Baseband transmissions

1. Parameters of the physical lines

- The physical lines are channels whose useful frequency band is limited only by their physical characteristics;
- Examples: copper twisted wires, coaxial cables, UTP cables
- They can be modeled as transmission media with distributed constants, characterized by the propagation exponent γ, attenuation a, phase shift b and characteristic impedance Z_c,:

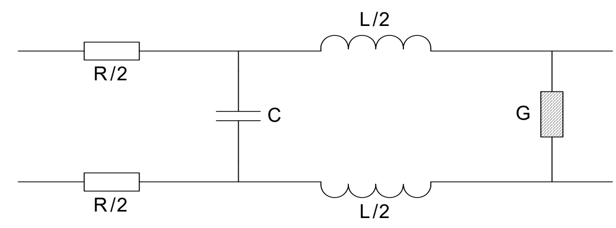


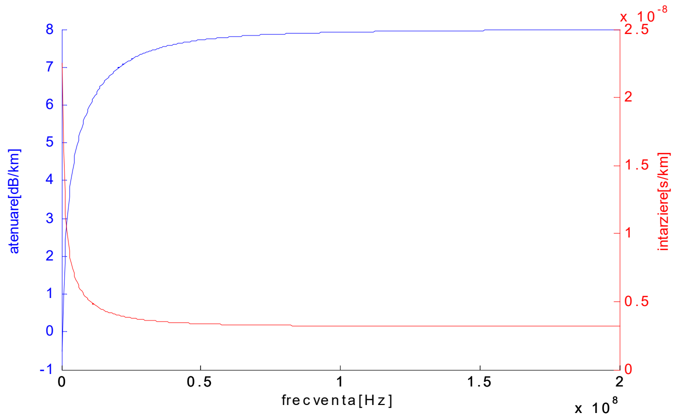
Figure 1. a. Equivalent scheme of the twisted wires

$$\gamma = \alpha(\omega) + j\beta(\omega); a = \alpha \cdot l; b = \beta \cdot l; Z_c(\omega) = \sqrt{\frac{R + j\omega L}{G + j\omega C}}; \qquad (1)$$

- in fig. 1, R, L, C and G (leakage conductance) are per length unit.
-

- The group delay time vs frequency $\tau_g(\omega)$ characteristic is defined as: $\tau_g(\omega) = d\varphi(\omega)/d\omega$ (1')
- Condition required to ensure constant a(f) and linear β(f) (or constant τ_g(f)) characteristics, is (2).a.
- Fulfillment of (2).a leads to an undistorted transmission of the data pulses on these channels.
- The values of attenuation and phase shift per length unity are obtained imposing (2).a in (1); we get (2).b and (2).c.

$$\frac{RC}{GL} = 1; (a) \Rightarrow \alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}; \quad (b)\, \beta = \omega\sqrt{LC}; (c) \qquad (2)$$



- Practically, condition (2).a cannot be fulfilled over a whole frequency band;
- the real characteristics look like figs. 1. b, c.

Figure 1 b. c. The a(f)-blue and τ_g(f)-red characteristics – UTP cable
for twisted copper wires – the slope of a(f) is steeper

- For medium frequency transmissions, the group-delay time distortion is negligible; only the attenuation distortion should be considered.

- The characteristic impedance Z_c, varies in frequency; its typical values range from 600 Ω at f < 4 kHz to 130 Ω at f in the domain of MHz;

- A special case is the DSL (Digital Subscriber Line –DSL) transmission;
- Due to the large diversity of employed cables, the ETSI standards include some cable parameters and a channel model that should be used when evaluating the attenuation vs. frequency;
- For a greater flexibility, the *Insertion Loss* is employed to specify the a(f) characteristic;
- The *Insertion Loss (IL)* is the ratio between the power dissipated by a generator into the load through the pair of twisted cables and the power dissipated by the same generator directly into the same load. It is dependent on the length and physical characteristics of the cable.
- ETSI characterizes the cables by their *electrical length*, i.e., the value of the insertion loss at a given frequency and by their Z_c;
- some brief considerations regarding the ETSI classification of the cables are presented in Annex 1.

2. Baseband Codes

2.1. General aspects

- The baseband transmission signifies the transmission of a signal in its original bandwidth.
- The baseband (or line) coding means the correspondence, acc. to a certain rule, between the electrical signal (with a finite set of values) and the symbols of the source's alphabet, e.g., binary "0" and "1").
- In terms of the number of levels of the associated electric signal, the BB codes can be split into:
  - *Unipolar* codes with two levels, out of which one differs from zero (e.g., TTL 0V and 5V)

- *Bipolar codes,* 2 levels, symmetrical to 0V(e.g., CMOS -12V and +12V)
- *Bipolar codes,* 3 levels: 0V and two values symmetrical to 0V (e.g., 0V, -AV and +AV)
- *Multilevel* codes – the PAMs – see next chapter

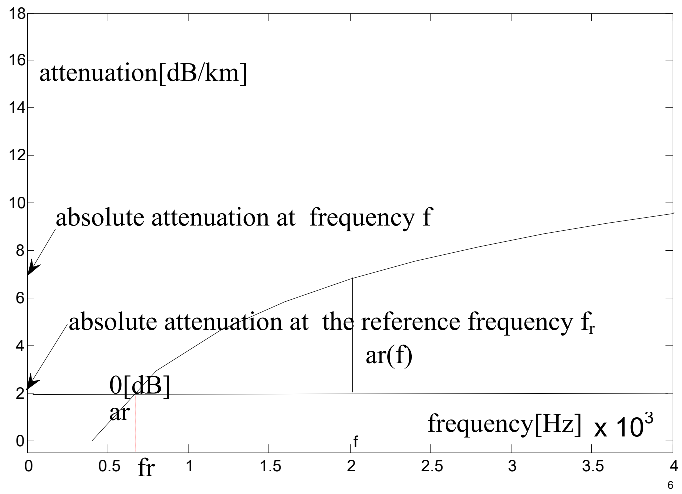
- the information-carrier signal is rectangular → simple structure and implementation, but some shortcomings:
  - very large frequency BW
  - a weak resilience to perturbations
- main aspects that should be considered when analyzing these transmissions:
  1. the distribution of the power spectral density of the transmitted signal
  2. the recovery and synchronization of the symbol (bit) clock at the receiving end
  3. bit-error rate (BER) performance
  4. implementation complexity

- the distribution of the power spectral density should be analyzed for the following reasons:

1) Due to the increase of attenuation with frequency, the Physical .Channels have a LP characteristic; their a(f) characteristic may be approximated by, see fig.1.b: $a(f) = K\cdot\sqrt{f}$; (3)

- a signal with large BW would be distorted because its spectral components would be attenuated differently. The larger the signal BW, the greater would be the difference between the attenuation of its extreme components, leading to an increased signal distortion.

2) the line-connection circuits, implemented with impulse transformers, do not allow the passage of the d.c. component and distort the very low frequency components, e.g. due to the saturation of the magnetic core.



- figure 2 shows the a(f) characteristic in absolute values;
- in practice, a relative attenuation characteristic, described by (4), is used:

$$a_r(f) = a_{abs}(f) - a_{abs}(f_{ref}); \quad (4)$$

- the power spectral distribution of the data signal should be modified prior to transmission; it should be concentrated into a BW as narrow as possible, placed as low as possible, but far enough from the d.c. component for the following reasons:
- the narrow BW leads to a reduced effect of the non-uniformity of the a(f) characteristic;
- the positioning on low frequencies would decrease the absolute attenuation of the signal;
- the "distance" kept from the d.c. component would cancel the effects of the line-connection circuits.

- the processing that transforms the data signal to fulfill, more or less, the above requirements, is the baseband encoding; the codes employed are also called transmission codes or line codes.
- the corresponding baseband decoding, performed at the receiving end, requires a locally generated clock signal that has to be synchronized with the received coded signal.
- the synchronization capability is another important parameter of the line codes. Long sequences containing identical bits lead to a synchronization loss; therefore, the baseband coded signal should have additional transitions inserted, so that no long sequence without transitions would be transmitted.

