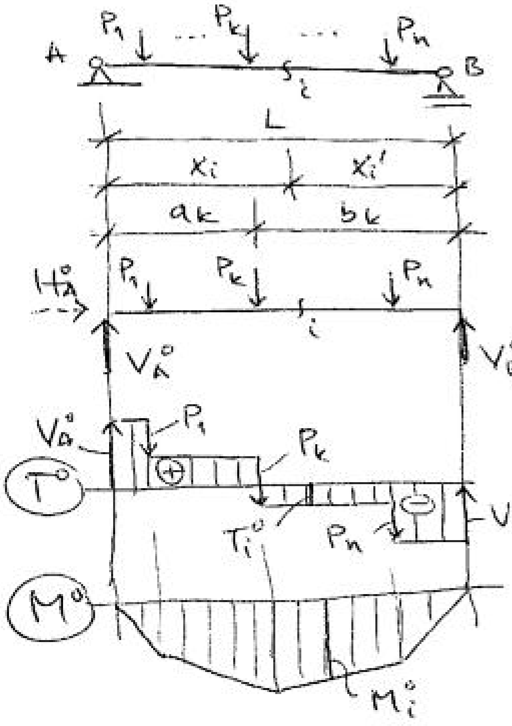
LECTURE 3

STRAIGHT BEAMS

A BEAM IS A STRUCTURAL MEMBER USED FOR BEARING LOADS AND USUALLY WORKS THROUGH BENDING MOMENT AND SHEAR FORCE. THE SIMPLY SUPPORTED BEAM IS THE SIMPLEST VARIANT.



IN ORDER TO COMPUTE THE REACTIONS, THEY SHALL BE EXHIBITED REPLACING THE SUPPORTS. THE VERTICAL COMPONENTS CAN BE CALCULATED:

$$V_A^\circ = \sum_A^B \frac{P_k \cdot b_k}{L}$$

$$V_B^\circ = \sum_A^B \frac{P_k \cdot a_k}{L}$$

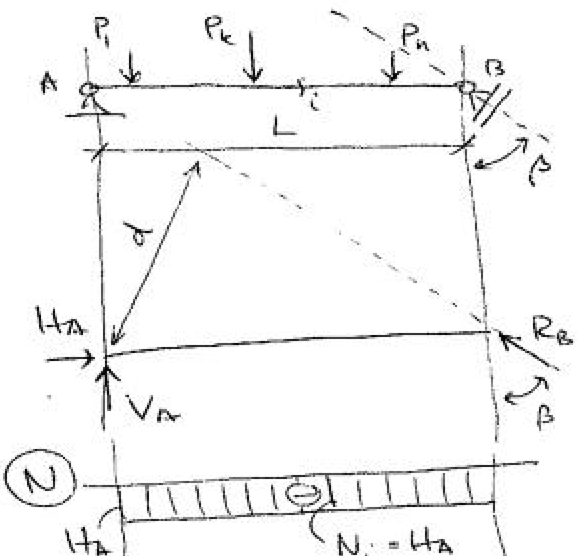
THE EFFORTS IN SECTION i CAN BE OBTAINED IN THE FOLLOWING WAY:

$$T_i^\circ = V_A^\circ - \sum_A^i P_k$$

$$M_i^\circ = V_A^\circ \cdot x_i - \sum_A^i P_k (x_i - a_k)$$

IF THERE ARE NO INCLINED LOADS, THERE WILL BE NO AXIAL FORCE ($N_i^\circ = 0$ AND $H_A^\circ = 0$).

IN CASE OF AN INCLINED SUPPORT (OR, IF THERE ARE INCLINED LOADS) THERE WILL BE ALSO A HORIZONTAL COMPONENT OF THE REACTIONS AND THERE WILL BE AXIAL FORCE IN THE BEAM. THE SHEAR FORCE AND THE BENDING MOMENT IN SECTION i WILL HAVE SIMILAR EXPRESSIONS, AS ABOVE.



$$R_B = \frac{1}{d} \sum_A^B P_k \cdot a_k$$

$$H_A = R_B \cdot \sin\beta$$

$$V_A = \sum_A^B \frac{P_k \cdot b_k}{L} = V_A^\circ$$

SO $T_i = T_i^\circ$

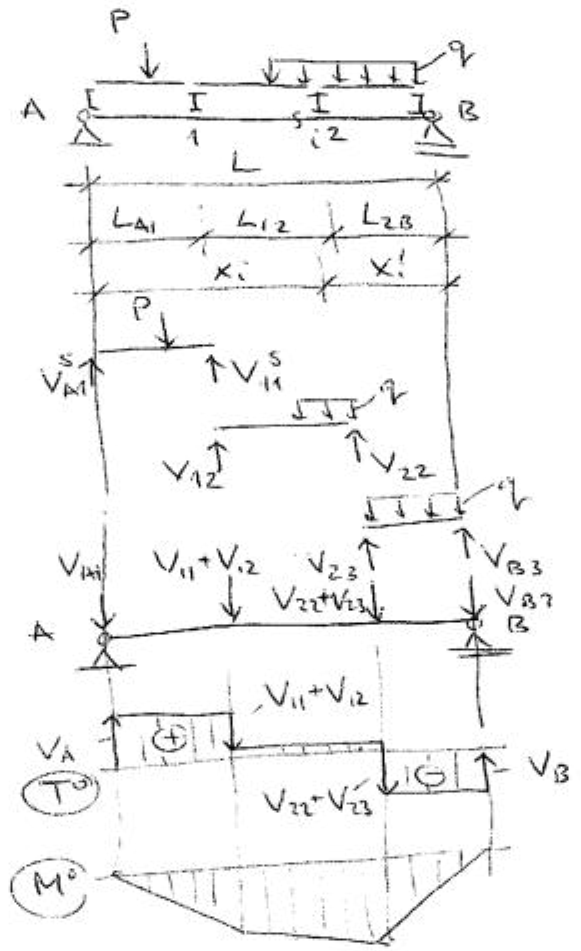
$M_i = M_i^\circ$

AND $N_i = H_A$ (IF THERE ARE NO INCLINED LOADS)

