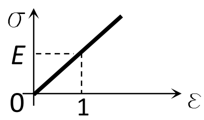
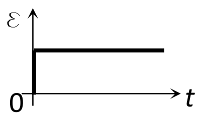
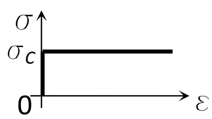
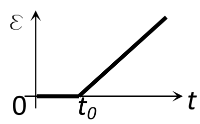
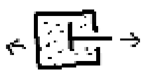
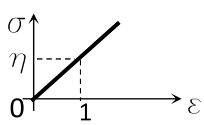
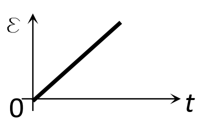
INTRODUCTION

Any construction must correspond to the intended purpose, also meeting safety criteria in use. The strength and stability of a construction results mainly from the proper composition and sizing of the load-bearing structure. The load-bearing structure is the cohesive assembly of the resistant elements, capable of safely taking over the loads. In the case of statics, the load-bearing structure is considered in the form of a skeleton made from bars, which by the links at their ends ensure the geometric invariability of the structure and the proper fixation to the ground.
The object of construction statics is the analysis of the response of load-bearing structures subjected to different actions, by determining the state of efforts and displacements on some wireframe models. Thanks to the schematizations and analysis methods used, statics has an important role in shaping the thinking of structural design engineers, offering the possibility of choosing structural variants appropriate to the destination and safety of structures and providing initial data for dimensioning.
The approximation of structural behaviour is primarily aimed through the simplicity of these models and methods. Addressing more refined (more accurate) models is also possible, but requires appropriate tools (such as computer-aided analysis). In this material, only some aspects related to the linear-elastic static analysis of planar beam structures will be discussed.

SCHEMATIZATIONS

Materials

The behaviour of building materials differs depending on their type and the mode and duration of the stress. The object of rheology is the study of the deformation of bodies, providing statics with approximate models for the physical-mechanical behaviour of building materials. Rheological properties are grouped into two main categories: fundamental (including elasticity, plasticity and viscosity) and technological (such as ductility, malleability, penetrability, etc.). In the following table, three linear rheological models (illustrating the mentioned fundamental rheological properties) and two ideal models (rigid and liquid) are summarized:

Name	Scheme	Equations of state	Characteristic curve	State curve	Use
Linear-Elastic (*Hooke*)		$\sigma = E \cdot \varepsilon$ $\varepsilon = \dfrac{\sigma}{E}$			Linear-elastic analysis
Note: the deformation starts with the load and it is reversible (disappears after cessation of loading).					
Linear-Plastic (*Saint-Venant*)		$\sigma = \sigma_C$ $\varepsilon \neq 0$			Plastic analysis
Note: the deformation occurs only when reaching the flow stress, after which it will increase linearly and will be remanent.					
Linear-Viscous (*Newton*)		$\sigma = \eta \cdot \dot{\varepsilon}$ $\varepsilon = \int \dfrac{\sigma}{\eta} dt$			Fluid mechanics
Note: the deformation will develop proportionally to the load.					

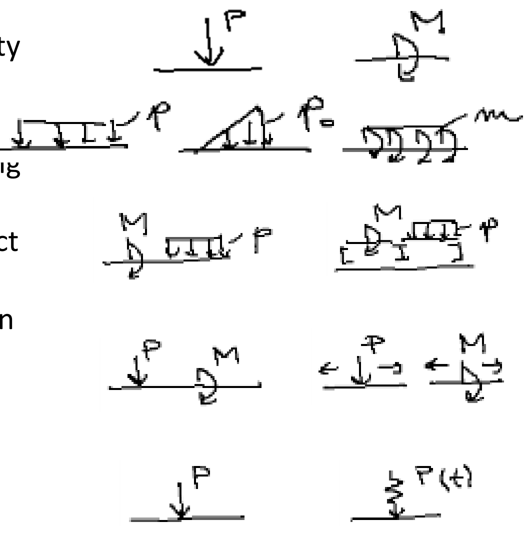
Ideal Rigid (*Euclid*)		$\sigma \neq 0$ $\varepsilon = 0$			Rigid body mechanics
Note: non-deformable body.					
Ideal Liquid (*Pascal*)		$\sigma = 0$ $\varepsilon \neq 0$			Fluid mechanics
Note: the deformation occurs even without any load.					

Given that building materials have the three fundamental properties (elasticity, plasticity, viscosity) in different proportions, it is usually necessary to use compound rheological models to approximate their actual behaviour. The definition of these compound rheological models is carried out by experimental tests in testing laboratories. In this part of statics, only the linear-elastic model (for the material of the "bars" that will make up the structural scheme) and the ideal rigid one (for the "land" to which the structure is fixed) will be used.