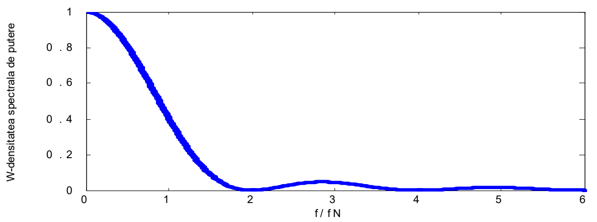
2.2 Baseband codes

- the BB codes modify the spectral distribution of the signal and help the synchronization of the bit clock.
- the spectrum of the coded signal would be positioned in the same BW and the carrier signal would be rectangular.
- the most employed BB codes are: the Non-Return to Zero (NRZ), biphasic, Miller, CMI, and AMI with its variants: BnZs, HDB3, 4B3T; the combination of MLT-3 and b4b5 is also used.
- the main parameters of these codes are:
- the BW which concentrates the most of the coded signal's power and its positioning on the absolute-frequency axis;
- the presence/absence of the d.c. component in spectrum of the coded signal;

- the synchronization capability;
- the encoding-decoding complexity.

The Non-Return-to-Zero (NRZ) Codes

- there is a class of NRZ codes, namely NRZ-L, NRZ-M, NRZ-S and NRZ-I codes.
- the NRZ-L (Level) encodes a “1” (Mark) with level “High”, and a “0” (Space) with level “Low”
- the NRZ-M (Mark) and NRZ-S encodes a Mark (Space) alternatively with levels High and Low, while the Space (Mark) are encoding by maintaining the level of the previously coded bit
- the NRZ-I encodes a “1” by a transition at the midpoint of the bit interval (HL or LH), while a “0” is encoded alternatively as a H or L during the whole bit period, or equivalently as HH or LL.
- if the two levels corresponding to H and L are positive and negative the codes are named bipolar codes, while if the L level corresponds to 0 volts, the codes are denoted as unipolar codes .
- the spectral densities of the unipolar NRZ codes are similar to the one presented in Figure 3, while the spectral densities of bipolar ones lack the d.c. component.



- the spectral distribution of the average power of the digital unipolar NRZ-M signal is shown in figure 3.

Figure 3. Power spectral density of a random unipolar NRZ digital signal

- most of the signal power is concentrated in the $[0; 2]\cdot f_N$ BW, $f_N = f_{bit}/2$; this BW should be received as correctly as possible.
- the data signal has its energy concentrated in the d.c. and low frequency components, which would be distorted by the line-connection circuits;
- the spectral distributions of the biphasic, CMI, Miller and AMI are presented in fig. 4.

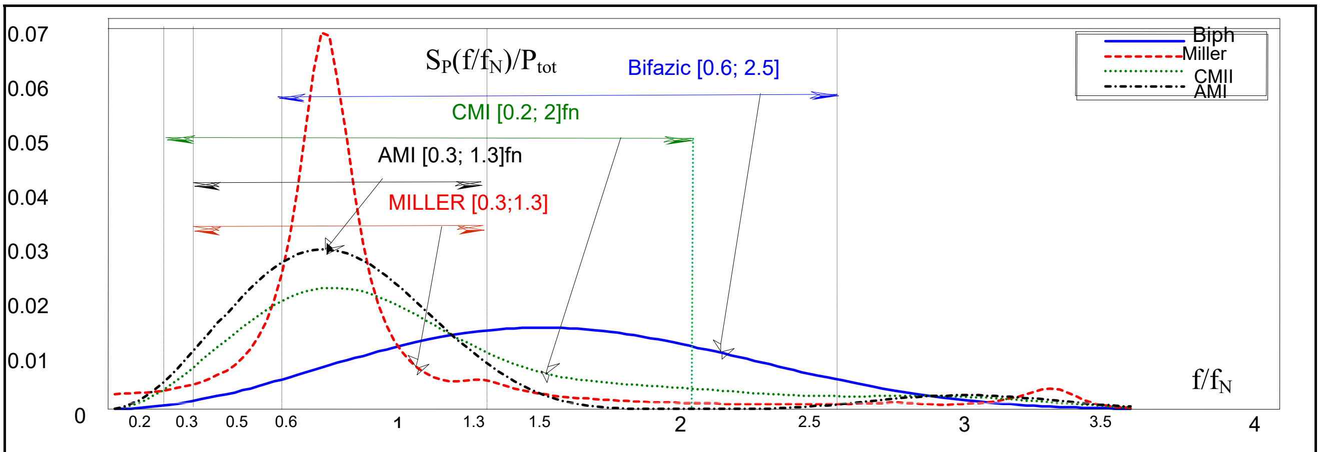


Figure 4. Power spectral densities of the biphasic-S, Miller, CMI and AMI codes – approximate representation

The Biphasic (Manchester) Codes

- the “parent” biphasic code, the biphasic-L, has the following encoding rule, see figure 5:

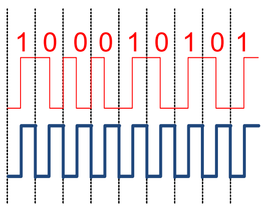


Figure 5. biphasic-L (Manchester) code – Encoding rule

-„1” (mark-M) -coded with a positive transition in the middle of the bit-period (LH);
-“0” (space-S) - coded with a negative transition in the middle of the bit-period (HL)
- this code exhibits an uncertainty of 180°, which requires a re-synchronization circuit- explanations: due to the line connection
- to avoid this, two differential variants were developed: biphasic-S and biphasic-M, also named differential Manchester codes.
- the **biphasic-S** has the following encoding rule:
  - a transition is inserted at every margin of the bit-period and, if the bit is “0”, an additional transition is inserted at the middle of the bit period – see figure 6 for a bipolar rectangular carrier.
- the **biphasic-M** has a similar encoding rule, but the additional transition is inserted in “1”-bit period.
- for the three biphasic codes, the minimum level duration would equal half bit-period, equivalent to a double output bit rate, leading to an increased frequency BW of the coded signal, see figure 4.

- considering the spectral components that contain about 80% of the coded signal's power, the useful FB of all three biphasic codes is $[0.6\ 2.5]\cdot f_N \approx [0.5, 2.5]\ f_N$
- the spectrum of this useful FB is approximately symmetrical around its central frequency $f_{cB} \approx 1.5\ f_N$
- this BW is approximately equal to the one of the non-coded sequence, but the distribution of the signal power within the BW is significantly modified;
- the d.c. component is removed and the power is concentrated in the higher-frequency components;
- the low frequency components have small amplitudes, decreasing the distorting effect of the line-connection circuit and simplifying its implementation
- the relatively large BW of this code, makes it unadvisable for high bit rates. On applications employing coaxial or UTP cables, the biphasic is employed for bit rates up to tens of Mbps due to the large BW and short lengths of these cables.
- the synchronization capability of the code is very good, due to the small maximum time interval between two subsequent transitions, which equals one bit period.
- since the initial state of the encoder could be "0" or "1", there are two biphasic-encoded sequences for the same information sequence.
- considering the encoding rule, the minimum level duration is half bit-period, or a period of a $2f_{bit}$ clock. This would require the synchronization at the receiving end of a local clock $f_{local} = 2f_{bit}$.
- If we consider the two half-periods, 2k+1 and 2k+2, of the (k+1)-th bit period, when the b_{k+1} bit is encoded (see figure 4), the biphasic-S can be expressed:

$$s_b(2k+1) = \bar{s}_b(2k);\ s_b(2k+2) = s_b(2k+1) \oplus \bar{b}_{k+1} \tag{5}$$

- the first expression shows the occurrence of a transition in the coded signal at the beginning of every bit-period; the second one, the occurrence of a transition at the middle of the bit-period if the encoded bit is "0".