IN CASE OF INCLINED LOADS THERE WILL BE A VARIATION OF THE AXIAL FORCE DIAGRAM (JUMPS IN CASE OF POINT LOADS).

IN CASE OF INCLINED STRAIGHT BEAMS, THE APPROACH IS SIMILAR TO THAT IN CASE OF BEAMS WITH INCLINED SUPPORT.

IN CASE OF INDIRECT APPLIED LOADS (APPLIED ON ADDITIONAL SYSTEMS THAT WILL TRANSFER THE LOADS ON THE BEAM):



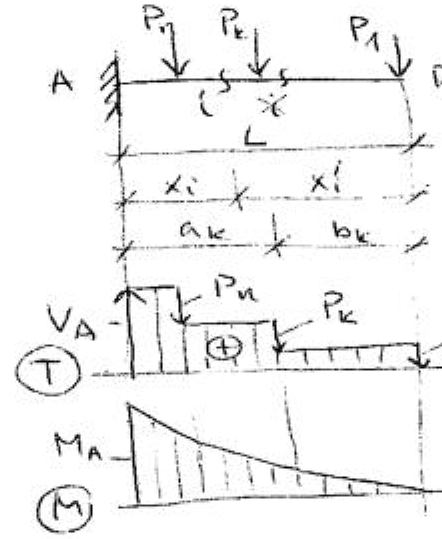
THE REACTIONS CAN BE OBTAINED AS DESCRIBED IN THE PREVIOUS INITIAL CASE.

THE LONGITUDINAL ELEMENTS ON WHICH THE LOADS ARE APPLIED CAN BE CONSIDERED AS SIMPLY SUPPORTED BEAMS, SO DETERMINING THE REACTIONS AT THEIR END POINTS IS EASY. THESE REACTIONS WILL BE CONSIDERED AS LOADS (WITH REVERSED DIRECTIONS) ON THE BEAM.

GETTING THE EFFORTS IN SECTION i WILL BE DONE LIKE IN THE INITIAL DISCUSSED CASE, FROM THESE LOADS.

TAKING INTO ACCOUNT THAT THE TRANSFER OF LOADS IS DONE THROUGH THE CONTACT ELEMENTS, THE BENDING MOMENT DIAGRAM WILL SHOW LINEAR VARIATIONS.

IN CASE OF STRAIGHT CANTILEVER BEAMS, THE EFFORT DIAGRAMS CAN BE DRAWN EVEN WITHOUT PREVIOUSLY COMPUTING THE REACTIONS, CONSIDERING THE APPLIED LOADS FROM THE FREE END TOWARDS THE FIXED END OF THE BEAM.



$$T_i = \sum_{B}^{i} P_k$$

$$M_i = \sum_{B}^{i} P_k \cdot (x_i' - b_k)$$

THE REACTIONS WILL RESULT AS:

$$V_A = \sum_{B}^{A} P_k$$

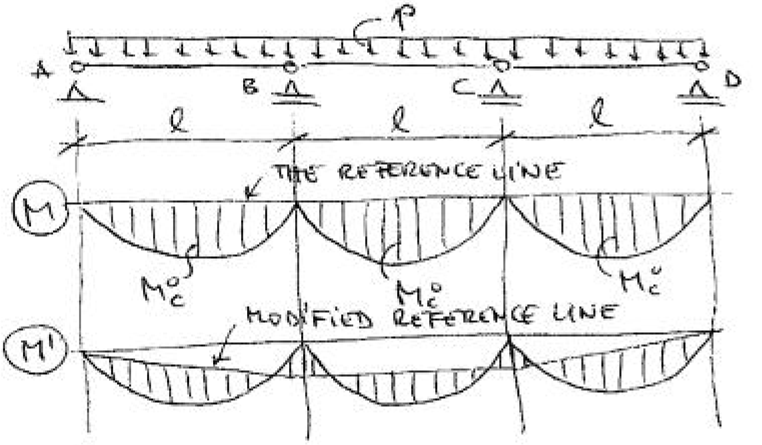
$$M_A = \sum_{B}^{A} P_k \cdot a_k$$

CANTILEVERED BEAMS WITH HINGES

THESE BEAMS ARE ALSO KNOWN AS "GERBER" BEAMS (UPON THE NAME OF JOHANN HEINRICH GOTTFRIED GERBER, A GERMAN RAILWAY STRUCTURAL ENGINEER, WHO PATENTED THEM IN THE 19-TH CENTURY).

THE BASE CONCEPT WAS ABOUT TRANSFORMING A SYSTEM MADE OF SIDE BY SIDE MOUNTED SIMPLY SUPPORTED BEAMS, BY MOVING THE HINGES (ARTICULATION POINTS), INTO A CANTILEVERED BEAM STRUCTURE.

