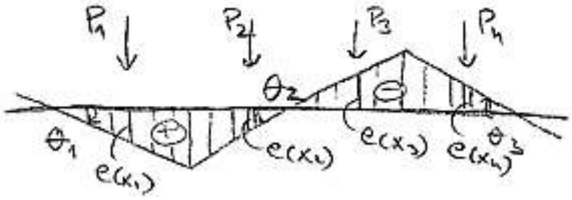
# ASSESSMENT OF MAXIMUM EFORTS

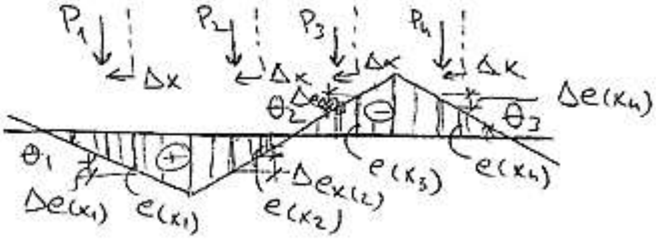
The ascertainment of the most unfavourable position in the case of a load by a mobile convoy of forces (point loads placed at invariable distances from each other) can be achieved by many tests, or, if influence diagrams are used, by a small number of tests. By static determined structures the influence diagrams have linear variations (they have polygonal outlines). In the case of a straight beam segment illustrated below, for any $E(x)$ link force in a chosen cross-section:




The value of the link force can be expressed by multiplying the intensity of the point loads in the convoy and the corresponding coefficients of influence (considering the linear superposition of effects):

$$E(x) = P_1 \cdot e(x_1) + P_2 \cdot e(x_2) - P_3 \cdot e(x_3) - P_4 \cdot e(x_4) = \sum P_i \cdot e(x_i)$$

If the convoy moves to the left with the $\Delta x$ increment from the previous position:




then the value of the link force in the chosen section will change by $\Delta E(x)$:

$$E(x) + \Delta E(x) = P_1(e(x_1) - \Delta e(x_1)) + P_2(e(x_2) + \Delta e(x_2)) - P_3(e(x_3) - \Delta e(x_3)) - P_4(e(x_4) + \Delta e(x_4))$$

The resulting $\Delta e(x_i)$ differences of the influence coefficients can be expressed using the θ rotation angles of the influence diagram, in the form $\Delta e(x_i) = \theta_i \cdot \Delta x$, and the above expression can be developed further as:

$$\begin{aligned} E(x) + \Delta E(x) &= P_1(e(x_1) - \theta_1 \cdot \Delta x) + P_2(e(x_2) - \theta_2 \cdot \Delta x) - P_3(e(x_3) + \theta_2 \cdot \Delta x) - \\ &\quad - P_4(e(x_4) + \theta_3 \cdot \Delta x) = \\ &= P_1 \cdot e(x_1) + P_2 \cdot e(x_2) - P_3 \cdot e(x_3) - P_4 \cdot e(x_4) - \\ &\quad - \Delta x (P_1 \cdot \theta_1 + P_2 \cdot \theta_2 + P_3 \cdot \theta_2 + P_4 \cdot \theta_3) = \\ &= E(x) - \underbrace{\Delta x \sum P_i \cdot \theta_i}_{\Delta E(x)} \end{aligned}$$

noting that the $\Delta E(x)$ difference will be represented by the negative term ”$-\Delta x \cdot \Sigma P_i \cdot \theta_i$”. Consequently, in order to achieve in the chosen cross-section a greater link force than in the previous case (before moving the force convoy by $\Delta x$ to the left), $\Delta E(x)$ should be positive, so ”$\Sigma P_i \cdot \theta_i$” should be negative.

<u>Conclusion:</u>

If for a certain position of the convoy of forces the ”$\Sigma P_i \cdot \theta_i < 0$” inequality results, then in order to obtain a more unfavourable position – a higher value of the $E(x)$ force in the cross-section – the convoy should be shifted to the left (until of the sign of inequality changes). Once the inequality sign has changed, the value of the $E(x)$ link force in the chosen cross-section will no longer increase by further shifting towards left the convoy, instead it will begin to decrease.