The Miller Code

- the mathematical expression of the Miller-encoding rule is complex [alex];
- a variant of the Miller code can be defined in a simpler manner using the biphasic-S encoding:

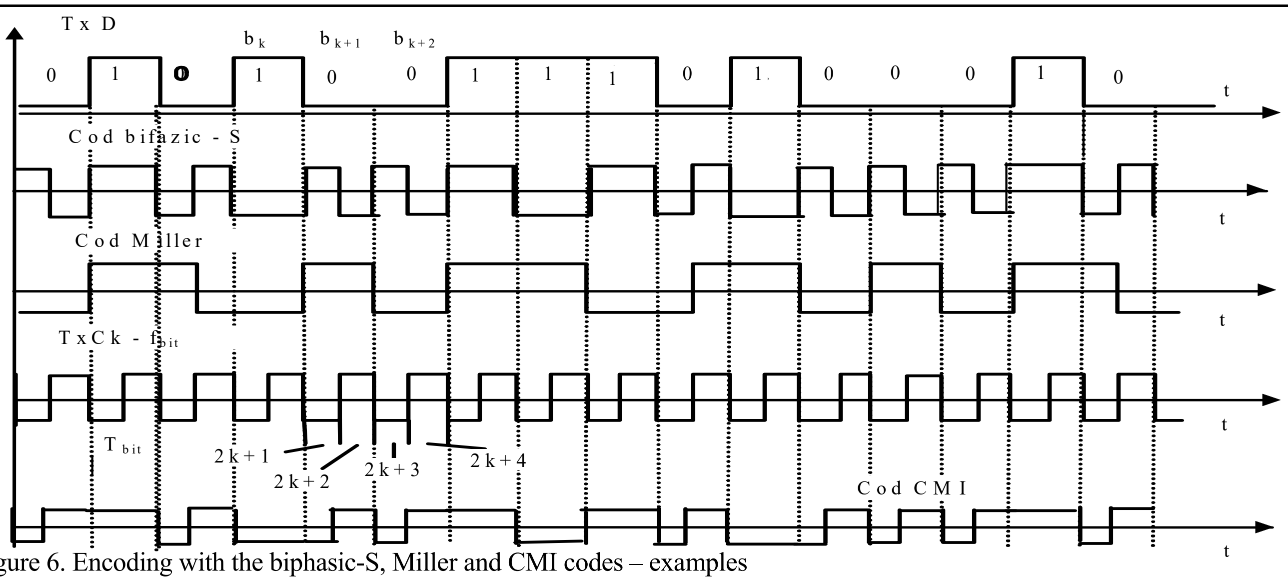


Figure 6. Encoding with the biphasic-S, Miller and CMI codes – examples

- every other transition is removed from the biphasic-encoded sequence – see figure 6

- the minimum level duration of the encoded signal is one bit-period;
- the maximum time interval between two transitions is two bit-periods, leading to the decrease of the BW occupied by the Miller-encoded signal.
- the Miller code concentrates the power within the $[0,3\ 1,3]\cdot f_N$ BW, see figures 4 and 7, considering the components that contain approximately 80% of the coded signal's power, while the central frequency is $f_{cM} \approx 0.8\ f_N$
- this decreases the distorting effects of the a(f) absolute value and non-uniformity, allowing the code's use for higher bit rates.
- but this code exhibits a d.c. component and low-frequency components, which make it unadvisable for low bit rates.
- figures 7 and 8 present in a comparative manner the effects of absolute attenuation and of the non-uniform a(f) characteristic upon the biphasic and Miller encoded signals.

- due to its narrower BW, the Miller-encoded sequence is less distorted by the a(f) characteristic and is less affected by disturbances.
- due to its positioning at lower frequencies, the Miller code is less attenuated by a(f) than the biphasic code
- the synchronization capability is good, i.e. the maximum interval between two subsequent transitions being of four encoded levels or two bit-periods.
- because there are two biphasic encoded sequences for every information sequence and the initial state of the Miller encoder could be „0" or „1" and considering that from the biphasic-encoded sequence we may eliminate either the positive or the negative transitions, we would have 8 Miller-encoded sequences, out of which pairs of two are identical, i.e. for the same information sequence there are four Miller-encoded sequences. It leads to the necessity of a resynchronization circuit in the receiver.

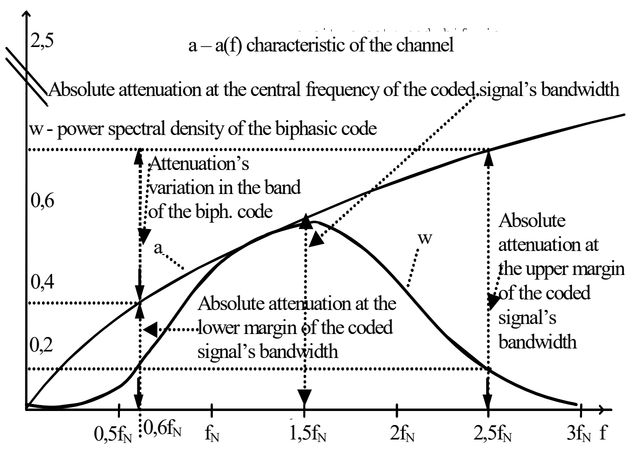


Figure 7. Effects of non-uniformity of the a(f) characteristic upon the biphasic-encoded signal

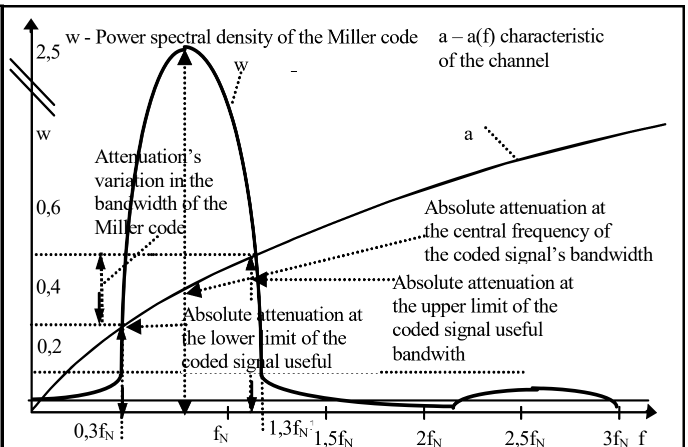


Figure 8. Effects of non-uniformity of the a(f) characteristic upon the Miller-encoded signal

The Coded Mark Inversion Code

- the encoding rule of the CMI code is, see figure 6:
    - the "0" bit is encoded with a rising transition at the middle of the bit-period (LH);
    - the "1" bit is encoded alternatively with "1" (+V) or "0"(–V), (LL or HH)
- the transition at the beginning of the bit-period will not occur for all data bits sequences;
- the "0" bits are encoded with a transition at the middle of the bit-period, inserting the information required by the bit-clock synchronization system at the receiving end
- the power spectral distribution of the CMI code is asymmetrical, see figure 4; the BW of the CMI code is approximately $[0,2\cdot f_N,\ 2\cdot f_N]$ by considering the components that contain 80% of the coded signal's power and the coded signal has no d.c. component

The AMI (Alternate Mark Inversion) Code

- the AMI code has the following encoding rule, see figure 9:
    - the "0" bit is encode with the level of 0 volts; - undesirable
    - the "1" bit is encoded alternatively with levels of +/- A for the whole bit-period.
- it is a 3-level code and the transmitted bit rate equals the information bit rate

