The ASK&PSK (QAM) combined modulation

- the independent employment of the ASK, see the lectures on PAM, or PSK, see the lectures on PSK, modulations for N ≥ 8 vectors provide great symbol-error probabilities.
- to ensure a high spectral efficiency, β_W = D/BW, for a BER below an imposed limit, a combined amplitude and phase modulation (ASK&PSK) is employed; it provides better SNR performances, i.e. ensures the same BER values at smaller SNR values, than the ASK or PSK modulations.
- the signal constellations employed are generated and demodulated by the quadrature amplitude modulation technique → the ASK&PSK constellations and modulations are also called QAM constellations and modulations, respectively.

The expression of the ASK&PSK modulated signal

- A+PSK – modulation where the amplitude and phase of the carrier signal belong each to a finite set of values, A and Φ.
- the values of the two parameters are kept constant during a symbol period T, being dictated by the modulating bit-combination (multi-bit) transmitted during that symbol period.
- the QAM signal is expressed by (1), where the amplitude and phase-shift during the k-th symbol period are denoted by A_k and Φ_k, V_0 is the amplitude of the carrier and V_r is the multiplier's reference voltage.

$$s_{QAM}(t) = \frac{V_0}{V_r} \sum_{k=-n}^{n} A_k \cdot \cos(\omega_c t + \Phi_k) \cdot u_T(t - kT); \tag{1}$$

- the signal (non-filtered) during a symbol period can be expressed as (2).a. Note that the I_k and Q_k coordinates are no longer independent, but fulfill condition (2).b:

$$\begin{aligned} s_{MAQ}(t) &= [V_0 A_k \cos\Phi_k u_T(t-kT)\cos\omega_c t - V_0 A_k \sin\Phi_k u_T(t-kT)\sin\omega_c t] / V_r = \\ &= I_k u_T(t-kT)\cos\omega_c t - Q_k u_T(t-kT)\sin\omega_c t; \quad \text{a.} \\ A_k^2 &= I_k^2 + Q_k^2; \quad \text{b.} \end{aligned} \tag{2}$$

- if $V_0 = V_r$, relation (1) represents the vector in polar coordinates A_k and Φ_k, and relation (2) expresses the same vector in Cartesian coordinates I_k and Q_k, within a system orthogonal axes composed of the two carrier signals, cosine (considered the phase-reference signal) and sine, the quadrature signal.
- the MAQ signal can also be expressed in a complex form as the real part of product between the complex modulating symbol c_k and the complex carrier $e^{j\omega t}$ as:

$$\begin{aligned} m_k &= I_k + jQ_k = A_k \cdot e^{j\Phi_k}; \text{ a.} \\ s_{PSK}(t) &= \mathrm{Re}\{(I_k + jQ_k)\cdot(\cos\omega_c t + j\sin\omega_c t)\} = \mathrm{Re}\{A_k \cdot e^{j(\omega_c t + \Phi_k)}\}; \quad \text{b} \end{aligned} \tag{3}$$

- expression (2) shows that the ASK&PSK signal can be expressed as the sum of two DSB-SC signals, see the LM lectures, modulated on two orthogonal carrier signals that have the same frequency.
- since the transmission is structured into symbol periods of duration T_s, during which the parameters of the modulated signal take constant values, the axes of the orthogonal system in the decomposition (2) are actually the signals $u_T(t\text{-}kT)\cdot V_0\cos\omega_c t$ and $u_T(t\text{-}kT)\cdot V_0\sin\omega_c t$.
- with these assumptions, the orthogonality of the two carrier signals is to be analyzed considering two aspects:
  - the integral over one carrier period of their product equals zero, meaning that the modulating signals of the two branches could be separated, see lectures on DPSK-QAM.
  - the integral of their product over one symbol period may be written as:

$$\frac{V_0^2}{V_r \cdot T_s} \int_0^{T_s} u_T^2(t - kT_s) \cdot \sin(\omega_c t) \cdot \cos(\omega_c t) \cdot dt = \frac{V_0^2}{V_r \cdot 2\pi} \cdot \frac{T_c}{T_s} \cdot \sin^2 2\pi \frac{f_c}{f_s} \in \left[0, \ \frac{V_0^2}{2 \cdot V_r \cdot 2\pi} \cdot \frac{T_c}{T_s}\right] \tag{4}$$

- expression (4) shows that the average value of the product of the two signals over a symbol period has a constant value, which depends only of the transmission parameters V_0, T_s and f_c and does not depend on the modulating signal; for $f_c >> f_s$ this integral is close to zero.
- these two signals compose the basis in the bi-dimensional space of the ASK&PSK modulated signals. Therefore, the value of the QAM demodulated signal, see later, is proportional to the modulating signal.
- the energy of the bi-dimensional modulating signal over a symbol period is expressed by (5), and depends of the modulating levels, assuming that $V_0 = V_r$.

$$E_s = \frac{A_k^2 \cdot V_0^2 \cdot T_s}{2 \cdot V_r^2} = \frac{(I_k^2 + Q_k^2)T_s}{2} \tag{5}$$

Types of ASK&PSK signal constellations. Constellation parameters

- the geometrical representation of the permitted sets of amplitudes and phase-shift values of the modulated signal is a Cartesian one, see (2); the axes Ox (I) and Oy (Q) represent the signals $u_T(t-kT)V_0\cos\omega_c t$, and $u_T(t-kT)\cdot V_0\sin\omega_c t$, respectively.
- the constellations could be classified according to the way the vectors are displaced in the I-Q plane
- the most employed signal constellations are shown in figure 1, A_0 being the elementary amplitude unit of the two modulating signals I_k and Q_k; A_0 could considered as a scaling factor

