LECTURE 5 PLANAR STRUCTURES WITH PINNED JOINTS (TRUSSES)

A PLANAR STRUCTURE WITH PINNED JOINTS (ALSO CALLED TRUSS) IS AN ASSEMBLY OF BEAMS CONNECTED THROUGH PINNED JOINTS (ALSO CALLED NODES) OR HINGES.

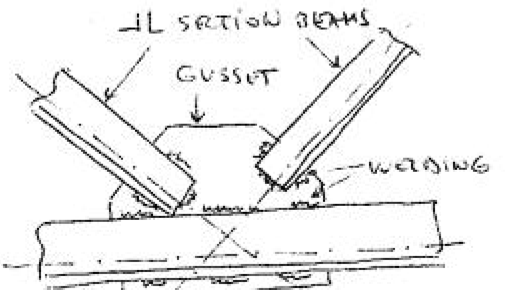
ASIDE THE ACCEPTED HYPOTHESIS OF LINEAR ELASTIC STRUCTURAL ANALYSIS, THERE ARE ALSO SOME OTHER ASSUMPTIONS TO BE CONSIDERED:

1) THE PINNED JOINTS (THE SO CALLED NODES, WHERE THE ENDS OF THE BEAMS ARE CONNECTED) ARE PERFECT (WITHOUT ANY FRICTION, ALLOWING FREE ROTATION OF THE BEAMS).

   IN REALITY SUCH CONNECTIONS ARE RARE, IN FACT THESE JOINTS ARE MADE BY WELDING, OR BY USING SCREWS, RIVETS AND GUSSETS (GUSSETS ARE MOUNTING PLATES USED BY JOINTS). AS A CONSEQUENCE, IN REALITY THERE ARE ALSO BENDING MOMENTS AT THE ENDS OF THE BEAMS, BUT THESE EFFORTS ARE IGNORED IN THE SIMPLIFYED STRUCTURAL ANALYSIS (CONSIDERING NEGLECTABLE THEIR EFFECT). THE BROAD LENGTH OF THE BEAMS IN RELATION TO THEIR SECTIONAL DIMENSIONS ALLOWS THE ACCEPTANCE OF THIS HYPOTHESIS.

2) THE AXIS OF THE BEAMS ARE MEETING IN THE THEORETICAL POINT OF ARTICULATION. AS A RESULT, THE AXIAL FORCES WILL BE CONCURRENT IN THAT POINT.

   IN REALITY THIS ASSUMPTION IS ALSO RARELY FULFILLED, ALTHOUGH BY PROPER CHOOSING OF THE GUSSET'S GEOMETRY AND POSITIONS OF BEAMS THERE CAN GET CLOSE ENOUGH TO THE HYPOTHETIC SITUATION (SEE THE FIGURE TO THE RIGHT).

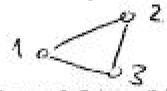


3) THE LOADS ARE APPLIED IN THE JOINTS (AS POINT LOADS). IN REALITY, NEITHER THIS HYPOTHESIS IS ACCURATE BECAUSE ALL PARTS HAVE SELF WEIGHT. BY USING SOME SECONDARY STRUCTURAL PARTS (STRUTS, BRACES, ETC.) WE CAN GET CLOSE ENOUGH TO THIS ASSUMPTION (EXCEPTING THE SELF WEIGHT OF THE BEAMS).

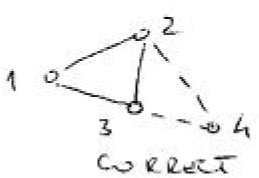
FROM THE ABOVE HYPOTHESIS WILL RESULT THAT THERE WILL BE ONLY AXIAL EFFORTS IN THE BEAMS OF A TRUSS AND THE DEFORMATIONS THAT WILL OCCUR WILL BE DUE TO THE MODIFICATION OF THE LENGTH OF THE BEAMS.

THE CONDITION OF GEOMETRIC INVARIABILITY AND OF FIXATION TO THE GROUND:

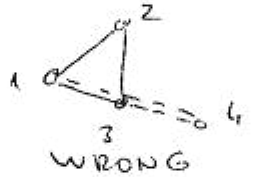
2 NODES ARE CONNECTED THROUGH A BEAM: 

3 NODES ARE CONNECTED BY 3 BEAMS: 

THE TRIANGLE IS THE SIMPLEST INVARIABLE GEOMETRIC SHAPE. ANY SUPPLEMENTAL NODE WILL REQUIRE FIXING WITH ANOTHER 2 BEAMS (BUT, PAY ATTENTION TO THEIR POSITION IN ORDER TO AVOID CRITICAL SHAPES).



CORRECT



WRONG

NOTE: THE CRITICAL SHAPE MEANS THAT THE EFFORTS CAN NOT BE DETERMINED (IT ALLOWS DISPLACEMENTS THAT WILL GENERATE AN UNCONTROLABLE VARIATION OF THE EFFORTS).

THE RELATION FOR GEOMETRIC INVARIABILITY CAN BE WRITTEN AS:

$$b + 3 = 2 \cdot n$$

nr. of beams nr. of nodes (joints)

THE ABOVE RELATION IS A NECESSARY CONDITION, BUT NOT ENOUGH, THE BEAMS SHOULD ALSO BE CORRECTLY POSITIONED (AVOIDING CRITICAL SHAPES AND MECHANISMS).

THE PREVIOUS RELATION CAN ALSO BE WRITTEN AS:

$$g = b + 3 - 2 \cdot n$$

IF $g = 0$, THAT MEANS THERE ARE THE MINIMAL REQUIRED NUMBER OF BEAMS. IF $g > 0$, THAT MEANS THERE ARE MORE BEAMS, THAN REQUIERED. IF $g < 0$, THAN WE HAVE A MECHANISM (DUE TO THE INSUFFICIENT NUMBER OF BEAMS).

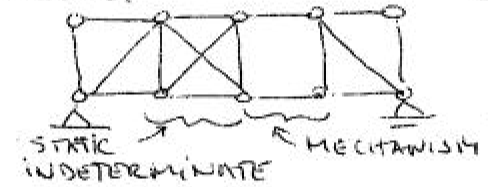
IN ORDER TO FIX A BODY TO THE GROUND, THE SAME PRINCIPLE APPLYES, BEING REQUIRED 3 LINKS (IN 2D). BY REFORMULATING THE ABOVE RELATION (MARKING WITH S THE SUPPORTS, EACH ONE REPRESENTING 3 LINKS):

$$g = b + s - 2 \cdot n$$

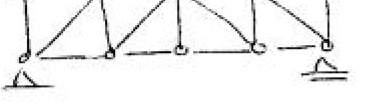
nr. of beams supports ($s = 3$) nr. of nodes

NOTE! ONE ADDITIONAL LINK CAN REPLACE A MISSING BEAM (THE LINK BETWEEN 2 NODES). IN SUCH CASES $s \geq 3$. IF $s \geq 3$ AND $g = 0$ THEN THE STRUCTURE IS GEOMETRIC INVARIABLE ONLY TOGETHER WITH THE SUPPORTS!