Loads

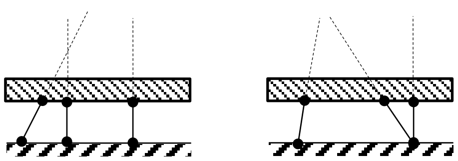
Any cause capable of generating states of mechanical stress in a construction is called action. Depending on their duration and frequency, in the design of structures the actions are classified into permanent actions (acting throughout the duration of the use of construction), temporary (acting during certain periods of time) and exceptional (those that occur less often, but with a special intensity). The modelling of the actions in statics is done through loads (forces, moments, temperature variations, support failures, imposed displacements, etc.). Loads can be classified according to several criteria, such as:

- by distribution: point loads (their intensity being applied at a point), distributed loads (their intensity being distributed over a distance or on a surface);
- by application: direct loads (applied to load-bearing elements), indirect loads (applied by means of secondary elements, which will transfer their effect to the load-bearing elements);
- by position: fixed loads (whose areas of application do not change), mobile loads (whose areas of application fall within a range within which it can have any position);
- by variation: static loads (constant intensity), dynamic loads (variable intensity).



Links

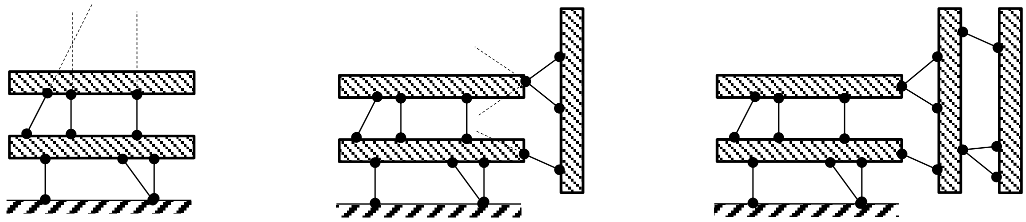
In order to properly secure a rigid body to the ground (in the case of a planar problem) 3 links are required, arranged in such a way as to avoid the appearance of instantaneous rotation centres. In the following illustrations, some correct and incorrect variants are presented:



Correct variants (without instantaneous centres of rotation there is no possibility of relative displacements).

Incorrect variants (with instantaneous centre at a point, allowing the relative rotation of the body, or instantaneous centre to infinity, allowing the relative translation of the body).

In case of positioning the links to connect 2 rigid bodies, the problem is similar to the previous case. For each additional rigid body added, another 3 correctly positioned simple links will be required.



In conclusion, the condition needed for the composition of a geometrically invariant system of bodies fixed to the ground can be written in the form:

$$3 \cdot b = l + s$$

where b is the number of bodies, l is the number of internal links (between the bodies) and s is the number of external links (with the ground). Using the above relation, the degree of determination for a body system can be expressed as:

$$g = l + s - 3 \cdot b$$

Depending on the value of this degree of determination, we can speak of statically indeterminate systems ($g > 0$, with more links than needed) or statically determined systems ($g = 0$, with as many links as necessary) if the links are arranged correctly, or mechanisms ($g < 0$, with fewer links than would be needed).

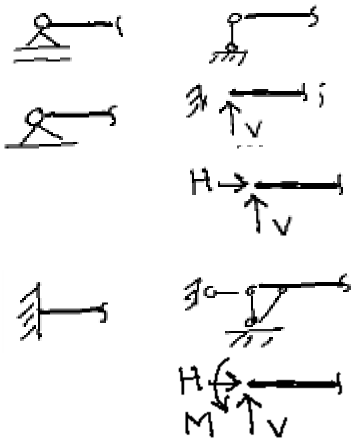
Structural schemes

Bars (represented by their axes) and connections (schematized links) are used for the composition of structural schemes. Their arrangement must be made in such a way as to ensure the geometric invariability of the structure and its fixing to the ground. In the composition of the structural scheme, two categories of schemes are used for connections: internal (between the bars that make up the structure) and external (for attaching the structure to the ground).

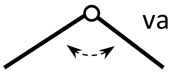
External links are needed to fix the structural scheme to the ground, also having the role of transferring the effects of loads to the ground (allowing the expression of reactions). The schemes of these connections are generically called "supports". In the case of planar structures these connections can be simple supports (allowing the relative rotation and translation of the attached end), articulated supports (allowing only the relative rotation of the attached end) or fixed supports (rigid grips, without the possibility of any relative displacement of the attached end):

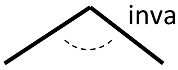
- bar end attached with a simple support to the ground
  (a single link, blocking the relative translation in the

direction of the link. It corresponds to a reaction force in the direction of the single link);
- bar end attached with an articulated support to the ground (two simple links, blocking the relative translation in both directions. It corresponds to two components of a reaction force, which can be decomposed / projected in convenient directions);
- bar end attached to a fixed support (three simple links, blocking the relative rotation and translations. The corresponding reactions will include bending moment in addition to the two force components / projections).