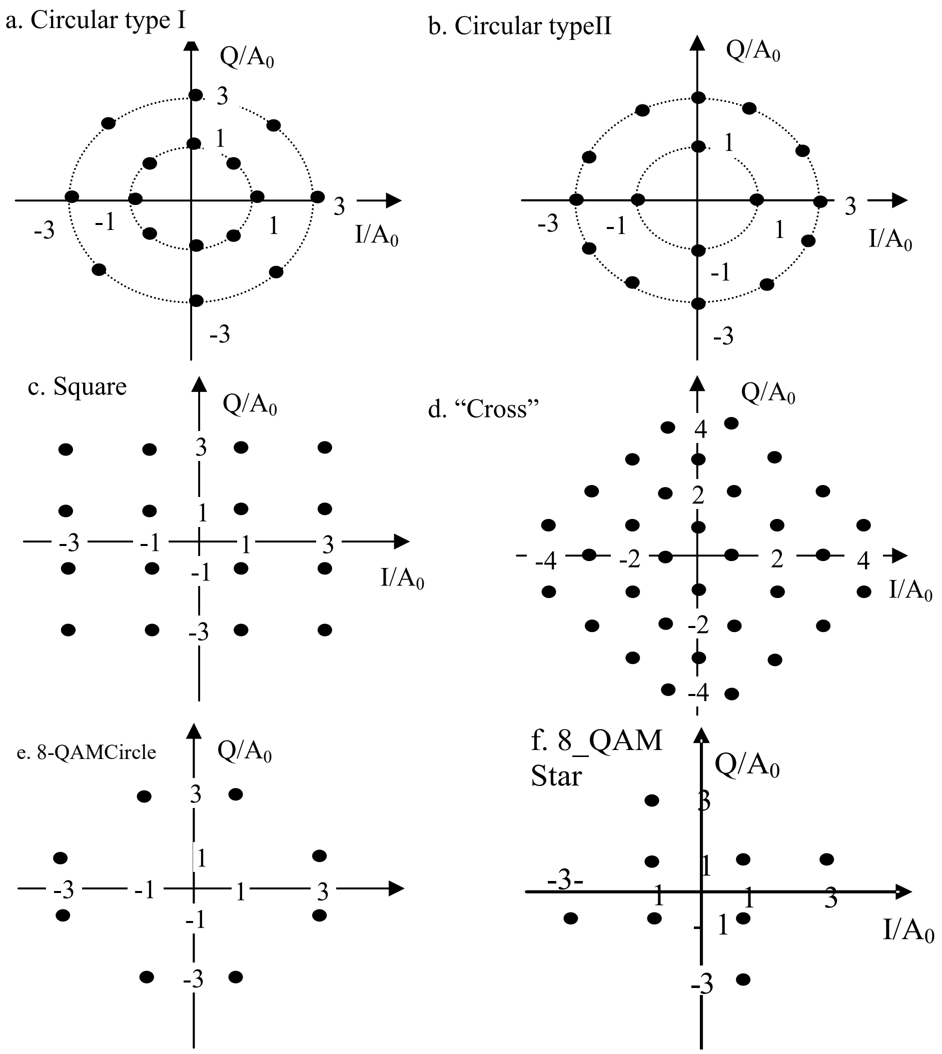


Figure 1 Main types of QAM constellations

- figure 1 shows the main types of constellations:
- there are two types of circular constellations, i.e., type I and II (a. and b.),
- square constellations, c.
- "cross" constellations, d.,
- the 8-QAM is a special case, two such constellations being also represented in fig. 1
- other types of constellations employed in special cases are shown in [fuqin].
- the power of the modulated signal during one symbol-period, (6), is computed using (5) and depends on the transmitted vector:

$$P_{sk} = (I_k^2 + Q_k^2)/2; \quad (6)$$

- the Euclidean distance between vectors, f_i and f_j, is expressed by (7):

$$d_E(f_i, f_j) = \sqrt{|f_i - f_j|^2} = \sqrt{(I_i - I_j)^2 + (Q_i - Q_j)^2}; \quad i \neq j; \quad i, j \in N; \quad (7)$$

- ***the parameters of the signal constellations*** are:

1. **The number of vectors within the constellation, N**.
2. **The number of bits/symbol, n**, which indicates the number of bits that are „carried" by a vector during a symbol period. The relation linking the two magnitudes is: $N = 2^n$ (8)

- the transmission bit rate D, is expressed, in terms of the „modulation" speed v_t, which numerically equals the symbol frequency f_s:

$$D\,[\text{bit/s}] = v_t\,[\text{symb/s}] \cdot n\,[\text{bit/symb}]; \quad (9)$$

3. **The average power of the constellation vectors** – is computed using (6), for harmonic carrier signals:

$$P_m = \frac{\sum_{k=1}^{N} P_{sk}}{N} = \frac{\sum_{k=1}^{N}(I_k^2 + Q_k^2)}{2N} - \textit{on the carrier signal}; \quad P_m = \frac{\sum_{k=1}^{N}(I_k^2 + Q_k^2)}{N} - \textit{in baseband}; \quad (10)$$

4. **The peak power of the constellation vectors** (11) – should as small as possible, for an imposed P_m value

$$P_v = \max P_{sk}; \; k \in \{1, \ldots, N\}; \quad (11)$$

5. **The PAPR factor** – the ratio between the peak and the average power

- it should be as close to unity as possible to decrease the amount of distortions inserted by the final radiofrequency amplifiers (see TD lectures). It is expressed logarithmically by:

$$PAPR = 10\lg(P_v / P_m)\,[\text{dB}]; \quad (12)$$

6. **The minimum Euclidean distance between the constellation vectors, Δ_0**, defined by:

$$\Delta_0 = \min (d(f_i, f_j)); \ i,j \ \varepsilon \ [1,\ldots,N]; \ i \neq j; \quad (13)$$

- Δ_0 – affects the symbol-error probability. The Δ_0 could be increased only within the limits imposed by keeping an acceptable value of the PAPR for an imposed value of P_m. → the value of Δ_0 is a trade-off between a large value, requested by the decrease of p_e, and a smaller value, requested by a smaller value of PAPR.
- the value of the PAPR factor, for imposed values of P_m and Δ_0, is also depending of the constellation shape.

