Frequency Modulation (FM)

- the FM modifies the frequency deviation of the modulated signal, around the carrier frequency, directly proportional to the voltage level of the modulating signal, while keeping the amplitude of the modulated signal to a constant value.
- the sign of frequency deviation might be the same as the one of the modulating signal or opposite
- considering the modulating signal: $g(t) = g_M \cdot f(t),\ f \in [-1, 1];$ (1)
- the expression of the momentary pulsation is:

$$\omega_i(t) = \omega_c + K_{FM} \cdot g_M \cdot f(t) = \omega_c + \Delta\omega_M \cdot f(t); \quad [K_{FM}] = rad/(s \cdot V) \tag{2}$$

- because the momentary phase of the modulated carrier is computed as the integral, with respect of time, of the momentary pulsation, and considering the V_0 the amplitude of the carrier signal, the FM signal is expressed by (3), where $\Delta\omega_M$ is the maximum frequency deviation allowed by the transmission:

$$s_{FM}(t) = V_0 \cos(\omega_c t + \Delta\omega_M \cdot \int_0^t f(\tau)d\tau) \tag{3}$$

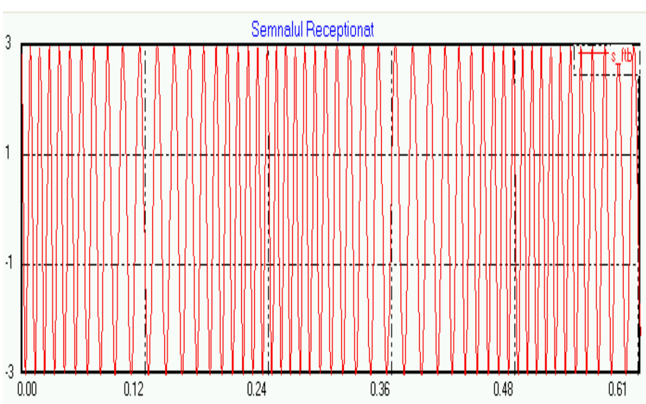
- for a periodical modulated signal the FM can be decomposed as (4), where $J_k(\beta)$ are the k-th order Bessel functions, while β indicates the modulation index, see [Ed.Nicolau].

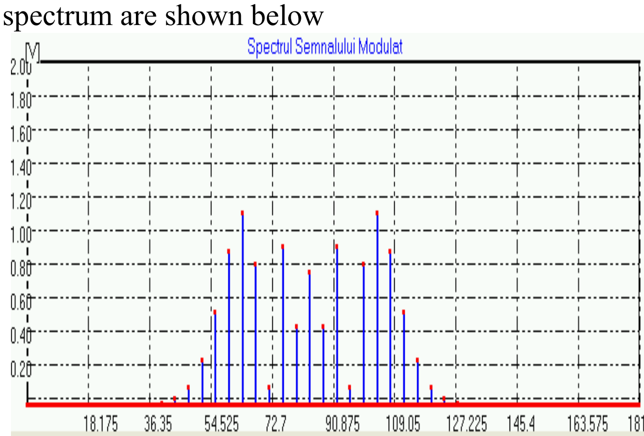
$$s_{FM}(t) = V_0 \cdot \sum_{k=-\infty}^{\infty} J_k(\beta) \cdot \cos(\omega_c t + k\omega_m t); \quad \beta_{tr} = \frac{\Delta\omega_M}{\omega_{mM}}; \tag{4}$$

- the bandwidth containing about 99% of the power of the FM signal is given by (5) and is centered on ω_c:

$$BW_{FM} = 2f_{mM}(1 + \beta + \sqrt{\beta}); \tag{5}$$

- for a modulating signal $g(t) = A\cos\omega_m t$ with $f_m = 4$ Hz, A = 2V, $\Delta\omega_M = 13$ Hz and for $f_c = 80$ Hz;, i.e. $\beta = 3.25$, $BW_{FM} = 40.25$ Hz; the modulated signal and its spectrum are shown below





FM modulated signal (left) and its spectrum (right) with the parameters mentioned above

- the most common example is the FM radio, with the parameters $f_{mM} = 15$ kHz, $\Delta f_M = 50$ kHz, $\beta = 3.33$, $BW_{FM} \approx 184.6$ kHz**;**

Generation of the FM signals

- several methods are described in reference [Ed. Nicolau]

The Armstrong method

- it is based on an approximation of the FM signal which is described by:

$$s_{FM}(t) = V_0 \cos[\omega_c t + \Delta\omega_M \int_0^t f(\tau)d\tau] = V_0 \cos(\omega_c t) \cdot \cos[\Delta\omega_M \int_0^t f(\tau)d\tau)] - V_0 \sin(\omega_c t) \cdot \sin[\Delta\omega_M \int_0^t f(\tau)d\tau)]$$

$$\text{for } \left|\Delta\omega_M \int_0^t f(\tau)d\tau\right| \le 0.2 \text{ rad} \Rightarrow \cos[\Delta\omega_M \int_0^t f(\tau)d\tau)] \approx 1 \text{ and } \sin[\Delta\omega_M \int_0^t f(\tau)d\tau)] \approx \Delta\omega_M \int_0^t f(\tau)d\tau) \Rightarrow \tag{6}$$

$$s_{FM}(t) \approx V_0 \cdot \cos\omega_c t - V_0 \cdot [\Delta\omega_M \cdot \int_0^t f(\tau)d\tau] \cdot \sin\omega_c t$$

- the conditions imposed in (6) to allow the approximations, lead to a very small modulation index, for a band-limited modulating signal $[\omega_{mm}, \omega_{mM}]$, i.e.:

$$\beta' = \frac{\Delta\omega_M}{\omega_{mM}} \le 0.5 \tag{7}$$

- in order to allow higher values of β, the signal is firstly modulated on an intermediate pulsation ω'_c (smaller than the channel carrier's pulsation ω_c) and with a modulation index β'(7), smaller than the requested β;
- then it is limited in amplitude and its Fourier decomposition is:

$$s_L(t) = \frac{4V_0}{\pi} \sum_{k=0} \frac{1}{2k+1} \sin\left[(2k+1)\omega'_c t + (2k+1)\Delta\omega'_M \int_0^t f(\tau)d\tau \right] \tag{8}$$

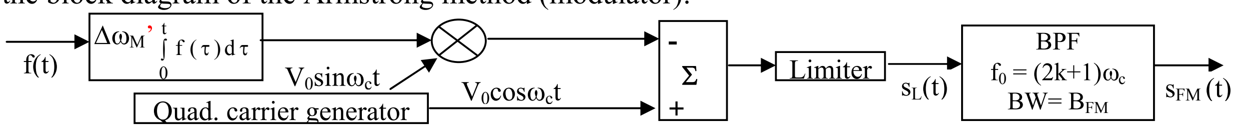
- imposing for the (2k+1)-th harmonic of (8), the (k+1) non-zero harmonic, the requirements:

$$(2k+1)\omega'_c = \omega_c; \quad (2k+1)\Delta\omega'_M = \Delta\omega_M \Rightarrow \quad (2k+1)\beta' = \beta \Leftrightarrow (2k+1)\frac{\Delta\omega_M'}{\omega_{mM}} = \frac{\Delta\omega_M}{\omega_{mM}} \tag{9}$$

- we get that the values of the intermediate pulsation ω_c' and of the modulation index β', used for the modulation described by (6). They are expressed by:

$$\omega'_c = \omega_c/(2k+1); \ \beta' = \beta/(2k+1) \leq 0.5 \tag{10}$$

- by BP-filtering the limited signal (8) with a filter centered on ω_c and with a passing band equaling BW_{FM}, the FM signal is modulated on the desired carrier frequency and has the desired modulation index.
- there should be noted that the factor (2k+1) should not be very high (< 11), because the amplitude of the corresponding harmonic in (8) might be too small
- the block diagram of the Armstrong method (modulator):



- another commonly used method is the control of the output frequency of a VCO by the amplitude of the modulating signal; considerations on this method are presented in the paragraph dedicated to *FM demodulation using a Phase-Locked Loop PLL*

Digital generation of FM signals

- the use of digital methods allows the computation of the cos(·) with an arbitrary accuracy
- a digital method to generate the modulated signal with non-linear modulations consists of two steps:

a. generation of a signal proportional to the momentary (instantaneous) phase of the modulated signal, this signal being the *argument* of the modulated signal. The momentary phase should be proportional to the modulating signal.

b. the computation of the cosine of the argument computed at step a.

