Demodulation of the PSK signals

- the DPSK signals are demodulated by digital methods because they are more stable and flexible
- the digital PSK demodulators are basically of two types

- demodulators that perform the demodulation by measuring the phase-shift of the modulated signal; these demodulators will be denoted as “direct digital DPSK demodulators
- demodulators that use the QAM technique to perform the demodulation of the (D)PSK signals

- the direct demodulators have the disadvantage of being specific only to DPSK and cannot be used for the demodulation of the A+PSK signals (see next chapter of the course) and therefore they are less and less used in modern equipment
- a brief description of a direct digital momentary-coherent demodulation method is presented in Annex 3 of this material and a brief description of the DPSK receiver which uses the “direct” demodulation is presented in Annex 4. *They are not required for the examination.*
- due to their greater versatility, the (D)PSK demodulators based on the QAM technique are widely used and therefore this material will focus on their description

Demodulation of DPSK signals using the QAM technique

- the demodulation of the DPSK signals by using the QAM technique may be performed in two ways:

- by employing LP filters to suppress the high-frequency spectral components;
- by using the Hilbert transform of the received signal to suppress the high-frequency components.

DPSK-QAM demodulator with LP filters

- using the QAM decomposition of the PSK signal presented in the first PSK lecture, see (24), the expression of the received DPSK signal becomes (28), where I’(t) and Q’(t) denote the filtered modulating signals affected by the channel perturbations and distortions:

$$s_{rPSK} = r(t) = I'(t)\cdot A\cos\omega_c t - Q'(t)\cdot A\sin\omega_c t = A\cdot \mathrm{Re}\left\{\left(I'(t)+Q'(t)\right)\cdot e^{j\omega_c t}\right\} \qquad (28)$$

- the demodulation of the DPSK signals may be accomplished using the QAM method, i.e. by using coherent DSB-SC demodulation, described by relations (5...8) of the lecture on QAM. It is included in the block diagram of the QAM-DPSK receiver shown in figure 10 below.
- rewriting equations (5,…8) of the QAM chapter for the DPSK signal expressed by (28) we get:

$$i_x(t) = \frac{r(t)A\cos\omega_L t}{K} = \frac{AI'(t)}{2K}[\cos\theta(t) + \cos(2\omega_c t + \theta(t))] - \frac{AQ'(t)}{2K}[-\sin\theta(t) + \sin(2\omega_c t + \theta(t))] \qquad (29)$$

$$c_x(t) = \frac{r(t)A\sin\omega_L t}{K} = \frac{AI'(t)}{2K}[-\sin\theta(t) + \sin(2\omega_c t + \theta(t))] + \frac{AQ'(t)}{2K}[\cos\theta(t) - \cos(2\omega_c t + \theta(t))] \qquad (30)$$

- by suppressing the spectral components centered on $2\omega_c$ with the LP filters, the output signals are:

$$i_F(t) = \frac{A}{2K}(I'(t)\cos\theta(t) + Q'(t)\sin\theta(t)) \;\rightarrow\; \frac{A}{2K}\cdot I'(t) \text{ for } \theta(t)\rightarrow 0; \quad f_{t-LP} > f_N(1+\alpha) \qquad (31)$$

$$c_F(t) = \frac{A}{2K}(-I'(t)\sin\theta(t) + Q'(t)\cos\theta(t)) \;\rightarrow\; \frac{A}{2K}\cdot Q'(t) \text{ for } \theta(t)\rightarrow 0; \quad f_{t-LP} > f_N(1+\alpha) \qquad (32)$$

- the QAM demodulation delivers the filtered modulating signals (31) and (32) affected by the channel’s perturbations and distortions.
- then, using the recovered symbol clock, these signals are probed at $t=kT_s$ to extract the modulating levels of the k-th symbol period I_k’’ and Q_k’’, which are affected by the channel and do not belong to the modulating alphabet. The complex baseband signal (QAM symbol), see (31),(32), after probing is:

$$m''_k(t) = I''_k + jQ''_k = I'_k\cdot\cos\Theta_k + Q'_k\cdot\sin\Theta_k + j\left(-I'_k\cdot\sin\Theta_k + Q'_k\cdot\cos\Theta_k\right) = m'_k\cdot e^{-j\Theta_k} \qquad (33)$$

- these signals are inserted in the decision block which delivers the two estimates I^*_k and Q^*_k of the transmitted levels, see (18); note that I^*_k and Q^*_k belong to the modulating alphabet.
- the decision of the estimated symbol can be made by two methods:

a. *hard decision* – it uses only the coordinates obtained in the current symbol period

- the decided levels I^*_k and Q^*_k should be the coordinates of the constellation-vector which is closest to the received vector (and hence it’s the most probable); therefore the decision block would compute the Euclidean distances between the received vector and the constellation’s vectors and store the coordinates of the vector placed at the minimum d_E from the received vector.

b. *differential decoding* – it uses the coordinates received in the current k-th and previous (k-1)-th symbol periods

- it computes the absolute phase in the current period: $\Phi_k^{"} = \arctan\left(Q_k^{"} / I_k^{"}\right)$ (34)

- then it computes the differential phase-shift between the current phase Φ_k'' and the absolute phase stored in the previous symbol period: $\Delta\Phi_k^{"} = \Phi_k^{"} - \Phi_{k-1}^{"}$ (35)

- finally it establishes the phase-shift $\Delta\Phi_i$ from the allowed set (alphabet) which is closest to the computed $\Delta\Phi_k$''.

- the differential decoding is less sensitive to the errors of the local carrier recovery, see (4) in the QAM lecture, because during each symbol period T_s, the computed phase shift $\Delta\Phi_k$ is affected only by the dynamic phase-offset accumulated during that symbol period, as shown below:

- the variation of phase offset of the local carrier over the k-th symbol period is,:

$$\Delta\Theta(kT_s) = \Theta(kT_s) - \Theta((k-1)T_s) = 2\pi \cdot df \cdot kT_s + \Theta_0 - 2\pi \cdot df \cdot (k-1)T_s - \Theta_0 = 2\pi \cdot df / f_s \quad (36)$$

- but for values of df much smaller than f_s, the phase offset of (36) is small compared to the phase-shifts $\Delta\Phi_k$ of the DPSK constellation. This way the $\Delta\Phi_k$ is affected only by the phase offset accumulated during one symbol period

- the decided symbol (either defined by its levels or by the phase-shift) undergoes then the operations listed below, which are needed to extract the estimated multibit. Thee three operations are the inverse operations of the ones performed by the transmitter's encoder, see (20), (18), (23):

- the bit-demapping, inverse to the bit mapping, which delievers the estimated $c_1^k c_0^k$ dibit; this operation can be implemented by table reading;
- the differential decoding, inverse of (23), delivers the estimated $b_1^k b_0^k$ dibit in binary-natural code:

$$(c_1^k c_0^k - c_1^{k-1} c_0^{k-1})_{\text{mod } 4} = b_1^k b_0^k; \quad (37)$$

- the natural-Gray conversion, which finally delivers the decided data dibit $a_1^k a_0^{k,}$ (inverse of (20).