7. **The spectral efficiency factor β_w**, represents the ratio between the transmission bit rate and the occupied BW of the modulated and filtered signal, (14).a.
- since the ASK&PSK modulated signals are filtered with an RC characteristic, defined by α, their BW is the one of the QPSK signals. The β_w factor of the QAM transmissions is computed using (14).b.

$$\beta_w = \frac{D}{BW} [\frac{bit/s}{Hz}]; a. \qquad \beta_w = \frac{v_t \cdot n}{f_s(1+\alpha)} = \frac{n}{1+\alpha} [\frac{bit/s}{Hz}]; b. \quad (14)$$

8. **The sensitivity (to perturbations) factor S** defined by (15) - is employed in literature as a qualitative measure of the robustness of a constellation against the channel distortions and perturbations. A constellation is more robust to the channel impairments if its S factor is smaller.

$$S = P_m/\Delta_0^2; \quad (15)$$

Defining the ASK&PSK constellations

- from (2) → the vectors are defined by the I_k and Q_k coordinates.
- each of the two quadrature carriers is ASK modulated, but the two modulating levels are connected by (3), since they define a bi-dimensional modulating signal ; to ensure an optimum spectrum, i.e. to ensure a DSB-SC on each carrier signal, the vector coordinates should have a null average value (see the LM lectures).
- the way the coordinates are generated is specific for each constellation type mentioned above.
- for **square constellations** with a form factor f = 1, i.e. all neighboring vectors are equidistant, the number of bits/symbol has to be even; their numbers of vectors and of bits/symbol are linked by:

$$N = 2^n = (2^{n/2})^2 = L^2; \ n = 2p; \quad (16)$$

- → for a square constellation the number of levels on each axis (I or Q) should be:

$$L = \sqrt{N} = 2^{n/2}; \quad (17)$$

- the L symmetrical levels, with a separation of $2A_0$ and a null average value, are obtained by:

$$I_i(i), Q_i(i) = (2i + 1 - L)A_0; \quad i = 0, 1,\ldots,L-1; \quad (18)$$

- applying (18) on each axis, the vector coordinates in a square constellation are the pairs (I_i, Q_i), i.e. the elements of the Cartesian product {$I_i(i)$ x $Q_i(i)$}.
- the minimum Euclidean distance between the vectors of a square constellation is: $\Delta_0=2A_0$; (19)
- the average power P_m of a signal modulated with the vectors of a square constellation is the sum of the average powers of the two signals modulated on quadrature carriers (20), where: n – the number of bits/symbol, A – the carriers' amplitude, V_r – the multiplier's reference voltage. Note that the two average powers P_I, P_Q, have equal values and P_m is computed considering the average power of the carrier, factor 1/2.

$$P_m = P_I + P_Q = \frac{2A_0^2(L^2-1)}{3} \cdot \frac{A^2}{2V_r^2} = \frac{A_0^2(2^n-1)}{3}; \quad A = V_r; \quad L^2 = N \quad (20)$$

- the peak power P_v of the modulated signal on the carrier signals is:

$$P_v = (I_{max}^2 + Q_{max}^2) \cdot \frac{A^2}{2V_r^2} = 2I_{max}^2 \cdot \frac{A^2}{2V_r^2} = (2^{\frac{n}{2}} - 1)^2 \cdot A_0^2; \quad A = V_r; \quad (21)$$

- the ratio P_v/P_m and the PAPR factor of the signals modulated with the vectors of a square constellation are:

$$PAPR = 10 \lg \frac{P_v}{P_m} = 10 \lg \frac{3 \cdot (2^{\frac{n}{2}} - 1)}{(2^{\frac{n}{2}} + 1)}; \quad (22)$$

- (22) shows that P_v/P_m (PAPR) increases with the increase of n, from 1,8 (2,55 dB) for n = 4 (16 QAM) up to 3 (4,77 dB), for n → ∞.
- the most employed square constellations with f = 1 are 16-QAM, $I_{max} = Q_{max} = +/-3A_0$, 64-QAM, $I_{max} = Q_{max} = +/-7A_0$, 256-QAM, $I_{max} = Q_{max} = +/-15A_0$ and 1024-QAM, $I_{max} = Q_{max} = +/-31A_0$. The square constellations have non-uniform versions, with f = 2, 3 and 4 (see the DT lectures) being used in the DVB-T and DVB-S.

- **the „cross" constellations** are obtained from square constellations with N' vectors, by removing the P vectors placed in the corners; their number of vectors N is not a perfect square, but it is an odd power of 2; the generation procedure of the „cross" constellations will be described in the DT lectures.
- the minimum Euclidean distance between two vectors would be:

$$\Delta_0 = \sqrt{2} \cdot A_0; \tag{23}$$

- the most used „cross" constellations are 32-QAM, $I_{max} = Q_{max} = +/-5A_0$ and 128-QAM, $I_{max} = Q_{max} = +/-9A_0$.
- P_v and P_m of the signals modulated with the „cross" constellations are computed using (11) and (10).
- the $N=2^3 = 8$ vector QAM constellations are a special case. Fig. 1 presents a square (e.) and a star (f.) 8-point constellation. Their average and peak powers should be computed using (10) and (11), while their PAPR is computed using (12).
- table 1 shows the parameters of the square, „cross" QAM and 8-QAM constellations, $N \leq 256$ for $A_0 = 1$.
- P_m – is computed for the modulated signals, including the ½ factor inserted by the carrier signals.
-PAPR[dB]=10lg(P_v/P_m), is shown in its logarithmic expression.