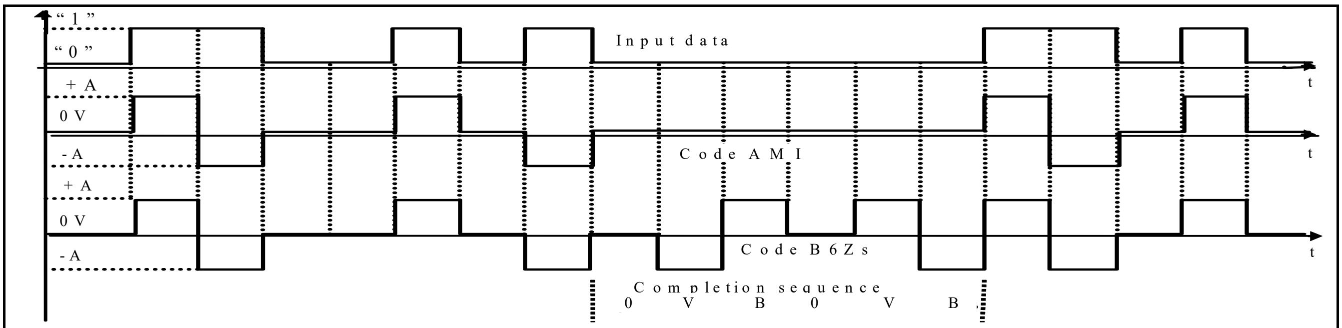


Figure 9. Encoding rules for the AMI and B6ZS codes

- the power spectral density is shown in figure 4; the code concentrates its power in the BW $[0,3\ 1,3]\cdot f_N$, considering only the spectral components containing 80% of the coded signal's power
- the AMI's BW is narrower than the one of biphasic code, there is no d.c. component and the energy is concentrated at higher frequencies than the energy of the Miller code.

- the synchronization capability of the code is very poor, since the long "0" sequences are encode without any transitions
- to remove this inconvenience the BnZS (Bipolar with n Zeros Substitution – used in US and in Europe) code was developed.

- the encoding rule is identical to the one of the AMI code, except for "0"-bit sequences longer or equal to n, a natural number. Should such a sequence occur, it is replaced by a *completion sequence*.

- for n = 6, the completion sequence is 0VB0VB, see figure 9:

  0 - "0"-bit coded with 0 volts
  V – "1"-bit coded with a +/- A level that violates the AMI encoding rule
  B - "1"-bit coded with a +/- A level that observes the AMI encoding rule

- the violations of the AMI encoding rule are employed for the identification of these sequences at the receiving end, where they should be replaced by n-bit "0" sequences.
- The power spectral density of the BnZs code is distributed similarly to the one of the AMI code
- other codes developed to improve the synchronization capability of the AMI code are the *HDB3 (high-density bipolar-3 zeros- used in Europe)* and *4B3T (4-binary, 3-ternary).*

The MLT (Multi Level Transmit))-3 and 4b5b Codes

- this concatenation of two codes is used in the Ethernet 100Base-TX transmissions at 100 Mbps
- the MLT3 used the bipolar NRZ data and encodes it according to the following rule, see Figure 10:

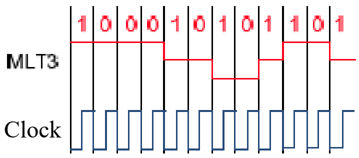


Figure 10 Encoding rule of MLT-3

- at every bit of "1", the output level jumps to the next level from the pattern "+V", "0", "-V"
- at every bit of "0", the output level keeps the level of the previous symbol period, i.e., the level transmitted during the previous bit of "1"

- This code has a poor synchronization capability because for long "0" input series the output signal has no transitions

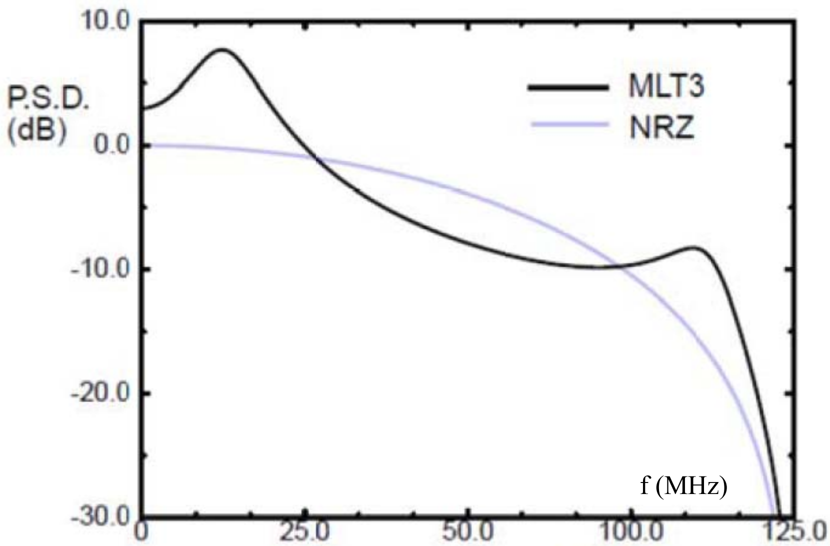
- to compensate for this shortcoming, the input data flow is precoded with the *b4b5 code,* see Table 1.

- Every group of 4 bits is replaced by a group of 5 bits by selecting 16 combinations with at least two bits of "1" out of the $2^5 = 32$ combinations; this leads to:

- a better synchronization capability, since only series of two consecutive "0" are provided
- a better resilience to transmission impairments, since there are only 16 valid 5-bit combinations, which is equivalent to inserting one check bit at every 4 information bits (i.e., a coding rate of 4/5)
- the actually transmitted bit rate increases from 100 Mbps to 125 Mbps; this is a disadvantage

Nr	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
4b	*0000*	*0001*	*0010*	*0011*	*0100*	*0101*	*0110*	*0111*	*1000*	*1001*	*1010*	*1011*	*1100*	*1101*	*1110*	*1111*
5b	*11110*	*01001*	*10100*	*10101*	*01010*	*01011*	*01110*	*01111*	*10010*	*10011*	*10110*	*10111*	*11010*	*11011*	*11100*	*11101*

Table 1. Encoding rule of the b4b5 code



- the symbol frequency of this transmission is f_s = 125/5= 25 MHz and the spectral components above this frequency are rather small.

Spectrum of b4b5-MLT-3 for Ethernet 100Base-TX

Implementation of the BB encoding – this section will be used in the laboratory classes and is presented in Annex 2

Decoding of the baseband codes

- *the biphasic-S code*
- the decoding of the biphasic–S code is based on the encoding equations (5).

$$s_b(2k+1)=\overline{s}_b(2k);\ s_b(2k+2)=s_b(2k+1)\oplus \overline{b}_{k+1} \tag{6}$$

- considering that bit b_{k+1} was encoded during the half-periods 2k+1 and 2k+2 of the bit-clock;
- using (3) and performing the additions for three half-periods separated by a bit-period, we get (7).
- the bit b_{k+1}, encoded during half-periods 2k+1 and 2k+2, is decoded during half-periods 2k+2 and 2k+3, inserting a half bit-period delay.
- the decoding of the next data bit b_{k+2} is started during the 2k+4 interval.

$$\begin{aligned}
&s_b(2k+2)\oplus s_b(2k)=s_b(2k+1)\oplus \overline{b}_{k+1}\oplus s_b(2k)=\overline{s}_b(2k)\oplus \overline{b}_{k+1}\oplus s_b(2k)=1\oplus \overline{b}_{k+1}=b_{k+1};\\
&s_b(2k+3)\oplus s_b(2k+1)=\overline{s}_b(2k+2)\oplus s_b(2k+1)=s_b(2k+1)\oplus b_{k+1}\oplus s_b(2k+1)=b_{k+1};\\
&s_b(2k+4)\oplus s_b(2k+2)=s_b(2k+3)\oplus \overline{b}_{k+2}\oplus s_b(2k+2)=\overline{s}_b(2k+2)\oplus \overline{b}_{k+2}\oplus s_b(2k+2)=1\oplus \overline{b}_{k+2}=b_{k+2};
\end{aligned} \tag{7}$$

- *the decoding of the Miller code* is accomplished by similar additions, that employ signals delayed with 1, 2, 3 and 4 half-bit periods.
- *the decoding of the CMI code* is performed by XOR-ing the coded signals delayed with a half-bit period.
- for all three codes, the signals resulted from the summation circuits should be “sampled” with a bit clock, with frequency f_{bit}, whose phase differs from code to code.
- the biphasic and CMI codes the sampling bit-clock should be shifted with 270º, compared to the bit clock obtained by dividing by 2 the f_{sincro} clock delivered by the dynamic synchronization circuit.
- for the Miller code the sampling clock has to be shifted with 90º compared to the same reference signal

Further considerations regarding the implementation of the BB decoding is presented in Annex 3 and will be used in the laboratory classes.