WRONG:



CORRECT:



CRITICAL SHAPE

THE STRUCTURE IS STATICALLY DETERMINED IF IT CONTAINS THE REQUIRED MINIMAL NUMBER OF LINKS FOR GEOMETRIC INVARIABILITY AND FOR FIXATION TO THE GROUND ($q = 0$).

THERE CAN BE WRITTEN 2 EQUATIONS EXPRESSING STATIC EQUILIBRIUM (IN 2D) FOR EACH NODE. THESE EQUATIONS ARE USUALLY WRITTEN CONSIDERING THE PROJECTIONS OF THE FORCES (ACCORDING TO THE AXIS OF A CONVENIANT REFERENCE SYSTEM).

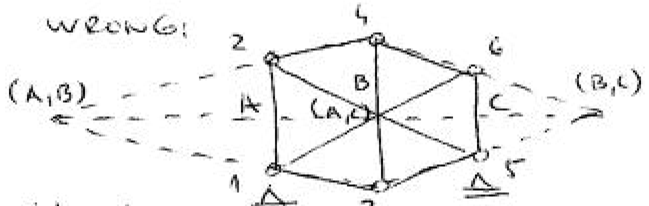
CONSIDERING THE $q = b + s - 2 \cdot n = 0$ RELATION, WE CAN WRITE: $2 \cdot n = b + s$

THE ABOVE RELATION SHOWS THAT, IF WE CAN WRITE FOR EACH NODE 2 EQUATIONS EXPRESSING EQUILIBRIUM, THAN WE CAN COMPUTE ALL THE UNKNOWN FORCES (EFFORTS).

NOTE: THE PROCEDURE CAN BE APPLIED ON INDIVIDUAL BEAMS, BUT ALSO ON GEOMETRIC INVARIABLE PARTS OF THE STRUCTURE (CONNECTED BODIES).

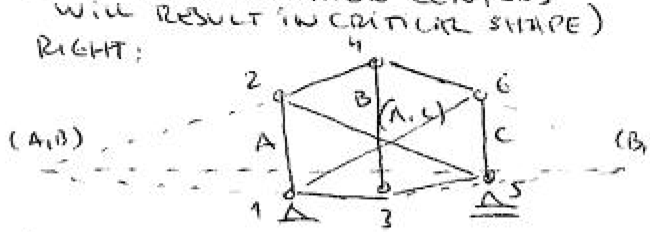
BEFORE STARTING TO SOLVE A PLANAR TRUSS, ATTENTION SHOULD BE PAID TO COMPOSE A PROPER STATICAL SCHEME, IN ORDER TO AVOID CRITICAL SHAPES AND MOVEABLE PARTS. IN CASE OF CRITICAL SHAPES THE STRUCTURAL RESPONSE WILL GO OUTSIDE THE BOUNDARY OF SMALL DISPLACEMENTS!

WRONG:



(COLLINEAR ROTATION CENTERS WILL RESULT IN CRITICAL SHAPE)

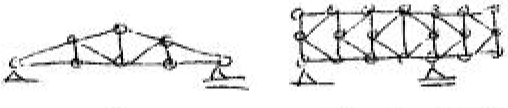
RIGHT:



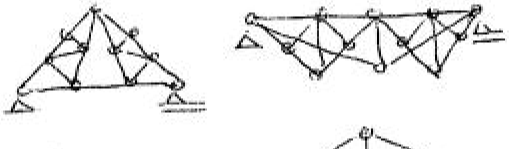
IN THE NEARBY SCHEME THE BEAMS BETWEEN JOINTS 1-2, 3-4 AND 5-6 CAN BE CONSIDERED RIGID BODIES CONNECTED BY PENDULUMS (TWO-FORCE MEMBERS). IN CASE OF COLLINEAR ROTATION CENTERS THERE WILL BE A CRITICAL SHAPE (SEE THE MECHANICS COURSE).

PLANAR STRUCTURES WITH PINNED JOINTS (TRUSSES) CAN BE CLASSIFIED AS:

SIMPLE STRUCTURES:



COMPOSED STRUCTURES: (COMBINED SIMPLE STRUCTURES)



COMPLEX STRUCTURES: (WICH ARE NOT IN THE PREVIOUS CATEGORIES)

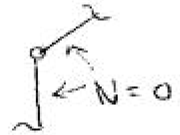
LECTURE 6 GETTING THE EFFORTS IN PLANAR TRUSSES

1) METHOD OF ISOLATING THE NODES

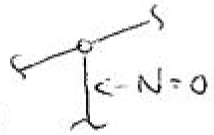
IT IS A PRACTICAL METHOD IN CASE OF SIMPLE STRUCTURES, PROVIDED THERE ARE NO MORE THAN 2 UNKOWNS AT EACH NODE, IT IS BASED ON THE ASSUMPTION THAT FOR EACH INDIVIDUAL NODE CAN BE 2 ECUATIONS WRITTEN FOR 2 UNKNOWN EFFORTS (CONSIDERING THE PROJECTED COMPONENTS OF THE FORCES, ACCORDING TO A SUITABLE REFERENCE SYSTEM).

NOTE: FOR CERTAIN LOAD CONFIGURATIONS INACTIVE (UNLOADED) BEAMS CAN BE IDENTIFIED.

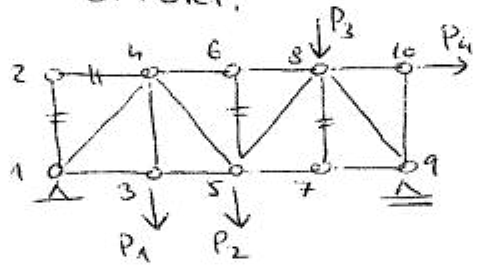
IN CASE OF 2 BEAMS JOINED IN AN UNLOADED NODE, IF THE DIRECTION OF THE BEAMS IS DIFFERENT, THEIR AXIAL EFFORTS WILL BE NULL.