- by uniformly sampling the FM signal (3) with a f_e sampling frequency (i.e. „reading the signal's values at t= $n \cdot T_e$ time instants), the values of the modulated signal at t= $n \cdot T_e$ is:

$$s_{FM}(nT_e) = V_0 \cos\left(\omega_c \cdot n \cdot T_e + \Delta\omega_M \cdot \int_0^{n \cdot T_e} f(\tau)d\tau \right) = V_0 \cos\left(\omega_c \cdot n \cdot T_e + \Delta\omega_M \cdot T_e \cdot \sum_{i=0}^{n} f(i \cdot T_e) \right) \tag{11}$$

- the momentary phase of the modulated signal at t= nT_e is:

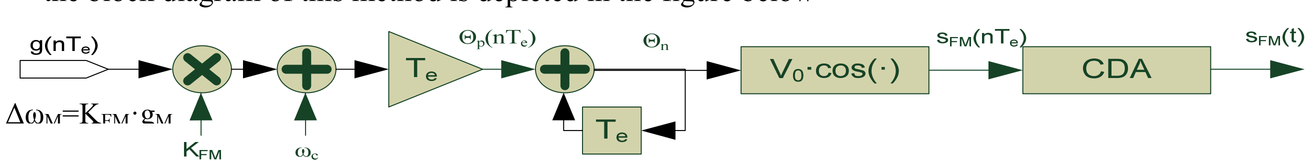
$$\begin{aligned} \Theta_n &= \omega_c \cdot n \cdot T_e + \Delta\omega_M \cdot T_e \cdot \sum_{i=0}^{n} f(i \cdot T_e) = T_e \cdot \left(\omega_c \cdot n + \Delta\omega_M \cdot \sum_{i=0}^{n} f(i \cdot T_e) \right) = \\ &= T_e \cdot \left(\omega_c \cdot (n-1) + \Delta\omega_M \cdot \sum_{i=0}^{n-1} f(i \cdot T_e) + \omega_c + \Delta\omega_M \cdot f(n \cdot T_e) \right) = \Theta_{n-1} + \Theta_p(n \cdot T_e) \end{aligned} \tag{12}$$

where by Θ_p we denoted the function which shows the variation of the momentary phase's value from one sample to the next one, i.e. during the n-th sampling period.

- the function $\Theta_p(\cdot)$ is directly proportional to the modulating signal:

$$\Theta_p(n \cdot T_e) = \omega_c \cdot T_e + \Delta\omega_M \cdot T_e \cdot f(n \cdot T_e) = \omega_c \cdot T_e + K_{FM} \cdot T_e \cdot g(n \cdot T_e) \tag{13}$$

- the block diagram of this method is depicted in the figure below



Block diagram of the FM modulator which generates digitally the momentary phase

Note: this method can be used only if the carrier frequency is relatively small (since it should observe the sampling theorem)

- for greater values of the carrier (central) frequency, in practical applications this signal is most often generated by using the QAM technique.
- Using the equality $\cos(a+b)=\cos(a)\cdot\cos(b)-\sin(a)\cdot\sin(b)$, the expression of the FM signal (3), can be rewritten as:

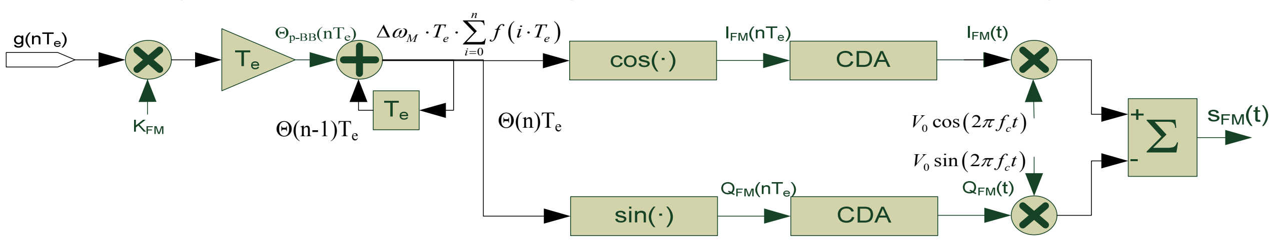
$$\begin{aligned} s_{FM}(t) &= V_0\cos\left(\omega_c t+\Delta\omega_M\cdot\int_0^t f(\tau)d\tau\right)= \\ &= V_0\cos\left(\Delta\omega_M\cdot\int_0^t f(\tau)d\tau\right)\cdot\cos(\omega_c t)-V_0\sin\left(\Delta\omega_M\cdot\int_0^t f(\tau)d\tau\right)\cdot\sin(\omega_c t)= \\ &= I_{FM}(t)\cdot V_0\cos(\omega_c t)-Q_{FM}(t)\cdot V_0\sin(\omega_c t) \end{aligned} \quad (14)$$

Note: the complex signal $s_{FM-BB}(t)=I_{FM}(t)+j\cdot Q_{FM}(t)$ is called *the complex envelope of the FM* signal

- by sampling the $I_{FM}(t)$ and $Q_{FM}(t)$ signals we get the discreet signals $I_{FM}(nT_e)$ and $Q_{FM}(nT_e)$;
- the phase-argument of the cosine and sine functions can be expressed in a recursive manner, similar to relations (11), (12) and (13), with a variable step which depends on the modulating signal:

$$\Theta_{p-I}(n\cdot T_e)=\Theta_{p-Q}(n\cdot T_e)=\Theta_{p-BB}(n\cdot T_e)=\Delta\omega_M\cdot T_e\cdot f(n\cdot T_e)=K_{FM}\cdot T_e\cdot g(n\cdot T_e) \quad (15)$$

- the block diagram of the modulator built using the QAM technique is shown in the figure below



QAM-based digital FM modulator

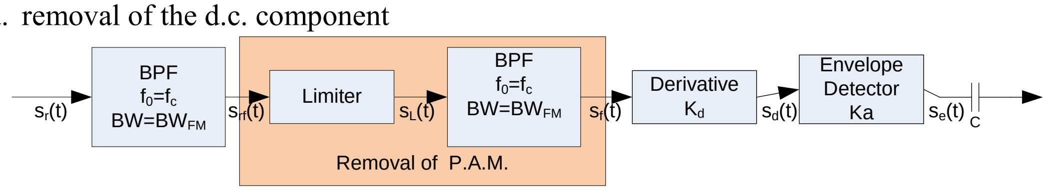
Note: the QAM based FM modulator requires a much smaller sampling frequency than the modulator that uses the digitally generation of the FM signal directly on the carrier frequency (usually $f_{mM} << f_c$)

- a digital method which generates directly the values of the cos(Θ_p) and sin(Θ_p) in a recursive manner will be presented in the lab classes

Demodulation of the FM signals – principles

- the demodulation of the FM signals consists of the following steps, see the block diagram below:

a. removal of the "parasitic" amplitude modulation (PMA) induced by the channel – the resulted signal has a constant amplitude
b. performing the derivative of the FM modulated signal – the resulting signal gets a DSB-C (AM) modulation, besides the existing FM
c. performing the envelope detection (or an non-coherent LM demodulation)
d. removal of the d.c. component