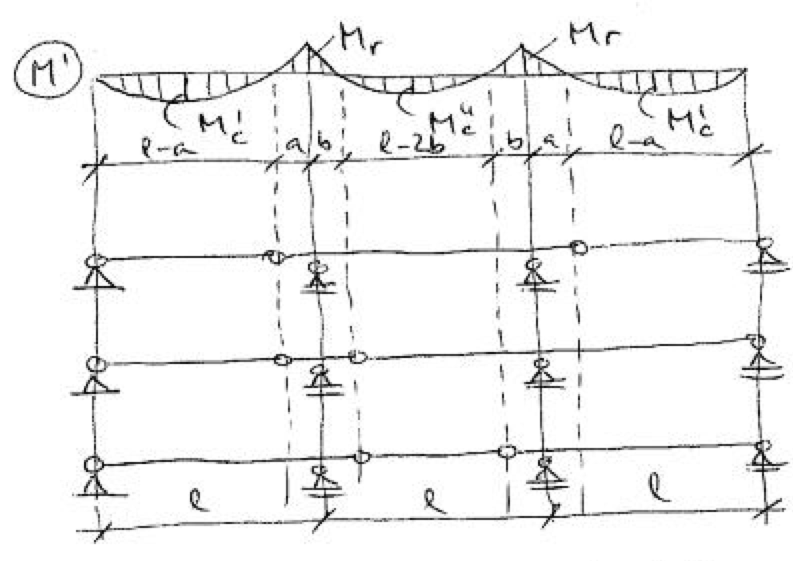
WE SHALL DISCUSS THE CASE OF SUCH A STRUCTURE COVERING THREE EQUAL OPENINGS (QUITE COMMON IN CASE OF BRIDGES).



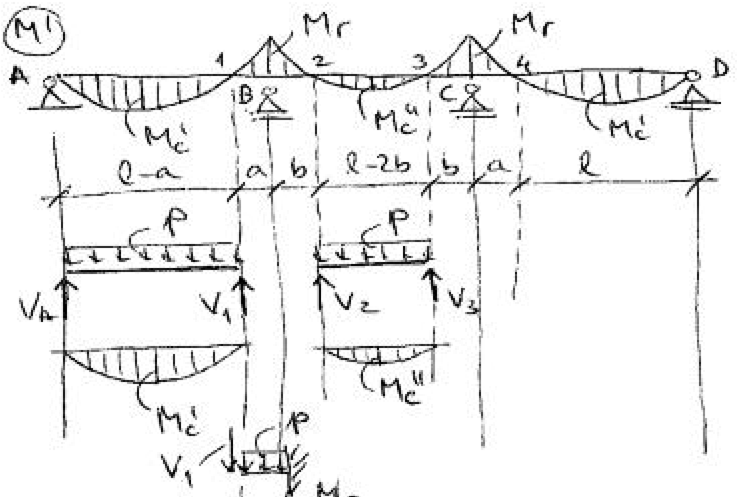
CONSIDERING A HORIZONTAL UNIFORMLY DISTRIBUTED VERTICAL LOAD (LIKE THE SELF-WEIGHT), THE BENDING MOMENT DIAGRAM WILL CONSIST OF THREE IDENTICAL PARABOLIC SHAPES, THE MAXIMAL VALUES IN THE MIDDLE OF THE FIELDS WILL BE:

$$M_c^o = \frac{p \cdot l^2}{8}$$

IN ORDER TO HAVE A MORE EFFICIENT USE OF THE MATERIAL IN THE CROSS SECTIONS ALL OVER THE BEAMS, THE BENDING MOMENT DIAGRAM SHOULD BE CHANGED BY MODIFYING THE REFERENCE LINE (LIKE IN THE FIGURE ABOVE), SO THAT THE PARTS NEAR THE INNER SUPPORTS COULD ALSO UNDERTAKE SOME BENDING. THIS THING CAN BE OBTAINED BY MOVING THE HINGES IN THOSE POINTS IN WHICH THE BENDING MOMENT DIAGRAM SHOULD CROSS THE REFERENCE LINE. OF COURSE, THERE ARE SEVERAL POSSIBILITIES, BUT NOT ALL OF THEM ARE SAFE AT THE SAME EXTENT.

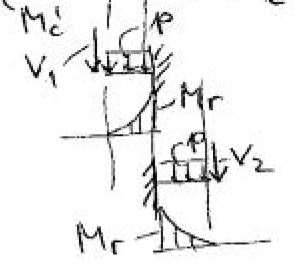


IN ORDER TO ASSESS THE SAFETY, IT MUST BE SEEN TO WHAT EXTENT THE ENTIRE STRUCTURE WOULD BE AFFECTED FOLLOWING THE DAMAGE OF A PART. FROM THE THREE ILLUSTRATED EXAMPLES, THE THIRD HAS THE HIGHEST SAFETY (BUT THIS DOES NOT EXCLUDE THE POSSIBILITY OF USING THE OTHER VARIANTS).



NEXT, IT WILL BE DISCUSSED HOW TO ESTABLISH THE POSITIONS OF THE HINGES UPON CERTAIN CRITERIA.

IT CAN BE NOTICED, THAT BOTH IN THE SIDE FIELDS AN IN THE MIDDLE THERE ARE SOME PORTIONS THAT CAN BE CONSIDERED SIMPLY SUPPORTED BEAMS ($l-a$ AND $l-2b$ SPAN) AND CANTILEVERS (a AND b).