By applying a similar reasoning, in the case of the movement of the force convoy to the right with a $\Delta x$ increment from the original position, there will be a "$\Delta E(x) = +\Delta x \cdot \Sigma P_i \cdot \theta_i$" variation in the $E(x)$ value of the link force, concluding that for a higher value of the effort, the "$\Sigma P_i \cdot \theta_i$" term should be positive.

<u>Conclusion:</u>

If for a certain position of the convoy of forces the "$\Sigma P_i \cdot \theta_i > 0$" inequality results, then in order to obtain a more unfavourable position – a higher value of the $E(x)$ force in the cross-section – the convoy should be shifted to the right (until of the sign of inequality changes). Once the inequality sign has changed, the value of the $E(x)$ force in the chosen cross-section will no longer increase by further shifting towards right the convoy, instead it will begin to decrease.

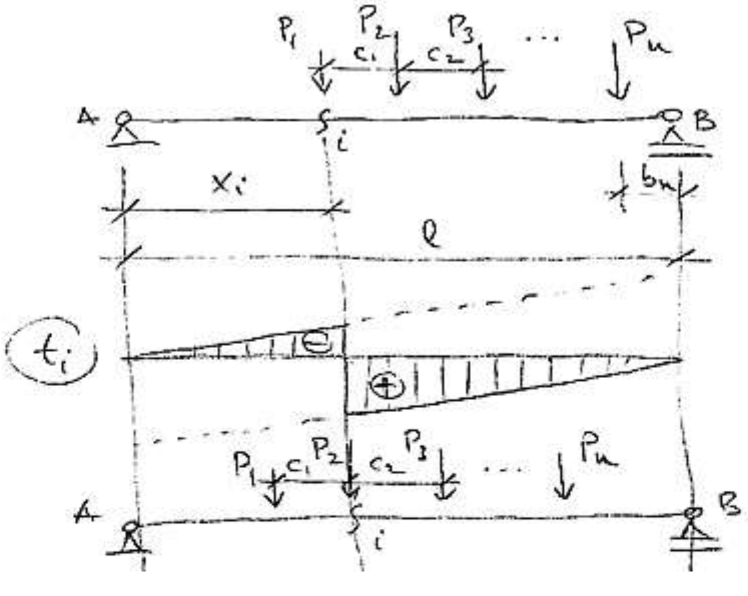
Upon the above two conclusions follows that, in order to obtain the maximum value of the $E(x)$ force in the chosen cross-section, the shifting direction for the load convoy is shown by the inequality sign:

$$\Sigma P_i \cdot \theta_i < 0 \quad \leftarrow \Delta x$$
$$\Sigma P_i \cdot \theta_i > 0 \quad \rightarrow \Delta x$$

Because the influence diagram has linear variations, the $\theta_i$ rotation angles on these segments will have constant values. In order to notice the change of the above inequality signs, the load components will be decisive, so the inequality signs will change only when a point load passes from one segment with linear variation to another (changing the partial result of the load convoy on the segments). It may happen that some point loads from the convoy get off the load line (or enter), in which case relative maximum forces will be obtained.

## MAXIMUM SHEAR FORCE IN A CROSS-SECTION

In the next figure it can be observed that, as the convoy approaches to cross-section $i$, the ordinates (influence coefficients) are higher in the $t_i$ diagram. The maximum shear force in cross-section $i$ is likely to be obtained when the $P_1$ point load from the convoy steps over the cross-section. However, it must be verified that the movement of the convoy to the left will not result in a higher cutting force ($P_1$ passing over the negative area of the influence diagram):




If the $P_1$ point load is positioned over cross-section $i$, the value of the shear force can be obtained from the $V_A$ reaction:

$$V_A = \frac{\sum_1^n P_k \cdot b_k}{l}$$

In case the convoy moves towards left and the $P_2$ point load arrives over cross-section $i$, the value of the $V_A$ reaction will be modified by:

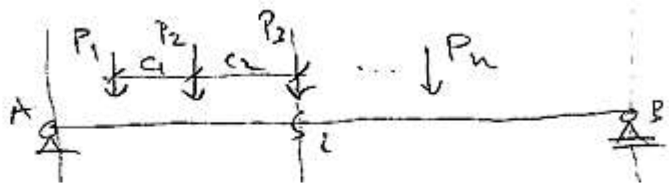
$$V_A + \Delta V_A = \frac{\sum_1^n P_k \cdot (b_k + c_1)}{l}$$
$$\Delta V_A = \frac{c_1}{l} \sum_1^n P_k$$