Block diagram of a baseband modem

- it is presented in figure 11

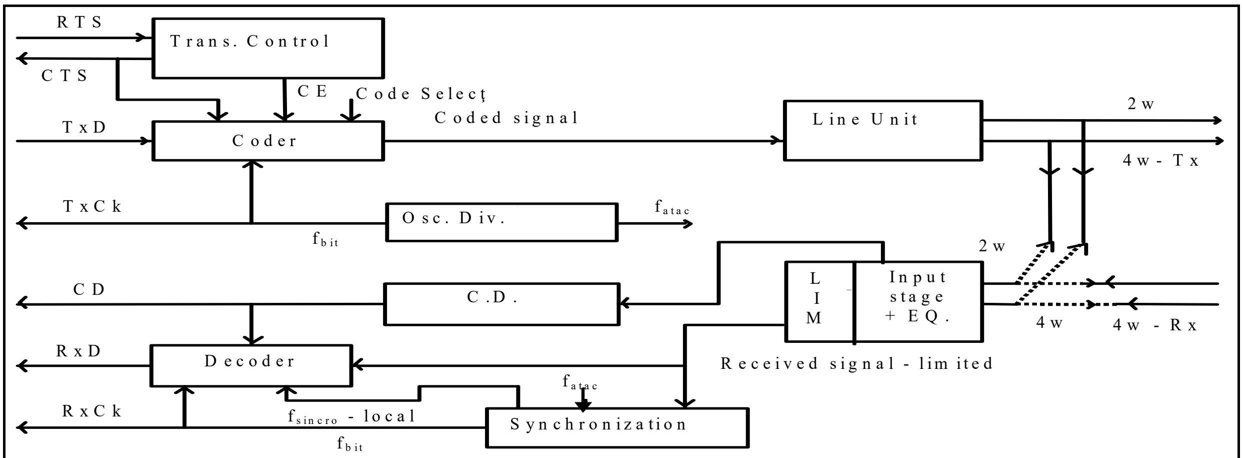


Figure 11. Simplified block diagram of a BB modem

- the TxD are delivered to the encoder, using the bit clock Tx Ck delivered by the Oscillator-Divider block. The desired BB code is selected by the internal control “Code Type”.
- the encoded signal is sent to the line unit which transfers the signal into the physical signal.
- the Transmission Control block is commanded by the RTS interface signal; it sends back to the computer the confirmation CTS signal, after the RTS/CTS time interval, while delivering to the encoder the synchronization sequence.
- depending on the 2w/4w connection, the transmission employs one or two pairs of twisted wires.
- for the 2w half-duplex operation, the transmission has priority, so the Transmission Control circuit disables the receiver’s input stage when the RTS signal is active. On the 4w full-duplex operation this conditioning is not applied.
- the input stage ensures the transfer from symmetrical to asymmetrical circuit, the amplification and filtering of the received signal and the equalization of the attenuation inserted by the wires.
- the signal is then limited and used, as a reference signal, by the synchronization circuit, which delivers the f_{sincro} clock, required by the demodulator and the receive clock RxCk.

- the limited signal is also sent to the decoder, together with the f_{sincro} clock, for the "extraction" of the received data RxD.
- the signal delivered by the input stage is employed by the Carrier Detector block, which monitors the level of the input signal and enables/disables the receiver if its level is higher/smaller than some preset levels; this bloc delivers to the computer the CD interface signal.
- the modem may contain circuits that implement the test loops, scramble-descrambler and circuits for the generation and analysis of the test data; these circuits were not inserted in fig. 12.

Error probability of the BB transmissions

- the frequency band available on the physical channels is usually larger than the useful BW of the BB-coded signals, therefore an input filtering is required for the improvement of SNR.
- the evaluation made below assumes that the receiver has an adaptable BP filter that changes the passing band according to the code employed
- the transmission codes presented in this chapter are either 2-level codes (+/-A; M = 2), i.e. biphasic, Miller and CMI, or 3-level codes (+/-A, 0V; M = 3), i.e. AMI, BnZs.
- the average signal power P_s, for the two types of codes is given by (8). a and (8). b, respectively, and the power of the Gaussian noise, with a N_0 power spectral density, at the output of the input filter with a BW = B_f is given by (8).c:

$$P_{s2} = A^2;\ a. \qquad P_{s3} = A^2/2;\ b. \qquad P_z = BW_f \cdot N_0 = \sigma^2;\ c. \tag{8}$$

- we assume that the received signal is affected by the a Gaussian noise of null mean and variance σ, which is added to the useful signal, i.e.:

$$r(t) = s_{tr}(t) + n(t) \tag{9}$$

- therefore, the probability that the received signal would equal *r* at the sampling moment, if the transmitted level was *m* is:

$$p(r\,|\,m) = \frac{1}{\sqrt{2\pi}\cdot\sigma}\exp\left(-\frac{(r-m)^2}{2\sigma^2}\right); \tag{10}$$

- since the decided levels are obtained based on the minimum Euclidean distance between the received level and the permitted levels, the symbol (bit)-error probability equals the occurrence probability of a noise amplitude which makes the level of the received signal to be closest to a permitted level that differs from the level that was transmitted in that symbol period.
- if within a symbol period, the level m_k is transmitted, with probability P_{mk}, then the probability to have that symbol mistaken after the decision is given by (11), where $N_{k,A0}$ is the number of permitted levels that are placed at a distance $2A_0$ from the level m_k and A_0 is half of the minimum distance between two transmitted levels of that code.

$$p_e = \sum_k \left(P_{mk}\cdot p\left(|r-m_k| > A_0\right)\cdot N_{k,A_0}\right); \tag{11}$$

- the probability $p\left(|r-m_k| > A_0\right)$ actually represents the occurrence probability of a noise-level greater than A_0 in the probing moment; this probability is:

$$p\left(|r-m_k| > A_0\right) = \frac{1}{\sqrt{2\pi}}\int_{A_0}^{\infty}\exp\left(-\frac{(r-m_k)^2}{2\sigma^2}\right)d(r-m_k) = Q\left(\frac{A_0}{\sigma}\right); \tag{12}$$

where

$$Q(t) = \frac{1}{\sqrt{2\pi}}\int_t^{\infty}\exp\left(-\frac{u^2}{2}\right)du; \tag{13}$$

- since the power of a Gaussian noise equals σ^2, the signal/noise ratio in linear expression is:

$$\rho_{2,3} = \frac{P_{s2,3}}{P_{z2,3}} = \frac{P_{s2,3}}{\sigma_{2,3}^2} \tag{14}$$

- knowing the for the 2-level codes $A_0 = A$ and for the 3-lvel codes $A_0 = A/2$ and using (8) we express the average powers of the two types of codes in terms of their respective A_0 as:

$$P_{s2} = A_0^2; \quad P_{s3} = 2A_0^2; \tag{15}$$

- if we express A_0 of each code in terms of its P_s from (15) and insert it (12) and then consider (14) we get for the 2-level and 3-level codes:

$$p\left(|r-m_k|>A_0\right)=Q\left(\sqrt{\rho}\right)\ for\ 2-level\ a.;\quad p\left(|r-m_k|>A_0\right)=Q\left(\sqrt{\frac{\rho}{2}}\right)\ for\ 3-level\ b.; \tag{16}$$

- Q is a strictly decreasing function, i.e. the error probability increases with the decrease of the signal/noise ratio ρ (in linear expression).
  - Recall that: $\text{SNR} = 10\lg(P_s/P_z)$; (17)
- the Q function is a strictly descending function, i.e. the BER increases with the decrease of the SNR.