CONSIDERING THE FIRST SPAN (A-1):

$$V_A = V_1 = p \cdot \frac{l-a}{2} \quad \text{AND} \quad M_c' = p \cdot \frac{(l-a)^2}{8}$$

ON THE NEXT PART (1-B):

$$M_r = V_1 \cdot a + p \cdot a \frac{a}{2} = p \cdot \frac{l-a}{2} \cdot a + p \cdot \frac{a^2}{2} =$$
$$= \frac{p \cdot a}{2}(l-a+a) = \frac{p}{2} a l$$

IN THE MIDDLE AREA: $V_2 = V_3 = p\,\frac{l-2b}{2}$ AND $M_c'' = p\,\frac{(l-2b)^2}{8}$

WHILE $M_r = V_2 \cdot b + p \cdot b \cdot \frac{b}{2} = p\,\frac{(l-2b)}{2} \cdot b + p \cdot \frac{b^2}{2} = \frac{p \cdot b}{2}(l-2b+b) = \frac{p}{2} b(l-b)$

OBSERVING THAT ABOVE THE „B" SUPPORT WE GOT TWO EXPRESSIONS FOR THE „M_r" BENDING MOMENT, WE CAN OBTAIN A RELATION BETWEEN THE VALUES OF THE a AND b DISTANCES:

$$\frac{p}{2} a \cdot l = \frac{p}{2} b(l-b) \quad \text{SO} \quad a = b\,\frac{l-b}{l} = b - \frac{b^2}{l}$$

THUS, BY CHOOSING A VALUE FOR THE b DISTANCE, WE SHALL GET A CORRESPONDING VALUE FOR THE a DISTANCE (AND VICE-VERSA).

IN CASE THAT WE WANT THE MAXIMAL VALUES OF THE BENDING MOMENTS IN THE SIDE FIELDS AND OVER THE SUPPORTS TO BE EQUAL, EG:

$$M_c' = M_r \quad \text{THAN} \quad p\,\frac{(l-a)^2}{8} = p\,\frac{a \cdot l}{2} \quad \text{SO} \quad l^2 - 2la + a^2 = 4al$$
$$\text{AND} \quad a^2 - 6al + l^2 = 0$$

AFTER SOLVING THIS EQUATION, THE PROPER VALUE FOR a WILL BE THE ONE BETWEEN $(0, l)$.

IN CASE THAT WE WANT EQUAL MAXIMAL VALUES IN ALL THREE FIELDS:

$$M_c' = M_c'' \quad \text{THAN} \quad p\,\frac{(l-a)^2}{8} = p\,\frac{(l-2b)^2}{8} \quad \text{SO} \quad a = 2b$$

IN CASE THAT WE WANT THE MAXIMAL VALUES OF THE BENDING MOMENTS IN THE MIDDLE FIELD AND OVER THE SUPPORTS TO BE EQUAL:

$$M_c'' = M_r \quad \text{THAN} \quad p\,\frac{(l-2b)^2}{8} = p\,\frac{b(l-b)}{2} \quad \text{SO} \quad l^2 - 4bl + 4b^2 = 4bl - 4b^2$$
$$\text{AND} \quad 8b^2 - 8bl + l^2 = 0$$

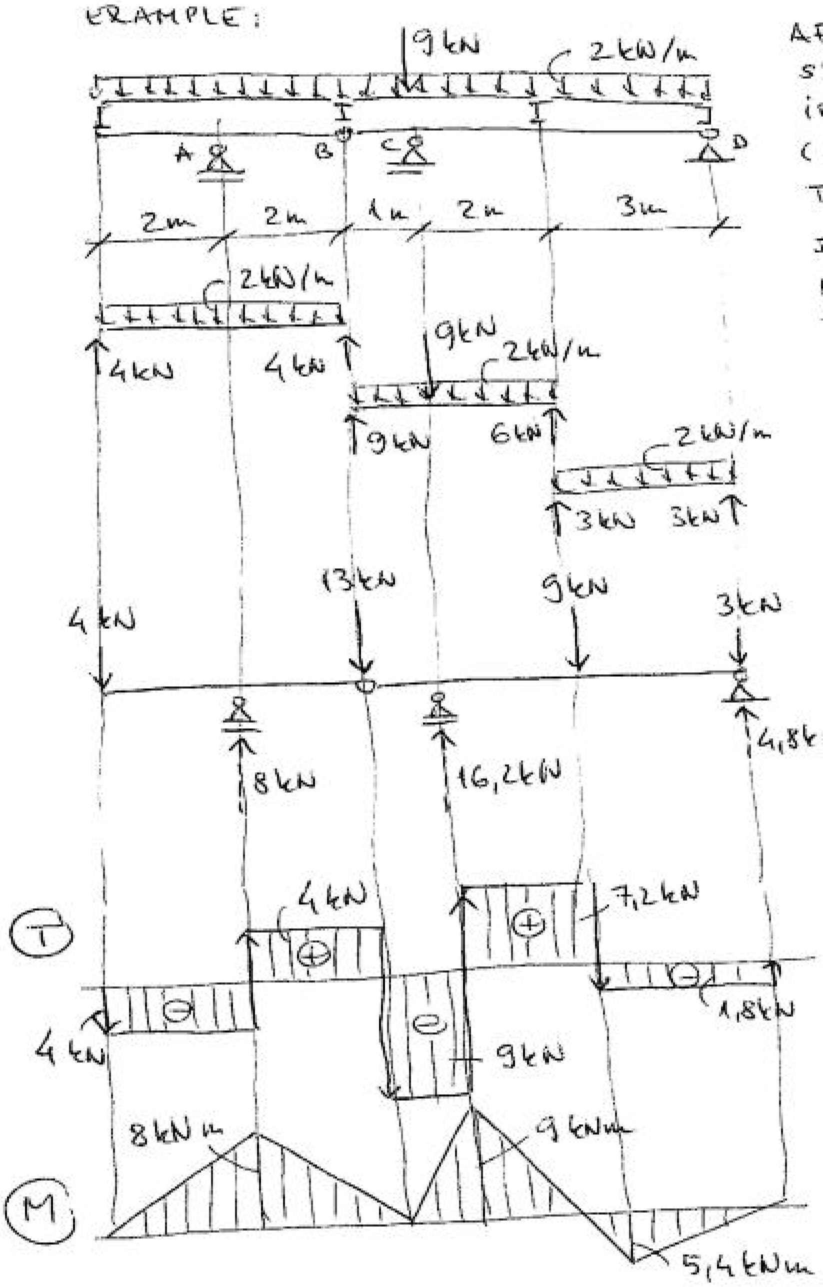
THE PROPER SOLUTION OF THE PREVIOUS EQUATION WILL BE THE VALUE OF b BETWEEN (0, l).