N-QAM	4	8-circle	8-star	16	32	64	128	256
n-bit/symb.	2	3	3	4	5	6	7	8
P_v	0,5	5	5	9	8,5	49	42,5	225
P_m	0,5	5	3	5	5	21	20,5	85
PAPR [dB]	0	0	2.21	2,6	2,3	3,7	3,3	4,22
Δ_0	√2	2 (2.41)	2	2	√2	2	√2	2
S	0,25	0.625	0.75	1,25	2,5	5,25	10,25	21,25

Table 1 - Parameters of the main square and „cross" QAM constellations
- table 1 also contains the parameters of the QPSK (DPSK-A4) constellation of radius $A_0 = 1$.

Comments:
- the vector placement in the „cross" constellations was set to decrease the P_m and P_v to values comparable to the ones of the inferior neighboring square constellation, while using integer coordinates, to frame the power level of the modulated signal within the limits for the transmission channels and to ensure the smallest possible PAPR factor, for the radio channels.
- fulfillment of this requirement leads to a Δ_0 which is √2 times smaller than the one of square constellations, fact that increases the error-sensitivity of the transmissions using „cross" constellations about two times, compared to the inferior neighboring square constellations.
- the 8-QAM circle has the same PAPR but smaller Δ_0 than 4-PSK, for the same P_m; the 8-QAM "star" constellation has a greater PAPR and a smaller Δ_0, which leads to a greater sensitivity to errors, i.e. greater S.
- the QPSK has the PAPR and error-sensitivity much smaller than those of the ASK&PSK constellations → it is more appropriate for low-quality radio channels or channels with non-linear HFPA, at the expense of significantly smaller bit rate.
- the circular constellations would be discussed in the DT lectures.

Bit-mapping in the QAM constellations
The Gray bit-mapping

- the multibit-vector mapping according to the Gray code ensures a Hamming distance of only one bit between the multibits corresponding to neighbouring vectors of the 16-QAM square constellation, see fig. 2.

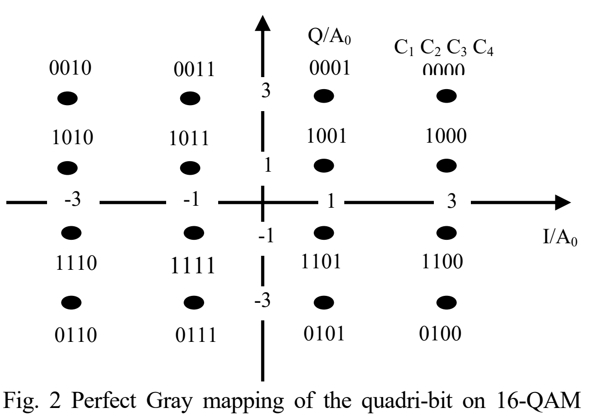


Fig. 2 Perfect Gray mapping of the quadri-bit on 16-QAM

- since the most probable errors are the replacement of one symbol with one of its neighbours → p_b (due to to perturbations) decreases significantly if this mapping is employed, see the PSK lectures.
- this mapping is a bi-dimensional one (it does not involve a previous Gray-natural conversion), since it is implemented in a tabular manner.
- square constellations allow for perfect Gray mapping;
- the „cross" and the type-II circular constellations do not allow for a perfect Gray mapping; for these constellations the average number of bit errors is slightly greater than 1, when a vector is mistaken for one of its neighbors.
- when the constellations are demodulated with the QAM method, see the DPSK-QAM lectures, the carrier recovery circuit may insert constant phase-shifts of k·90º (it is also valid for the A+PSK constellations) → the reception of a vector shifted with k·90º leads to the demodulation of a multibit that may contain up to n-1 bits differing from the transmitted ones;

- example; in fig. 2 the shift of 90º of vector (3, -1) → the received vector is (1, 3) → three bits differing from the ones of the correct vectors.
- the effect of this phase-shift is the increase of p_b for the same SNR → the main shortcoming of the Gray mapping → this mapping is not employed in most of the A+PSK constellation used in single-carrier transmissions because of the k·90º rotations specific to the QAM technique, when no pilot signals are used for the carrier-recovery in demodulation.

Bit-mapping to ensure the „k·90º rotationally invariant" constellations

- to compensate for the bit-errors generated by the k·90º rotations inserted by the local carrier-recovery circuit, some special methods of bit-mapping which would accomplish the so-called **„k·90º rotationally invariant constellations"** were developed.
- regardless of the error of the demodulated vector, if this error is inserted only by a k·90º rotation, the demodulated bits would be the ones of the transmitted vector, assuming that the errors inserted by the channel and other processing are neglected.
- formally, the bits of a multibit are divided into two groups: a first group composed of the first two bits and a second group composed of the rest of (n-2) of the multibit.
-*the first two bits* define the quadrant where the vector lies and, since the k·90º rotations involve the change of the quadrant, these bits are *differentially encoded* before the mapping, in the transmitter, and *differentially decoded*, after the demodulation, decision and demapping, in the receiver.
- the differentially encoded bits are mapped to the quadrants according to the Gray rule, the dibits allocated to neighbouring quadrants differing only by one bit.
- *the other n-2 bits* are mapped in different ways, according to the number of vectors in the constellation and employment/non-employment of a forward error correction. For the non-coded modulations, these bits are also mapped using a Gray mapping, which is independent of the first two bits.
- most of the standardized applications for band-limited channels employ constellations with N ≥ 32 vectors together with a error-correcting code, generating the so-called „coded modulations". The coded modulations, together with the specific bit-mapping techniques employed, will be dealt with in the coded-modulations chapter of the DT lectures.
- the only A+PSK constellations that were employed without an error-correcting code and only on vocal telephone channels are those with N ≤ 16; the construction of a rotationally invariant to k·90º constellation will be shown on a square 16-QAM constellation, see fig.3, where the framed numbers {0,...,15} are only labels that identify the vectors.
- the invariance to k·90º rotations is ensured by two operations:

a. *Differential encoding of the first two bits of the quadbit*

- this operation inserts a phase shift of k·90º between two consecutive symbols, see the DPSK-QAM lectures (14, .15), but, as opposed to the QPSK (4-PSK), it is performed in tabular manner, see table 2 [V.32]; the b_1^k b_2^k bits are the first two bits of the current data quadbit, the c_1^{k-1} c_2^{k-1} are the two bits differentially encoded in the previous symbol period and c_1^k c_2^k are the two bit differentially encoded in the current symbol period.
- the generating rule of table 2 is actually equivalent to the Gray-natural conversion composed with the differential encoding of the dibit, i.e. the modulo 4 summation.

Current input $b_1^k b_2^k$	Previous output $c_1^{k-1} c_2^{k-1}$	Current output $c_1^k c_2^k$	Current input $b_1^k b_2^k$	Previous output $c_1^{k-1} c_2^{k-1}$	Current output $c_1^k c_2^k$	Current input $b_1^k b_2^k$	Previous output $c_1^{k-1} c_2^{k-1}$	Current output $c_1^k c_2^k$	Current input $b_1^k b_2^k$	Previous output $c_1^{k-1} c_2^{k-1}$	Current output $c_1^k c_2^k$
00	00	01	01	00	00	10	00	11	11	00	10
00	01	11	01	01	01	10	01	10	11	01	00
00	10	00	01	10	10	10	10	01	11	10	11
00	11	10	01	11	11	10	11	00	11	11	01

Table 2
The table for differential encoding of the first two bits of 16-QAM

- the c_1^k c_2^k bits are the same for all vectors within a quadrant and the dibit-quadrant allocation is made according to Gray rule.
- the operation ensures the invariance of the first two bits to the k·90º rotations, by differential encoding, as well as a decreased number of bit errors at the change of the quadrant, due to phase errors differing of k·90º, by the Gray mapping of the bits that define the quadrant.

b. *The Gray mapping within a quadrant and its k·90º rotation at the quadrant change*

- the last two bits of the quadribit select the vector inside each quadrant
- the four two-bit combinations are Gray-mapped on the four vectors to ensure a minimum p_b when the vector is turned into one of its neighbours.
- the Gray mapping of these bits is rotated with 90º, for each different quadrant, in the same sense we cover

the four quadrants, see fig. 3.

- by this rotation of the Gray mapping, the last two bits mapped on the vectors with the same modulus but differing by k·90º phase-shifts, are identical.

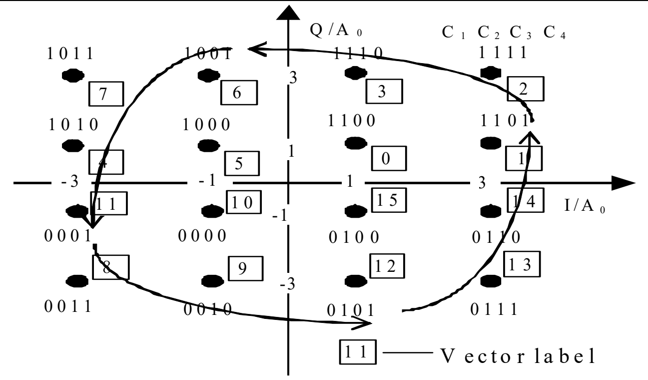


Figure 3. Quadribit mapping for a 16-QAM constellation invariant to k·90° rotations

- **example**: in fig. 3 the {1, 6, 11, 12} vectors, shifted with 90º from each other, have identical last two bits, namely „01". So, at a k·90º rotation these bits would not be in error, though the demodulated vector is different from the transmitted one.
- the rotation of the Gray mapping of the last two bits, when passing from a quadrant to another, ensures the invariance to k·90º phase-shifts of these bits. This method can be used on larger constellations to ensure the invariance of the last (n-2) bits to k·90º rotations.
- the example presented in table 3 shows that de differential encoding-decoding of the first two bits also ensures their invariance to k·90º rotations.

$b_1^k\ b_2^k\ b_3^k\ b_4^k$	$c_1^{k-1}\ c_2^{k-1}$	$c_1^k\ c_2^k\ c_3^k\ c_4^k$	F_k	F'_k	$c'^k_1\ c'^k_2\ c'^k_3\ c'^k_4$	$C'^{k-1}_1\ c'^{k-1}_2$	$b'^k_1\ b'^k_2\ b'^k_3\ b'^k_4$
01(01)	00	00(01)	11	6	10(01)	00	11(01)
11(10)	00	10(10)	4	3	11(10)	10	11(10)
10(00)	10	01(00)	15	10	00(00)	11	10(00)
00(11)	01	11(11)	2	13	01(11)	00	00(11)
01(01)	11	11(01)	1	12	01(01)	01	01(01)

Table 3 Example: invariance of 16-QAM to the 270º rotation by differential encoding-decoding