- the block diagram of a QAM-demodulator for the 4-DPSK signals, which employs LP filters, is shown in figure 10. The signals employed by the carrier recovery will be described later in this chapter, while and symbol-clock recovery methods will be studied in the chapter dedicated to A+PSK; the scheme does not include the ouput parallel-series converter, which is controlled by the bit-clock.

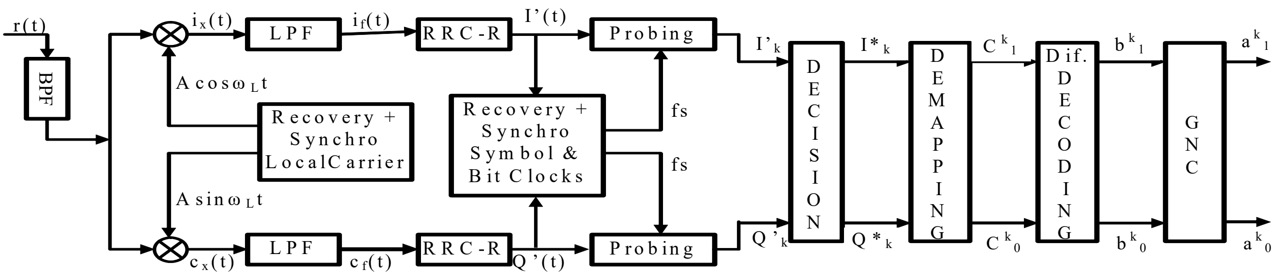


Figure 10. Block scheme of the QAM-demodulator for the 4-DPSK signals; variant with LP filters

- if the mapping-demapping is performed by look-up table, the N-G conversion is not required

- the employment of the LP filters inserts a group-delay distortion which can affect significantly the performances, by inducing ISI. This shortcoming is circumvented by the *DPSK-QAM demodulator with Hilbert transform*, which will be discussed in the IVth year at the Data Transmissions course

Demodulation of the other DPSK constellations with the QAM Technique

Constellations A2 and B2

- since the PSK signals modulated with these constellations are one-dimensional signals, see (25), their demodulation would require only the employment of one arm of the QAM demodulator.

- therefore their demodulation would be performed similarly to the one described above for QPSK, except for the differential precoding-decoding that are performed as mod 2 operations on one bit.

- this differential precoding-decoding ensures only the cancellation of the k·180º rotations inserted by the carrier recovery. The k·90º rotations can be compensated by using both arms of the QAM receiver.

Constellation B4

- QAM generation of B4 requires a modulo-8 differential decoding on 3 bits.

- the c_2c_1 dibit obtained after the after demapping is transformed in the $c_2c_1c_0$ tribit by adding c_0 = "1".

After the differential decoding only the two most significant bits are employed in the final processing.

Constellation A8

- the QAM demodulation of the A8 constellation is implemented similarly as the one of A4, with the difference that it requires a modulo-8 differential decoding on the three bits

Recovery and synchronization of the local carrier - the DDCR method

- to recover and synchronize the local carrier, instead of the classical PLL circuit which determines the phase-error by comparing an external phase-reference signal to the locally generated signal, the QAM receiver determines directly the phase error using:

$$e_k(t) = I'_k \cdot Q^*_k + Q'_k \cdot I^*_k = (I'^2_k + Q'^2_k) \cdot \sin\theta(t); \qquad (38)$$

- the relation (38) can be derived using (31) and (32); I'_k and Q'_k represent the values of $I'(t)$, $Q'(t)$, the signals at the demodulator's outputs, at $t = kT_s$, i.e. in the sampling moments; I^*_k, Q^*_k denote the decided levels during the k^{th} symbol period. Since this method employs the decided levels, it is called „Decision Directed Carrier Recovery – DDCR".
- the block diagram of the receiver which employs this method is shown in figure 11; it shows that the PLL closes across the demodulator and the hard-decision block.

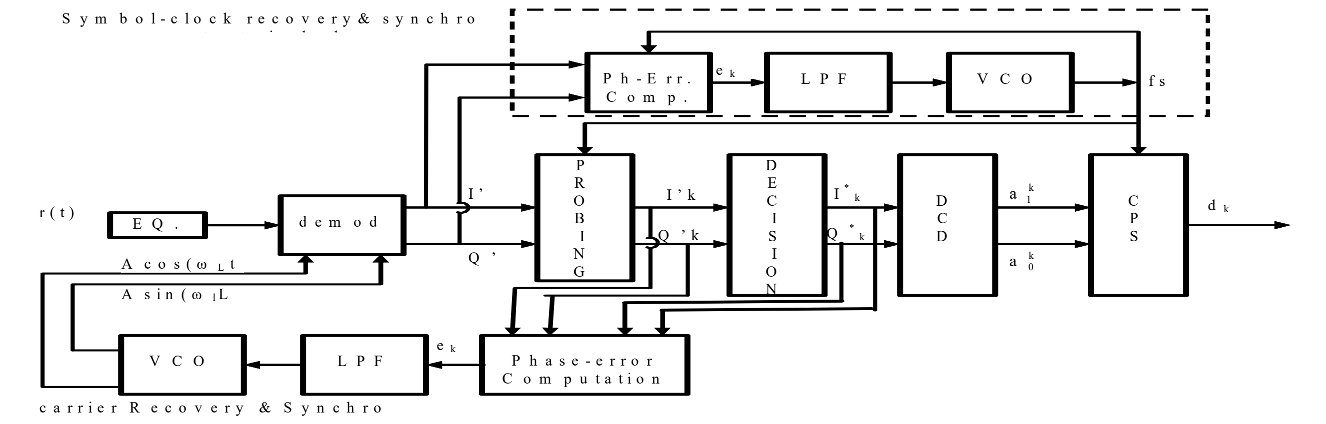


Figure 11. Block diagram of the QAM receiver for the DPSK signals

- for small values of the phase-shift, the error-voltage $e_k(t)$ may be considered directly proportional to the magnitude of the phase-shift. For greater values of the phase-shift, but still $\Theta(t) \in [-\pi/2, \pi/2]$, the error-voltage is no longer directly proportional to the phase-shift, due to the variation law of the sine function, but the sign of the error-voltage still follows the sign of the phase-shift. - see blackboard
- therefore, an analog PLL with a proportional phase control would insert errors because, for great phase-shifts, the error-voltage is no longer directly proportional to the phase-shift.
- a digital PLL with a constant phase-step, see the bit (symbol)-clock synchronization in the BB codes lecture notes, should be employed instead. It is controlled only by the sign of the error-voltage, which is the same as the one of the phase-shift for $\Theta(t) \in [-\pi/2, \pi/2]$
- if the value of $\theta(t) \in (-\pi, -\pi/2)$ or $(+\pi/2, +\pi)$, the PLL will change the phase of the local carrier so that the error voltage would be minimized; this leads to the occurrence of a constant phase-shift of $-/+\ \pi$.
- since some PLL circuits insert a $\pi/2$ phase-shift between the local and the received carriers (the reference one), a phase shift of $k \cdot \pi/2$, $k \neq 0$, may occur, which cannot be removed by the carrier recovery circuit. This phase-shift is also called „$k \cdot \pi/2$ uncertainty"
- because this uncertainty is constant during a transmission, it is removed by the differential encoding-decoding employed to generate the differential phase-modulation.