THERE IS NO WAY TO OBTAIN EQUAL VALUES FOR ALL THE BENDING MOMENTS (IN BOTH SIDE FIELDS, IN THE MIDDLE FIELD AND OVER THE TWO SUPPORTS) WITHOUT MODIFYING THE l DISTANCES (KEEPING THE ENTIRE SPAN 3·l).

INDIRECT APPLIED LOADS

LIKE IN CASE OF STRAIGHT BEAMS (DISCUSSED BEFORE), THE TRANSFER OF THE LOADS WILL TAKE PLACE ONLY WHERE THERE IS CONTACT BETWEEN THE SECONDARY SYSTEM AND MAIN STRUCTURE.

EXAMPLE:

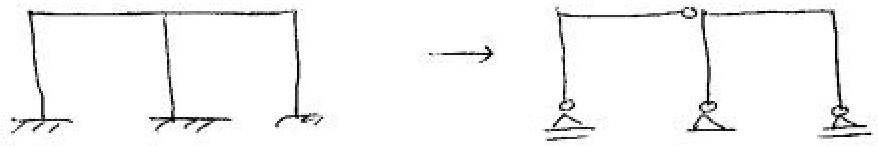


AFTER SOLVING THE SECONDARY SYSTEM, THE RESULTING REACTIONS SHALL BE POINT LOADS (WITH REVERSED DIRECTIONS) ON THE MAIN STRUCTURE.

FROM THESE LOADS CAN THE REACTIONS BE OBTAINED (IN THE SUPPORTS), THEN, AFTER KNOWING ALL THE EXTERIOR FORCES, THE EFFORTS CAN BE CALCULATED IN ORDER TO DRAW THE CORRESPONDING DIAGRAMS.

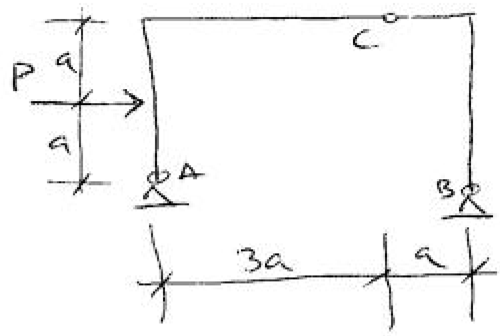
PLANAR FRAMES

FRAMES ARE STRUCTURES COMPOSED OF BEAMS, USUALLY WITH RIGID (FIXED) JOINTS. THE STATICALLY DETERMINED VARIANTS ARE QUITE RARE IN REALITY, ALTHOUGH THESE ARE AT THE BASE OF SOLVING STATICALLY UNDETERMINED FRAMES.
THE BELOW ILLUSTRATED STATICALLY UNDETERMINED FRAME CAN BE TRANSFORMED IN SEVERAL VARIANTS OF STATICALLY DETERMINED STRUCTURAL SCHEMES:

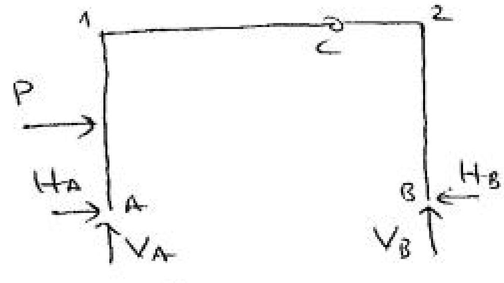


THE DRAWING OF THE EFFORT DIAGRAMS CAN BE DONE ACCORDING TO THE DISCUSSIONS FROM THE STRAIGHT BEAMS (AFTER ASSESSING THE REACTIONS). FOR EACH ISOLATED JOINT (OR CROSS SECTION) THREE EQUATIONS CAN BE WRITTEN IN ORDER TO EXPRESS THE STATIC EQUILIBRIUM (TWO EQUATIONS FOR THE PROJECTED COMPONENTS OF THE FORCES AND ANOTHER THIRD EQUATION FOR THE BENDING MOMENT).