IN CASE OF AN UNLOADED NODE IN WHICH ARE 3 BEAMS JOINED, IF 2 BEAMS HAVE THE SAME DIRECTION THE 3RD BEAM WILL HAVE NO AXIAL EFFORT.



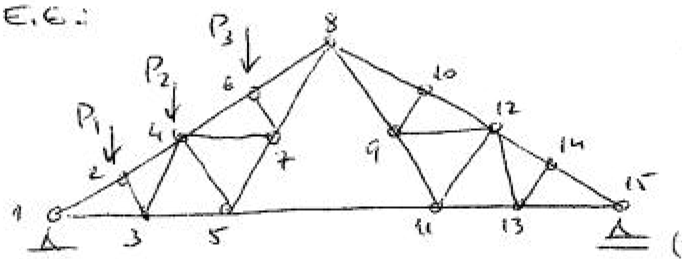
E.G.:



BEAMS WITHOUT AXIAL EFFORT: 1-2
2-4
5-6
7-8

IN CASE OF A COMPOSED STRUCTURE, GOING FROM NODE TO NODE CAN BE DONE BY JUMPING, SEARCHING FOR THE NODES WITH ONLY 2 UNKNOWNS (EFFORTS).

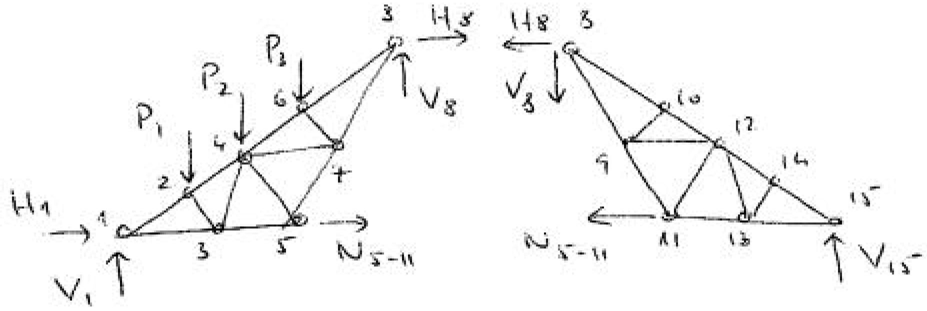
E.G.:



IN CASE OF THIS STRUCTURE NODES 2, 3 AND 6 HAVE 3 UNKNOWNS EACH, WHILE NODE 5 HAS 4 UNKNOWNS AND NODE 4 HAS 5 UNKNOWNS (NR. UNKNOWNS = NUMBER OF CONNECTED BEAMS)

SOLVING THIS STRUCTURE CAN START WITH NODE 1, THEN ISOLATING NODE 2, AFTERWARDS NODE 3.

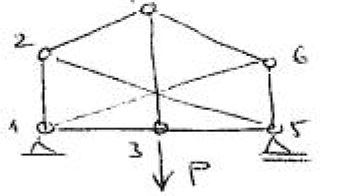
ANOTHER EFFICIENT APPROACH IS THE DECOMPOSITION OF THE STRUCTURE IN SIMPLE TRUSSES (SEE NEXT FIGURE).



ON THE RIGHT PART OF THE STRUCTURE, BEAMS 9-10 AND 13-14 WILL HAVE NO EFFORT. SUCH OBSERVATION CAN ALSO HELP IN SOLVING THE STRUCTURE.

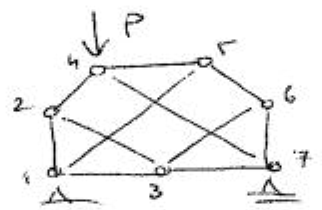
IN CASE OF COMPLEX STRUCTURES, TOGETHER WITH THE PREVIOUS OBSERVATIONS AND REMARKS, THE SYMMETRY AND ANTISYMMETRY CAN BE USED.

E.G.:

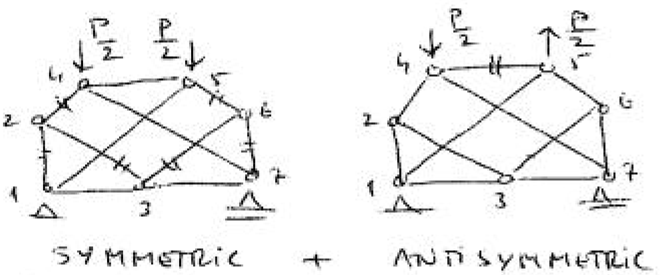


SINCE THE STRUCTURAL SCHEME IS SYMMETRICAL, THE EFFORTS SHOULD ALSO BE SYMMETRICAL. FIRST, THE EFFORT IN BEAM 3-4 CAN BE FOUND ($N_{3-4} = P$), THAN STARTING FROM NODE 4 THE METHOD OF ISOLATING NODES CAN BE APPLIED.

IN THE NEXT CASE, THE STRUCTURE CAN BE DECOMPOSED IN A SYMMETRIC AND IN AN ANTISYMMETRIC VARIANT:



AT THE ANTISYMMETRIC VARIANT IT CAN BE OBSERVED, THAT BEAM 4-5 CAN NOT HAVE IN THE SAME TIME COMPRESSION AND TENSION, SO IT WILL HAVE NO EFFORT



SYMMETRIC (SYMMETRICAL EFFORTS) + ANTISYMMETRIC (ANTISYMMETRICAL EFFORTS)

AT THE SYMMETRIC PART THE PROJECTED FORCES FROM NODE 3 CAN NOT HAVE VERTICAL (AND SYMMETRIC) COMPONENTS. THUS, BEAMS 2-3 AND 3-6 SHALL HAVE NO EFFORT. AS A CONSEQUENCE, NEITHER SHALL HAVE BEAMS 1-2, 2-4 AND 5-6, 6-7.

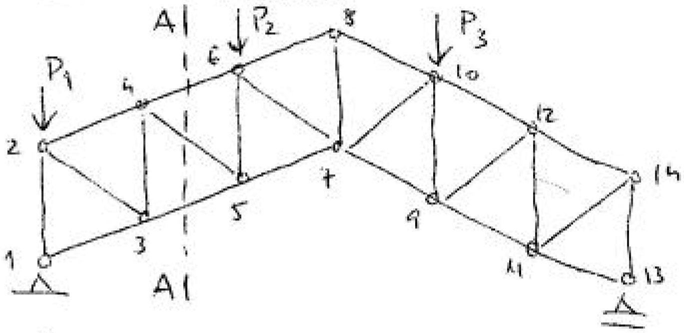
AFTER THESE OBSERVATIONS, THE METHOD OF ISOLATING NODES CAN BE APPLIED ON BOTH PARTS (SYMMETRIC AND ANTISYMMETRIC), SOLVING THEM SEPARATELY, THE FINAL EFFORTS WILL BE OBTAINED BY ADDING THE VALUES RESULTED FROM THE SEPARATE SOLUTIONS, FOR EACH BEAM, ONE BY ONE.