Block diagram of a FM demodulator principle

- the input band-pass filter is meant to increase the SNR of the received signal

a. *Removal of the "parasitic" amplitude modulation*

- the received signal is given by (16); note the variable amplitude A(t):

$$s_{rf}(t)=A(t)\cos(\omega_c t+\Delta\omega_M\int_0^t f(\tau)d\tau); \quad (16)$$

- the removal of the P.A.M. consists of two steps: - limiting the signal, followed by a BP filtering
- the limited signal is:

$$s_L(t) = \frac{4V}{\pi}\sum_{k=0}\frac{1}{2k+1}\sin[(2k+1)(\omega_c t + \Delta\omega_M \int_0^t f(\tau)d\tau)]; \quad (17)$$

- the second BP filter retains only the first harmonic of (17), resulting the FM signal with a constant amplitude (envelope):

$$s_f(t) = \frac{4V}{\pi}\sin(\omega_c t + \Delta\omega_M \int_0^t f(\tau)d\tau)]; \quad (18)$$

- to allow the filtering, the spectra around the ω_c and $3\omega_c$ should not overlap, i.e.:

$$3\,\omega_c - BW(3\Delta\omega_M)/2 - \omega_{mM} > \omega_c + BW(\Delta\omega_M)/2 + \omega_{mM}; \rightarrow \ \omega_c > BW(3\Delta\omega_M)/2 + BW(\Delta\omega_M)/2 + \omega_{mM}; \quad (19)$$

Note: two issues should be mentioned with respect to (19):

- the maximum frequency deviation of the FM spectrum centered on the (2k+1)-th harmonic of the limited signal has the value $\Delta\omega_{M-(2k+1)} = (2k+1)\ \Delta\omega_M$,, see (17); this leads to a modulation index of the FM on this harmonic of $\beta_{2k+1} = (2k+1)\beta$.
- the bandwidth of the the FM on this harmonic should be computed using this value of this β_{2k+1} modulation index in (5), which is $BW_{2k+1} = \left(2f_{mM}1 + \beta_{2k+1} + \sqrt{\beta_{2k+1}}\right)$

b. ***Performing the derivative of the FM modulated signal***

- the derivative of the FM signal is intended to insert an additional DSC-C modulation, besides the FM one:

$$s_d(t) = k_d\frac{4V}{\pi}(\omega_c + \Delta\omega_M f(t)) \cdot \cos(\omega_c t + \Delta\omega_M \int_0^t f(\tau)d\tau)]; \quad (20)$$

- (20) correlated to (17) show that each spectral component of an FM signal would also be DSB-C modulated and the bandwidth of this AM modulation equals +/- ω_{mM}; this is why the the frequency bandwidths considered in (19) are expanded with ω_{mM}.

c. ***Envelope detection***

- it extracts a signal proportional to the envelope of (20), in the baseband, accompanied by a d.c. component:

$$s_o(t) = k_d \cdot k_e\frac{4V}{\pi}(\omega_c + \Delta\omega_M f(t)) = k_d \cdot k_e\frac{4V}{\pi}\omega_c + k_d \cdot k_e\frac{4V}{\pi}\Delta\omega_M f(t)) \quad (21)$$

d. ***Removal of the d.c. component***

- the capacitor C together with the Z_{in} of the audio amplifier act like a HP filter delivering an output signal proportional with the modulating one:

$$s_o(t) = k_d \cdot k_e\frac{4V}{\pi}\Delta\omega_M f(t)) \quad (22)$$

- HOMEWORK: Compute the effects of the removal of the P.A.M. block from the demodulator, upon the demodulated signal

- the derivative and the envelope detector may be performed by several methods each; therefore there exist more variants of demodulators according to the combination of methods employed.

Methods of performing the derivative of the FM signal

1. ***employing a circuit that performs directly the derivative*** – the Clarck-Hess demodulator (see ref. Ed.Nicolau) – it is employed in FM "classical" radio receivers
2. ***the delay-derivative method*** – it is employed the TV analogue receivers for the sound signal demodulation

- this method is based on the approximation of the derivative of a signal:

$$\frac{du(t)}{dt} \approx \frac{u(t) - u(t - t_0)}{t_0}; \quad \text{for } t_0 \text{ small} \quad (23)$$

- this method is detailed in [Ed. Nicolau] and a brief description is presented in *Annex 1* of this material

Envelope detection

- basically, it could be performed by several methods

1. Synchronous (coherent) detection – this method is detailed in [Ed. Nicolau] and briefly described in *Annex 2* of this material
2. by using an average detector (non-coherent), [Ed. Nicolau], briefly described in *Annex 3* of this material
3. by using a peak detector (non-coherent), [Ed. Nicolau], briefly described in *Annex 4* of this material

- a comparison between the performances of these three detectors is presented in *Annex 5* of this material

Principles of practical schemes of analogue FM demodulators

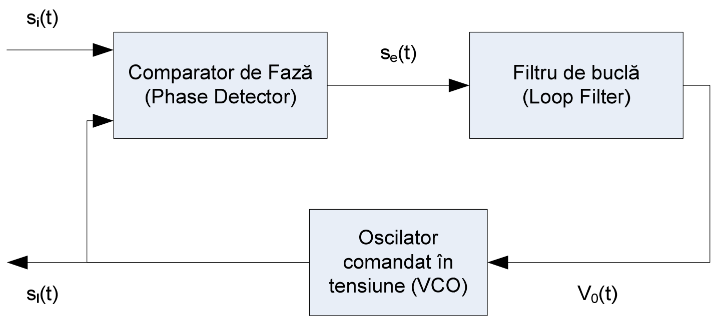
- three practical schemes are most often employed for the analogue demodulation of the FM signals:

1. the frequency discriminator
2. the ratio detector
3. the delay-derivative and coherent detection demodulator (DD-CD)

- the first two are dealt with in [Ed.Nicolau], while the third is described in *Annex 6* of this material

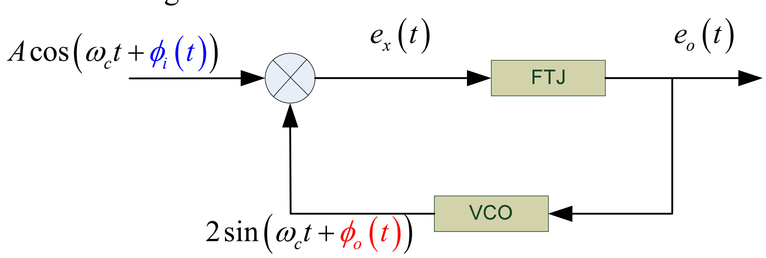
FM demodulation using a Phase-Locked Loop PLL

- PLL is a group of functional blocks that generates a signal $s_l(t)$ (output) whose phase is synchronized (aligned) to the phase of a reference signal $s_i(t)$ (input). Its block diagram is presented in the figure below.