Summarizing, the synchronization of the local carrier is accomplished in two steps:

- the extraction of the phase-reference signal, denoted as „recovery", which is accomplished in figure 11 by the error-voltage circuit and by the LPF;
- actual synchronization of the local carrier (actually two quadrature carriers), which is performed by the PLL (with its VCO) in figure 11. This operation may be performed either by the digital PLL, see BB codes, if a constant phase-step (in terms of the sign of the error-voltage) is desired, or by an analog PLL (VCO), if the phase of the local carrier(s) is to be changed proportionally to the error-voltage.
- though it exhibits good performances, this carrier recovery method assumes an almost perfect synchronization of the local symbol-clock, employed to probe the correctly the I_k^* and Q_k^* levels.

- if the local symbol-clock is not correctly recovered (and synchronized), → the I_k^* and Q_k^* levels may take wrong values, → this leads to a wrong error-voltage provided by (38) → which leads to an incorrect demodulation (wrong carrier synchronization), see (31) and (32) *for $\theta(t) \neq 0$, → the symbol-clock recovery is affected; so the receiver might enter into „vicious circle".-*

- therefore this carrier recovery method may be employed only if the symbol-clock is recovered by a method whose performances do not depend of the quality of the local carrier recovery. Otherwise, the carrier recovery should be accomplished by methods which are not affected by the symbol-clock synchronization. Such a method will be described in the Data Transmissions course in the IV^{th} year

Recovery and synchronization of the local symbol and bit clocks in the receiver

- the RC-filtered signal at the $t = kT_s$ probing moments, when ISI = 0, is acquired on the positive edges of the local symbol-clock.
- the local symbol-clock should be synchronized with the received modulated signal, i.e. the positive edges of the local f_s should be synchronous with moments the momentary phase reaches its maxima
- to accomplish this task, two steps should be performed:

• the extraction (recovery) of synchronization information (criterion) from the modulated signal (operation called symbol-clock recovery); this ensures the phase reference required by the phase comparator of the dynamic synchronization

• the synchronization of a local signal clock, with frequency f_s.

- for the PSK modulated signal, the phase-reference signal cannot be the modulated signal, because it is phase modulated.
- some of the methods used for the symbol-clock recovery are using the effects of the RC-filtering upon the modulated signal. The principle of operation of such a method is described in Annex 5.

Signal-delay method for the symbol-clock recovery

- the method consists in the limitation of received signal, followed by the addition of the limited signal to its $T_s/2$-delayed version.

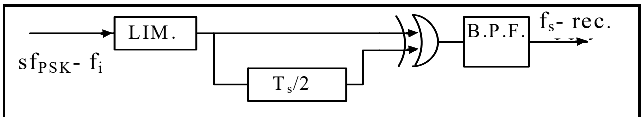


Figure 12. Block diagram of the f_s recovery circuit using the signal-delay method

- if the carrier frequency f_i observes (39), then the phase-difference between the two signals will be 90º and the spectrum of the resulted signal would contain a component on the f_s frequency.

$$f_i = (2k+1)\cdot f_s / 2; \qquad (39)$$

- the component on f_s is filtered by the BPF and limited, generating the recovered symbol-clock, see block diagram in fig. 12.
- the method is simple and is not affected by the envelope variations of the received signal. Still, the delay inserted by the BPF, at $f = f_s$, should be taken into account.

Other methods used for the symbol-clock recovery

- one of the best symbol-clock recovery method is the *energetic* method, which has the advantage of being independent of the quality of the carrier's signal recovery – it will be discussed at the DT course
- the symbol clock can also be recovered by using a *modified version of the DDCR* method described above; this method will be presented in the lectures on the A+PSK modulations.

Synchronization of the symbol and bit clocks

- the signal provided by the symbol-clock recovery circuit is used as phase reference by a clock synchronization circuit, be it either an analog PLL or a digital dynamic synchronization circuit
- because many applications require more flexible value of the phase-step of the digital synchronization circuit, the value of the phase-step should take values equaling $\Delta\Phi_p=360^o/N$, with N a natural number, $N \neq 2^n$. Such a circuit is described in Annex1 of the 2^{nd} lecture on BB codes

Considerations regarding the effects of the frequency deviations inserted by the channel

- the modems also have to cope with a frequency deviation Δf_{imax}, inserted by the channel;
- this frequency deviation that occurs upon the carrier frequency also affects the phase of the received signal and the one of the symbol clock.
- the frequency deviation is the "weak" point of the PSK modulation; it can support a maximum deviation with a reasonable decrease of noise performances; but, if the frequency deviation is larger than the permitted value, the error-performance decreases drastically**.**

Variants of the QPSK modulation

- as previously shown in the previuos DPSK lecture and above the absolute phase of the (D)QPSK signals modulated with type-A constellations exhibits phase-shifts of 180º during a symbol-period.
- the filtering of these signals with an (R)RC characteristic inserts an amplitude modulation, see the previous lecture on PSK. Recall that the amplitude of the filtered-modulated signal A(t) has minimum values, occurring close to t = kT+T/2, that depend of the phase-shift $\Delta\Phi_k$ as shown in (40).

$$A_{min}(t) = A\left|\cos\frac{\Delta\Phi_k}{2}\right|; \quad A = \sqrt{I_k^2 + Q_k^2}; \tag{40}$$

- for $\Delta\Phi_k$ = 180º the amplitude would take values around zero and its trip would be the maximum possible, from 0 to A during half a symbol-period.
- but in radio transmissions employing non-linear RF amplifiers, the characteristics AM-AM and AM-PM of these amplifiers, generate the „spreading" of the frequency spectrum of the modulated signal.
- this leads to the spectral regrowth, i.e. the remaking of the side spectral lobes that are positioned outside the permitted band, which have been filtered before the final amplification; it also distorts the spectral components inside the useful bandwidth. The spectral regrowth, which is illustrated in fig. 13 below, will be discussed in the Data Transmissions course in the IVth year.