EXAMPLE:



AT FIRST, THE REACTIONS SHOULD BE EXHIBITED IN ORDER TO CALCULATE THEIR VALUES:



$\sum M_B = 0 \quad V_A \cdot 4 \cdot a + P \cdot a = 0 \quad V_A = -\frac{P}{4}$ (THE MINUS SIGN SHOWS THAT IT WILL BE REVERSED AS DIRECTION)

$\sum M_A = 0 \quad V_B \cdot 4 \cdot a - P \cdot a = 0 \quad V_B = \frac{P}{4}$

$\sum M_C^{LEFT} = 0 \quad V_A \cdot 3 \cdot a - H_A \cdot 2 \cdot a - P \cdot a = 0$

$H_A = \frac{3}{2} V_A - \frac{P}{2} = -\frac{3}{2} \cdot \frac{P}{4} - \frac{P}{2} = -\frac{7}{8} P$ (AGAIN, REVERSED DIRECTION)

$\sum M_C^{RIGHT} = 0 \quad H_B \cdot 2 \cdot a - V_B \cdot a = 0 \quad H_B = \frac{V_B}{2} = \frac{P}{8}$

CHECKING THE RESULTED VALUES:

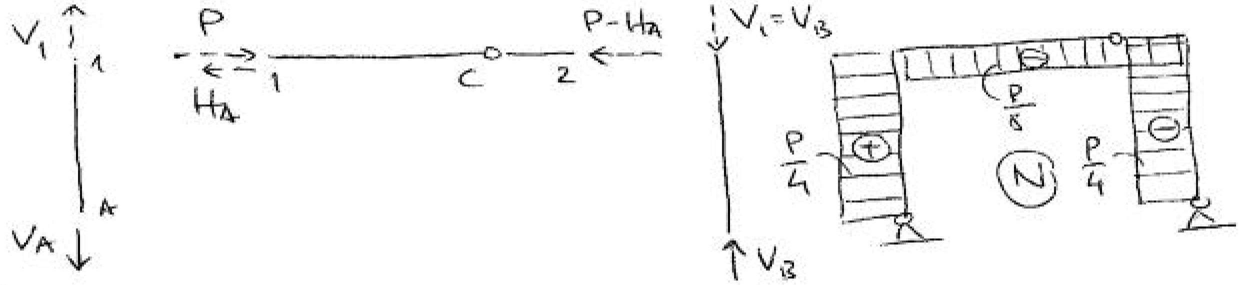
$\sum X = 0 \quad P = -H_A + H_B \quad P = \frac{7}{8} P + \frac{P}{8}$ ✓

$\sum Z = 0 \quad V_A + V_B = 0 \quad -\frac{P}{4} + \frac{P}{4} = 0$ ✓

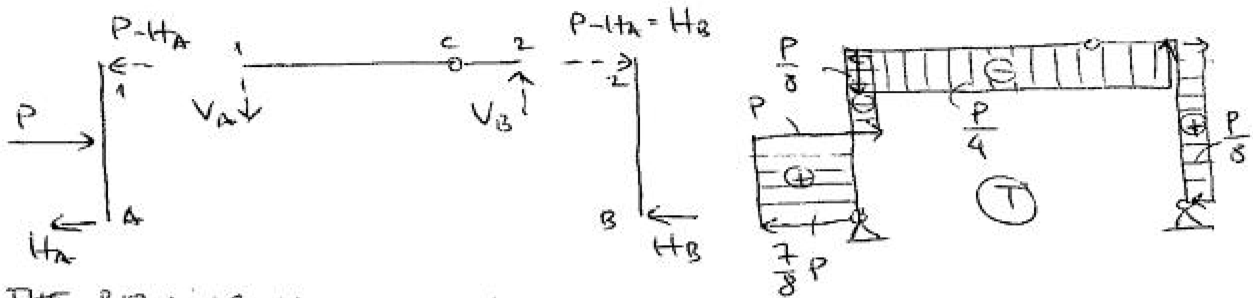
A GOING THROUGH DIRECTION CAN BE CHOOSEN IN ORDER TO DRAW

THE EFFORT DIAGRAMS (E.G. A-1, 1-C-2 AND 2-B).

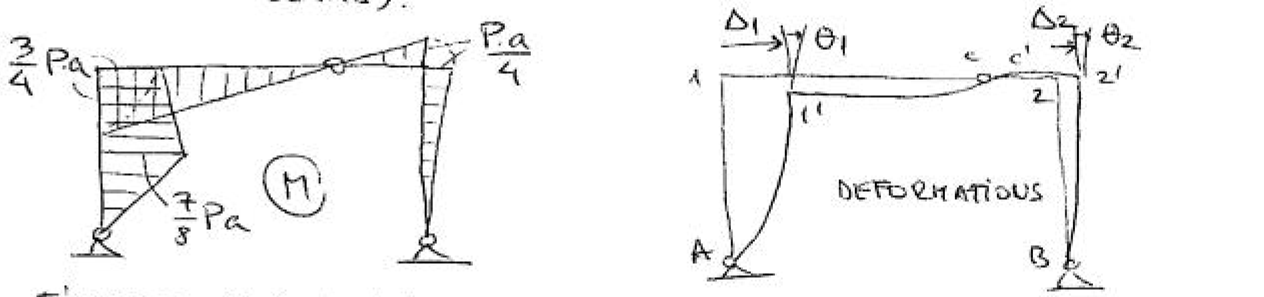
FOR THE A-1 COLUMN THE AXIAL FORCE WILL RESULT FROM THE V_A REACTION, FOR THE 1-C-2 HORIZONTAL PART FROM P AND H_A, WHILE REDUCING THE FORCES TO JOINT 2, IN THE 2-B COLUMN WE SHOULD GET AN AXIAL FORCE EQUAL TO V_B:



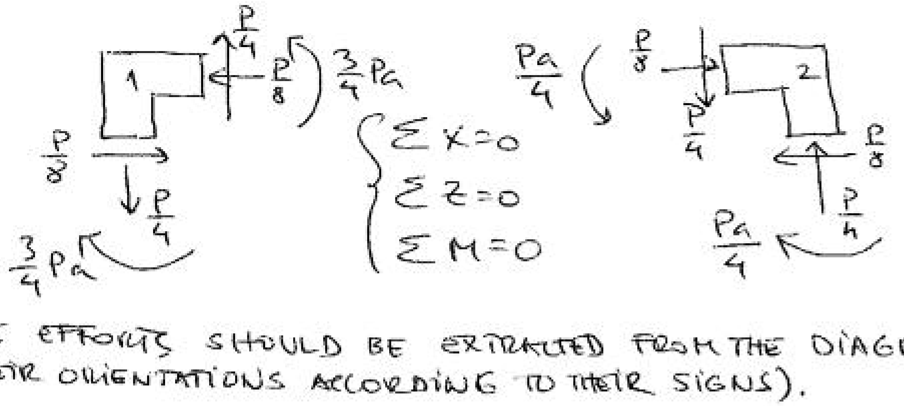
IN A SIMILAR MANNER WILL BE DRAWN THE SHEAR FORCE DIAGRAM, TAKING INTO ACCOUNT THE NORMAL FORCES TO THE AXIS OF THE BEAMS:



THE BENDING MOMENT DIAGRAM WILL ALSO SUGGEST THE DEFORMED SHAPE OF THE FRAME (GIVEN THAT IT IS DRAWN ON THE TENSIONED SIDE OF THE BEAMS):



FINALLY, IT IS ADVISABLE TO CHECK EACH JOINT FOR EQUILIBRIUM!

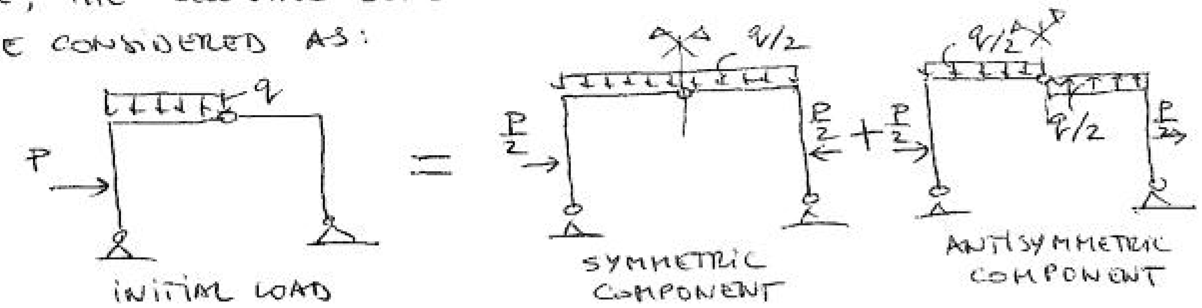


THE EFFORTS SHOULD BE EXTRACTED FROM THE DIAGRAMS (KNOWING THEIR ORIENTATIONS ACCORDING TO THEIR SIGNS).

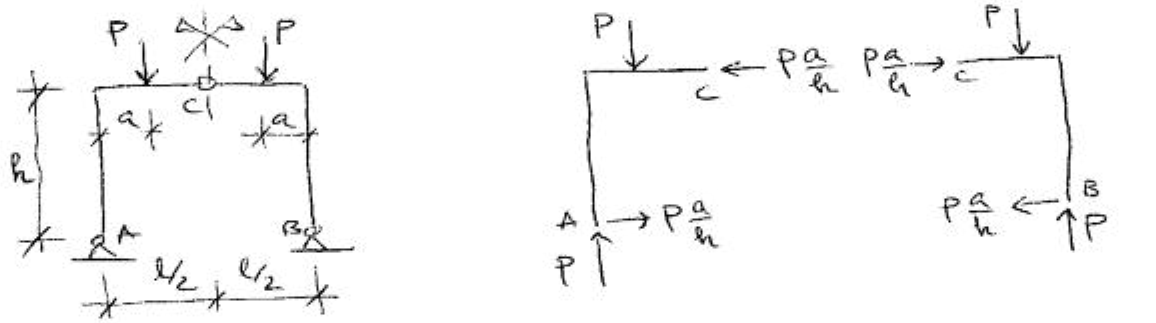
THE USE OF SYMMETRY

MANY STRUCTURES ARE SYMMETRICAL (FOR VARIOUS REASONS), WHICH SIMPLIFIES THEIR ANALYSIS. THE SYMMETRY OF A STRUCTURE (WITH RESPECT TO AN AXIS) MEANS BOTH THE SYMMETRY OF THE GEOMETRY AND THE CHARACTERISTICS OF THE BEAMS (CROSS SECTIONS AND MATERIALS), AS WELL AS OF THE JOINTS AND SUPPORTS.