Block diagram of a PLL

- the Phase Detector provides an output signal, named error-voltage, which is proportional (ideally) to the phase difference (offset) betwen its two input signals $s_l(t)$ and $s_i(t)$.
- the Loop Filter retains the low-frequency components of the error voltage (removing the the high frequency components inserted by noise, other interferences or by processing), thus providing a control-voltage that changes the momentary frequency of the local Voltage-Controlled Oscilator (VCO). The system reaches the „equilibrium" when the control voltage is constant, meaning that the momentary frequency and phase of the locally generated signal $s_l(t)$ equals the frequency and phase of the refference input signal $s_i(t)$.
- the block diagram of a FM demodulator built with a PLL circuit is presented below:



FM demodulator with PLL

- the error-voltage $e_x(t)$ is obtained by multiplying the received FM signal (input) to the locally generated VCO signal:

$$e_x(t)=A\cos\left(\omega_c t+\phi_i(t)\right)\cdot 2\sin\left(\omega_c t+\phi_o(t)\right)=A\sin\left(2\omega_c t+\phi_i(t)+\phi_o(t)\right)+A\sin\left(\phi_i(t)-\phi_o(t)\right) \quad (24)$$

- the LP filter attenuates the components around $2\omega_c$ thus providing the control-voltage:

$$e_o(t)=A\sin\left(\phi_i(t)-\phi_o(t)\right)=A\sin\left(\phi_e(t)\right) \quad (25)$$

where $\phi_e(t)$ denotes the phase-offset between the input (refference) and output (locally generated) signals:

$$\phi_e(t)=\phi_i(t)-\phi_o(t) \quad (26)$$

- since the received (refference) signal is an FM signal, by using the phase of (3), we get:

$$\phi_i(t)=\Delta\omega_M\cdot\int_0^t f(\tau)\,d\tau \quad (27)$$

- substituting (27) in (26) we get:

$$\phi_o(t)=\Delta\omega_M\cdot\int_0^t f(\tau)\,d\tau-\phi_e(t) \quad (28)$$

- the VCO is actually a frequency modulator whose output is a signal whose momentary frequency varies around ω_c (the free-running frequency, being set to ω_c) proportionally to the amplitude level of the control voltage.
- the pulsation of the VCO's output is:

$$\omega_l=\omega_c+K_{FM}\cdot e_o(t) \quad (29)$$

- knowing that momentary pulsation is the derivative of the momentary phase, i.e.,: $\phi_o'(t) = K_{FM} \cdot e_o(t)$ (30)

- we get that: $$e_o(t) = \frac{1}{K_{FM}} \phi_o'(t) \quad (31)$$

- replacing now (28) in (31), the control-voltage becomes:

$$\begin{aligned} e_o(t) &= \frac{1}{K_{FM}} \left(\Delta\omega_M \cdot \int_0^t f(\tau)d\tau - \phi_e(t) \right)' = \frac{1}{K_{FM}} \left(\Delta\omega_M \cdot \int_0^t f(\tau)d\tau \right)' + \frac{1}{K_{FM}} \phi_e'(t) = \\ &= \frac{\Delta\omega_M}{K_{FM}} f(t) + \frac{1}{K_{FM}} \phi_e'(t) \approx \frac{\Delta\omega_M}{K_{FM}} f(t) \end{aligned} \quad (32)$$

- relation (32) shows that the filtered error-voltage (the control voltage) is proportional to the the frequency deviation around the carrier frequency; but, for FM signals, the frequency deviation around the carrier frequency is proportional to the amplitude level of the modulating signal, see (2).
- the approximation made in (32) is acceptable if the control voltage has low variations with time, i.e. if the filter attenuates the very high components of the error-voltage.
- the FM modulation can be accomplished by „breaking" the loop between the PhD and the loop-filter
- if we set the „free-running" frequency to f_c, and $g_M \cdot K_{FM} = \Delta\omega_M$ and insert the modulating signal $g(t) = g_M \cdot f(t)$ in the loop filter, whose cut-off frequency is set to be equal to ω_{mM}, the momentary pulsation at the VCO's output would be:

$$\omega_l(t) = \omega_c + K_{FM} \cdot g_M \cdot f(t) = \omega_c + \Delta\omega_M \cdot f(t) \quad (33)$$

- then using (3), the expression of the signal generated by the VCO would be (34), where V_0 is the amplitude of the carrier

$$s_{FM}(t) = V_0 \cos(\omega_c t + \Delta\omega_M \cdot \int_0^t f(\tau)d\tau) \quad (34)$$

- some more considerations regarding the *PLL* circuits will be discussed in the chapter dedicated to bit synchronization in baseband coding.

Principle of Digital Demodulation of FM

- the FM modulated signal is expressed as:

$$s_{FM}(t) = V_0 \cos\left(\omega_c t + \Delta\omega_M \cdot \int_0^t f(\tau)d\tau \right) \quad (35)$$

- considering the trigonometrical identity:

$$\cos(a+b) = \cos(a)\cdot\cos(b) - \sin(a)\cdot\sin(b) \quad (36)$$

expression (35) may be rewritten as QAM decomposition as:

$$\begin{aligned} s_{FM}(t) &= V_0 \cos\left(\Delta\omega_M \cdot \int_0^t f(\tau)d\tau \right) \cdot \cos(\omega_c t) - V_0 \sin\left(\Delta\omega_M \cdot \int_0^t f(\tau)d\tau \right) \cdot \sin(\omega_c t) = \\ &= I_{FM}(t) \cdot \cos(\omega_c t) - Q_{FM}(t) \cdot \sin(\omega_c t) \end{aligned} \quad (37)$$

- by means of a QAM demodulator, see the second figure on page 12 and relations (15)-(21) in the lecture dedicated to LM demodulation, the components $I_{FM}(t)$ and $Q_{FM}(t)$ are extracted as:

$$I_{FM}(t) = V_0 \cos\left(\Delta\omega_M \cdot \int_0^t f(\tau)d\tau \right); \quad Q_{FM}(t) = -V_0 \sin\left(\Delta\omega_M \cdot \int_0^t f(\tau)d\tau \right); \quad (38)$$

- these signals are the real and imaginary parts of the *FM* baseband complex envelope:

$$s_{FM-BB}(t) = I_{FM}(t) + jQ_{FM}(t) = V_0 \cos\left(\Delta\omega_M \cdot \int_0^t f(\tau)d\tau \right) - jV_0 \sin\left(\Delta\omega_M \cdot \int_0^t f(\tau)d\tau \right) = V_0 e^{j\Delta\omega_M \cdot \int_0^t f(\tau)d\tau} \quad (39)$$

- the $I_{FM}(t)$ and $Q_{FM}(t)$signals are converted in digital signals by analog-digital converters (CAD) with a sampling period T_e, i.e, $t = nT_e$:

$$s_{FM-BB}\left(nT_e\right) = I_{FM}\left(nT_e\right) + jQ_{FM}\left(nT_e\right) = V_0 e^{j\Delta\omega_M \sum_{x=0}^{n} f\left(xT_e\right)} \tag{40}$$

- the phase variation of the complex envelope is actually the phase variation of the *FM* signal which is induced by the modulating signal $f(xT_e)$ and is expressed as: - explain the sum in the righthand expression

$$\phi(nT_e) = \measuredangle\left(s_{FM-BB}\left(nT_e\right)\right) = \tan^{-1}\left(\frac{Q_{FM}\left(nT_e\right)}{I_{FM}\left(nT_e\right)}\right) = \Delta\omega_M \sum_{x=0}^{n} f\left(xT_e\right) \tag{41}$$

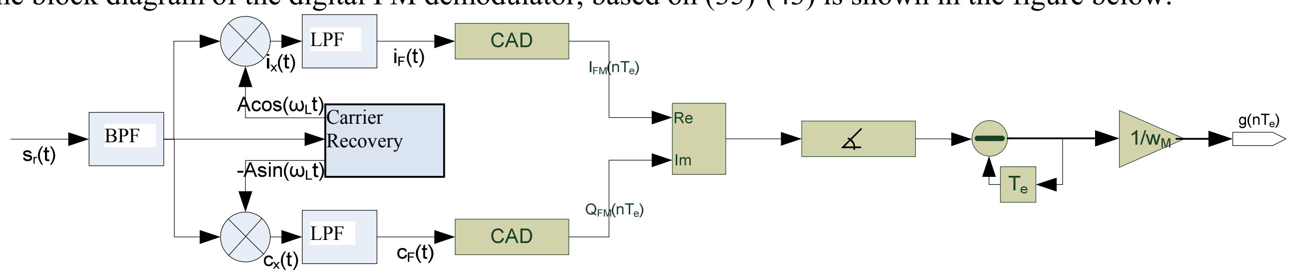
- expression (41) may be rewritten as:

$$\phi(nT_e) = \Delta\omega_M \sum_{x=0}^{n} f\left(xT_e\right) = \Delta\omega_M \sum_{x=0}^{n-1} f\left(xT_e\right) + \Delta\omega_M f\left(nT_e\right) = \phi((n-1)T_e) + \Delta\omega_M f\left(nT_e\right) \tag{42}$$

- using (42), the curent sample $f(nT_e)$ of the modulating signal at $t = nT_e$ can be computed recursively, from the succesive samples, by using (43), where $\Phi(0) = 0$:

$$f\left(nT_e\right) = \frac{\phi(nT_e) - \phi((n-1)T_e)}{\Delta\omega_M} \tag{43}$$

- the block diagram of the digital FM demodulator, based on (35)-(43) is shown in the figure below:



Block diagram of a FM digital demodulator

- the effects of the incorrect recovery of the local carrier signal are similar to the ones presented in the LM second lecture, relations (18).