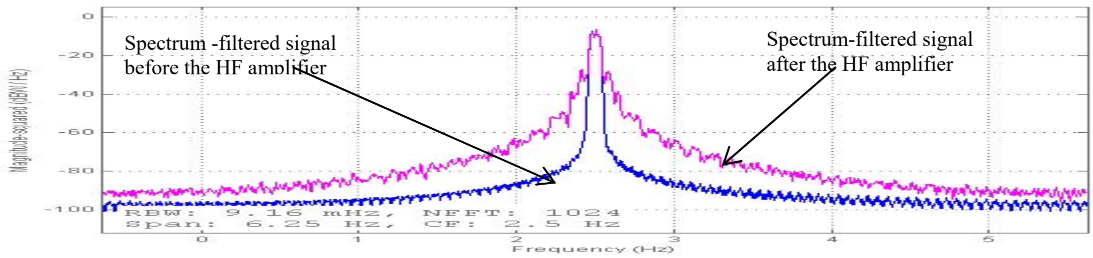


Figure 13 Illustration of the spectral regrowth

- therefore the DPSK or QPSK signals using type A constellations are recommended to be used only in transmissions that employ linear amplifiers.
- for transmissions using non-linear amplifiers, i.e. radio links, there were developed variants of QPSK which have side lobes as small as possible and exhibit small amplitude variations of the filtered signals.
- *these requirements can be fulfilled if the absolute phase of the modulated signal would not exhibit shifts of 180º, and so the amplitude of the modulated signal would not vary between A and 0.*
- another important parameter of the modulated signal affecting its behaviour when passed through a non-linear RF amplifier, is the PAPR (Peak-to-Average Power Ratio during a symbol period).
- to decrease the spectral regrowth, PAPR should be as close as possible to 1 (0 dB).
- the variants of QPSK that fulfill these requirements are Offset QPSK (OQPSK) and π/4-QPSK.

Offset QPSK – OQPSK

- to avoid the 180º phase shifts from one symbol to another that occur in the QPSK, the OQPSK does not change the values of the modulating levels I_k and Q_k in the same time instant, i.e. at the beginning of the symbol period. Instead, the modulating moments of the I_k and Q_k levels are delayed to one another with half a symbol-period, as shown in figure 14.
- this way the absolute 180º phase shifts phase are obtained in two steps, see fig.15 for the absolute phase shift from 45º to 225º:

➢ in the first half of T_s the vector (-√2/2;+ √2/2) is transmitted, generating a 90º phase-shift;
➢ during the second half of T_s the (-√2/2;- √2/2) vector is sent, generating a second 90º phase-shift

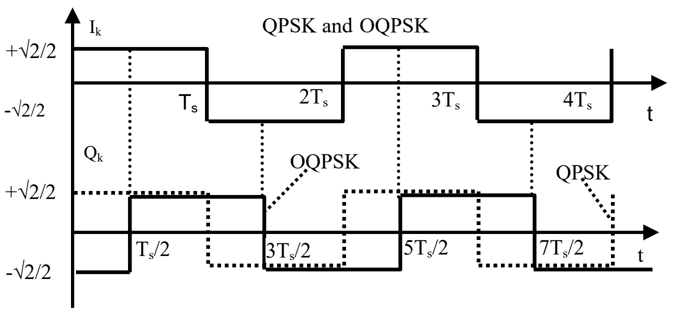


Fig. 14. Relative positioning of the I_k and Q_k modulating levels, to generate the QPSK and OQPSK modulations

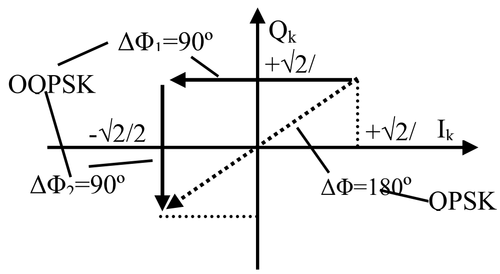


Fig. 15 Generation of the 180º phase-shifts by QPSK and OQPSK

- by avoiding the 180º phase-shift, the amplitude of the modulated signal would not reach zero and the amplitude variation during one T_s is much smaller.
- the PAPR value is also much smaller than the one of QPSK
- the block diagram of the OQPSK transmitter is almost identical to the one of the QPSK transmitter, see fig. 9
- the only difference is that the I_k and Q_k levels are delivered to the filters at time instants separated by half of T_s, i.e. there is one symbol-clock for each branch. The Q-arm symbol-clock is inverted reffered to the one used on the I-arm.
- the demodulation is similar to the QPSK one, see figures 10 and 11, but two symbol-clocks are employed for probing the demodulated signals on the two arms; the two clocks are phase-shifted with 180º by inverting the clock on the quadrature branch.
- note that probing in two different moments increases the symbol-error probability of the OQPSK, compared to the one of QPSK.
- the frequency band of the OQPSK signal is the same as the one of the QPSK signal and its spectrum looks similar, see figures 3 and 4, but the side lobes have smaller levels. This fact makes the spectral regrowth performed by the non-linear RF amplifiers generate smaller side lobes; due to this feature the OQPSK, is employed in radio transmissions that include non-linear RF amplifiers.

π/4-QPSK modulation

- the 180º phase-shifts of the absolute phase during one T_s can also be avoided by using the four possible phase-shifts of $(2p + 1)\cdot 45^o$, similarly to the B4 constellation, see fig. 2.

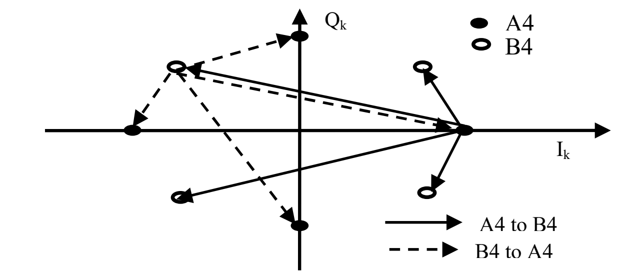
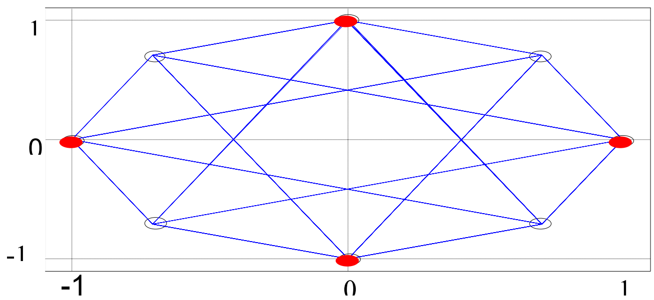


Figure 16 .The possible trajectories of the absolute-phase of the π/4-QPSK starting from a vector of A4 and B4

- the absolute phase would pass from constellation A4 to B4 or vice versa in every symbol period, as shown in figure 15 .
- the amplitude of the current vector would not reach the null value at any time instant.
- figure 16 only shows the possible trajectories starting from one vector (0º) of A4 and from one vector (135º) of B4.