Considerations regarding the BER of 2-level codes

- the subsequent considerations assume that the average power of the received coded signal is A^2, for 2-level codes (+/- A), the power spectral density of the noise equals N_0, and the noise power in the useful band of code x equals $N_0 \cdot BW_x = \sigma_x^2$.
- for the 2-level codes the half of the minimum distance between two levels equals $A_0 = A$, i.e. the decision threshold is placed at 0.
- replacing (16).a in (11) and taking into account that each level has only one neighbour at distance $2A_0$ and that all levels are equiprobable, we get the average error-probability of a level (symbol), which equals the bit-error probability of a bit, as:

$$p_{e2} = 0.5\cdot Q\left(A_0\sqrt{\frac{\rho}{A_0^2}}\right)\cdot 1 + 0.5\cdot Q\left(A_0\sqrt{\frac{\rho}{A_0^2}}\right)\cdot 1 = Q\left(\sqrt{\rho}\right) \tag{18}$$

- considering that the useful BW of the biphasic, Miller and CMI codes are:

$$B_{biph} \approx 2f_N;\ B_{Mil} \approx 1f_N;\ B_{CMI} \approx 1{,}8f_N \tag{19}$$

-the SNR values, rated to the one of the biphasic code, are:

$$\rho_{biph} = \frac{A_r^2}{2f_N \cdot N_0};\ \rho_{Mil} = \frac{A_r^2}{1\cdot f_N \cdot N_0} = 2\cdot\rho_{biph};\rightarrow \text{SNR}_{Mil} = \text{SNR}_{biph}[\text{dB}] + 3\text{dB};$$
$$\rho_{CMI} = \frac{A_r^2}{1{,}8\cdot f_N\cdot N_0} = 1{,}11\cdot\rho_{biph};\rightarrow \text{SNR}_{CMI} = \text{SNR}_{biph}[\text{dB}] + 0{,}46\text{dB}; \tag{20}$$

- and the BER is computed using (18), for M = 2 : the approximate expression is obtained by using the first term of the Taylor series decomposition of Q function

$$p_{e2} = Q\left(\sqrt{\rho}\right) \approx \frac{e^{-\frac{\rho}{2}}}{\sqrt{2}\cdot\sqrt{\pi\rho}}; \tag{21}$$

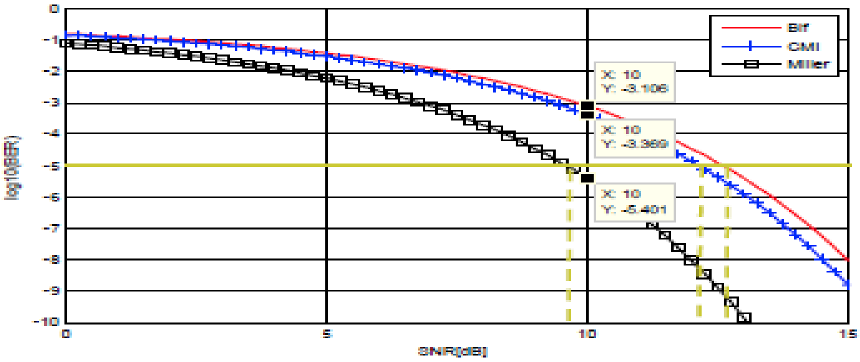
- considerations regarding the Q(t) function and its approximation will be discussed in the PSK chapter.
- equations (19) and (20) show that, for the same values of N_0 and of the received signal's power level, the SNR values of the Miller and CMI codes are greater with 3 dB and 0.46 dB respectively, than the SNR ensured by the biphasic code. Since the Q function is strictly descending, the BERs ensured by the Miller and CMI codes are smaller than the one provided by the biphasic code.
- to compare the levels of the received signal and of N_0 which ensure the same value of BER for the Biphasic and Miller codes, we should first note that, due to the bijectivity of the Q and square-root functions, we may write:

$$p_{ebif} = p_{eMill} \Leftrightarrow Q(\sqrt{\rho_{bif}}) = Q(\sqrt{\rho_{Mill}}) \Leftrightarrow \rho_{bif} = \rho_{Mill} \tag{22}$$

- the two ratios ρ may be expressed in terms of energy-per-bit/power-spectral-density ratio, E_b/N_0; relation (23) shows the computation for the Miller code and presents the final result for the CMI code as well:

$$\frac{P_{rbif}}{N_{0bif}\cdot LB_{bif}} = \frac{P_{rMill}}{N_{0Mill}\cdot LB_{Mill}} \Leftrightarrow \frac{P_{rbif}}{N_{0bif}\cdot f_{bit}} = \frac{P_{rMill}}{N_{0Mill}\cdot 0{,}5\cdot f_{bit}} \Leftrightarrow \frac{E_{b-bif}}{N_{0bif}} = 2\cdot\frac{E_{b-Mill}}{N_{0Mill}} \Leftrightarrow$$
$$\Leftrightarrow \frac{E_{b-bif}}{N_{0bif}}[\text{dB}] = \frac{E_{b-Mill}}{N_{0Mill}}[\text{dB}] + 3\text{dB};\quad \text{where } \frac{E_b}{N_0} = \frac{P_r}{N_0\cdot f_{bit}}; \tag{23}$$
$$\frac{E_{b-bif}}{N_{0bif}}[\text{dB}] = \frac{E_{b-CMI}}{N_{0CMI}}[\text{dB}] + 0{,}46\text{dB}$$

- relation (23) shows that, in order to ensure the same BER at the same bit rate D, the Miller and CMI codes allow the decrease of the received signal's power and/or the increase of the N_0, , so that their E_b/N_0 ratios would be smaller with 3 dB, and 0.46 dB respectively, than the E_b/N_0 ratio required by the biphasic code.



- Figure 12 shows the BER vs. SNR curves of the Biphasic-S, CMI and Miller codes

Considerations regarding the BER of 3-level codes

- for the AMI code, the received levels $+A$ and $-A$ have occurrence probabilities equalling 0.25, while the 0 level has the occurrence probability 0.5 (since the „1" and „0" bits are equiprobable). Therefore the decision thresholds are set at $+/- A/2$, so for these codes $A_0 = A/2$.
- the 0 level has two neighbours at a distance equalling $2A_0$, while the other two levels have only one neighbour at the same minimum distance.
- based on the above considerations and replacing (16).b in (12), the error-probability of the 3-level AMI codes can be expressed as (24), where ρ denotes the *SNR* of the used 3-level code in linear expression:

$$p_{e3} = \frac{1}{4}Q\left(\sqrt{\frac{\rho}{2}}\right)\cdot 1 + \frac{1}{2}Q\left(\sqrt{\frac{\rho}{2}}\right)\cdot 2 + \frac{1}{4}Q\left(\sqrt{\frac{\rho}{2}}\right)\cdot 1 = \frac{3}{2}Q\left(\sqrt{\frac{\rho}{2}}\right) \quad (24)$$

- considering that $BW_{AMI} \approx f_N$ and a bit rate with $f_{bit} = 2f_N$, the SNR would be:

$$\rho_{AMI} = \frac{A^2}{2 \cdot N_0 \cdot f_N}; \quad (25)$$

- the BER of AMI is computed using (24) and (25)for ρ_{AMI}
- the factor 1/2 under the square-root in (24) is generated by the fact the that the code has 3 levels in the same range of amplitude [-A, +A] which makes A_0 to be equal to $A/2$.
- to express *AMI*'s signal/noise ratio in a manner similar to (18), we define an "equivalent" signal/noise ratio for AMI:

$$\rho'_{AMI} = \frac{1}{2} \cdot \rho_{AMI} \quad (26)$$

- we express the ρ'_{AMI} in terms of the signal/noise ratio of the biphasic code:

$$\rho'_{AMI} = \frac{1}{2} \cdot \rho_{AMI} = \frac{1}{2} \cdot \frac{A^2}{2 \cdot N_0 \cdot f_N} = \frac{1}{2} \cdot \rho_{bif}; \Leftrightarrow SNR'_{AMI}[dB] = SNR_{bif}[dB] - 3\,dB; \quad (27)$$

