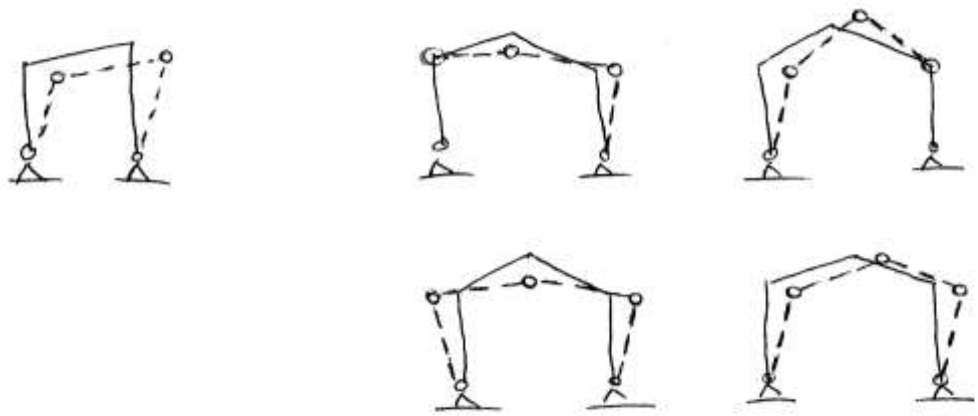
USING THE PRINCIPLE OF THE VIRTUAL MECHANICAL WORK

(Figures from the lecture, as seen)

$$L = \vec{Force} \times \vec{Distance}$$
$$(P, M) \quad (\delta, \theta)$$

$$L = \sum \vec{P}_i \cdot \vec{\delta}_i + \sum \vec{M}_j \cdot \vec{\theta}_j$$

Transforming a structural scheme in a kinematic scheme:

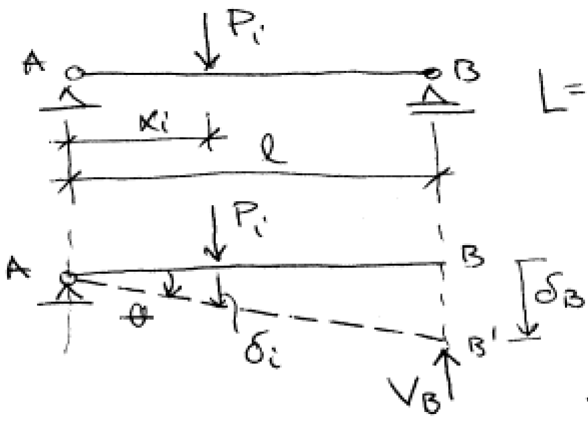


One released connection will result in one degree of kinematic freedom, allowing one virtual displacement. If a system of equilibrium forces acts on a mechanism, then no real kinematic displacement occurs.
Any force traveling through a virtual displacement will generate Virtual Mechanical Work. A structure loaded with a system of forces is in equilibrium, if the mechanical work developed by the loads traveling through virtual displacements (compatible with the connections of the system) is zero. Thus, the general form of the equilibrium equation is:

$$\sum \vec{P}_i \cdot \vec{\delta}_i + \sum \vec{M}_j \cdot \vec{\theta}_j = \sum P_i \cdot \delta_i^c + \sum M_j \cdot \theta_j^c = 0$$

Applying the Principle of Virtual Mechanical Work for the reactions:

Since, only external forces can generate mechanical work, the corresponding connection to the unknown reaction should be released and replaced by the reaction.



$$L = -V_B \cdot \delta_B + \sum P_i \cdot \delta_i = 0$$

$$V_B = \frac{\sum P_i \cdot \delta_i}{\delta_B}$$

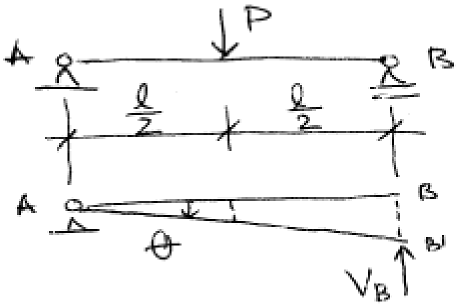
$$\delta_B = \theta \cdot l$$

$$\delta_i = \theta \cdot x_i$$

$$V_B = \frac{\sum P_i \cdot \cancel{\theta} \cdot x_i}{\cancel{\theta} \cdot l} = \frac{\sum P_i \cdot x_i}{l}$$