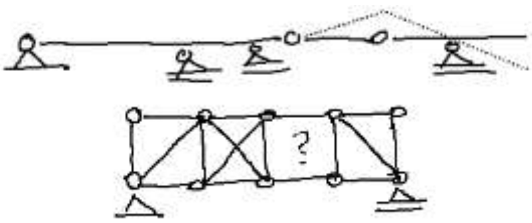


The internal connections are at the nodes and in the case of planar structures they can be articulated (allowing the relative rotation of the bar ends, without friction) or rigid (the angle between the tangents of the bar ends being invariable):

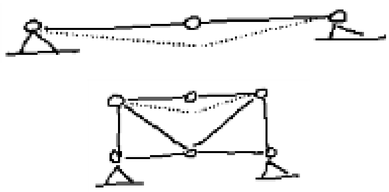

variable angle, articulated node


invariable angle, rigid joint

An articulated node (or pinned joint) represents two simple links (two components / projections of the connecting force will correspond), while a rigid joint means three simple links (the corresponding connecting forces will consist of bending moment and two components / projections of force). The main role of internal connections is to ensure the geometric invariability of the structural scheme, but they also contribute to the transfer of the effects of loads through the structure (through the connection forces).
In case of more complex analyses, more advanced schematics can also be considered (e.g. plastic joints or partially rigid joints, flexible connections, etc.), but only those that will be used in this introductory part of statics have been presented.
Care must be taken when placing connections (internal and external) to avoid parts that cannot take loads (movable parts, partial mechanisms) and critical systems (parts with uncontrollable displacement and effort).

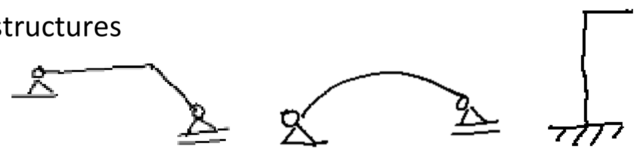


Schemes with movable parts (partial mechanisms).



Critical systems (a small vertical displacement of the central node would lead to large variations in axial stress in the adjacent horizontal bars).

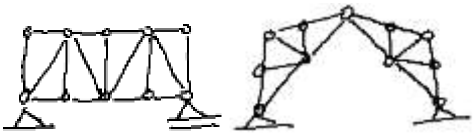
Depending on the composition and the likely states of stress, the schemes of planar structures can be classified into the following categories:
- beams: straight, bended, curved, etc. (structures usually subjected to bending);



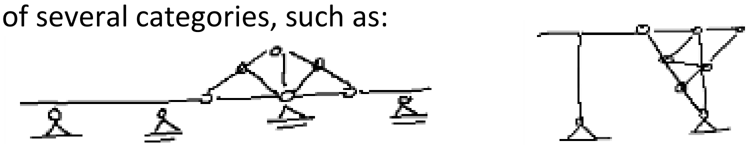
- arches (structures usually subjected to compression);

- planar frames, composed of different oriented bars connected by joints (structures usually subjected to bending);

- planar trusses or latticed structures, composed of different oriented bars with pinned connections / articulated nodes (structures subjected to axial forces);



Of course, the above classification being relative, there is also the possibility of drawing up structural schemes containing parts of several categories, such as:



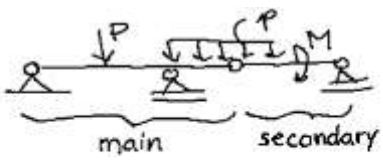
STATIC EQUILIBRIUM

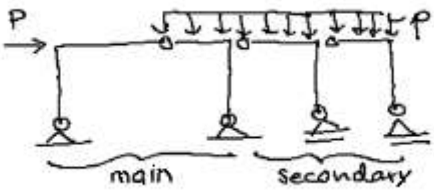
Defining a statics problem is done by building calculation schemes. These schemes are composed of structural schemes on which the considered scheme for the loads is represented too. In order to apply the conditions of static equilibrium, all the forces acting on the structure must be known. The expression of static equilibrium is achieved by determining the resultant of all the forces acting on the structure, assuming that the value of this resultant is null. The conditions for static equilibrium can be expressed in several ways:

- analytically (by equations for the components of forces projected in two distinct directions and by equations of moments);
- using vectors (by summing all vectors and reducing the torsor relative to a chosen point);
- graphically (by the proportional graphical representation of the forces, according to their orientations, forming a funicular polygon).