2) METHOD OF CROSS SECTIONS

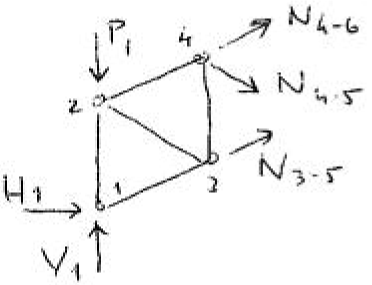
THIS METHOD IS USED FOR ALL STATICALLY DETERMINED STRUCTURES. FOR SOLVING PLANAR TRUSSES, IT IS USUALLY APPLIED WHEN THE EFFORTS IN SOME CHOSEN BEAMS SHOULD BE DIRECTLY OBTAINED (BUT OFF COURSE, IT CAN ALSO BE USED TO SOLVE A TRUSS BY GETTING THE EFFORTS IN ALL THE BEAMS).

BY MAKING CROSS SECTIONS THROUGH THE STRUCTURE THE STATIC EQUILIBRIUM EQUATIONS CAN BE WRITTEN FOR THE REVEALED FORCES, OBTAINING THEIR VALUES.

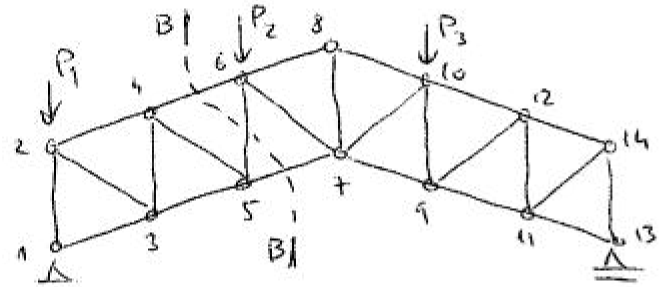
EXEMPLIFICATION:



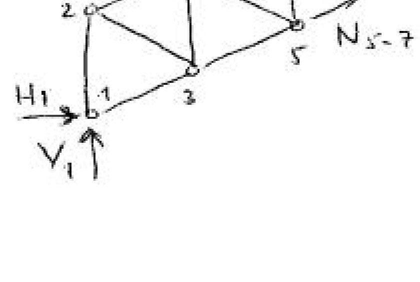
TAKING INTO ACCOUNT SECTION A-A, THE UNKNOWN FORCES (EFFORTS) CAN BE REVEALED:



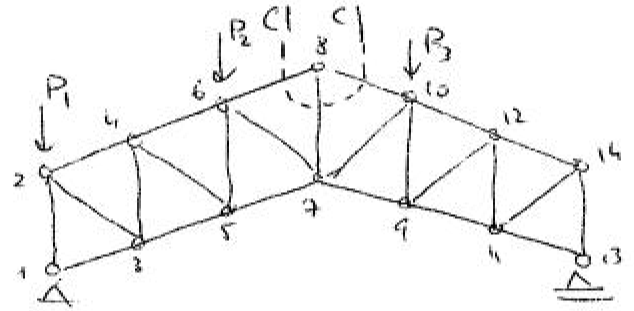
IN ORDER TO GET THE VALUE OF THE N_{3-5} EFFORT, A STATIC EQUILIBRIUM EQUATION SHALL BE WRITTEN FOR THE 4TH NODE ($\Sigma M_{(4)}=0$) OBSERVING THAT N_{4-6} AND N_{4-5} ARE PASSING THROUGH NODE 4. IN A SIMILAR MANNER, BY EXPRESSING THE BENDING MOMENT FOR NODE 5 (THROUGH WHICH N_{3-5} AND N_{4-5} ARE GOING THROUGH) THE VALUE OF N_{4-6} CAN BE OBTAINED ($\Sigma M_{(5)}=0$). TO OBTAIN THE VALUE OF N_{4-5}, AN EQUILIBRIUM EQUATION FOR THE PROJECTED COMPONENTS OF THE FORCES CAN BE USED (EITHER $\Sigma H=0$ OR $\Sigma V=0$). IF $\Sigma H=0$ IS USED, THAN $\Sigma V=0$ CAN BE APPLIED TO CHECK THE RESULTS (AND VICE-VERSA).



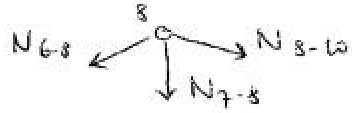
IN ORDER TO GET THE VALUE OF THE EFFORT IN A VERTICAL BEAM, A B-B TYPE SECTION CAN BE CONSIDERED:



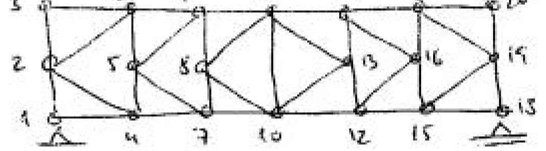
KNOWING ALREADY THE VALUES OF N_{4-6} AND N_{5-7} (BY APPLYING A-A TYPE SECTIONS), THE VALUE OF N_{5-6} WILL RESULT FROM $\Sigma V=0$.



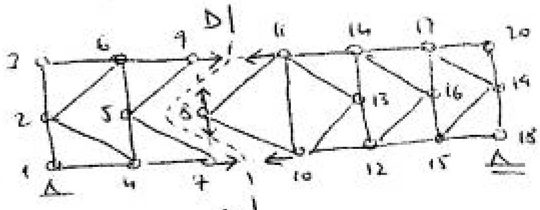
CONSIDERING C – C TYPE CROSS SECTION, THE N_{7-8} CAN BE OBTAINED IF N_{6-8} AND N_{8-10} ARE ALREADY KNOWN:



A DIFFERENT SITUATION MAY OCCUR WHEN YOU CAN NOT CUT ONLY 2 BEAMS, AS IT IS THE CASE OF K SHAPED BRACINGS:

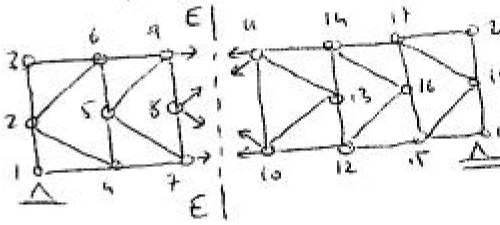


IN CASE OF SUCH TRUSSES THE FOLLOWING CROSS SECTIONS CAN BE CONSIDERED IN ORDER TO GET THE EFFORTS:



THE D-D TYPE CROSS SECTIONS WILL ALLOW TO OBTAIN THE EFFORTS IN THE UPPER AND LOWER HORIZONTAL BEAMS (N_{9-11} AND N_{7-10} IN THE NEARBY FIGURE) BY EXPRESSING THE BENDING MOMENT IN THE CORRESPONDING NODES ($\Sigma M_{(7)} = 0$ FOR N_{9-11}, $\Sigma M_{(9)} = 0$ FOR N_{7-10}).