Signal to noise ratio performances of the FM

- we consider that the input FM signal has the power P_s and the additive Gaussian noise has the power $P_N = N_0 \cdot B_{FM}$, both after the P.AM.R.;
- the SNR performances of the FM are evaluated by the ratio between the values of the signal/noise ratio at the output of the demodulator and the signal to noise ratio at its input, see (44).
- the S/N ratio in linear expression is denoted by ρ; its logarithmic expression is denoted by SNR [dB]

$$\eta = \frac{\rho_o}{\rho_i}; \quad \rho_o = \frac{P_{so}}{P_{No}}; \quad \rho_i = \frac{P_{si}}{P_{Ni}} = \frac{V_0^2 \widetilde{f^2(t)}}{2N_0 BW_{FM}}; \tag{44}$$

- recalling that the input FM signal is:

$$s_i(t) = V(t)\cos(\omega_c t + \Delta\omega_M \int_0^t f(\tau)d\tau) = V(t)\cos(\omega_c t + \Phi(t)) \tag{45}$$

- the noise signal can be expressed in a similar form:

$$n_i(t) = R(t)\cos(\omega_c t + \Psi(t)) \tag{46}$$

- because the signal at the input of the demodulator is the sum of $s_{FM}(t)$ and the noise signal, it may be expressed as:

$$\begin{gathered} s_r(t) = s_{FM}(t) + n_i(t) = V(t)\sin(\omega_c t + \Phi(t)) + R(t)\cos(\omega_c t + \Psi(t) = A(t)\cos(\omega_c t + \varphi_r(t)); \\ \varphi_r(t) = \Phi(t) + \mathrm{arctg}\frac{U(t)}{V_0}; \quad U(t) = R(t)\sin(\Psi(t) - \Phi(t)); \end{gathered} \tag{47}$$

- due to the P.A.M.R. block, the resultant amplitude A(t) will become constant, denoted by A
- the FM demodulation will extract only a voltage $U_{LF}(t)$ proportional to the frequency variation of the resultant signal around the carrier's (central) frequency, that represents the output signal $s_o(t)$:

$$U_{LF}(t) = A\frac{d\varphi_r(t)}{dt} = A\frac{d[\Phi(t) + \mathrm{arctg}\frac{R(t)\sin(\Psi(t)-\Phi(t))}{V_0}]}{dt}; \quad (48)$$

- in terms of the values of SNR_i, the output signal $U_{LF}(t)$ should be analyzed for two “extreme” cases:

1. SNR_i very small, i.e. $R(t) >> V_0 \rightarrow SNR_i < 0$ dB

- in this case the probability of the noise amplitude R(t) to be higher than V_0 is almost 1, leading to an output signal that has no term which would be proportional to the modulating signal, see ref. Ed. Nicolau, but only terms that “are controlled” by noise:

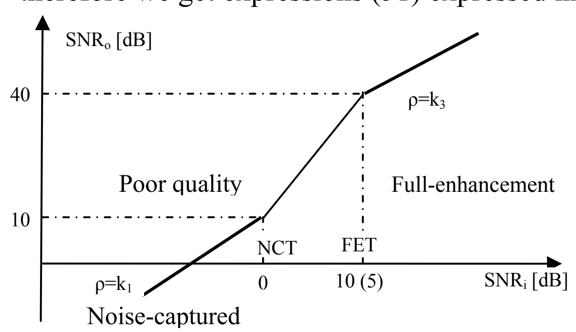
$$P(R(t) > V_0) \approx 1 \Rightarrow U_{LF}(t) = A\frac{d\varphi_r(t)}{dt} = A\frac{d[\mathrm{arctg}\frac{R(t)\sin(\Psi(t)-\Phi(t))}{V_0}]}{dt}; \quad (49)$$

- therefore, in this SNR_i domain, the reception is “captured” by noise, and the $SNR_i = 0$ dB threshold is called “the noise-captured” threshold, NCT.
- for $SNR_i = 0$ dB, the corresponding $SNR_0 = 10$ dB and, for $SNR_i < 0$ dB, SNR_0 is approximated by:

$$SNR_0\,[dB] = 10\,dB + k_1 \cdot SNR_i\,[dB]; \quad (50)$$

2. SNR_i high, i.e. *$R(t) << V_0 \rightarrow SNR_i > 10$ dB (5 dB)*

- in this case for $SNR_i \geq 10$ dB (5 dB), $P(R(t) < V_0) > 0.996$ and the SNR at the output is significantly higher than the one at the input;
- therefore we get expressions (51) expressed in linear and logarithmic scales:



$$P(R(t) < V_0) \approx 1 \Rightarrow \eta = \frac{\rho_o}{\rho_i} = k_3 \cdot 3 \cdot \beta^2 \cdot \frac{BW_{FM}}{2 \cdot f_{mM}}; \quad \text{for } \rho_i > 10 \Rightarrow$$

$$SNR_0[dB] = 8.5dB + 10\lg(3 \cdot \beta^2 \cdot \frac{BW_{FM}}{2 \cdot f_{mM}}) + SNRi\,[dB]$$

$$\text{for } SNR_i > 10dB \quad (51)$$

- in the logarithmic expression of (51), the value of k_3, i.e. the final value of the SNR_o corresponding to the (0,10) (10, ...) dB interval, was computed for $\beta = 3{,}3$
- this phenomenon is called “full-enhancement” and the $SNR_i = 10$ dB (5 dB) is called the full-enhancement threshold, FET.
- the FET = 10 dB for demodulators that have no control loop, like the ones presented in the Annexes
- the FET = 5 dB for demodulators that contain a control loop, like the PLL-based one, presented above
- for SNR_i ranging between the two thresholds ($0<SNR_i<10$dB), the SNR_o is multiplied by a different factor k_2, see adjoining figure.
- in this third domain, $SNR_i \in [0, 10]$ dB, the demodulated signal is one of poor quality, but still it is not captured by noise.
- some considerations regarding the *Emphasizing and de-emphasizing in FM transmissions* are presented in Annex 4 of this material

Considerations regarding the frequency downwards-translation of the LM and FM signals

- the FM modulated signals are transmitted on different carrier signals; to perform the demodulation, the FM receiver should change the central frequency of the two band-pass filters (while keeping the pass-band constant!) and modify the parameters of the derivative circuit and of the envelope detectors.
- these requirements would involve considerable technological problems and would considerably complicate the implementation.
- therefore, the demodulation is performed on a fixed frequency, called intermediate frequency f_i =(10.7 MHz).
- the translation of the modulated signal from a carrier frequency f_c to the intermediate frequency f_i is accomplished in two steps:
- *multiplication of the received signal with a locally generated translation (cosine) signal, whose frequency is denoted f_t:*

$$s_x(t) = [V_0\cos(\omega_c t + \Phi(t)) \cdot A\cos\omega_t t]/V_{ref} = V'\cos(\omega_c t + \Phi(t) - \omega_t t) + V'\cos(\omega_c t + \Phi(t) + \omega_t t); \quad (52)$$

- *band pass-filtering that retains only the bandwidth centered on f_i*
- because f_i is smaller than f_c, the f_t should be higher than f_i; therefore the second term is centered on high

frequencies and would be cancelled by the band-pass filtering, leaving only the translated signal $s_t(t)$, the first term of (52).

