorithm**

$\Delta_0 = 0.2$

$\Delta_e = 0.2$

$b_0 = 0$

$b_1 \neq b_0 \Rightarrow \Delta_1 = \Delta_0 = 0.2$

$b_2 = b_1 \Rightarrow \Delta_2 = \Delta_1 + \Delta_e = 0.4$

$b_3 = b_2 \Rightarrow \Delta_3 = \Delta_2 + \Delta_e = 0.6$

$b_4 \neq b_3 \Rightarrow \Delta_4 = \Delta_3 - \Delta_e = 0.4$

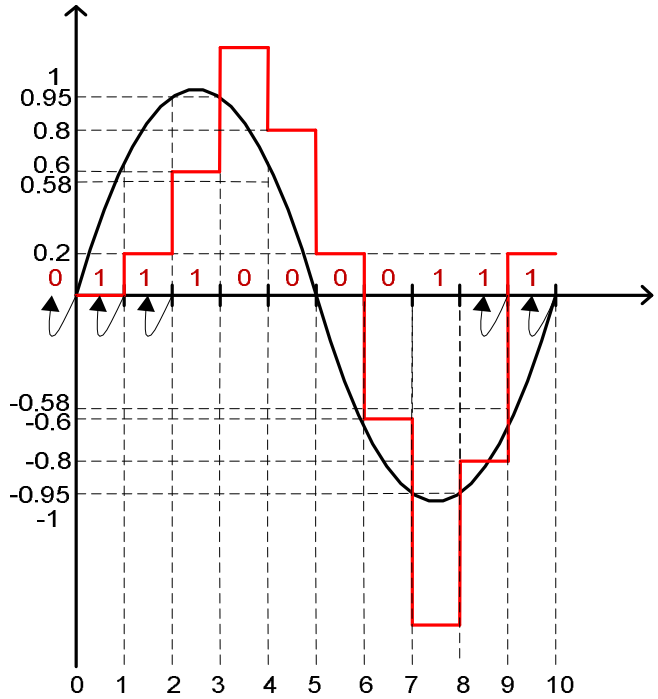
$b_5 = b_4 \Rightarrow \Delta_5 = \Delta_4 + \Delta_e = 0.6$

$b_6 = b_5 \Rightarrow \Delta_6 = \Delta_5 + \Delta_e = 0.8$

$b_7 = b_6 \Rightarrow \Delta_7 = \Delta_6 + \Delta_e = 1$

$b_8 \neq b_7 \Rightarrow \Delta_8 = \Delta_7 - \Delta_e = 0.8$

$b_9 = b_8 \Rightarrow \Delta_8 = \Delta_7 + \Delta_e = 1$



a. h.

- **The Jayant algorithm**

$\Delta_0 = 0.2$

$p = 1.2$

$b_0 = 0$

$b_1 \neq b_0 \Rightarrow \Delta_1 = \Delta_0 = 0.2$

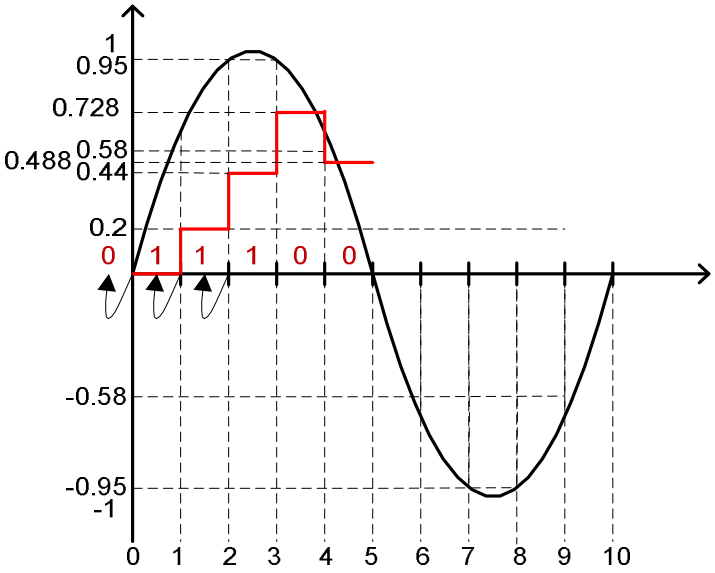
$b_2 = b_1 \Rightarrow \Delta_2 = \Delta_1 * p = 0.24$

$b_3 = b_2 \Rightarrow \Delta_3 = \Delta_2 * p = 0.288$

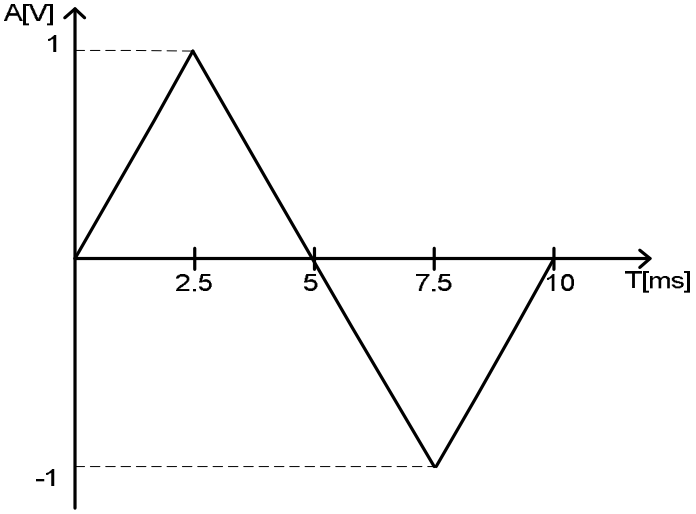
$b_4 \neq b_3 \Rightarrow \Delta_4 = \Delta_3 / p = 0.24$

$b_5 = b_4 \Rightarrow \Delta_4 = \Delta_3 * p = 0.288$

a.s.o.



2. It is given the signal in the figure. The sampling frequency is $f_e = 10f_s$ (f_s – frequency of the signal)



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