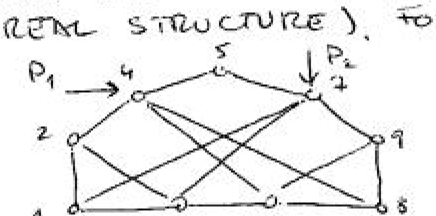
AFTER OBTAINING THE EFFORTS IN THE HORIZONTAL BEAMS, IN ORDER TO GET THE EFFORTS IN THE DIAGONAL BRACINGS, TYPE E-E SECTIONS CAN BE CONSIDERED.



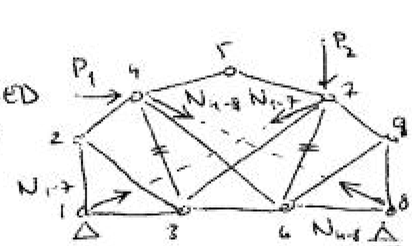
FINALLY, FOR THE VERTICAL BEAMS THE EFFORTS CAN BE OBTAINED BY THE NODE ISOLATION METHOD.

3) METHOD OF REPLACING BEAMS

THIS METHOD IS SPECIFIC FOR COMPLEX STRUCTURES. THE BASE IDEA IS TO TRANSFORM THE STRUCTURAL SCHEME IN ONE, THAT COULD BE EASYER SOLVED. FOR THIS PURPOSE, INSTEAD OF SOME CHOOSEN BEAMS THEIR EFFORTS WILL BE MARKED AS UNKNOWN LOADING FORCES (BUT WITH KNOWN DIRECTIONS), WHILE BETWEEN SOME UNCONNECTED NODES ADDITIONAL BEAMS ARE INSERTED (KNOWING, THAT THOSE BEAMS CAN NOT HAVE EFFORTS, SINCE THEY ARE NOT PARTS OF THE REAL STRUCTURE). FOR EXAMPLE:



CAN BE TRANSFORMED AS IT IS SHOWN:



AS IT CAN BE OBSERVED, THE BRACINGS BETWEEN NODES 4-8 AND 1-7 WERE ELIMINATED (THEY ARE REPLACED BY THEIR EFFORTS, AS LOADING FORCES WITH UNKNOWN VALUES), WHILE TWO ADDITIONAL BRACINGS WERE INSERTED BETWEEN NODES 3-4 AND 6-7 (KNOWING THAT THESE ADDITIONAL BEAMS HAVE NO EFFORTS, SINCE THEY WERE NOT PART OF THE STRUCTURE). THE TRANSFORMED STRUCTURE CAN BE SOLVED MUCH EASILY, EVEN BY THE METHOD OF ISOLATING NODES.

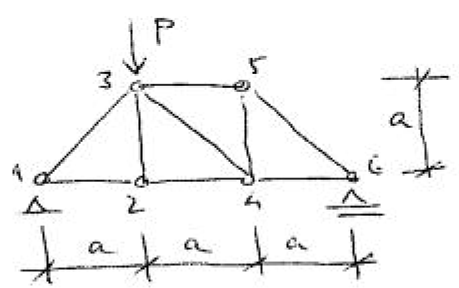
CARE SHOULD BE TAKEN WHEN APPLYING THIS METHOD, IN ORDER TO OBTAIN A PROPERLY TRANSFORMED STRUCTURE (GEOMETRIC INVARIABLE AND FIXED TO THE GROUND).

4) GRAPHIC METHOD

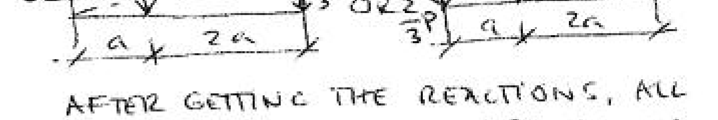
THIS IS PROBABLY THE MOST ANCIENT SOLVING METHOD FOR TRUSSES (IF FACT, THE STRUCTURAL SCHEME OF TRUSSES IS ORIGINATING FROM THIS METHOD). THE PROCEDURE USES THE CREMONA DIAGRAM, BY CONSTRUCTING THE GRAPHICAL SCHEME OF ALL FORCES.

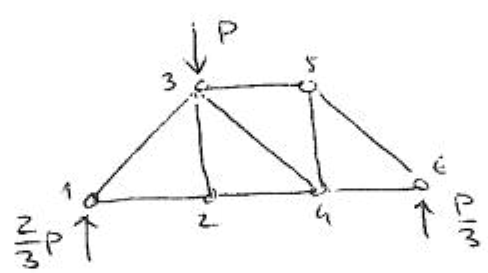
NOTE: CREMONA IS NOT ONLY THE NAME OF A PERSON (THERE IS ALSO AN ITALIAN VILLAGE WITH THIS NAME), BUT THE NAME OF A MECHANICAL TOOL USED TO REDRAW AT DIFFERENT SCALE (GREATER OR SMALLER) THROUGH RETRACING A DRAWING.
THE POLYGONAL SHAPE RESULTING FROM THE GRAPHICAL REPRESENTATION OF ALL THE FORCES SHOULD BE CLOSED IN ORDER TO ERPRESS THE EQUILIBRIUM.

EREMPLIFICATION:



IN ORDER TO OBTAIN THE REACTIONS, THE ANALYTICAL METHOD CAN ALSO BE APPLYED. IN THIS CASE THE GRAPHIC METHOD WILL BE USED BY DISTRIBUTING THE LOAD ON THE SIMPLE SUPPORTED BEAM LIKE BODY.