- if we impose: $$|\omega_c - \omega_t| = \omega_i ; \quad (53)$$

then we have two alternatives:

$$\begin{aligned} &\omega_c > \omega_t \Rightarrow \omega_t = \omega_c - \omega_i \quad \text{and } s_t(t) = k_f V' \cos(\omega_i t + \Phi(t)); \quad \text{a.} \\ &\omega_c < \omega_t \Rightarrow \omega_t = \omega_c + \omega_i \quad \text{and } s_t(t) = k_f V' \cos(\omega_i t - \Phi(t)); \quad \text{b.} \end{aligned} \quad (54)$$

- alternative a. employs a smaller translation frequency and does not change the sign of the translated signal's phase, while alternative b. changes the sign of this phase (the cosine function is an even function) and employs a greater translation frequency.
- the BP filter should have the following parameters:

$$f_0 = f_i \text{ and } \Delta\omega = B_{FM} \quad (55)$$

- the condition required by the spectra of (44) to be separable can be easily derived
- since the analog multiplier is not available for high frequencies, the multiplication can be performed by an unbalanced or a balanced chopper; the chopper is described in the first LM lecture notes, (44)-(45) and (47)-(48).
- the assembly chopper + BP filter, implemented by a parallel RLC tuned circuit, is named as "mixer"-see the first LM lecture
- this frequency translation method is used in FM receivers on an intermediate frequency f_i = 10.7 MHz
Some considerations regarding the choice of this value of f_i will be discussed the FM-seminar
- the frequency translation method is similarly used for the AM transmissions, but for a f_i = 455 kHz

Homework:

1. **How many signal carriers are downwards-translated using the method described above? Mathematical justification.**
2. **Derive the equations that describe the upward frequency translation, from the intermediate frequency f_i to the channel-carrier frequency f_c; how many values could f_t have and when is the phase of the modulated signal changed by translation?**

Annexes

Annex 1 – not required for the exam

The delay-derivative method – it is employed the TV analogue receivers for the sound signal demodulation

- this method is based on the approximation of the derivative of a signal:

$$\frac{du(t)}{dt} \approx \frac{u(t)-u(t-t_0)}{t_0}; \quad \text{for } t_0 \text{ small} \tag{A.1}$$

- this method is detailed in [Ed. Nicolau] and a brief description is presented in *Annex 1* of this material

- denoting by $s_{rf}(t)$ the received signal and by $s_h(t-t_0)$ its version delayed by t_0, the signal $s_d(t)$ after the derivative is expressed by (A.2) (the block diagram is included in the first figure of Annex 2).

$$\begin{aligned} s_d(t) &\approx s_{rf}(t) - s_h(t-t_0) = V_0 \sin[\omega_c t + \Delta\omega_M \int_0^t f(\tau)d\tau] - V_0 \sin[\omega_c t + \Delta\omega_M \int_0^{t-t_0} f(\tau)d\tau - \omega_c t_0] = \\ &= 2V_0 \sin\left[\frac{\Delta\omega_M}{2}\int_{t-t_0}^{t} f(\tau)d\tau + \frac{\omega_c t_0}{2}\right]\cdot\cos\left[\omega_c t + \Delta\omega_M \int_0^{t-t_0} f(\tau)d\tau + \frac{\Delta\omega_M}{2}\int_{t-t_0}^{t} f(\tau)d\tau - \frac{\omega_c t_0}{2}\right] \end{aligned} \tag{A.2}$$

- the $s_d(t)$ signal has two factors: the first (A) is a baseband frequency signal and represents a DSB-C modulation added to the FM signal; the second (B) is centered on the carrier and represent the FM modulation.
- the envelope detection that follows the derivative will "suppress" this second factor
- since the factor A is not directly proportional to the modulating signal f(t), it requires further processing
- if we impose that: $$t_0 < 2/\omega_{mM}; \tag{A.3}$$

  then the integral can be approximated by:

$$\int_{t-t_0}^{t} f(\tau)d\tau \approx f(t-\frac{t_0}{2})\cdot t_0; \tag{A4}$$

- using (A.4) the factor A of (A.2) becomes:

$$A = 2V_0 \sin[\frac{\Delta\omega_M}{2}\int_{t-t_0}^{t} f(\tau)d\tau + \frac{\omega_c t_0}{2}] \approx 2V_0 \sin[\frac{\Delta\omega_M}{2}\cdot f(t-t_0/2)\cdot t_0 + \frac{\omega_c t_0}{2}]; \tag{A.5}$$

- if we impose now that:

$$\begin{aligned} &\alpha = \left|(\Delta\omega_M/2)\cdot f(t-t_0/2)\cdot t_0\right| \le 0.2 \text{ radians}; \Rightarrow \sin\alpha \approx \alpha \text{ and } \cos\alpha \approx 1; \text{ a.} \\ &\omega_c t_0 = \pi/2 \Rightarrow t_0 = 1/4f_c; \text{ b.} \end{aligned} \tag{A.6}$$

- then factor A of (A.2) becomes, see (A.6):

$$A \approx 2V_0 \cdot \frac{\Delta\omega_M t_0}{2}\cdot f(t-t_0/2)\cdot\cos\frac{\pi}{4} + 2V_0\cdot\sin\frac{\pi}{4} = \sqrt{2}\cdot V_0\cdot[\frac{\Delta\omega_M t_0}{2}\cdot f(t-t_0/2)+1]; \tag{A.7}$$

- replacing now (A.7) in (A.2) we get the expression of the derivative signal $s_d(t)$, obtained by this method:

$$\begin{aligned} s_d(t) &\approx \sqrt{2}\cdot V_0\cdot[\frac{\Delta\omega_M t_0}{2}\cdot f(t-t_0/2)+1]\cdot\cos[\omega_c t + \Delta\omega_M \int_0^{t-t_0} f(\tau)d\tau) + \frac{\Delta\omega_M}{2}\int_{t-t_0}^{t} f(\tau)d\tau - \frac{\pi}{4}] \approx \\ &\approx \sqrt{2}\cdot V_0\cdot[\frac{\Delta\omega_M t_0}{2}\cdot f(t-t_0/2)+1]\cdot\cos[\omega_c t + \Delta\omega_M \int_0^{t} f(\tau)d\tau) - \frac{\pi}{4}] \end{aligned} \tag{A.8}$$

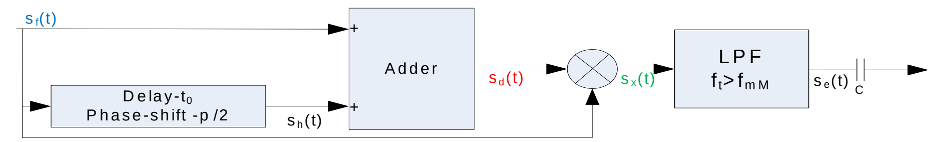
- the signal after the derivative has a DSB-C modulation which is directly proportional to the modulating signal; the carrier, which is FM modulated would be removed by the envelope detection;
  - to remove the (+1) term, that would lead to a d.c. component after the envelope detection, a balanced version of this method is described in [Ed.Nicolau]; an High-Pass output filtering could also be employed

Annex 2 –not required for the exam

Synchronous (Coherent) Product Detection – the block diagram is included in the figure below, together with the delay-derivative method

- it involves the multiplication of the $s_d(t)$ signal (24) with the received signal $s_f(t)$ (13), generating the signal $s_x(t)$; then this signal is LP filtered and the d.c. component (if any) is removed, as shown in the figure below

- considering the $s_f(t)$ (18), repeated here for convenience: $$s_f(t) = \frac{4V}{\pi}\sin(\omega_c t + \Delta\omega_M \int_0^t f(\tau)d\tau) \tag{18}$$