Choosing the most advantageous variant of the above is made depending on the type of structure, the nature of the loads and after the user experience.

Structures may consist of load-bearing elements and non-load-bearing elements (i.e. ”worn” elements, which can take on loads that they transfer to load-bearing elements or which contribute only to stability). There are situations in which the presence of main and secondary parts must be taken into account. The main parts are made up of the load-bearing elements that can take over the loads themselves, directly transferring their effect to the ground, while the secondary parts rather ensure the geometric invariability of the structure, leaning on the main parts:





In order to solve calculation schemes containing main parts and secondary parts, the secondary parts are usually isolated first (to determine the connection forces between them and the main parts, then projecting these connection forces in reversed direction on the remaining main parts).

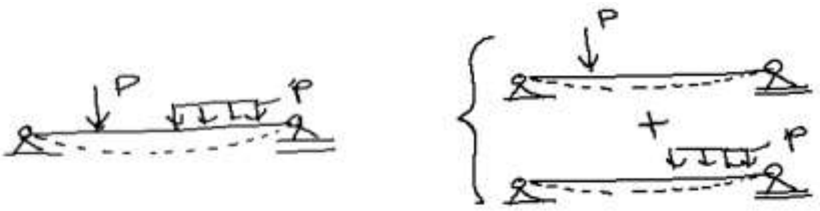
The static equilibrium may be expressed for the whole structural scheme (if it is intended to determine the reactions) or for any part of the structure (if the aim is to obtain the connection forces or efforts). To express equilibrium over the entire structure, the ground links of the structure are replaced by the corresponding connection forces (reactions). Similarly, in order to express the balance of a part of the structure, the links with the rest of the structure of this part are replaced by the corresponding connection forces. If some bars are sectioned, internal forces (efforts, i.e. stresses) will result).
If the static equilibrium conditions are not sufficient to express equilibrium, they may be supplemented by conditions of compatibility of links, displacements, etc., or with conditions of material continuity, thus leading to conditions of elastic equilibrium. There is also a more general way of expressing the equilibrium conditions, by using the principle of virtual mechanical work.

Fundamental hypotheses

In linear elastic static analysis, in addition to the already known assumptions from mechanics and strength of material (material homogeneity, flatness and orthogonality of sections, etc.), there are three fundamental hypotheses:

- loads appear together with the structure (there is no initial moment without loads);
- all the forces are conservative (they do not change their point and direction of action);
- the material of the bars is continuous, homogeneous and isotropic, behaving according to the linear-elastic model (Hooke's law applies).).

These hypotheses are valid in the range of small displacements, considering that the positions resulting from the action of the loads of different points in the structure are very close to their initial position. Thus, the static equilibrium conditions are expressed on the initial (undistorted) shape of the structure, ignoring the displacements caused by loads (although it is possible to determine these displacements after performing the static analysis) and considering infinitely rigid the ground.
As this is a linear analysis carried out in the range of small displacements, the proportionality between the loads and the displacements they produce is accepted. Consequently, linear overlapping of effects can be applied at any time, breaking down the more complicated loads into simpler, separately treated components, and the end result being obtained by summing these effects (as shown in the figure below). Combinations of loading cases can also be made to determine the most unfavourable ones.



Efforts (stresses)

The section method is the one that can express the equilibrium of any part of a structure (after creating a discontinuity and replacing the suppressed links with their effect). Following the virtual sectioning of a bar (axes), the internal forces that will be highlighted (replacing the effect of the suppressed links) are called efforts or stresses.

In the case of a planar orthogonal reference system (considering the possibility of two distinct translations and one rotation), these inner forces may be:

- axial force, marked with N (oriented along the axis of the bar, considered positive if it pulls in front of the section);
- shear force, marked with T or Q (oriented perpendicular to the axis of the bar, considered positive if it acts on the face of the section clockwise);
- bending moment, marked with M (oriented perpendicular to the plane formed by axial stress and shear force).

In the case of flat structures normally loaded on their plane, torsion moment may also occur, marked with M_t (oriented along the axis of the bar, considered positive if it turns clockwise according to the “screw” rule).

In order to represent the stress state in the bars, the stress values in the characteristic points are determined (at the points where the loads act and in the nodes), followed by the graphical representation of the way in which the stress values vary along the axes of the bars that make up the structural scheme. These graphs are called effort diagrams or stress diagrams. On the diagrams that represent along the axes of the bars of a structure the variation of the axial effort, respectively the variation of the shear force, the positive and negative signs of the efforts are marked, while in the case of the bending moment diagrams the values are represented on the stretched side of the bars.