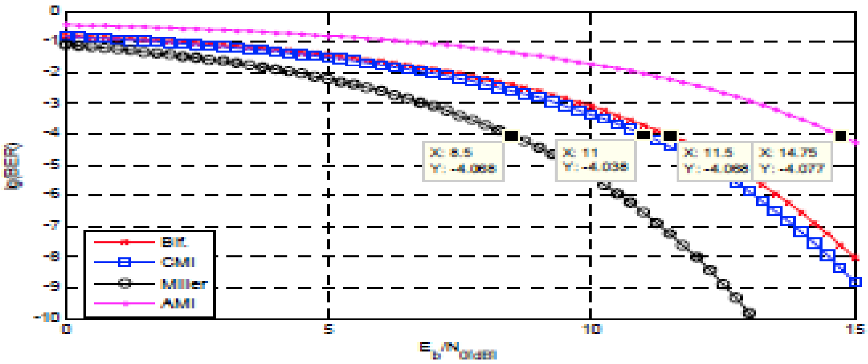
- equations (23), (25) and (27) show that, at the same values of N_0 and received amplitude levels $+/-A$, and for the same bit rate, the value of the signal/noise ratio of the ternary codes (AMI type) is smaller with 3 dB, than the one of the biphasic code. This value should be furthermore increased with about 0.5 dB to compensate the effect of the 1.5 factor from (24). Since the Q function is strictly descending, the BERs ensured by the AMI-type codes are greater than the BER provided by the biphasic code.
- the considerations above are also valid for the *BnZs* and *HDB3* codes
- to compare the levels of the received signal and of N_0 which would ensure the same value of BER for the two codes, we should first note that, due to the bijectivity of the Q and square-root functions, we may write (28), where the factor 3/2 was approximated by 1:

$$p_{ebif} = p_{eAMI} \Leftrightarrow Q(\sqrt{\rho_{bif}}) = Q(\sqrt{\rho'_{AMI}}) \Leftrightarrow \rho_{bif} = \rho'_{AMI} \quad (28)$$

- the two ratios ρ may be expressed in terms of energy-per-bit/power-spectral-density ratio, E_b/N_0:

$$\frac{P_{rbif}}{N_{0bif} \cdot LB_{bif}} = \frac{1}{2} \cdot \frac{P_{rAMI}}{N_{0AMI} \cdot LB_{AMI}} \Leftrightarrow \frac{A_{rbif}^2}{N_{0bif} \cdot f_{bit}} = \frac{1}{2} \cdot \frac{A_{rAMI}^2}{2 \cdot N_{0AMI} \cdot f_{bit}/2} \Leftrightarrow \frac{E_{b-bif}}{N_{0bif}} = \frac{1}{2} \cdot \frac{E_{b-AMI}}{N_{0AMI}} \Leftrightarrow$$
$$\Leftrightarrow \frac{E_{b-AMI}}{N_{0AMI}}[dB] = \frac{E_{b-bif}}{N_{0bif}}[dB] + 3dB; \quad \text{where } \frac{E_b}{N_0} = \frac{P_r}{N_0 \cdot f_{bit}} \quad (29)$$

- relation (29) shows that, in order to ensure the same BER for the same bit rate D, the AMI-type codes require the increase of the received signal's power and/or the decrease of the N_0, so that the E_b/N_0 ratio would be greater with 3 dB than the E_b/N_0 ratio required by the biphasic code. This value should be furthermore increased with about 0.5 dB to compensate the effect of the 1.5 factor from (24).
- finally, the AMI-type ternary codes require a E_b/N_0 higher with 3.5 dB than the biphasic code to ensure the same BER, at the same bit rate, if the BW of the input filter is modified according the code used.



- Figure 13 shows the BER vs. E_b/N_0 of AMI. compared to those of Biphasic-S, CMI and Miller

- table 2 presents the useful BW and the relative E_b/N_0 values, rated to the one of the biphasic code, required by the BB codes to ensure the same BER, if the input filter bandwidth B_f is modified according to the employed code.

Code	Biphasic M = 2	Miller, M = 2	CMI; M = 2	AMI, BnZs, HDB3; M = 3
BW_f	$\approx 2 \cdot f_N$	$\approx 1 \cdot f_N$	$\approx 1{,}8 \cdot f_N$	$\approx 1 \cdot f_N$
E_b/N_0 [dB]	E_b/N_{0bif}	$E_b/N_{0\ bif}$ – 3 dB	$E_b/N_{0\ bif}$ – 0,46 dB	E_b/N_{0bif} + 3,5 dB

Table 2 Useful bandwidths (rated to f_N) and E_b/N_0 values (relative to the Biphasic code) of Miller CMI and AMI

- the increase of the number of levels of the coded signal (M = 3), when the maxim level is kept constant (+/-A) to keep the peak power constant, leads to a smaller distance between two neighboring levels, from 2A for codes with M = 2, to A for codes with M = 3, and to a smaller average power of the coded signal. These facts explain the increase of the E_b/N_0 required by the M= 3 codes to ensure the same BER as the codes with M = 2.
- the BER computation for the MLT-3 code is more complex and is beyond the scope of this course;
- the filtering of the BB-coded signal, in order to match the useful BW of the received code, might exhibit adverse effects which should be removed by the filtering itself and by subsequent processing.
- the computations above consider only the power of the received coded signal. But, due to their different positioning of the absolute-frequency axis and to the different attenuation inserted by the cable for different frequencies, the signals coded with these codes suffer different attenuations; therefore, in the assumption of the same transmitted power, the received power is depending on the code employed.
- the computation of the received power in terms of transmitted power and the effects of the different attenuations inserted by the cable will be dealt with in the seminar classes.

References

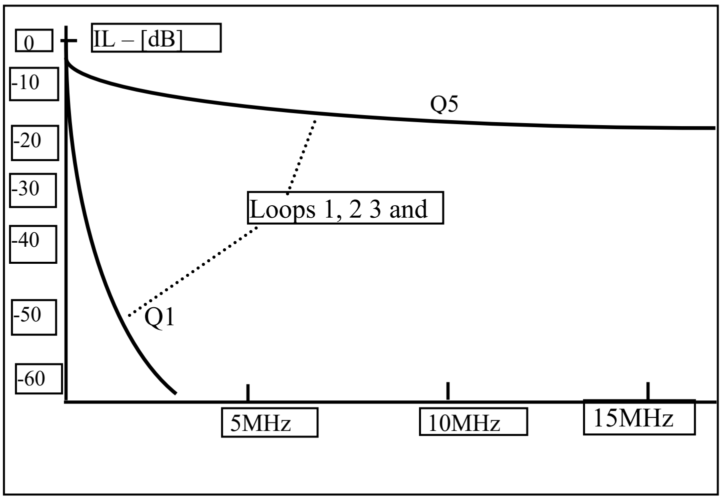
Nicolae Dumitru Alexandru, Gunter Morgenstern; Digital Line Codes and Spectral Shaping, Matrix Rom București 1998

Ha H. Nguyen, Ed Shwedyk; A First Course in Digital Communications – Cambridge University Press 2009

Annex 1

Brief description of the ETSI classification of copper cables - not required for the exam

- ETSI characterizes the cables by their *electrical length*, i.e., the value of the insertion loss at a given frequency and by their Z_c;



- It has defined five types (groups) of test cables (loops), Q1,…,Q5, by defining their electric length IL(f) – see figure A.1, for approximate representation.
- Each group contains 4 types of loops having the same electrical length, but different Z_c.
- the physical lengths of the Q1 test loops vary between 1990 and 2100 m; the ones of Q5 have between 250 and 300 m in physical length.
- the perturbations that occur on these physical channels are the Gaussian noise, impulse noise and crosstalk.