Figure 16. Possible phase shifts produced by the π/4-QPSK



- the phase-shifts of this kind can be implemented in a QAM absolute-phase modulator by using alternatively the mapping tables of the A4 and B4 constellations, as shown by the complete diagram of the phase-shifts performed by *π/4-QPSK*, which is presented in figure 17;

Fig. 17 Complete diagram of the phase-shifts performed by *π/4-QPSK*

- this is accomplished by a 3-bit addressing, where the LSB is alternatively “0” or “1”. see notes
- the amplitude variation of the QPSK, OQPSK and π/4-QPSK are presented in figure 18 below

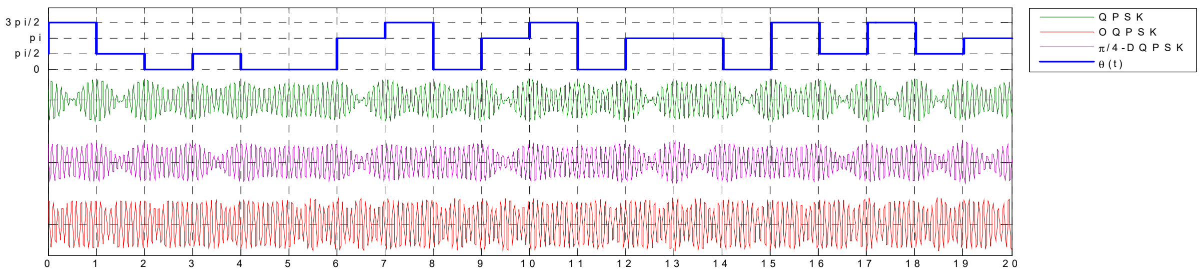


Figure 18. Envelope variations vs. time of QPSK, OQPSK and π/4-QPSK

- as shown in fig.18, the QPSK modulated signal has the greatest amplitude variations, the π/4-QPSK has smaller amplitude variations, while OQPSK has the smallest amplitude variations.
- recall that as the amplitude variations of the modulated signal get smaller, the non-linear RF amplifier distorts less the amplified modulated signal
- the PAPR of π/4-QPSK decreases to 3.2 dB and OQPSK has 2.6 dB, compared to the 4 dB of QPSK

- but the better resiliencies of OQPSK and π/4-QPSK at the effects of the RF amplifier are accompanied by slightly increased BER values as discussed in the next paragraph.
- the frequency spectrum of π/4-QPSK is similar to the ones of QPSK and OQPSK and the amplitudes of the side lobes are smaller than the ones of QPSK, but greater than the ones of OQPSK. The bandwidth of the main spectral lobe is equal to the one of QPSK, see (3).

Error-performances of the PSK modulation

- the error performances of the PSK are analyzed only in terms of the channel noise; the other distortions and perturbations are either included in the noise-power or are considered null, assuming that they are cancelled by the equalizing circuits.
- the computations use the following assumptions:

1. the noise is a Gaussian one with a power spectral density N_0 and variance σ, i.e. its power at the demodulator's input equals σ^2 and its BW equals the useful BW of the modulated signal.
2. the global filtering characteristic is raised-cosine, which is equally split between the transmitter and receiver. We assume ISI = 0 in the probing moments, provided that the symbol-clock is correctly recovered in the receiver.
3. The M phase-shifts are equidistant and equiprobable, since the symbol values are independent.

- the noise vector Z is added to the attenuated transmitted one S, giving the received vector R, fig. 19.

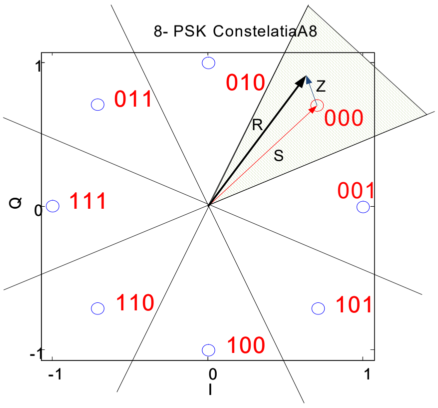


Fig. 19. Vectorial composition of the received signal

- to get a correct decision after the demodulation, the R should remain inside the area delimited by the angles φ = ((2k-1)π/N and φ = (2k+1)π/N) of the unitary circle.
- all symbol-error probabilities are expressed using the „error-complementary function" erfc(t), defined by (41) ;
- the Taylor-series decomposition of this function is also expressed by (41); for small values of the argument, retaining of the first term leads to a reasonable accuracy approximation.

$$\mathrm{erfc}(t) = \frac{2}{\sqrt{\pi}} \int_t^{\infty} e^{-u^2} du \approx \frac{e^{-t^2}}{t\sqrt{\pi}} \left(1 - \frac{1}{2t^2} + \frac{3}{4t^4} - \ldots\right); \quad (41)$$

- using the erfc-function we define the Q(t), see (42), which is employed for a more direct expression of both symbol and bit-error probabilities.

$$Q(t) = \frac{1}{2}\mathrm{erfc}\left(\frac{t}{\sqrt{2}}\right) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} e^{-\frac{u^2}{2}} du \approx \frac{e^{-\frac{t^2}{2}}}{t\sqrt{2\pi}} \left(1 - \frac{1}{t^2} + \frac{3}{t^4} - \ldots\right); \quad (42)$$

- the signal-to-noise ratio ρ at the receiver's input is expressed by (43).a for a cosine carrier, A denoting the signal amplitude and σ the noise variance. The noise power can also be expressed as (43).b. in terms of its power spectral density N_0 and bandwidth.

$$\rho = \frac{P_s}{P_z} = \frac{A^2}{2\sigma^2} = \left(\frac{A}{\sqrt{2}\cdot\sigma}\right)^2 ;\text{a.} \quad P_z = N_0 \cdot f_s (1+\alpha);\text{b.} \quad (43)$$

- but the decisions are made using the signal's values in the probing moments , which are baseband signals, and so the value of the signal/noise ratio that should be used is expressed by (44),

$$\frac{A^2}{\sigma^2} = \left(\frac{A}{\sigma}\right)^2 = 2\rho \quad (44)$$

- since the input signal to noise ratio is a direct "measure" of the quality of the received signal, it will be employed as the argument in the following computations. The SNR will denote the value of ρ in dB.
- in some analyses shown in literature the noise power is computed in (43) and in (44) for a bandwidth equaling f_s considering an ideal frequency characteristic of RC filter.

Symbol-error probability (SER) of 2-PSK

- computations presented in literature derive the symbol-error probability p_{e2}, in terms of the signal amplitude A and noise variance σ, as (45); the curve p_{e2} vs. SNR is presented in figure 20.

$$p_{e2}(\rho) = Q\left(+\frac{A}{\sigma}\right) = Q(\sqrt{2\rho}) \approx \frac{e^{-\rho}}{\sqrt{2\pi\rho}}; \quad (45)$$

- the second term of the Taylor-series of (42) is smaller than 5% of the first term, for SNRs > 6 dB.