- consider that the first two bits of the modulating quadribits over five symbol periods b_i^k, i = 1,...,4, col. 1 of table 3, are differentially encoded according to table 2, using the first two bits encoded in the previous symbol period, $c_1^{k-1}\ c_2^{k-1}$, col. 2; we assume that $c_1^{-1}\ c_2^{-1} = 00$.
- the encoded bits, together with the last two non-coded bits compose the quadribits c_i^k, i = 1,...,4, which is mapped on the constellation, col. 3, generating the vectors F_k, see fig. 3, during the 5 symbol periods, col.4.
- since they are not differentially encoded, the last two bits fulfill the relation:

$$c_3^k\ c_4^k = b_3^k\ b_4^k; \tag{24}$$

- the F_k vectors suffer a 3·90º rotation in the positive trigonometric sense → the F'_k vectors, col. 5.
- col. 6 contains the demodulated quadribits c'^k_i, i = 1,...,4, from the F'_k vectors, during each symbol period. The last two bits $c'^k_3\ c'^k_4$ are identical to the transmitted ones and, since they observe (24), they would be identical to the last two bit of the quadribit delivered to the receiving computer, col. 8.
- col. 7 contains the first two bits, $c'^{k-1}_1\ c'^{k-1}_2$, of the quadribit resulted from the vector demodulated during the previous symbol period. By using the first two bits of the quadribit demodulated during the current symbol period, $c'^k_1\ c'^k_2$, and the first two bits obtained from the F'_{k-1} vector, $c'^{k-1}_1\ c'^{k-1}_2$, the differential decoding deliver the first two bits of the current quadribit, $b'^{k-1}_1\ b'^{k-1}_2$ (col. 8), which are identical to the $b_1^{k-1}\ b_2^{k-1}$ bits of the modulating quadribit, except for the first transmitted symbol. The differential de coding is performed using table 2.
- the first symbol is in error because it was not differentially encoded.
- this method can be employed for all non-coded constellations that exhibit symmetries with respect to the two coordinate axes and are generated using the QAM approach.
- in most practical applications the QAM constellations are combined with error-correcting (FEC) codes; in these cases the bit-mapping and the invariance are ensured by methods that take into account the properties of the employed FEC code

Filtering the ASK&PSK signals

- the global filtering of the ASK&PSK signals, required to limit the modulated signal's bandwidth is accomplished with an RC characteristic with a α roll-off factor, which ensures a null ISI in the probing moments.
- this characteristic is equally split between the transmitter and receiver to ensure better performances in the presence of noise, so that the signal is filtered with an RRC characteristic at the transmitting end.

- *filtering could be accomplished in two ways:*

1. by filtering the modulating I_k and Q_k signals with a LP-RRC characteristic. Thus, the frequency band of the filtered modulating signal would be:

$$FB = [0,\ f_N(1+\alpha)]; \tag{25}$$

2. by filtering the modulated ASK&PSK signal with a BP-RRC. So, the filtered signal's FB and BW are:

$$FB = [f_p - f_N(1+\alpha),\ f_p + f_N(1+\alpha)] \text{ a.}; \quad BW = f_s(1+\alpha) \text{ b}; \tag{26}$$

- considering expression (2) of the modulated ASK&PSK signal, its expression after filtering becomes:

$$s_{QAM}(t) = I(t)\cdot\cos\omega_p t - Q(t)\cdot\sin\omega_p t; \tag{27}$$

- the LP filtering of the modulating levels requires two filters, for the I_k, Q_k signals, of relatively small orders.
- the BP filtering of the modulated signal requires only one filter of a greater order.
- for the ASK +PSK constellations generated by the QAM technique, the LP filtering of the two modulating signals is preferred.
- the spectral efficiency of the QAM modulations is computed using the BW of the filtered modulated signal (27), which is not depending of the constellation employed, and the transmissions bit rate (28), and is expressed by (29).

$$D = f_s\cdot n = f_s\cdot \mathrm{ld}N; \tag{28}$$

$$\beta_w = \frac{D}{BW} = \frac{f_s\cdot \mathrm{ld}N}{f_s\cdot(1+\alpha)} = \frac{\mathrm{ld}N}{(1+\alpha)}\left[\frac{\text{bit}/\text{s}}{\text{Hz}}\right]; \tag{29}$$

- since the BW is the same, regardless the constellation employed, *the spectral efficiency increases (it is better!) with the increase of the constellation.*
- this involves the decrease of Δ_0, because P_m has to be kept approximately constant, → increase of p_e. →
- the employment of QAM constellations with great N ensures an efficient use of the bandwidth, but imposes the insertion of some error-correcting codes, of some channel equalizers and requires higher quality channels, to ensure a small p_b.
- the modems employing these constellations are considered spectrally efficient modems.

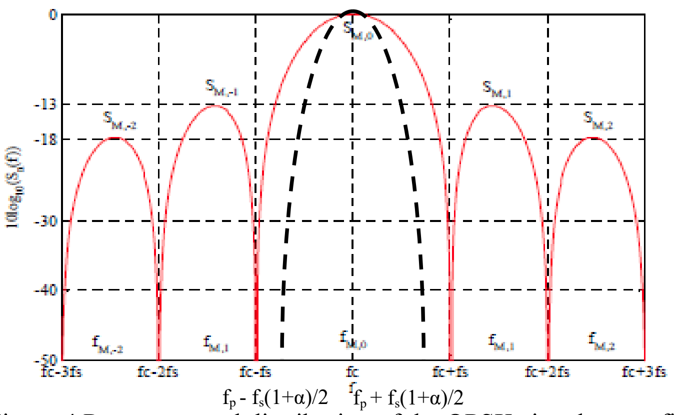
The spectrum of the A+PSK modulated signals

- the A+PSK modulated signals are expressed as a sum of two ASK modulated signals (2), on carriers of the same frequency, and the modulating levels of the two ASK signals have null average values → the expression of their power spectral density is obtained by adding the expressions of the power spectral densities of the two ASK signals
- applying the relations that define the power spectral density and average power of the ASK signal, to the QAM signal composed of the ASK signals on axes I and Q, we get its average power (20) and power spectral density (30), computed for non-filtered modulated signal.
- (30) shows that *the spectrum shape is not depending of constellation employed, as long as it has null-average coordinates;* only the amplitudes of the side lobes are depending on constellation's average power.