- the coaxial cables exhibit a similar behavior, but the increase of the attenuation with frequency is significantly slower, leading to a larger usable frequency bandwidth. The levels of the above mentioned distortions are significantly smaller.

Annex 2

***Implementation of the BB encoding** – not required for the exam this section will be used in the laboratory classes*

- the encoding of the BB codes can be implemented using the mathematical relations that describe the encoded signal, e.g. relation (3) for the biphase code.
- the implementation can be significantly simplified if the some particularities of the encoding rules are considered.
- due to the different numbers of levels of the encoded signals and to the different durations of a level in the coded signals, half of the bit-period or a bit-period, the encoding of the biphase, Miller and CMI codes should be approached in a different manner, as opposed to the encoding of the AMI-type codes.
- the biphase-S, and CMI have a common rule of encoding the "1"-bit, namely by alternating the level H and L. This can be implemented with J-K flip-flop, clocked by the bit clock, which has on its J and K inputs the data signal.
- the two codes also transmit the bit clock signal, during the "0"-bit
- the CMI code encodes the « 0 » bit by the bit-clock signal.
- the biphase-S encodes the "0"-bit either by the bit-clock or by the inverted bit-clock, depending on the level (H or L) which encoded the previous "1"-bit.
- based on these considerations a common encoder for the biphase-S and CMI codes might be built using a J-K flip-flop and an electronic switch; an additional switch should be used to select the desired code.
- to ensure the bipolar carrier, C-MOS circuits that operate using the +/- V levels should be employed.
- the Miller can be generated by dividing by (in frequency) the biphase-S coded signal.

HOMEWORK: Design the electric diagram of an encoder for the three codes with C-MOS digital circuits using the considerations mentioned above.

- the encoding of AMI should consider that the coded signal has three amplitude levels.
- It could be generated using a level converter that operates similarly to a 2-bit D/A converter and whose sign bit is toggled alternatively, for the "1" input bits.
- another approach employs a J-K flip-flop for the encoding of "1" (see considerations above) and a switch that has an input to the electrical ground, for the "0"-bit, controlled by the input data.
- for the encoding of the BnZS codes, prior to the AMI encoder there should be inserted a circuit that identifies the n-bit "0" series and replaces them by the completion (synchro) sequence of the code.

Annex 3

Implementation of the BB decoding *– **not required for the exam; this section will be used in the laboratory classes***

- *the biphasic-S code*
- the decoding of the biphasic–S code is based on the encoding equations (5).
- considering that bit b_{k+1} was encoded during the half-periods 2k+1 and 2k+2 of the bit-clock;
- using (3) and performing the additions for three half-periods separated by a bit-period, we get (7).
- the bit b_{k+1}, encoded during half-periods 2k+1 and 2k+2, is decoded during half-periods 2k+2 and 2k+3, inserting a half bit-period delay.
- the decoding of the next data bit b_{k+2} is started during the 2k+4 interval.

$$\begin{aligned} &s_b(2k+2)\oplus s_b(2k) = s_b(2k+1)\oplus \bar{b}_{k+1}\oplus s_b(2k) = \bar{s}_b(2k)\oplus \bar{b}_{k+1}\oplus s_b(2k) = 1\oplus \bar{b}_{k+1} = b_{k+1};\\ &s_b(2k+3)\oplus s_b(2k+1) = \bar{s}_b(2k+2)\oplus s_b(2k+1) = s_b(2k+1)\oplus b_{k+1}\oplus s_b(2k+1) = b_{k+1};\\ &s_b(2k+4)\oplus s_b(2k+2) = s_b(2k+3)\oplus \bar{b}_{k+2}\oplus s_b(2k+2) = \bar{s}_b(2k+2)\oplus \bar{b}_{k+2}\oplus s_b(2k+2) = 1\oplus \bar{b}_{k+2} = b_{k+2}; \end{aligned} \tag{30}$$

- *the decoding of the Miller code* is accomplished by similar additions, that employ signals delayed with 1, 2, 3 and 4 bit half-bit periods.
- *the decoding of the CMI code* is performed by XOR-ing the coded signals delayed with a half-bit period.
- for all three codes, the signals resulted from the summation circuits should be “sampled” with a bit clock, with frequency f_{bit}, whose phase differs from code to code.
- the biphasic and CMI codes the sampling bit-clock should be shifted with 270º, compared to the bit clock obtained by dividing by 2 the f_{sincro} clock delivered by the dynamic synchronization circuit.
- for the Miller code the sampling clock has to be shifted with 90º compared to the same reference signal
- the block diagram of a decoder for the biphase-S, Miller and CMI codes is shown in figure A2.a.
- the K_1 switch selects the decoded signal; the K_2 switch selects the sampling clock, both depending of the code to be decoded
- *the decoding of the AMI* and *BnZs* codes employs the fact that the “1” bit is encoded with levels of modulus A, and the “0” bit is encoded with a level of 0 volts.
- the *decoding of the AMI code* requires the 2-wave rectification of the received coded signal which is sampled with a locally-recovered bit clock.
- then a comparison with +A/2 threshold performs the decision of the received bit, see figure A2.b. Before the comparison, an AGC ensures a constant level of the received signal.
- the *decoding of the BnZs and HDB3 codes* require the insertion, besides the AMI decoding, of a dedicated circuit that recognizes the synchronization sequence, inserted at the transmission end, and replace with n or 4 bits of "0".
- the recovery of the symbol clock in AMI decoders might be performed using the” energetic” method – see the QAM lectures later.

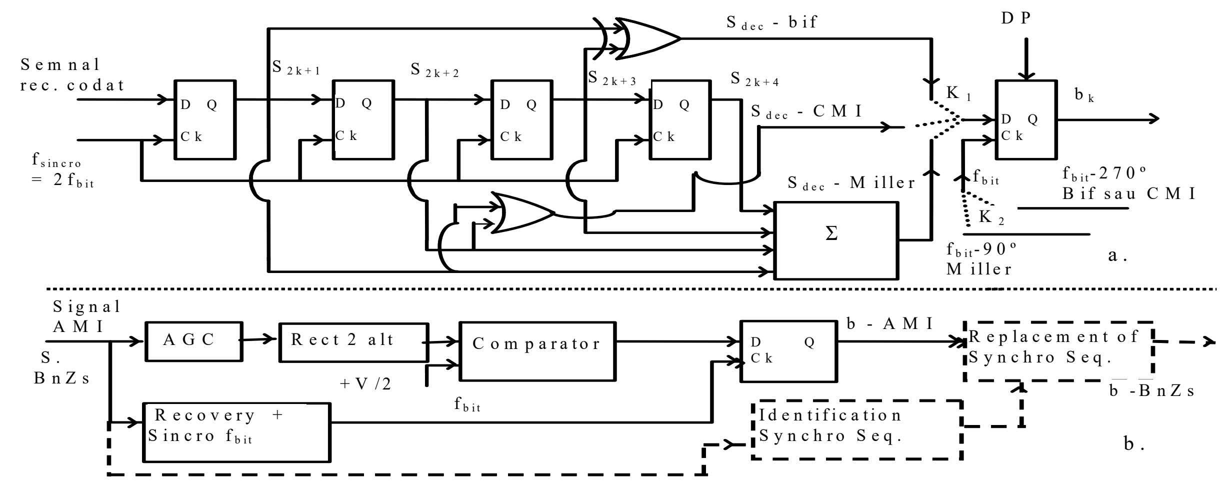


Fig. A2 a. Block diagram of the biphase-S, CMI and Miller decoders; b. Block diagram of the AMI and BnZs decoders