This allows for the approximation of p_{e2} with the third expression in (45).
- note that the SNR required by 2-PSK to ensure $p_e = 1\cdot10^{-5}$ approximately equals 9.5 dB

Symbol-error probability of 4-PSK

- for the 4-PSK, the correct-decision areas of unity circle are bounded by the angles (2k+1)π/4.
- after computations similar to the 2-PSK case, the expression of SER of the 4-PSK, p_{e4} is:

$$p_{e4} = 1-[1-Q(\sqrt{\rho})]^2 \approx 2Q(\sqrt{\rho}) \approx \text{erfc}\sqrt{\frac{\rho}{2}} \approx \frac{\sqrt{2}\cdot e^{-\frac{\rho}{2}}}{\sqrt{\pi\rho}}; \text{ for SNR} > 6 \text{ dB} \Rightarrow Q^2(\sqrt{\rho}) \approx 0; \quad (46)$$

- using the binomial expansion, p_{e4} may be approximated by 2Q(√ρ), since the term Q^2(√ρ),, second term of (46) is negligible for SNR > 6 dB. Then, using (42) p_{e4} may be approximated by the third term of (46), again for SNR>6 dB. The p_e vs. SNR of 4-PSK is also represented in figure 20.
- the SNR required by the 4-PSK to ensure the same p_e is 3 dB higher than the one needed by 2-PSK. This is equivalent to an increase of p_e at the same SNR and is caused by the decrease of the minimum d_E between two neighboring vectors, from d_{min2} = 2A, for 2-PSK, to d_{min4} = √2·A, for 4-PSK; A being the constellation radius.

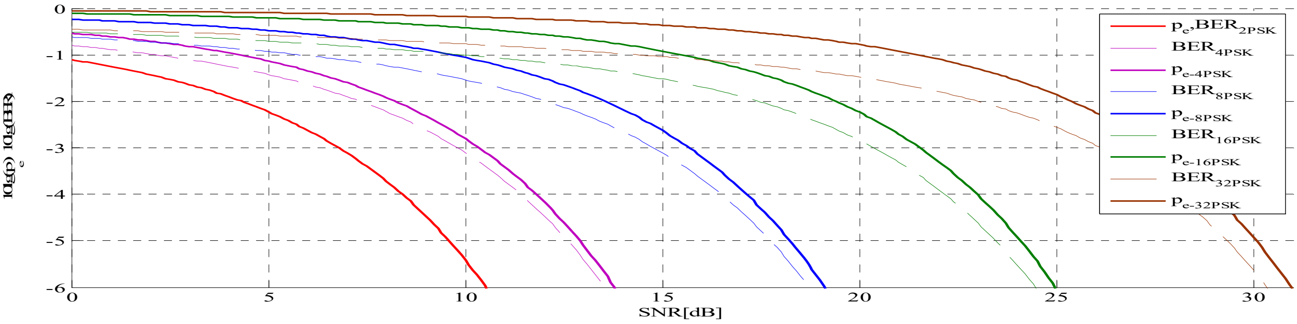


Figure 20. p_{e2}, p_{e4}, p_{e8}, p_{e16}, p_{e32} (continous lines) and BER_2, BER_4 BER_8, BER_{16}, BER_{32}, (daseh lines) vs. SNR

Symbol-error probability of the N-PSK

- considering the number of phase-shifts N = 2^p, p being the number of bits/symbol, the phase-shifts are ΔΦ = k2π/N; hence, the correct decision area of a vector is bounded by the angles ((2k-1)π/N, (2k+1)π/N), see fig. 18.
- the p_e is expressed in terms of ρ by the first two expressions of (47); using the Taylor-series expansion, as in the two previous cases, we get an approximate expression of $p_{e\text{-}N}$ as the third term of (47). The p_e vs. SNR of 8-PSK, 16-PSK and 32 PSK are presented in fig. 20.

$$p_{eN} \approx \text{erfc}\left(\sqrt{\rho}\sin\frac{\pi}{N}\right) = 2Q\left(\sqrt{2\rho\sin^2\frac{\pi}{N}}\right) \approx \frac{e^{-\rho\sin^2\frac{\pi}{N}}}{\sqrt{\pi\rho}\sin\frac{\pi}{N}} \quad (47)$$

- knowing that the minimum distance between the vectors of a N-PSK constellation equals (48).a and the average power is $P_m = A^2/2$, the symbol-error probability can be expressed also as (48).b

$$d_m^2(N) = 4A^2\cdot\sin^2(\pi/N) \quad .a \quad \Rightarrow \quad p_{eN} = 2Q\left(\sqrt{\frac{d_m^2}{4\sigma^2}}\right) \quad .b \quad (48)$$

- the higher N, the greater is the SNR needed to ensure a given error-probability; this is due to the decrease of the minimum Euclidean distance between the constellation vectors.
- when N doubles, the SNR increases required by PSK constellations to ensure a given symbol error-probability p_{e0} are shown in table 5. The value of the SNR_0 needed by 2-PSK for a desired p_{e0} is computed with (45).

Constellation	2-PSK	4-PSK	8-PSK	16-PSK	32-PSK
SNR [dB]	SNR_0	SNR_0+3	SNR_0+8.3	SNR_0+14.2	SNR_0+20.2
ΔSNR [dB]	-	3	5.3	5.9	6

Table 5. SNR values required by PSK constellations to ensure a certain p_e

- as a rule of thumb, we may say that, for N > 8, when a constellation is doubled, the SNR required to ensure a given p_e is 6 dB greater than the one needed by the non-doubled constellation.

Bit-error probability of the PSK modulations

- the computation of the bit-error probability, BER, should take into account two aspects:

- each symbol „carries” $p = \log_2 N$ bits;
- due to the Gray-mapping, the most probable symbol-errors, when a symbol is mistaken for one of its neighbors, lead to only one error-bit. Because the symbol-errors that lead to more than one error-bit have low probabilities, at least for medium and high SNRs, we may say that for these SNRs, the BER decreases p times, compared to the symbol-error rate, p_e. This is justified by:

$$BER=N_{be}/N_{br}\approx(N_{se}\cdot 1)/(N_{sr}\cdot p)=p_e/p \qquad (49)$$

- so, the BER of 2-PSK,..., N-PSK may be approximated by:

$$BER_N \approx p_{eN}/\log_2 N; \;\rightarrow\; BER_2 = p_{e2};\;\; BER_4 \approx p_{e4}/2;\;\;\; BER_8 \approx p_{e8}/3; ..., BER_N \approx p_{eN}/p \qquad (50)$$

- the constellations B2 and B4 ensure the same p_e and BER as their counterparts A2 and A4.
- the BER vs. SNR curves of the APSK modulations, for N = 2, 4, 8, 16 and 32 are represented in figure 20, as well.

Symbol and bit error probabilities of the DPSK modulation

- the symbol-error probability of the DPSK depends essentially of the type of demodulation used.
- if the decision upon each received symbol is made independently every symbol period, followed by the demapping and differential decoding at multibit level, then *the coherent-differential demodulation, i.e. the QAM demodulation with differential encoding-decoding, would have a double p_e compared to the APSK with the same N;*
- indeed, if a symbol is mistaken, its absolute phase becomes phase-reference for the next symbol period and, due to the phase-subtraction, the phase difference of the next symbol period is also mistaken; so, the phase-shifts of two consecutive symbol-periods are mistaken leading to a double error-probability, compared to the APSK. The above considerations expressed in phase-shifts are equivalent to the differential encoding and decoding, see (22), in the first lecture, and (37). Therefore, as a rule of thumb we may say that:

$$p_{eN\text{-}DPSK} = 2 \cdot p_{eN\text{-}PSK}; \qquad (51)$$

- the bit error probability of the DPSK is computed in the same manner as the one the APSK, using its symbol-error probability, see (50).

