

## Problems for PAM, Data Filtering and PSK

### PAM

1. Knowing that the average power of a PAM signal with  $M = 16$  levels has to be  $P_{M=16} = 1,224$ , derive the relation that expresses the amplitude levels only in terms of the level's index.
2. Knowing that the highest amplitude level of a PAM transmission with  $M=32$  is  $A_M = 4.65$ , compute the values of the levels having the minimum modulus and the average power of the PAM signal.
3. Knowing that the average power of an M-PAM transmission is  $P_{av}=0.86$  and the maximum allowed level is  $A_M=1.5$ , compute the transmission bit rate, if  $f_s = 9.6$  kHz.
4. A M-PAM transmission has the average power  $P_{av}=30.6$  and the value of the second positive level equals  $A_2=1.8$ . Compute the transmission bit rate if the first spectral null in the transmission spectrum occurs at  $f=4.8$  kHz.
5. Consider a correlator-based PAM demodulator, as shown in fig. 3, part 2 of the PAM lecture notes. Indicate a practical method to implement the multiplication of the received signal to the time-window  $u_T(t-kT_s)$
6. Explain using fig. 4 and expressions (19)-(22) from the PAM lecture notes, why the employment of the Gray mapping ensures a bit-error rate smaller than the one ensured by the mapping according to the natural binary code.
7. The symbol-error probability of a 2-PSK modulation is expressed by  $p_{e-2PSK} = Q(\sqrt{2\rho})$  and it equals  $p_{e-2PSK} \approx 1 \cdot 10^{-5}$  for  $SNR = 9.5$  dB, ( $SNR [dB] = 10 \lg \rho$ ). Compute the SNR value for which the symbol-error probability of PAM with  $M=2$  levels equals  $p_{e-2PAM} = 1 \cdot 10^{-5}$ . Use the expression of the PAM symbol-error probability derived in the lecture notes.
8. Compute with how many decibels should the SNR be increased to ensure  $p_e = 10^{-5}$  for a PAM transmission with  $M=4$  levels, w.r.t. the SNR value required by the PAM with  $M=2$  levels to ensure the same value of the symbol-error probability  $p_e$ , (values computed in problem 7). Generalize for  $M=2^n$ , where  $n$  denotes the number of bits/symbol PAM. Show that these expressions are also valid for the ASK transmission.
9. Point out the difference between the methods used to demodulate the non-filtered PAM and RC-filtered PAM signals exposed in the lecture notes, and explain the reasons that generate this difference.

### Problems and questions regarding the Data Signals Filtering

1. Define the roll-off factor and explain the reason for its use.
2. Indicate the values of the received signal that are used in the demodulation process of a signal filtered with the RC characteristic. Justify the values chosen.
3. What are the differences between the impulse response of the RC characteristic and the one of the RRC characteristic, both having the same roll-factor  $\alpha$ .
4. What is maximum frequency bandwidth of a DSB-SC signal filtered with a RC- $\alpha$  characteristic, if the modulating signal has a symbol frequency equaling  $f_s$ ?
5. Enumerate the effects of the RC-filtering upon the PSK signal.
6. Explain in qualitative manner how does the momentary frequency deviation of a PSK signal vary after the RC-filtering, w.r.t. the symbol period. What factor affects the amplitude of the momentary frequency's deviation?
7. Explain in qualitative manner how does the amplitude of a PSK signal vary after the RC-filtering, w.r.t. the symbol period. What factor affects the amplitude of the amplitude variation?
8. Explain why the phase-shift  $\Delta\Phi=0^\circ$  should be avoided in DPSK transmissions filtered with a RC characteristic.

## PSK - QAM

**I.** Consider a  $D = 3600$  bps DPSK transmission over the vocal telephone channel (300 – 3400) Hz with an SNR smaller than 20 dB.