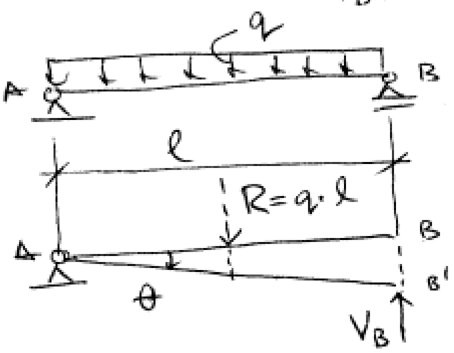
The virtual displacement was considered on the reversed direction of the unknown force, thus it is easier to obtain the value of the reaction (by changing it's position in the equation, with respect to the "=" sign)…

Examples:



$$L = -V_B\cdot\theta\cdot l + P\cdot\theta\cdot\frac{l}{2} = 0$$

$$V_B = P\cdot\frac{l}{2}\cdot\frac{1}{l} = \frac{P}{2}$$



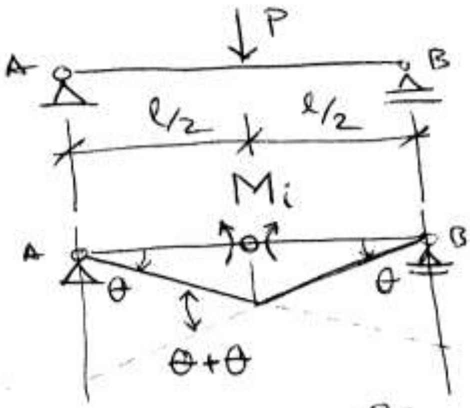
$$L = -V_B\cdot\theta\cdot l + \underbrace{q\cdot l}_{R}\cdot\theta\cdot\frac{l}{2}$$

$$V_B = \frac{q\cdot l}{2}$$

Applying the Principle of Virtual Mechanical Work for the inner forces:

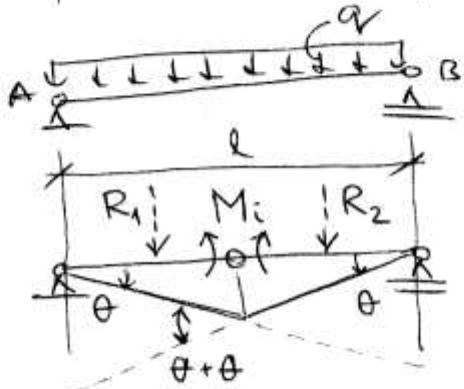
Since, only external forces can generate mechanical work, the corresponding connection to the unknown effort should be released and replaced by the force.

Examples:



$$L = -M_i\,(\theta+\theta) + P\cdot\frac{l}{2}\cdot\theta = 0$$

$$M_i = \frac{P\cdot l}{4}$$



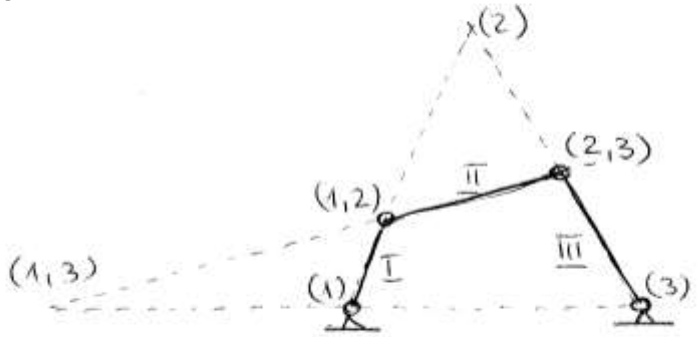
$$L = -M_i(\theta+\theta) + R_1\cdot\frac{l}{4}\theta + R_2\cdot\frac{l}{4}\theta$$

$$L = -M_i\cdot 2\theta + \frac{q\cdot l}{2}\theta\left(\frac{l}{4}+\frac{l}{4}\right) =$$