Considerations regarding the error-performances of the OQPSK and π/4-QPSKmodulations

- in the assumption of perfect carrier and symbol-clock synchronization, OQPSK and π/4-QPSK provide slightly greater symbol-error probability than QPSK (4-PSK).

Conclusions

- the DPSK with $N \leq 8$, ensures a good spectral efficiency of up to β_w = 3 bit/s/ Hz see notes on the blackboard,
- DPSK ensures a very good resilience to noise and amplitude hits, especially for N = 2 or N = 4.
- the APSK is very sensitive to the frequency deviations, while DPSK is less sensitive to such impairments, depending on the demodulation method that is used. Still, the frequency deviations decrease, to some extent, the noise margins of the symbol-decision.
- the DPSK with $N \geq 16$, require high SNRs to ensure low symbol-error probabilities, due to the significant decrease of the minimum Euclidean distances of the constellations.
- the bit-error probabilities of DPSK with $N \geq 16$ increase significantly, due to the large number of bits/symbol; even at medium SNRs the approximation of (50) is not very accurate.
- for data transmissions with $p > 3$, the DPSK is replaced by the combined amplitude and phase modulation A+PSK (don’t confuse it with Absolute PSK, APSK), see lectures on A+PSK to follow.

Applications of the PSK modulation

- the DPSK implemented by the QAM technique and having N=4, i.e. QPSK and its variants, is one of the most employed modulations in digital transmissions, due to its resilience to noise and distortions.
- in modems designed for satellite and radio channels including non-linear RF amplifiers, , e.g. sensor networks, Bluetooth, the QPSK is replaced by its variants OQPSK or π/4-QPSK.
- the 8- PSK constellation is used in some particular cases, while larger PSK constellations were replaced in most applications by the A+PSK constellations.

Annex 3

Momentary-coherent demodulation of the DPSK signals – basic considerations – not required for the exam

- the demodulation is performed on the received signal, translated on an intermediate frequency, f_i, for reasons to be shown below.
- the momentary character comes from the fact that the demodulator employs only the phase "read" in the probing moments;
- the coherent character comes from the fact that the demodulator employs a local carrier of frequency $8 \cdot f_i$ called "synchro-carrier", which is synchronized to the received signal.
- because the phase-shifts of the modulating constellations are $\Delta\Phi_n = k \cdot 45^o$, then the absolute phase of the carrier, at the probing moment, is an integer multiple of 45°
- the absolute phase of the synchrocarrier is is "read" at the probing moments $t = kT_s$, when ISI = 0 and the momentary phase reaches its nominal value;
- the demodulator acts like Φ/D converter which has the cuanta of 45°, (a period of $8 \cdot f_i$ is 1/8 of the period of f_i) and therefore the absolute phase of the received signal probed at $t = kT_s$ is, see figure A3.1:

$$\Phi_N = C_N \cdot 180^o + B_N \cdot 90^o + A_N \cdot 45^o = (C_N \cdot 2^2 + B_N \cdot 2^1 + \cdot A_N \cdot 2^0) \cdot 45^o \qquad (A3.1)$$

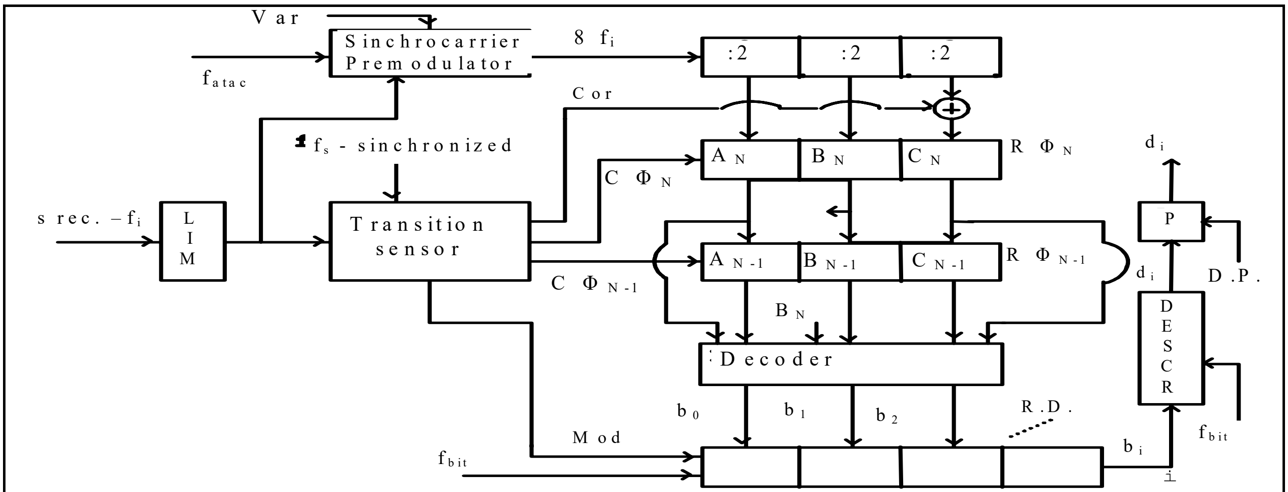


Figure A3.1. Block diagram of the momentary coherent-differential DPS demodulator ("with phase subtraction")

- the sensor-block ST indicates the probing moment (with a delay!) and gives the command of reading the value of the of the 8-counter into the new-phase memory, Φ_N.
- it is expressed on three bits, so the phase-shift is stored as a $K_N \cdot 45^o$ multiple in the Φ_N memory.
- the bit A will signify the 45°, bit B the 90° and C the 180° phase-shifts.
- the "previous-phase" register Φ_{N-1}, contains the value of the 8-conter read in the previous probing moment, which is the absolute phase during the previous symbol-period, also stored as $K_{N-1} \cdot 45^o$.
- the decoder performs the subtraction of the two subsequent absolute-phases of the synchro-carrier, Φ_N and Φ_{N-1}, delivering the phase-shift during the N-th symbol-period. Because the phase-shifts are represented as multiples of 45°, their subtraction is equivalent to the subtraction of these numbers, represented on three bits:

$$\Delta\Phi_N = \Phi_N - \Phi_{N-1} = K_N \cdot 45^o - K_{N-1} \cdot 45^o = \Delta K_N \cdot 45^o; \qquad (A3.2)$$

- the decoder also performs the natural-Gray conversion, the inverse of the operation performed by the GNC in the transmitter.
- then the data are serialized by the parallel-series register RD.
- note that the evacuation of the last bit is performed during the demodulation of the next symbol. Therefore, the RD should an additional cell, compared to the number of bits/symbol.
- the output data are descrambled and delivered as Rx data if the CD signal is active, gate P.
- the difference performed acc. to (A2.2) makes this demodulator a *differential* one
- the delay of the reading moment, referred to the ideal probing moment, can alter the read value for two reasons:

- the sampled phase-value is no longer the nominal one; it decreases the more the actual probing is made later, due to the variation characteristic of the momentary phase, see fig. 8;

- ISI is no longer null in the actual probing moments; its value increases with the delay of the actual probing moment.