$$S_{QAM}(f) = (P_{mI} + P_{mQ})\cdot T\cdot\left(\frac{\sin\frac{\pi(f-f_c)}{f_s}}{\frac{\pi(f-f_c)}{f_s}}\right)^2 = P_m\cdot T\cdot\left(\frac{\sin\frac{\pi(f-f_c)}{f_s}}{\frac{\pi(f-f_c)}{f_s}}\right)^2; \tag{30}$$

- → the shape of the power spectral density of the ASK&PSK signal is similar to the QPSK modulated signal, see fig.4, for the same symbol frequency f_s. The main lobe is ranging between $f_c - f_s$ and $f_c + f_s$.
- if the signal is filtered with an RRC characteristic of factor α, see the lecture notes on Filtering the digital data, → its power spectral density is expressed by (31) and represented in figure 5 for f_s =0.33 kHz, α=0.5.

$$S_{QAM}(f) = P_m\cdot T\cdot(X_\alpha^{1/2}(f))^2 = P_m\cdot T\cdot X_\alpha(f); \tag{31}$$



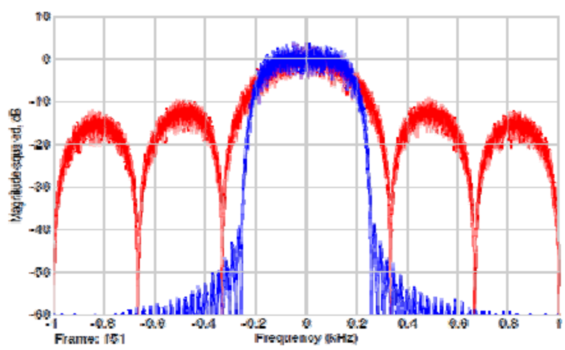


Figure 4 Power spectral distribution of the QPSK signal – non-filtered and RRC filtered - left
Figure 5 Power spectra of non-filtered and RRC-filtered PSK signals - snapshot - right

Generation of the A+PSK modulated signals

- the general method to produce the signals modulated with the ASK&PSK constellation is the QAM technique, see the QPSK lectures. The block diagram of such a modulator is shown in figure 5, for N = 16.
- regardless the number of bits/symbol, the first two bits of the multibit are differentially encoded.
- then, the n bits are mapped on the N constellation vectors, by a tabular generation of the (I_k,Q_k) coordinates.
- the RRC filtering is performed in the baseband by LP filters (FFE), delivering the continuous I(t) and Q(t) modulating signals that are modulated on the quadrature carriers.

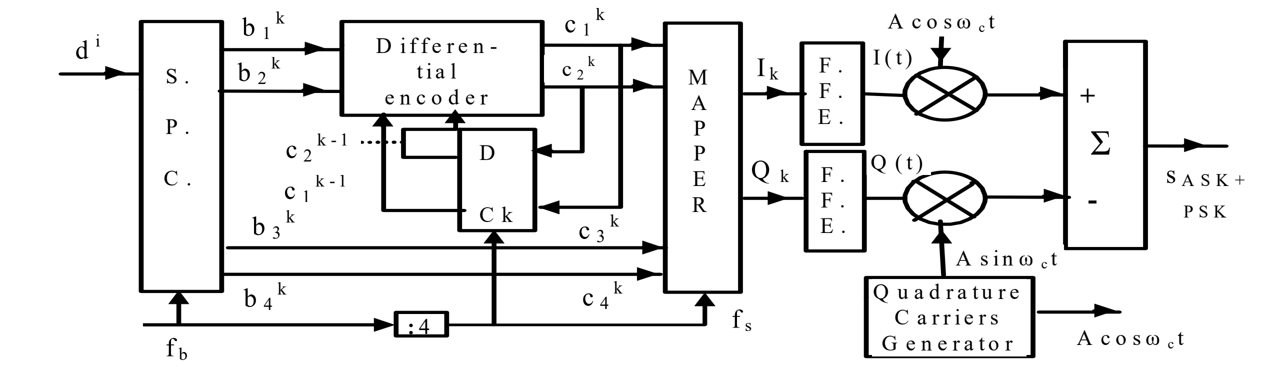


Figure 5. Block diagram of the ASK&PSK modulator built with QAM technique; n = 4

- these signals are then subtracted, generating the ASK&PSK modulated signal.
- the differential encoding and mapping are performed in the rhythm of the symbol clock f_s, which is obtained by dividing by n the bit clock f_b.
- *the square, „cross" and type-II circular constellations could be easily generated only by this approach.*
- current research evaluates the possibility to use Offset QAM modulations in transmissions that include a non-linear HP RF amplifiers, e.g., mobile phones, by examining the trade-off between the slightly increased BER due to noise vs. the distortions inserted by the non-linear amplifier and the small out-of-band spectral components.

Demodulation of the A+PSK signals

- the most employed ASK&PSK demodulation is the QAM technique, due to the property of these signals to be expressed under the QAM form, see (2).
- there are two variants of QAM demodulators:
- QAM demodulators that employ LP filters on each branch, see the DPSK-QAM lectures;
- QAM demodulators that employ the Hilbert transform of the received signal, see DT lectures
- the block diagram of the LPF variant adapted for the demodulation of the ASK +PSK signals is shown in fig. 6 for constellations with N=16. The figure does not show the signals employed by the carrier-recovery circuit, because they differ according to the recovery method employed, see DPSK-QAM and DT lectures.