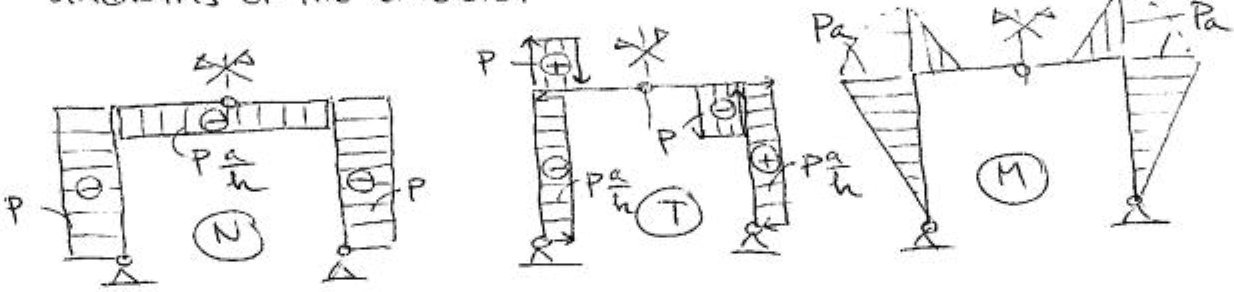
ANY LOAD APPLIED ON A SYMMETRICAL STRUCTURE CAN BE DECOMPOSED IN A SYMMETRICAL AND ANTISYMMETRIC COMPONENT, AND THE EFFECTS OF THESE COMPONENTS CAN BE SUPERIMPOSED LINEARLY IN ORDER TO OBTAIN THE EFFORTS IN THE BEAMS OF THE STRUCTURE. FOR EXAMPLE, THE FOLLOWING LOAD APPLIED ON THE SYMMETRIC FRAME CAN BE CONSIDERED AS:



IN CASE OF SYMMETRIC LOAD APPLIED ON A SYMMETRICAL STRUCTURE: EXHIBITING ALL THE FORCES (REACTIONS):

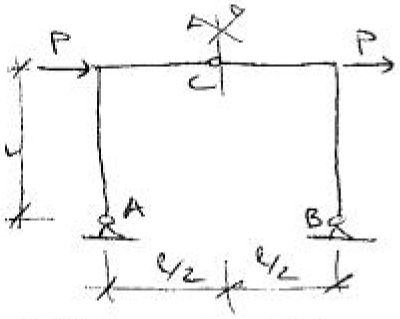


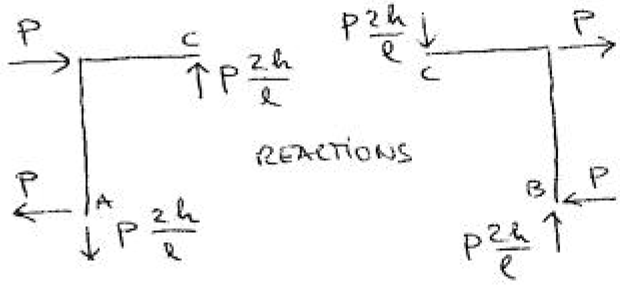
DIAGRAMS OF THE EFFORTS:



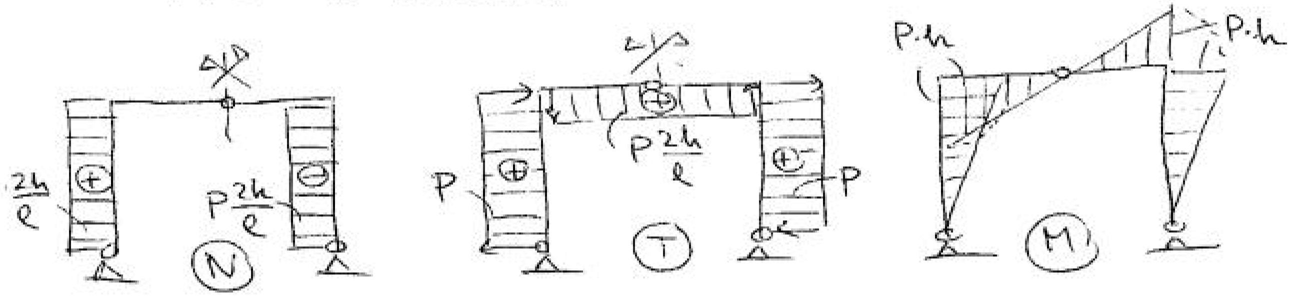
AS CAN BE OBSERVED, THE REACTIONS WILL RESULT SYMMETRIC AND AS A CONSEQUENCE THE N AND M DIAGRAMS WILL BE SYMMETRIC, WHILE THE T WILL BE ANTISYMMETRIC.

IN CASE OF ANTISYMMETRIC LOAD APPLIED ON A SYMMETRICAL STRUCTURE (PRESENTED NEXT) THE REACTIONS WILL RESULT ANTISYMMETRIC, SO THE N AND M DIAGRAMS WILL ALSO BE ANTISYMMETRIC, WHILE THE T WILL BE SYMMETRIC.





DIAGRAMS OF THE EFFORTS:



AS CONCLUSION, BY TAKING ADVANTAGE OF THE STRUCTURE'S SYMMETRY, IN CASE OF ANY LOAD DECOMPOSED IN A SYMMETRIC AND ANTISYMMETRIC COMPONENTS, THE EFFORTS CAN BE ASSESSED ON HALF OF THE STRUCTURE, AFTER WHICH THEY CAN BE TRASPOSED ON THE OTHER HALF TOO.