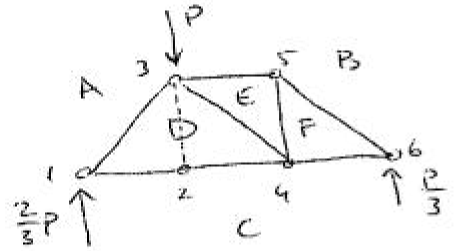




AFTER GETTING THE REACTIONS, ALL THE EXTERIOR FORCES ARE KNOWN. ALTHOUGH THE VALUES OF THE INNER FORCES ARE STILL UNKNOWN, THEIR DIRECTIONS WILL BE CORRESPONDING WITH THE DIRECTIONS OF THE BRACES.

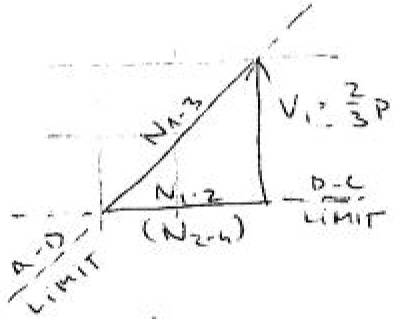
FROM THE ABOVE SCHEME CAN BE OBSERVED THAT THE 2-3 BEAM CAN NOT HAVE EFFORT (ALSO, THAT $N_{1-2} = N_{2-4}$).

THE FIELDS BETWEEN THE FORCES WILL BE MARKED WITH LETTERS, TO HELP ILLUSTRATING THE SOLVING. FIELD A SHALL BE BETWEEN THE V_1 REACTION AND THE P FORCE, FIELD B FROM THE P FORCE UNTIL THE V_6 REACTION, WHILE BETWEEN THE TWO REACTIONS (V_1 AND V_6) THERE WILL BE THE C FIELD. IN A SIMILAR WAY THE INTERIOR FIELDS ARE ALSO MARKED



(SINCE $N_{2-3}=0$, BETWEEN THE NODES MARKED WITH 1, 2, 4 AND 3 THERE WILL BE ONE FIELD, MARKED AS D). THE DECOMPOSITION OF THE FORCES WILL BE REPRESENTED BY USING THESE NOTATIONS. THE VALUES OF THE EFFORTS WILL RESULT GEOMETRICALLY, BY TRACING THE DIRECTIONS OF THE EXTERIOR AND INTERIOR FORCES. THE OBTAINED DIAGRAM WILL BE ACCURATE IF THE GRAPHICAL CONTOUR WILL BE CLOSED.

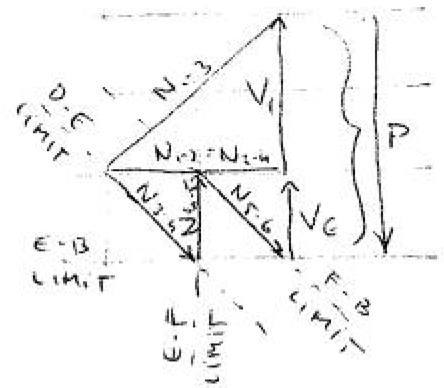
STARTING WITH THE $V_1 = \frac{2}{3}P$ REACTION THE LINES BETWEEN THE A-D AND D-C FIELDS ARE DRAWN. THE VALUES OF THE N_{1-2} AND N_{1-3} EFFORTS ARE GIVEN BY THE INTERSECTIONS.



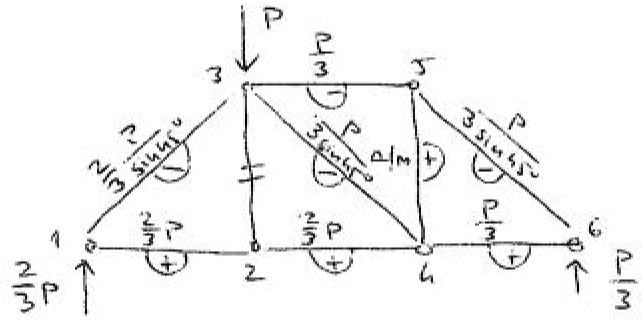
$$N_{1-2} = \frac{2}{3}P = N_{2-4} \quad \text{WHILE} \quad N_{1-3} = \frac{2}{3}P \cdot \frac{1}{\sin 45^\circ}$$

THE ORIENTATION OF THE FORCES WILL RESULT FROM THE GOING THROUGH DIRECTION, FROM POINT TO POINT.

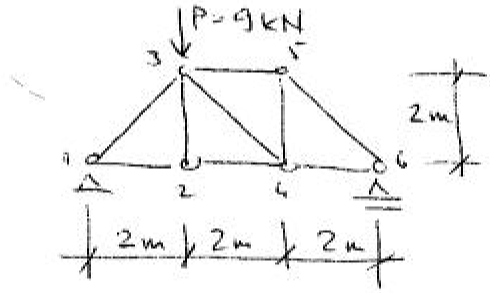
BY DRAWING THE LINE BETWEEN FIELDS D-E AND CROSSING IT WITH THE LINE BETWEEN E-B (HORIZONTAL, AT A $\frac{P}{3}$ DISTANCE FROM THE D-C LINE) WILL RESULT N_{3-4}. THEN, FROM THE VERTICAL INTERSECTED WITH THE D-C LINE WILL RESULT N_{4-5}. FROM THIS LAST CROSSING POINT WILL BE DRAWN THE F-B LINE (THE DIRECTION OF N_{5-6}). THE VERTICAL SEGMENT REMAINING UPWARD TILL THE STARTING POINT OF V_1 SHOULD BE THE V_6 REACTION. THE SUM OF V_1 AND V_6 SHOULD EQUAL P.



FINALLY THE OBTAINED VALUES OF THE EFFORTS CAN BE REPRESENTED ON THE STRUCTURAL SCHEME (AS IT WAS PREVIOUSLY MENTIONED, THE SENSE OF EACH FORCE WILL RESULT BY GOING THROUGH THE GRAPHICAL SOLVING).