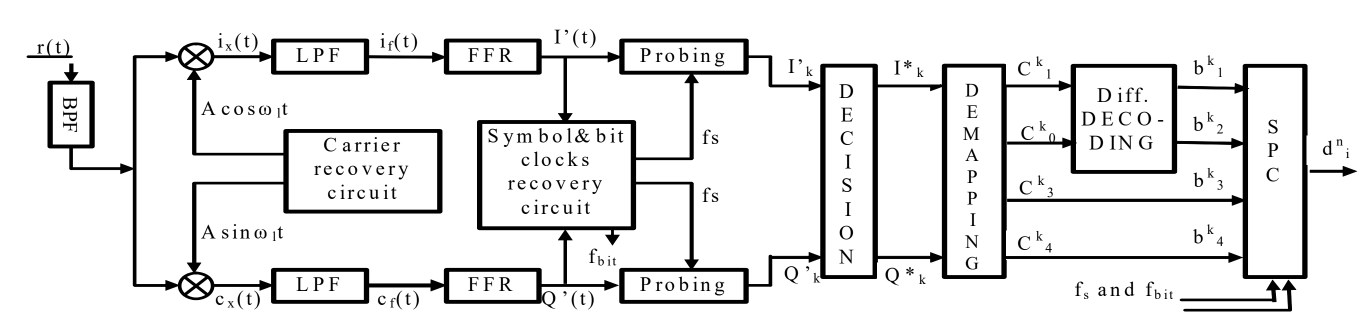


Figure 6. Block diagram of the ASK+PSK demodulator using the QAM technique; the LPF variant

- the considerations presented in the DPSK-QAM lectures regarding the LPF variant the QAM demodulator still hold for the demodulation of the ASK+PSK signals.
- the expression of the received A+PSK signal is expressed by (32), where I'(t) and Q'(t) denote the filtered modulating signals affected by the channel perturbations and distortions:

$$s_{rPSK} = r(t) = I'(t)\cdot A cos\omega_c t - Q'(t)\cdot A sin\omega_c t = A\cdot \mathrm{Re}\left\{\left(I'(t)+Q'(t)\right)\cdot e^{j\omega_c t}\right\} \quad (32)$$

- the expressions of the signals at the outputs of the multipliers on the two arms are:
- rewriting equations (5,…8) of the QAM chapter for the A+PSK signal we get:

$$i_x(t) = \frac{r(t)A\cos\omega_L t}{K} = \frac{AI'(t)}{2K}[\cos\theta(t) + \cos(2\omega_c t + \theta(t))] - \frac{AQ'(t)}{2K}[-\sin\theta(t) + \sin(2\omega_c t + \theta(t))] \quad (33)$$

$$c_x(t) = \frac{r(t)A\sin\omega_L t}{K} = \frac{AI'(t)}{2K}[-\sin\theta(t) + \sin(2\omega_c t + \theta(t))] + \frac{AQ'(t)}{2K}[\cos\theta(t) - \cos(2\omega_c t + \theta(t))] \quad (34)$$

- by suppressing the spectral components centered on $2\omega_c$ with the LP filters, the output signals are:

$$i_F(t) = \frac{A}{2K}(I'(t)\cos\theta(t) + Q'(t)\sin\theta(t)) \rightarrow \frac{A}{2K}\cdot I'(t) \text{ for } \theta(t) \rightarrow 0; \quad f_{t-LP} > f_N(1+\alpha) \quad (35)$$

$$c_F(t) = \frac{A}{2K}(-I'(t)\sin\theta(t) + Q'(t)\cos\theta(t)) \rightarrow \frac{A}{2K}\cdot Q'(t) \text{ for } \theta(t) \rightarrow 0; \quad f_{t-LP} > f_N(1+\alpha) \quad (36)$$

- the QAM demodulation delivers the filtered modulating signals (35) and (36) affected by the channel's perturbations and distortions.
- then, using the recovered symbol clock, these signals are probed at $t=kT_s$ to extract the modulating levels of the k-th symbol period I_k'' and Q_k'', which are affected by the channel and do not belong to the modulating alphabet. The complex baseband signal (QAM symbol), see (35),(36), after probing is:

$$m''_k(t) = I''_k + jQ''_k = I'_k\cdot\cos\Theta_k + Q'_k\cdot\sin\Theta_k + j\left(-I'_k\cdot\sin\Theta_k + Q'_k\cdot\cos\Theta_k\right) = m'_k\cdot e^{-j\Theta_k} \quad (37)$$

- these signals are inserted in the decision block which delivers the two estimates I^*_k and Q^*_k of the transmitted levels, see (18); note that I^*_k and Q^*_k belong to the modulating alphabet.
- in this case the decoding (demapping) of the decided levels I_k* and Q_k* is performed in a tabular manner, using a table coordinate pair → quadribit c_i^k, which is built by using figure 3.
- then, the first two bits are differentially decoded using table generated from table 2 as follows: columns 3, 6, 9, 12 are interchanged with columns 1, 4, 7, 10, respectively. Having the c_1^{k-1} c_2^{k-1} dibits, demodulated from the previous symbol, and the c_1^k c_2^k dibits, demodulated from the current symbol, the values of current dibit b_1^k b_2^k are read.
- the symbol-clock recovery method could be different from the one employed for the DPSK-QAM, which is appropriate for constellations that contain vectors of the same modulus, see DT lectures.
- the demodulators presented or mentioned above could be employed for all ASK+PSK modulations, but involve recovery and synchronization of a local carrier, since they are coherent demodulators. Their extension to constellations with a greater number of vectors only requires the employment of different mapping-demapping tables and S/P and P/S converters with n and (n+1) cells, respectively and different frequency of the bit-clock, which should observe $f_{bit} = n\cdot f_s$