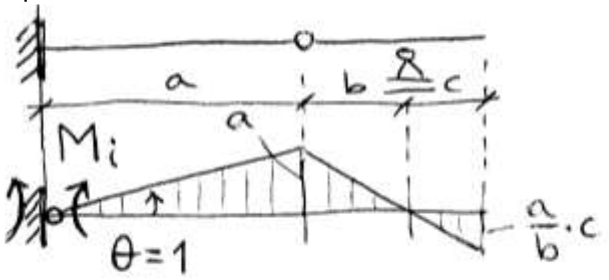
$$= -M_i\cdot 2\theta + \frac{q\cdot l}{2}\theta\,\frac{2l}{4} = 0$$

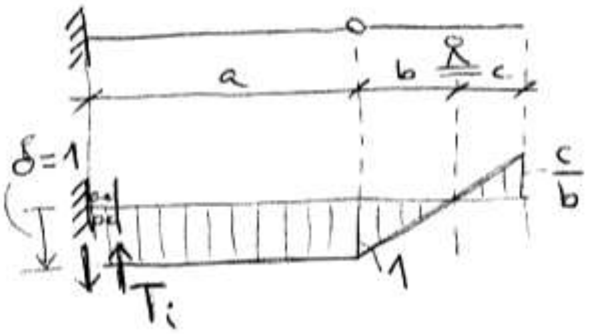
$$M_i = \frac{q\cdot l^2}{8}$$

Drawing displacement diagrams:

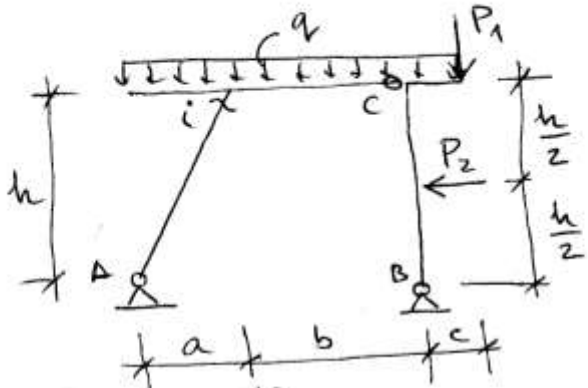


Examples:

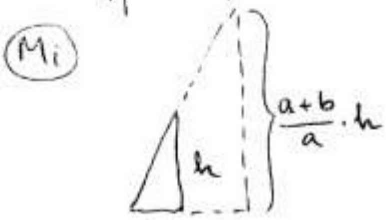


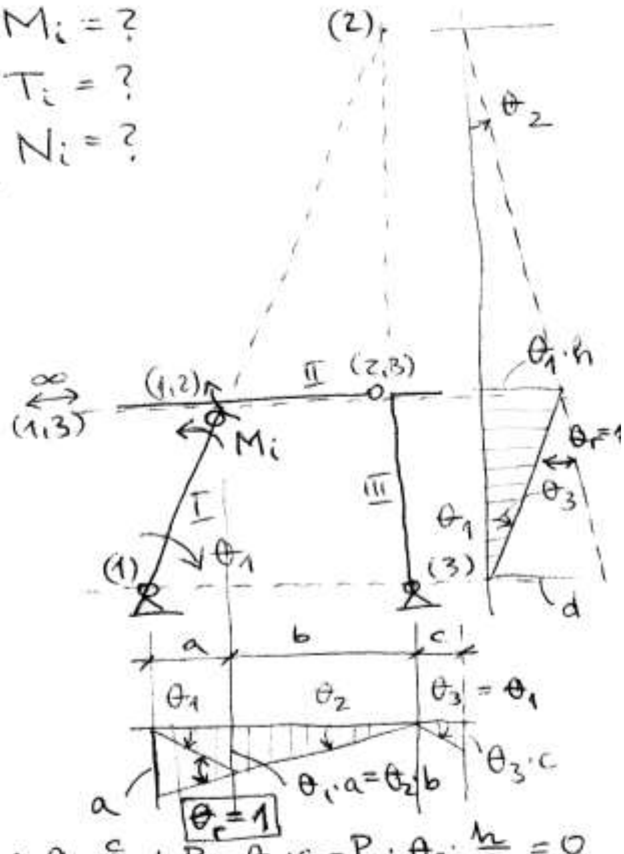


Sample problem solution:



$$\begin{cases} M_i = ? \\ T_i = ? \\ N_i = ? \end{cases}$$





$\theta_2 = \frac{a}{a+b}$; $\theta_1 \cdot a = \theta_2 \cdot b$

$\theta_1 = \frac{b}{a+b}$; $\theta_3 = \theta_1$ $(1,3) \to \infty$

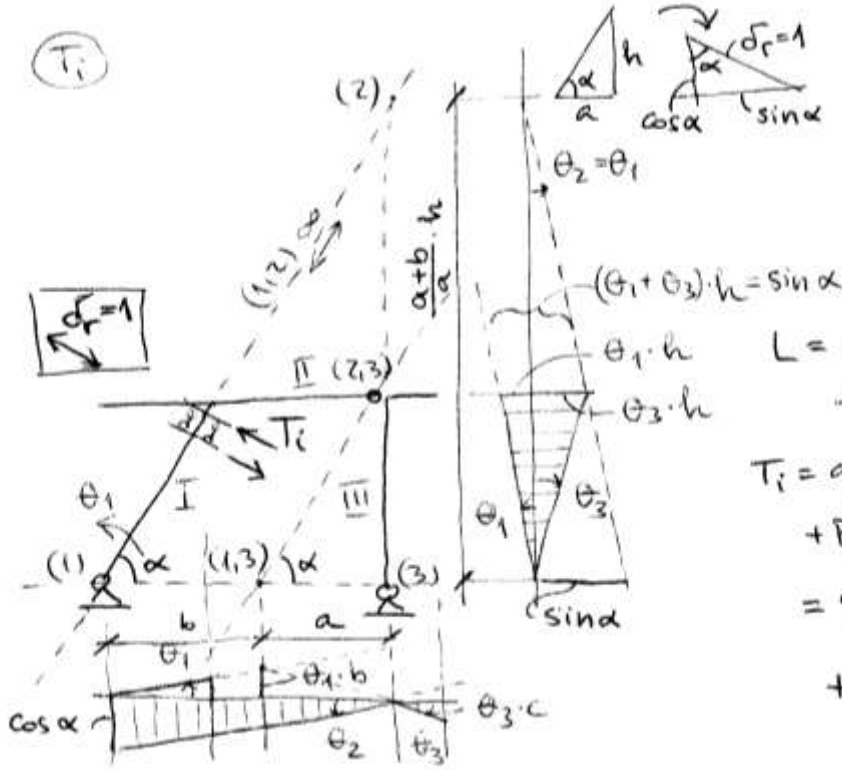
$d = \theta_2 \cdot \frac{a+b}{a} \cdot h = h$

$$L = -M_i \cdot 1 + q(a+b) \cdot \theta_2 \cdot \frac{a+b}{2} + q \cdot c \cdot \theta_3 \cdot \frac{c}{2} + P_1 \cdot \theta_3 \cdot c - P_2 \cdot \theta_3 \cdot \frac{h}{2} = 0$$

$$M_i = q(a+b)\frac{a}{a+b} \cdot \frac{a+b}{2} + q \cdot c \cdot \frac{b}{a+b} \cdot \frac{c}{2} + P_1 \cdot \frac{b}{a+b} \cdot c - P_2 \cdot \frac{b}{a+b} \cdot \frac{h}{2} =$$

$$= q\left[\frac{a(a+b)}{2} + \frac{bc^2}{2(a+b)}\right] + P_1 \cdot \frac{bc}{a+b} - P_2 \cdot \frac{b \cdot h}{2(a+b)}$$

(T_i)