SOLVING THE SAME PROBLEM BY THE METHOD OF ISOLATING NODES (WITH NUMERICAL VALUES):



OBTAINING THE REACTIONS:

$\Sigma M_1 = 0 \quad 2 \cdot P - 6 \cdot V_6 = 0 \quad V_6 = 3\,kN$

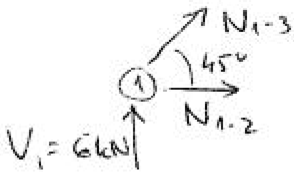
$\Sigma M_6 = 0 \quad 4 \cdot P - 6 \cdot V_1 = 0 \quad V_1 = 6\,kN$

CHECKING THE VALUES:

$\Sigma H = 0; \quad \Sigma V = 0 \quad V_1 + V_6 = P$ ✓

ISOLATING THE NODES, ONE BY ONE, STARTING FROM LEFT:

NODE 1:



THE N_{1-3} FORCE CAN BE DECOMPOSED IN A VERTICAL AND HORIZONTAL COMPONENT. WRITING THE EQUATIONS FOR THESE PROJECTED COMPONENTS, EXPRESSING THE EQUILIBRIUM IN NODE 1:

$\Sigma V_{(1)} = 0 \quad V_1 + N_{1-3} \cdot \sin 45^\circ = 0$

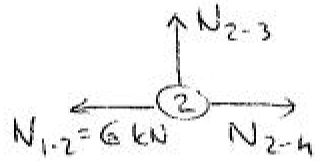
$$N_{1-3} = -\frac{V_1}{\sin 45^\circ} = -\frac{6}{0{,}707} = -8{,}49\,kN$$
(COMPRESSION)

CONSIDERING THE RESULTED ORIENTATION FOR N_{1-3} (WITH OPPOSITE DIRECTION):

$\Sigma H_{(1)} = 0 \quad N_{1-2} - N_{1-3} \cdot \sin 45^\circ = 0$

$$N_{1-2} = 8{,}49 \cdot 0{,}707 = 6\,kN$$
(TENSION)

NODE 2:

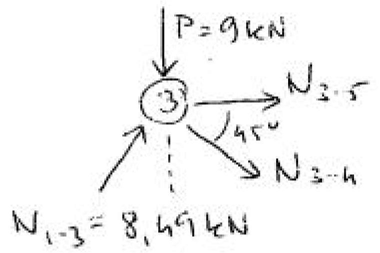


THE SIMILAR PROCEDURE, AS BY NODE 1:

$\Sigma V_{(2)} = 0 \quad N_{2-3} = 0$

$\Sigma H_{(2)} = 0 \quad N_{2-4} = N_{1-2} = 6\,kN$ (TENSION)

NODE 3:



SINCE $N_{2-3} = 0$ THE BEAM BETWEEN NODES 2 AND 3 CAN BE IGNORED

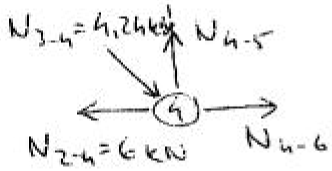
$\Sigma V_{(3)} = 0 \quad -P + N_{1-3} \cdot \sin 45^\circ - N_{3-4} \sin 45^\circ = 0$

$$N_{3-4} = N_{1-3} - \frac{P}{\sin 45^\circ} = 8{,}49 - \frac{9}{0{,}707} = -4{,}24\,kN$$
(COMPRESSION)

$\Sigma H_{(3)} = 0 \quad N_{1-3} \cdot \sin 45^\circ + N_{3-4} \cdot \sin 45^\circ + N_{3-5} = 0$

$$N_{3-5} = -(N_{1-3} + N_{3-4}) \cdot \sin 45^\circ = -(8{,}49 - 4{,}24) \cdot 0{,}707 = -3\,kN$$
(COMPRESSION)

NODE 4:



THE KNOWN EFFORTS ARE ILLUSTRATED WITH THEIR REAL DIRECTION ON THE NODE.

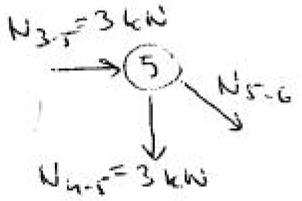
$\Sigma V_{(4)}=0 \quad -N_{3-4}\cdot\sin 45^\circ + N_{4-5}=0$

$N_{4-5}=N_{3-4}\cdot\sin 45^\circ = 4{,}24\cdot 0{,}707 = 3kN$ (TENSION)

$\Sigma H_{(4)}=0 \quad -N_{2-4}+N_{3-4}\sin 45^\circ + N_{4-6}=0$

$N_{4-6}=N_{2-4}-N_{3-4}\cdot\sin 45^\circ =$
$= 6 - 4{,}24\cdot 0{,}707 = 3kN$ (TENSION)

NODE 5:



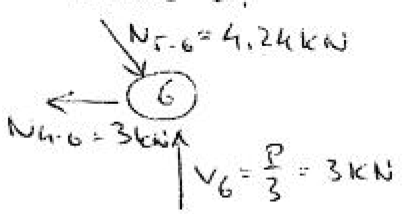
THE ONLY UNKNOWN IS N_{5-6}:

$\Sigma V_{(5)}=0 \quad -N_{4-5}-N_{5-6}\cdot\sin 45^\circ = 0$

$N_{5-6}=-\dfrac{N_{4-5}}{\sin 45^\circ} = -\dfrac{3}{0{,}707} = -4{,}24kN$ (COMPRESSION)

FROM THE HORIZONTAL PROJECTION THE SAME VALUE SHOULD BE OBTAINED.

NODE 6:



AT THE LAST NODE THE ABOVE CALCULATED RESULTS CAN BE VERIFIED:

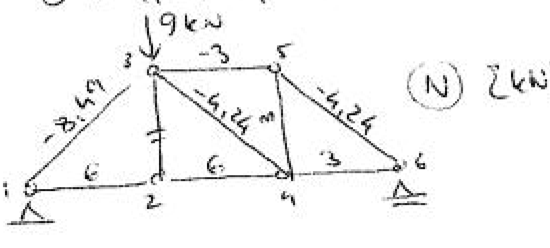
$\Sigma V_{(6)}=0 \quad -N_{5-6}\cdot\sin 45^\circ + V_6 = 0$

$-4{,}24\cdot 0{,}707 + 3 = 0$ ✓

$\Sigma H_{(6)}=0 \quad -N_{46}+N_{5-6}\cdot\sin 45^\circ = 0$

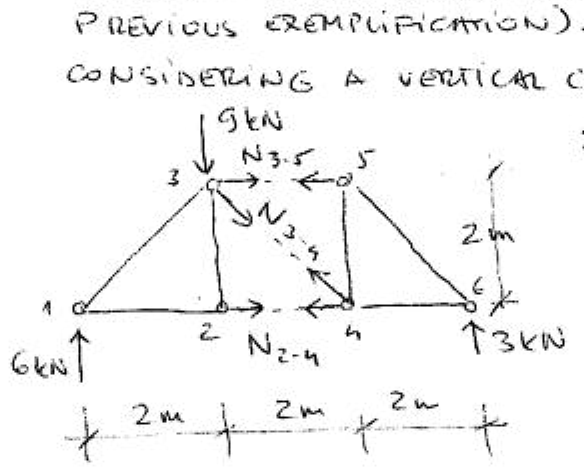
$-3 + 4{,}24\cdot 0{,}707 = 0$ ✓

THUS, THE SOLUTION WILL LOOK LIKE ILLUSTRATED IN THE FIGURE AT RIGHT:



SOLVING THE SAME PROBLEM WITH THE METHOD OF CROSS SECTIONS (THE REACTIONS ARE KNOWN ALREADY FROM THE PREVIOUS EXEMPLIFICATION).