- this delay has two basic causes:

- the occurrence moment of the nominal phase-shift in the modulated signal, when it is limited; to decrease this delay, the signal should be demodulated on an intermediate frequency f_i that is significantly higher than the symbol f_s, e.g. $f_i/f_s = 10.5$;
- the "jitter" of the symbol clock, which indicates the probing moment, which is inserted by the dynamic synchro circuit; therefore, the step-phase of the dynamic synchro circuit should be as small as possible.

- this demodulator also has coherent non-differential and non-coherent differential versions;
- note that the error-performances of the non-coherent differential demodulation are poorer than the ones of the coherent differential demodulation.
- detailed presentations of the three DPSK digital demodulators can be found in literature.

Demodulation of the DPSK constellations smaller than A8 using the digital demodulator – not required for the exam

- for the A4 constellation, only the B and C lines of the demodulator should be employed because the phase-shifts are $\Delta\Phi_N = K_N \cdot 90^o$, $K_N = 2p$, the LSB being always zero. For A2, since $\Delta\Phi_N = K_N \cdot 180^o$, $K_N = 4p$, only the MSB should be employed and the demodulation would be performed on line C.

- the demodulation of the B4 constellation, with $\Delta\Phi_N = (2p+1) \cdot 45^o$, requires first a rotation of 45^o, in the opposite direction to the one performed at the modulation. This is equivalent to subtracting one unity from the probed value and can be performed by disabling a transition of the synchrocarrier at the beginning of each symbol period (operation called premodulation of the synchrocarrier). Then the demodulation is similar to the A4 constellation. Similarly, the demodulation of B2 requires a premodulation of 90^o, and then the demodulation is done according to the A2 case.

Annex 4

Block diagram of the PSK receiver that uses the direct demodulation on the intermediate frequency – **not required for examination**

- it is presented in figure A4.1

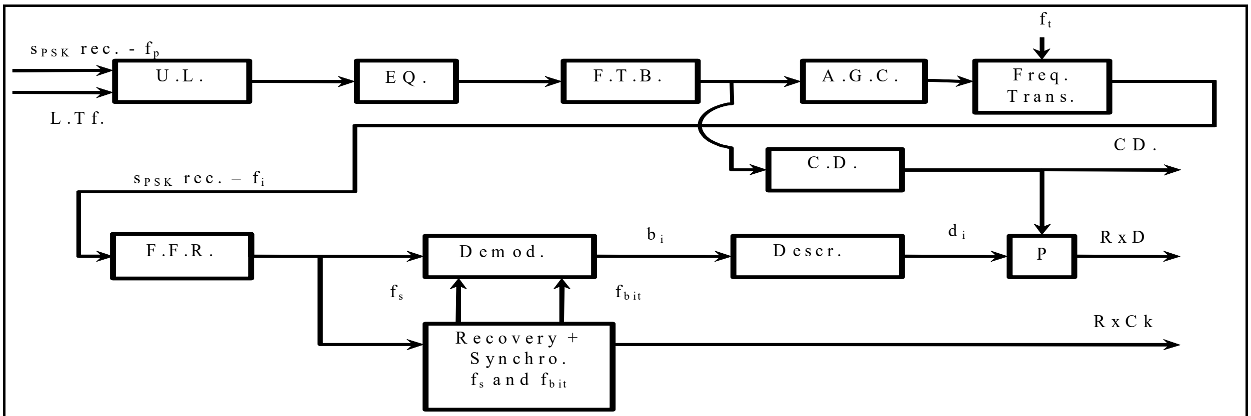


Figure A4.1. Block diagram of the PSK receiver with direct demodulation on intermediate frequency

- the received modulated signal on the channel-carrier $s_{r\text{-}PSK}\text{-}f_p$, is equalized by the EQ, is BP filtered and then is inserted into the AGC circuit which ensures a constant level of the received signal, see Data Transmissions lectures, required by the symbol-clock recovery circuit.

- after the translation on the intermediate frequency, TRANS.FRECV., the signal is filtered with the shaping RRC-filter, a BP filter.

- then the signal follows three paths:

- to the symbol-clock recovery and synchronization, which also delivers the bit-clock RxCk to the demodulator and to the computer;
- to the demodulator that delivers the demodulated data b_i, which are fed in the descrambler Descr; it "rebuilds" the initial data d_i that are enabled by the CD by means of the P gate.
- to the carrier-detector CD circuit; it compares the signal-level with the minimum accepted level and enables/disables the receiver by validating the data and generating the CD interface-signal. The CD signal is also employed in the fast synchronization of the symbol-clock and synchro-carrier signals.

Annex 5
Recovery and Synchronization of the Symbol-clock and Bit-clock in PSK Transmissions
Method using the effects of the RC-filtering

Envelope-method for the symbol-clock recovery – principle of operation

- as shown in figure 7, the envelope of the PSK modulated signal A(t) has minima approximately synchronous with the negative edges of the f_s and maxima approximately synchronous with the positive ones, the probing moments
- because the maxima are relatively flat, their accurate detection is difficult; therefore, a more accurate detection can be performed for the minima of the envelope.
- the values of the minima are computed using (14.b), i.e. $I_{min} = \left| \cos(\Delta\Phi_k/2 \right|$, for all $\Delta\Phi_n = k\cdot 45^o$, employed by the A2, A4, A8 and B2, B4 constellations, see table 1.
- note that the constellations containing $\Delta\Phi_n = 0^o$, do not have any envelope and momentary frequency variations during the whole symbol-period, and so no synchronization information is available. A series of such symbols may lead to the loss of f_s synchronization, generating a burst of error-bits.

$\Delta\Phi_n$	0º A2,A4, A8	45º A8; B4	90º A2, A8; B2	135º A8; B4	180º-A2 A2, A4, A8	225º A8, B4	270º A4, A8; B4	315º A8; B4
I_{min}	1	0,92	0,707	0,382	0	0,382	0,707	0,92

Table 5. Values of minima of the PSK envelope for the phase-shifts employed

- to compensate this disadvantage, the B-type constellations, B2 or B4, were developed, to avoid the $\Delta\Phi_n = 0^o$. A B8 variant would involve a positive rotation of $45^o/2 = 22{,}5^o$. But the minimum value of the envelope would be:

$$I_{min} = \cos(22{,}5^o/2) = 0.98; \qquad \text{(A.5.1)}$$

- the envelope variation would be of only 2% of its maximum value, hard to extract accurately in the presence of noise. Therefore, the B8 constellation is not employed. To avoid the long series of 0º phase-shifts, the input data are randomized using a scrambler, see the Data transmissions lectures. After the demodulation, the original data sequence is restored by a descrambling circuit.