Between the two positions, the difference of the shear force in cross-section $i$ will be:

$$\Delta T_i = \Delta V_A - P_1 = \frac{c_1}{l}\sum_1^n P_k - P_1$$

The second position is more unfavourable if:

$$\Delta T_i > 0 \rightarrow \frac{c_1}{l}\sum_1^n P_k - P_1 > 0$$ or, otherwise expressed: $$\frac{P_1}{c_1} < \frac{\sum_1^n P_k}{l}$$




Moving the force convoy further to the left, until $P_3$ reaches cross-section $i$, results another shear force difference:

$$\Delta T_i' = \Delta V_A' - P_2 = \frac{c_2}{l}\sum_1^n P_k - P_2$$

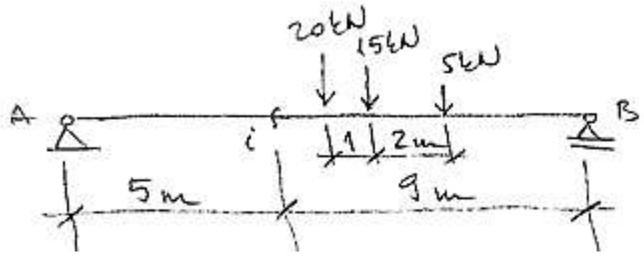
This position will be more disadvantageous than the previous ones if: $\Delta T_i' > 0$, meaning:

$$\frac{P_2}{c_2} < \frac{\sum_1^n P_k}{l}$$

Conclusion:

By placing successively the $P_1$, $P_2$ … point loads from the convoy over the $i$ cross-section and expressing for each case the inequality relations, the utmost disadvantageous convoy position will be obtained in case of that point load which changes the inequality.

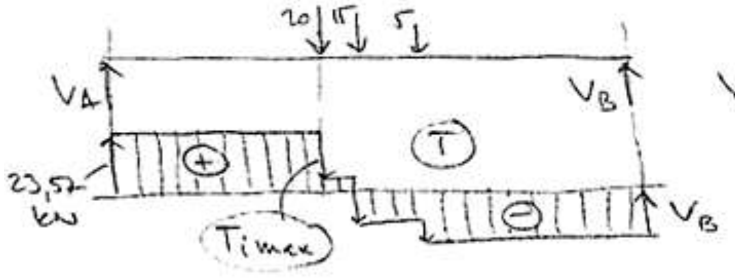
Example of use:




$P_1 = 20kN$ $\quad c_1 = 1m$
$P_2 = 15kN$ $\quad c_2 = 2m$
$P_3 = 5kN$

If the $P_1$ = 20 kN force steps over cross-section $i$, the inequality sign will point "to the right" (meaning that the convoy should not be moved to the left), so $T_{i\,max}$ occurs when force $P_1$ steps over section $i$:

$$\frac{P_1}{c_1} = \frac{20}{1} = 20 > \frac{\sum P_k}{l} = \frac{20+15+5}{14} = \frac{40}{14} = 2{,}86 \quad \rightarrow T_{i\,max}$$