Block diagram of the FM demodulator that uses the delay-derivative and coherent envelope detection (DD-CD)

- then using $s_d(t)$, (A.8), the multiplier's output $s_x(t)$ is:

$$s_x(t)=\sqrt{2}V_0[\frac{\Delta\omega_M t_0}{2}\cdot f(t-t_0/2)+1]\cdot\cos[\omega_c t+\Delta\omega_M\int_0^t f(\tau)d\tau-\frac{\pi}{4}]\cdot\frac{4V}{\pi}\sin[\omega_c t+\Delta\omega_M\int_0^t f(\tau)d\tau]=$$
$$=k_a k_d V_0[\frac{\Delta\omega_M t_0}{2}\cdot f(t-t_0/2)+1]\cdot[\frac{1}{4}-\frac{1}{4}\cos(2\omega_c t+2\Delta\omega_M\int_0^t f(\tau)d\tau)+\frac{1}{4}\sin(2\omega_c t+2\Delta\omega_M\int_0^t f(\tau)d\tau)] \quad (A.9)$$

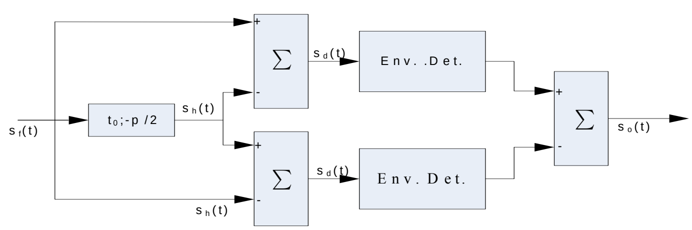
- the multiplied signal has a baseband spectrum (low-frequency) and a spectrum centered around $2\omega_c$
- the high-frequency spectrum is attenuated by the LP filter and the signal the output of the LP filter, $s_a(t)$, is:

$$s_a(t)=k_a k_d V_0[\frac{\Delta\omega_M t_0}{2}\cdot f(t-t_0/2)+1]=k_a k_d V_0\cdot\frac{\Delta\omega_M t_0}{2}\cdot f(t-t_0/2)+k_a k_d V_0 \quad (A.10)$$

- the d.c. component (the second term of (A.10) is removed by C and Z_{in} and the output signal is:

$$s_o(t)=k_a k_d V_0\cdot\frac{\Delta\omega_M t_0}{2}\cdot f(t-t_0/2) \quad (A.11)$$

- the circuit shown in the figure above is an unbalanced version, which delivers a d.c. component; its suppression, by HP filtering might alter the low-frequency components of the modulating signal.



- therefore, a balanced version of the demodulator is used; its block scheme shown in the adjoining figure employs two envelope detectors.

Balanced version of the DD-CD FM demodulator

- the output voltage is:

$$s_o(t)=k_a\cdot k_a\cdot V_0\cdot\Delta\omega_M f(t-t_0/2) \quad (A.12)$$

- the output voltage is doubled and this scheme does not deliver a d.c. component, so an HP filter is not needed
- the filtering condition required by the synchronous detection is:

$$\omega_{mM}<2\omega_c-B_{FM}(2\Delta\omega_M)/2-\omega_{mM}\rightarrow\omega_c>B_{FM}(2\Delta\omega_M)/4+\omega_{mM}; \quad (A.13)$$

- a comparison between the performances provided by the three envelope detection methods is presented in *Annex 5* of this material

Annex 3- not required for the exam

Non-coherent detection employing an averaging envelope detector

- this method performs the envelope detection of the output signal of the derivative circuit $s_d(t)$, see (18) or (A.2).
- recalling that the ideal diode behaves like an interrupter, controlled by the $s_d(t)$, the output signal, $s_r(t)$, is the product of the interruption function times the input signal $s_d(t)$.
- if the $f_i(t)$ is expressed by its Fourier series decomposition, relation (44) in the first LM lecture notes, the signal at the diode's output, $s_r(t)$, is expressed by (A.14)

$$\text{denote}\quad X(t)=\Delta\omega_M\int_0^t f(\tau)d\tau-\frac{\pi}{4};$$
$$s_r(t)=s_d(t)f_i(s_d(t)=k_d\frac{4V}{\pi}[\omega_c+\Delta\omega_M f(t)]\cdot\cos[\omega_c t+X(t)]\cdot$$
$$\cdot\left\{\frac{1}{2}+\frac{2}{\pi}\sin\left[\omega_c(t-\frac{T_c}{4})+X(t)\right]+\frac{2}{3\pi}\sin\left[3\omega_c(t-\frac{T_c}{4})+3X(t)\right]\cdots\right\}= \quad (A.14)$$
$$=\frac{1}{2}s_d(t)-k_d\frac{8V}{\pi^2}[\omega_c+\Delta\omega_M f(t)]\cos^2[\omega_c t+X(t)]+k_d\frac{2}{3\pi}s_d(t)\sin\left[3\omega_c(t-\frac{T_c}{4})+3X(t)\right]+\cdots=$$
$$=-k_d\frac{4V}{\pi^2}[\omega_c+\Delta\omega_M f(t)]-k_d\frac{4V}{\pi^2}[\omega_c+\Delta\omega_M f(t)]\cdot\cos[2\omega_c t+2X(t)]+\frac{1}{2}s_d(t)+A(t);$$

- term A(t) contains spectral components centered around $2\omega_c$, $4\omega_c$ and the other harmonics of ω_c resulted from term 3 in the 3rd row of (A.14)
- after the low-pass filtering performed by the R_0C_0 group (fig. pg. 11 AM lecture), only the baseband

components, i.e., the firs term in (A.14), are retained, i.e. the output signal of the envelope detector is:

$$s_a(t) = k_d \frac{4V}{\pi^2}(\omega_c + \Delta\omega_M f(t)) = k_d k_a V\omega_c + k_d k_a V\Delta\omega_M f(t)); \quad (A.15)$$

- the filtering condition is:

$$\omega_c - B_{FM}(\Delta\omega_M)/2 - \omega_{mM} > \omega_{mM} \rightarrow \omega_c > B_{FM}/2 + 2\omega_{mM}; \quad (A.16)$$

- after the removal of the d.c. component the output signal is:

$$s_o(t) = k_d k_a V\Delta\omega_M f(t)); \quad (A.17)$$

- this method cannot be used with the balanced delay-derivative circuit

***Annex 4** - not required for the exam*

Non-coherent detection employing a peak envelope detector

- the analysis of this envelope detector is complex and can be found in [Ed. Nicolau]; the considerations presented in pages 11-12 of the AM lectures hold except for the conditions imposed to ensure the proper behavior of the detector, i.e.:

$$|f_c - \Delta f_M| > 100 f_{mM}; \quad \text{to ensure that the detected signal follows the envelope;} \quad (A.18)$$

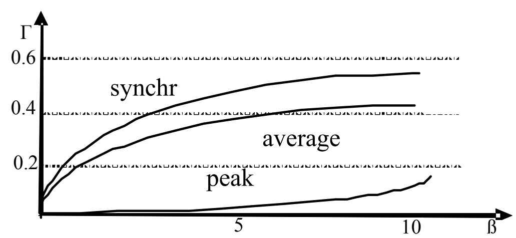
- cannot be used with the balanced delay-derivative circuit

Annex 5

Comparison between the performances of the envelope detectors

-the three envelope detectors have different operation limits, in terms of the ratio between the carrier ω_c and the maximum frequency deviation $\Delta\omega_M$. Recalling that:

$$\beta = \frac{\Delta f_M}{f_{mM}}; B_{FM}(k\Delta\omega_M) = 2f_{mM}(1 + k\beta + \sqrt{k\beta}); \text{ and denoting } \Gamma = \frac{\Delta f_M}{f_c}; \quad (A.19)$$

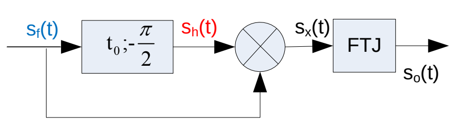


- the analytical expressions of Γ vs. β for the three envelope detectors can be derived using relations (A.6), (A.8), (A.9), (A.10).
- they are approximately represented in the left-hand figure
- for a correct operation, the parameters Γ and β of the studied detector should be placed on its corresponding curve
- the synchronous detector allows the demodulation for values the maximum frequency deviation which might rise to 0.6 of the carrier frequency, (with the increase of the modulation index and of the bandwidth);
- the averaging envelope detector allows the $\Delta\omega_M$ to be as high as 0.5 of ω_c;
- the peak detector allows only for a 0.2 value of Γ.
- in cheap commercial receivers, the Δf_M = 50 kHz and f_{int} = 10.7 MHz allow for the employment of this detector for $\beta = 3{,}3$.