$\theta_2 = \theta_1 \quad (1,2)\to\infty$

$\theta_2 = \frac{\cos\alpha}{a+b}$

$\theta_1 \cdot b = \theta_3 \cdot a;$

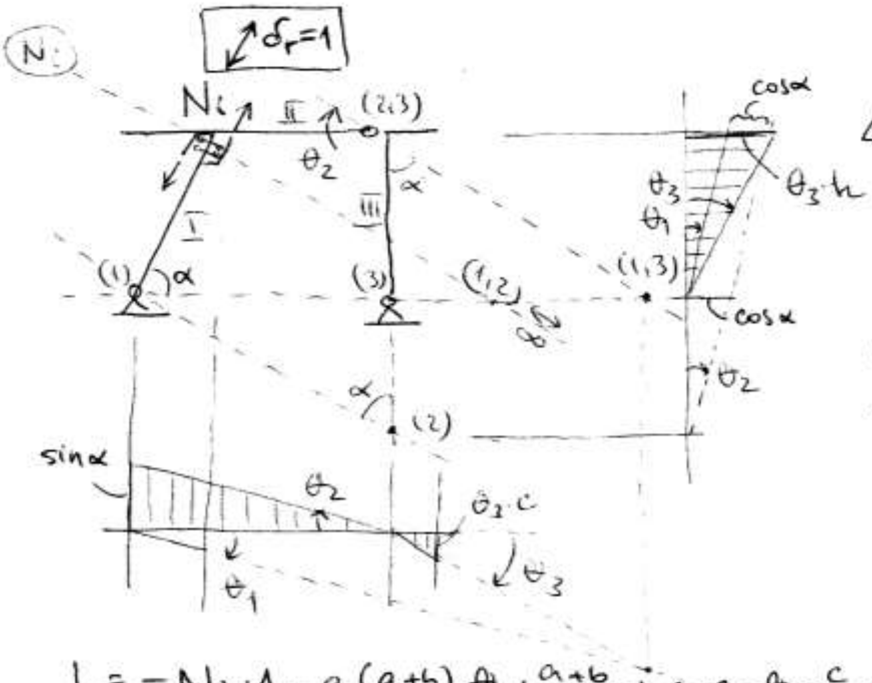
$\theta_3 = \theta_1 \cdot \frac{b}{a} = \frac{\cos\alpha}{a+b}\cdot\frac{b}{a}$

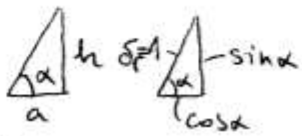
$$L = -T_i\cdot 1 + q(a+b)\,\theta_2 \cdot \frac{a+b}{2} + q\cdot c\cdot\theta_3\cdot\frac{c}{2} + P_1\cdot\theta_3\cdot c - P_2\cdot\theta_3\cdot\frac{h}{2} = 0$$

$$T_i = q(a+b)\frac{\cos\alpha}{a+b}\cdot\frac{a+b}{2} + q\cdot c\cdot\frac{\cos\alpha}{a+b}\cdot\frac{b}{a}\cdot\frac{c}{2} + P_1\cdot\frac{\cos\alpha}{a+b}\cdot\frac{b}{a}\cdot c - P_2\frac{\cos\alpha}{a+b}\cdot\frac{b}{a}\cdot\frac{h}{2} =$$

$$= \frac{q\cdot\cos\alpha}{2}\left[a+b+\frac{b\cdot c^2}{a(a+b)}\right] + \frac{\cos\alpha}{a+b}\cdot\frac{b}{a}\left(P_1\cdot c - P_2\cdot\frac{h}{2}\right)$$

(N_i)





$\theta_1=\theta_2 \quad (1,2)\to\infty$

$\theta_2 = \frac{\sin\alpha}{a+b}$

$\theta_3\cdot h = \theta_1\cdot h + \cos\alpha$

$\theta_3 = \theta_1 + \frac{\cos\alpha}{h} = \frac{\sin\alpha}{a+b}+\frac{\cos\alpha}{h}$

$$L = -N_i\cdot 1 - q(a+b)\cdot\theta_2\cdot\frac{a+b}{2} + q\cdot c\cdot\theta_3\cdot\frac{c}{2} + P_1\cdot\theta_3\cdot c - P_2\cdot\theta_3\cdot\frac{h}{2} = 0$$

$$N_i = -q(a+b)\frac{\sin\alpha}{a+b}\cdot\frac{a+b}{2} + q\cdot\frac{c^2}{2}\left(\frac{\sin\alpha}{a+b}+\frac{\cos\alpha}{h}\right) + P_1\cdot c\left(\frac{\sin\alpha}{a+b}+\frac{\cos\alpha}{h}\right) -$$

$$- P_2\cdot\frac{h}{2}\left(\frac{\sin\alpha}{a+b}+\frac{\cos\alpha}{h}\right) = -q\frac{(a+b)\sin\alpha}{2} + \left(\frac{\sin\alpha}{a+b}+\frac{\cos\alpha}{h}\right)\left(q\frac{c^2}{2}+P_1\cdot c - P_2\cdot\frac{h}{2}\right)$$