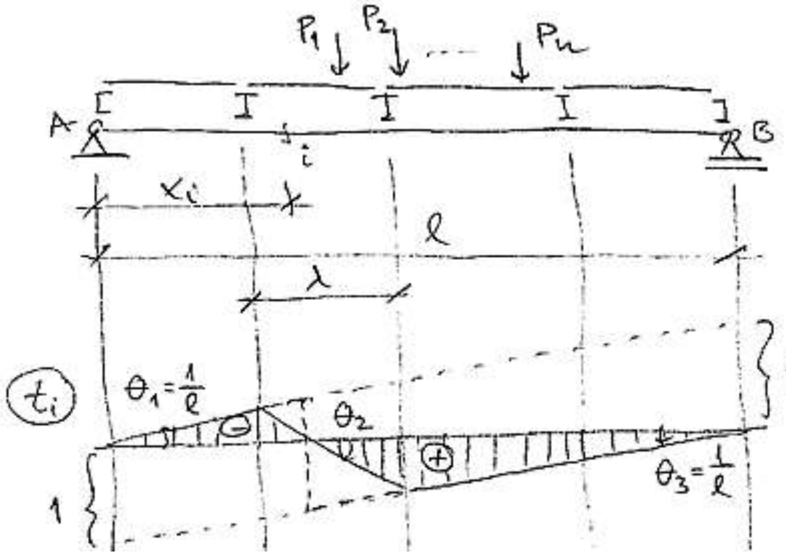



$$V_A = \frac{20\cdot 9 + 15\cdot 8 + 5\cdot 6}{14} = 23{,}57\ kN$$

$$T_{i\,max} = V_A$$

Indirectly applied loads

In the case of indirect loads, the passage of a point load beyond the section will not produce a sudden variation of the cutting force (the effect is treated on the panel). In the following figure the force convoy is placed over the positive area of the influence diagram (force $P_2$ placed above the spacer piece to the right of section $i$):



The resulting rotation angles will be: $\theta_1 = \theta_3 = \frac{1}{l}$

$$\theta_2 = \frac{1}{\lambda} - \frac{1}{l}$$

And will be obtained:

$$\left(\frac{1}{\lambda} - \frac{1}{l}\right)\sum_1^1 P_k - \frac{1}{l}\sum_2^n P_k < 0$$

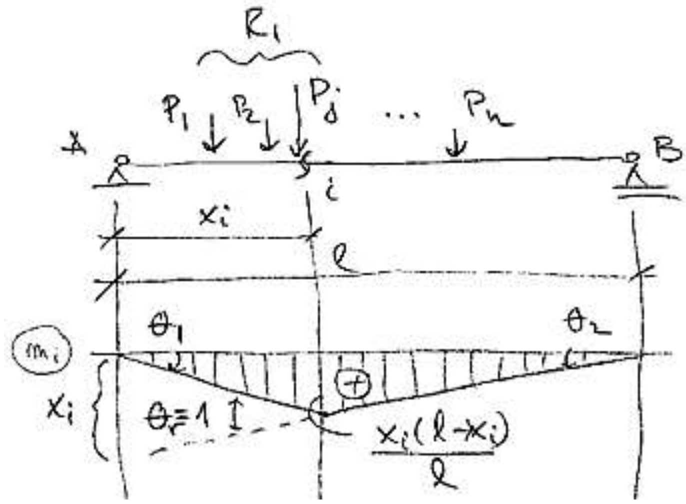
so

$$\frac{\sum_\lambda P_k}{\lambda} < \frac{\sum_l P_k}{l}$$

In the above inequality relationship, on the left are the point loads over the $\lambda$ panel and on the right side the point loads over the whole beam. The sign of inequality indicates the shifting direction for the load convoy in order to achieve a more unfavourable position.
The position leading to the obtaining of $T_{i\ max}$ in panel $\lambda$ is that corresponding to the placing of a point load next to the spacer piece to the right of the cross-section $i$, if that force entering the panel changes the sign of inequality.

## MAXIMUM BENDING MOMENT IN A CROSS-SECTION

Considering the bending moment in a cross-section on a beam, the influence diagram has only positive values. For the most unfavourable situation, the greatest forces should act in the area with the highest influence coefficients (maximum ordinates) of the influence diagram. The rotation angles of the segments will be: $\theta_1 = \frac{l-x_i}{l}$ ; $\theta_2 = -\frac{x_i}{l}$



If the $P_j$ point load from the convoy acts over cross-section $i$, than the general criteria for obtaining the most unfavourable position ($M_{i\ max}$) will be:

$\Sigma\, P_j \cdot \theta_j > 0$ and $\Sigma\, P_j \cdot \theta_j < 0$ simultaneously.

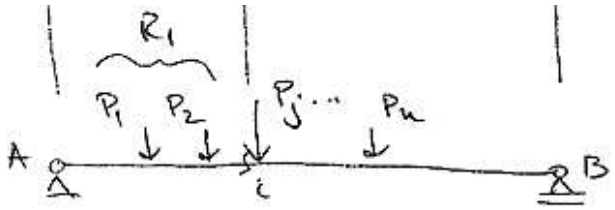
Considering the $P_j$ point load quite near left to the cross-section and marking with $R_1$ and $R_2$ the partial resultants of the convoy forces:

$$\underbrace{R_1}_{\sum_1^j P_k} \cdot \theta_1 + \underbrace{R_2}_{\sum_{j+1}^n P_k} \cdot \theta_2 > 0$$

meaning

$$\sum_1^j P_k \frac{l-x_i}{l} - \sum_{j+1}^n P_k \frac{x_i}{l} > 0$$

thus $\dfrac{\sum_1^j P_k}{x} > \dfrac{\sum_{j+1}^n P_k}{l-x}$




Considering the $P_j$ point load quite near right to the cross-section:

$$\underbrace{R_1}_{\sum_{1}^{j-1} P_k}\cdot\theta_1 + \underbrace{R_2}_{\sum_{j}^{m} P_k}\cdot\theta_2 < 0$$

so $$\frac{\sum_{1}^{j-1} P_k}{x} < \frac{\sum_{j}^{m} P_k}{l-x}$$

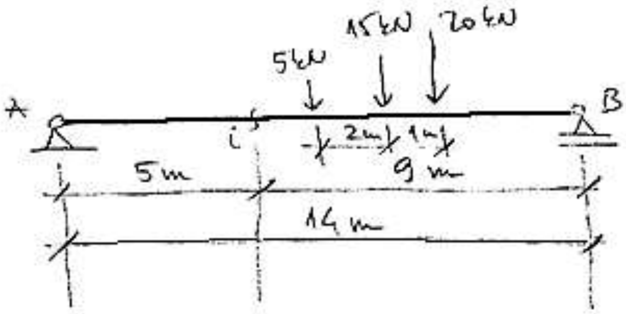
Observation:
The shifting direction for the load convoy is indicated by the two inequality relations

$$\frac{\sum_{1}^{j} P_k}{x} > \frac{\sum_{j+1}^{m} P_k}{l-x}$$ and $$\frac{\sum_{1}^{j-1} P_k}{x} < \frac{\sum_{j}^{m} P_k}{l-x}$$ in the following suggestive way:

| | | | |
|---|---|---|---|
| $>$ | $>$ | $\mapsto$ | The convoy should be shifted to right. |
| $<$ | $<$ | $\leftarrow$ | The convoy should be shifted to left. |
| $>$ | $<$ | $M_{i\,max}$ | Is the position from which $M_{i\,max}$ results. |

Conclusion:
If the above 2 inequalities are met simultaneously, the position of the convoy with the $P_j$ point load over the cross-section $i$ leads to the occurrence of $M_{i\,max}$.

Example of use:




$$\frac{P_1}{x} = \frac{5}{5} = 1 < \frac{P_2+P_3}{l-x} = \frac{15+20}{9} = 3,89$$

$$\frac{P_1+P_2}{x} = \frac{5+15}{5} = 4 > \frac{P_3}{l-x} = \frac{20}{9} = 2,22$$

$$\frac{P_1+P_2}{x} > \frac{P_3}{l-x} \quad M_{i\,max} \quad \frac{P_1}{x} < \frac{P_2+P_3}{l-x}$$

$$P_j = P_2$$

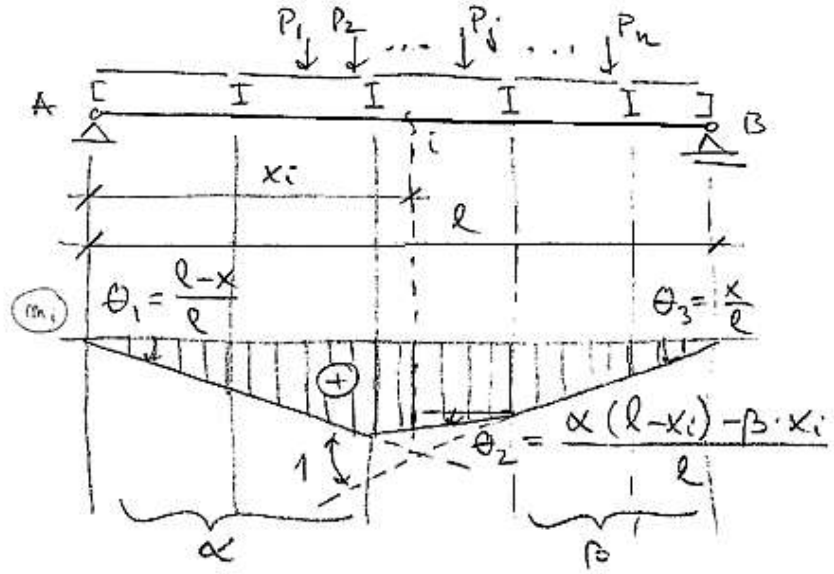
$$V_A = \frac{5\cdot 11 + 15\cdot 9 + 20\cdot 8}{14} = 25\ \text{kN}$$

$$M_{i\,max} = V_A\cdot 5 - P_1\cdot 2 = 25\cdot 5 - 5\cdot 2 = 115\ \text{kN}\cdot\text{m}$$