Annex 6

The optimized DD-CD FM demodulator

- it is employed in the demodulation of the FM sound signal of the analog TV transmissions
- it combines the delay-based derivative of the FM signal, see (A.1) to (A.8), with the synchronous detection, see (A.9) to (A.13).
- starting from the first figure of Annex 2, which describes the principle of the method, a simpler scheme was derived, which combines the derivative with the multiplication, as shown below



Optimized variant of the DD-CD demodulator

- using the expressions of $s_{rf}(t)$ and $s_h(t)$ as given in (A.2), the $s_x(t)$ is:

$$s_x(t) = s_{rf}(t)\cdot s_h(t-t_0) = \frac{V_0^2}{V_{ref}}\sin[\omega_c t + \Delta\omega_M \int_0^t f(\tau)d\tau]\cdot \sin[\omega_c t + \Delta\omega_M \int_0^{t-t_0} f(\tau)d\tau - \omega_c t_0] = \quad (A.20)$$

$$= -\frac{V_0^2}{V_{ref}}\sin[\omega_c t + \Delta\omega_M \int_0^{t-t_0} f(\tau)d\tau + \Delta\omega_M \int_{t-t_0}^{t} f(\tau)d\tau]\cdot \cos[\omega_c t + \Delta\omega_M \int_0^{t-t_0} f(\tau)d\tau]; \text{ for } \omega_c t_0 = \frac{\pi}{2};$$

- using the notations shown in the first row of the equation (A.21), the signal $s_x(t)$ may be expressed as:

$$\alpha = \omega_c t + \Delta\omega_M \int_0^{t-t_0} f(\tau)d\tau; \quad \gamma = \Delta\omega_M \int_{t-t_0}^{t} f(\tau)d\tau \tag{A.21}$$

$$\Leftrightarrow s_x(t) = -V\sin(\alpha+\gamma)\cdot\cos\alpha = -\frac{V}{2}\cos\gamma\cdot\sin 2\alpha - V\sin\gamma\cdot\cos^2\alpha =$$

$$= -\frac{V}{2}[\sin\gamma + \cos\gamma\cdot\sin 2\alpha + \sin\gamma\cdot\cos 2\alpha] = -\frac{V}{2}\sin\gamma - \frac{V}{2}\sin(2\alpha+\gamma);$$

- the first term of the final expression has the frequency spectrum at low frequencies, while the second has its spectrum centered around $2\omega_c$.
- imposing the separation condition:

$$2\omega_c - B(2\Delta\omega_M)/2 - \omega_{mM} > \omega_{mM} \leftrightarrow \omega_c > B(\Delta\omega_M)/4 + \omega_{mM}; \tag{A.22}$$

the envelope detection suppresses, by the LP filtering, the spectrum centered around the carrier's 2nd harmonic
- the output signal is:

$$\Rightarrow s_a(t) = -K_{LP}\frac{V}{2}\sin\gamma = -K_{LP}\frac{V}{2}\sin(\Delta\omega_M\cdot\int_{t-t_0}^{t} f(\tau)d\tau); \tag{A.23}$$

- imposing condition (A.3), for the approximation of integral (A.4) and condition (A.6) for the approximation of the sine with its argument, we get:

$$\text{if } t_0 < 2/\omega_{mM} \Rightarrow \int_{t-t_0}^{t} f(\tau)d\tau \approx f(t-\frac{t_0}{2})\cdot t_0; \tag{A.24}$$

$$\gamma = \left|(\Delta\omega_M/2)\cdot f(t-t_0/2)\cdot t_0\right| \le 0.2 \text{ radians}; \Rightarrow \sin\gamma \approx \gamma \quad \text{a.} \quad \omega_c t_0 = \pi/2 \Rightarrow t_0 = 1/4f_c; \text{ b.}$$

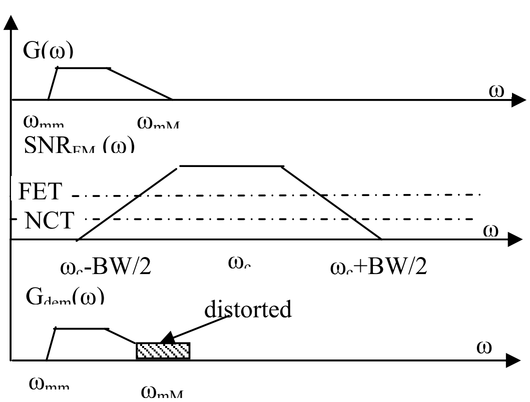
- the $s_a(t)$ becomes:

$$s_a(t) = -k_d\cdot\frac{\Delta\omega_M\cdot t_0}{2}\cdot f(t-\frac{t_0}{2}) = -k_d\cdot k_e\cdot f(t-\frac{t_0}{2}) = s_0(t); \tag{A.25}$$

- the output voltage contains no additional d.c. component and is directly proportional to the modulating signal; the processing inserts a delay.

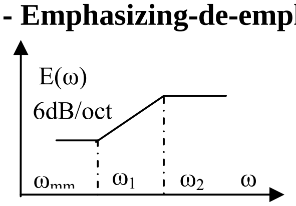
Annex 7

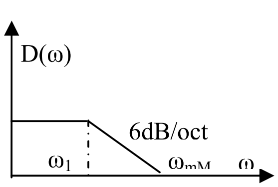
Emphasizing and de-emphasizing in FM transmissions



Necessity:
- some complex modulating signals, like music, may have high components of low and middle frequencies, while having small components on high frequencies.
- this would lead to different SNR_i (ω) values at the input of the demodulator, see the adjoining figure.
- while for low and middle frequencies the SNR_{LM} would be above FET, the SNR_H would be below the NCT. This leads to a very large $SINR_o$ for middle and low frequencies, while the high frequencies are "captured by noise", giving a significantly distorted demodulated signal.

- Emphasizing-de-emphasizing:





- before modulation, the modulating signal has its high-frequencies boosted by a HP filter; this filtering is called emphasizing. So, at demodulator's input the SNR of high-frequencies is above FET and the demodulated signal is no longer captured by noise.
- Still, this processing distorts the spectrum of the original signal, so the spectrum of the demodulated signal should be restored by a LP filtering, called de-emphasizing.
- the frequency characteristics of the two filters employed by the commercial FM transmissions are shown in the adjoining figure. The cut-off frequencies employed are f_l=2.1 kHz and f_2= f_{mM}=15 kHz.
- the effect of the emphasizing-de-emphasizing upon the quality of the demodulated signal is measured by the ratio between the power of noise the for the transmission

employing E-D, $P_{Noe\text{-}d}$, and the power of the noise for the transmission that does not employ E-D, P_{No}; both powers are evaluated at the demodulator's output. The expression of this ratio is given in (A.26); for the commercial FM transmissions $\lambda = 13$ dB.

$$\lambda = \frac{P_{Noe-d}}{P_{No}} = \frac{\omega_{mM}^2}{3\omega_1^2}; \tag{A.26}$$