- 1) Establish the transmission parameters ( $f_c$ ,  $n$ ,  $f_s$ ,  $\alpha$ , bandwidth), so that a minimum symbol error probability could be ensured. Consider several possible variants.
- 2) What is the maximum bit rate  $D_M$  that could be transmitted on this channel if the symbol-error probability  $p_e \leq 1 \cdot 10^{-5}$ ?
- 3) Compute then value of  $D_M$  if the SNR decreases to 15 dB and BER should be smaller than  $5 \cdot 10^{-6}$
- 4) Write the operational equations and the block diagram of the DPSK transmitter implemented using the QAM, for the transmission that has the parameters established at points 1) and 2). Indicate the multibit – (I, Q) mapping rule as well as the type and cut-off frequencies of the employed filters.
- 5) Write the operational equations and draw the block diagram of the DPSK receiver based on the QAM approach for the transmission of 1) and 2), indicating the filters' parameters.
- 6) What modifications should be made in the transmitter and receiver of 3) and 4) to provide smaller bit rates, if the filters are not modified?

**II. Consider** an  $R = 3600$  bps bit rate that has to be transmitted across the lower half of the vocal telephone channel (300, 1800 Hz) by using a DPSK transmission.

- a) Compute the parameters of the modulation involved ( $f_s$ ,  $n$ ,  $f_c$ ,  $\alpha$ ) in order to ensure the lowest bit-error probability.
- b) Draw the block diagram of the transmitter having the parameters computed at a). Explain the role of each block.
- c) Compute the bandwidth and the central (cut-off) frequency of each of the filters involved.

**III.** A DPSK A8 transmitter modulates the signal on the intermediate frequency  $f_i$  using the QAM-based modulator and transmits it on the carrier  $f_c > f_i$ , by using a translation frequency  $f_t > f_c$ .

- a) Draw the block diagram of the transmitter. Indicate how the systematic error inserted by the frequency translation is compensated in the transmitter
- b) Compute the values of the tribits that should be fed into the mapping block if the phase shifts should be behind.
- c) If the data sequence at the transmitter's input is 001, 111, 101, 110, 100, 010, 011, 001, and the initial absolute phase of the carrier equals  $30^\circ$ , compute the values of the absolute phase of the modulated signal for the 8 symbol periods
- d) If  $f_c = 10$  kHz,  $\alpha = 0.75$ ,  $D = 19200$  bps, compute the frequency band of the modulated signal

**IV.** A DPSK transmission should provide the bit rates  $D = 9600$ ,  $4800$  and  $2400$  bps in the frequency band [300, 3300] Hz.

- a) Derive the transmission parameters ( $n$ ,  $f_s$ ,  $f_c$ ,  $\alpha$ ) which would ensure minimum symbol-error probabilities for the three data rates and allow an as simple as possible switch between the three bit rates. Indicate the constellations used.
- b) Assuming that 2-PSK provides  $p_e = 10^{-5}$  at SNR = 9.5 dB, compute the SNR values at which the same error-probability is ensured for the three bit rates.

**V.** The DPSK receiver that demodulates the signals with the parameters set at problem IV.a performs the demodulation on the intermediate frequency  $f_i = 13.2$  kHz.

- a) If the translation frequency used to translate the received signal on  $f_i$  is  $f_t = 15$  kHz, show by computation what systematic error occurs? Indicate a method to compensate this error in the receiver
- b) Compute the minimum value of the  $f_{\text{atack}}$  frequency of the dynamic synchro circuit which synchronizes the symbol clock ( $f_s$ ) so that the maximum phase error inserted at the probing of the received signal, after its translation on  $f_i$ , would be  $< 5\%$  of the minimum phase shift of the DPSK modulations established at IV.a.
- c) Compute the maximum time interval required by the initial synchronization of the symbol clock ( $f_s$ ), if there is no fast synchro circuit.

**VI.** Consider a DPSK-A4 transmission generated by the QAM, technique, having the parameters  $f_c$ ,  $f_s$  and  $\alpha$ .

- a) Draw the block diagram and the operational equations of the QAM transmitter that generates the modulated signal, indicating the mapping rule and the parameters of the employed filters. Point out how is the differential phase modulation obtained.
- b) If after the G-N conversion that 01, 11, 10, 00, dibits were modulated and the local carrier in the QAM receiver is synchronized with a  $+90^\circ$  phase-offset, compute the number of wrongly QAM-demodulated bits (before the N-G conversion)
- c) How should the QAM transmitter be modified to provide the transmission with the same parameters, but using the O-QPSK modulation? But the receiver?