CONSIDERING A VERTICAL CROSS SECTION:



$\Sigma M^L_{(3)}=0 \quad 6\cdot 2 - N_{2-4}\cdot 2 = 0$

$N_{2-4}=6kN$ (TENSION)

THE EQUILIBRIUM IN NODE 3 CAN BE EXPRESSED ALSO BY CONSIDERING THE RIGHT SIDE:

$\Sigma M^R_{(3)}=0 \quad -N_{2-4}\cdot 2 + 3\cdot(2+2)=0$

$N_{2-4}=6kN$ (TENSION)

AS IN THE PREVIOUS CASE.

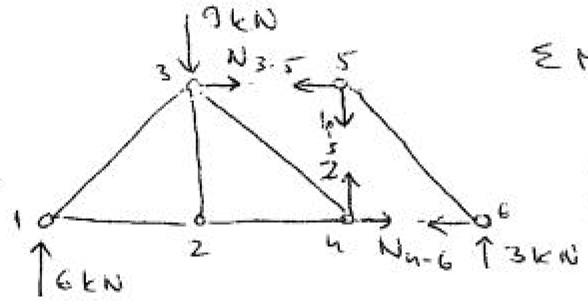
CONSIDERING THE LEFT BODY: $\sum M_{(4)}^{L}=0$ $\quad 6\cdot 4-9\cdot 2+N_{3\text{-}5}\cdot 2=0$

$N_{3\text{-}5}=-3\,kN$ (TENSION)

OR, THE RIGHT SIDE: $\sum M_{(4)}^{R}=0$ $\quad 3\cdot 2+N_{3\text{-}5}\cdot 2=0$

WILL LEAD TO THE SAME RESULT.

IN ORDER TO OBTAIN THE EFFORTS IN THE LATERAL HORIZONTAL BEAMS, A VERTICAL OR A DIAGONAL CROSS SECTION CAN BE CONSIDERED. IF A DIAGONAL CROSS SECTION IS CONSIDERED, THEN:



$\sum M_{(5)}^{R}=0$ $\quad 3\cdot 2-N_{4\text{-}6}\cdot 2=0$

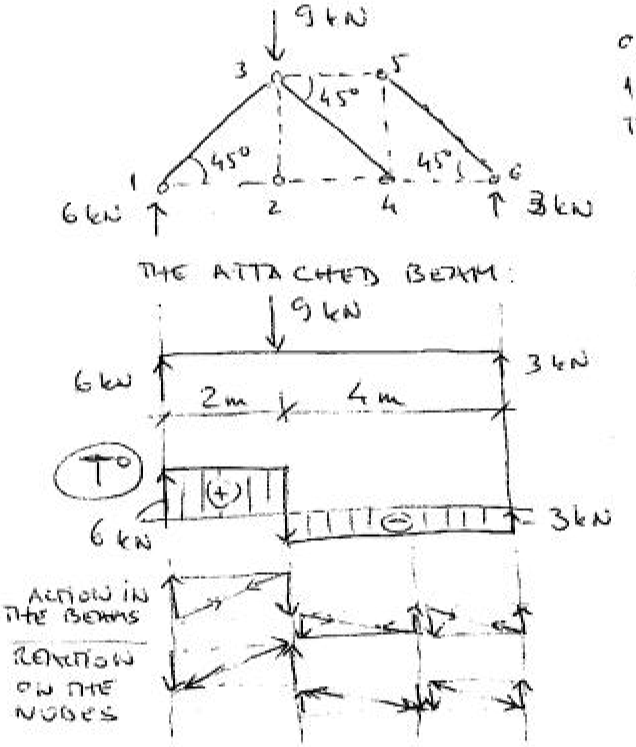
$N_{4\text{-}6}=3\,kN$ (TENSION)

OBVIOUSLY, $N_{3\text{-}5}$ COULD ALSO BE OBTAINED BY CONSIDERING SUCH A DIAGONAL SECTIONING.

FROM THE ABOVE CROSS SECTION THE EFFORT IN THE VERTICAL BRACING CAN ALSO BE OBTAINED; IF $N_{3\text{-}5}$ IS ALREADY KNOWN:

$$\sum M_{(6)}^{R}=0 \qquad N_{3\text{-}5}\cdot 2-N_{4\text{-}5}\cdot 2=0 \qquad N_{4\text{-}5}=3\,kN \text{ (TENSION)}$$

IN ORDER TO OBTAIN $N_{1\text{-}2}$ AND $N_{2\text{-}3}$, SIMILAR PROCEDURES CAN BE APPLIED. ALTHOUGH THE EFFORTS IN THE DIAGONAL BRACINGS CAN ALSO BE OBTAINED IN A SIMILAR MANNER, WE SHALL APPLY A DIFFERENT APPROACH. BY CONSIDERING AN ATTACHED BEAM IT CAN BE OBSERVED THAT THE SHEAR FORCE CAN NOT BE TAKEN BY VERTICAL AND HORIZONTAL BRACES, ONLY THE DIAGONAL ONES WILL WORK FOR THE SHEAR FORCE.



ON THE FIRST 2 m LONG SEGMENT THE 1-3 DIAGONAL WILL WORK (THE DIRECTION OF $N_{1\text{-}3}$ WILL RESULT FROM (T°)):

$$N_{1\text{-}3}=\frac{6}{0{,}707}=8{,}49\,kN \text{ COMPRESSION}$$

ON THE MIDDLE 2 m WIDE ZONE THE 3-4 DIAGONAL HAS A REVERSED ORIENTATION, BUT THE (T°) IS ALSO NEGATIVE (ROTATING COUNTERCLOCKWISE):

$$N_{3\text{-}4}=\frac{3}{0{,}707}=4{,}24\,kN \text{ (COMPRESSION)}$$

WHILE ON THE RIGHT SEGMENT:

$$N_{5\text{-}6}=\frac{3}{0{,}707}=4{,}24\,kN \text{ (COMPRESSION)}$$