As can be seen, $M_{i\,max}$ resulted for the position of point load $P_2$ = 15 kN over section $i$.

Indirectly applied loads

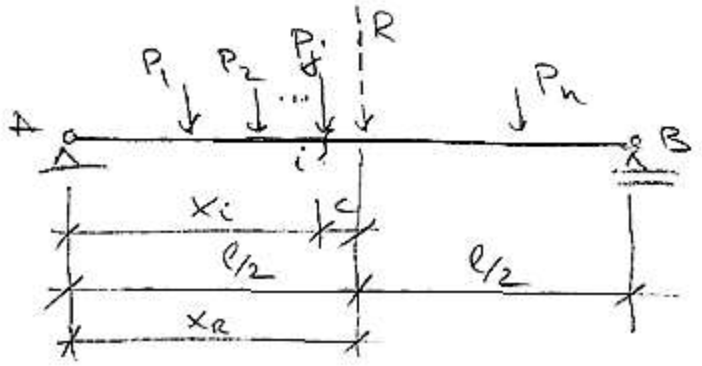
In the case of indirect loads, the passage of a point load beyond the section will not produce a sudden variation of the cutting force (the effect is treated on the panel). In the illustrated figure, the section being located within the range of a panel, the general criterion shall be applied using the trapezoidal contour of the influence diagram on the panel portion:



In the figure next to it was noted with $\alpha$ the number of panels preceding the panel in which section $i$ is located, and with $\beta$ the number of panels following the panel in which section $i$ is located (for the case considered $\alpha$ = 2 and $\beta$ = 2).

## MAXIMUM MAXIMORUM BENDING MOMENT

The "maximum maximorum" bending moment means the greatest bending moment that can occur in a straight beam loaded with a mobile convoy of forces. Its determination involves two aspects: the position of the cross-section (also called the "dangerous section") in which this maximum maximorum moment will appear, i.e. the value of the highest maximum moment.



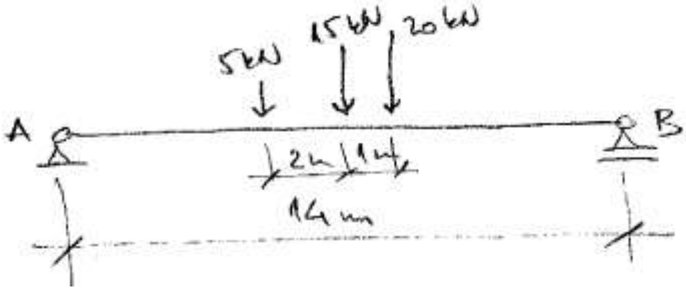
Considering the illustrated scheme in which $R$ represents the resultant force of the mobile convoy and the dangerous cross-section was marked with $i$ (under point load $P_j$, the closest one to the resultant), by positioning the resultant near the middle of the beam, the $V_A = \frac{R(l - x_R)}{l}$ reaction can be obtained. With this value, the $M_i$ bending moment can be calculated (knowing the $x_R = x_i + c$ distance), considering that it is also the maximum one: $M_{max} = V_A \cdot (x_R - c) - M_j$ (where $M_j$ represents the sum of the moments given by the forces from left to $P_j$). If this moment is really the maximum maximorum, then $\frac{dM_{max\,max}}{dx_R} = 0$ . Thus,

$$M_{max\,max} = \frac{R(l - x_R)}{l}(x_R - c) - M_j = \frac{R}{l}(l \cdot x_R - l \cdot c - x_R^2 + x_R \cdot c) - M_j$$

$$\frac{dM_{max\,max}}{dx_R} = \frac{R}{l}(l - 2x_R + c) = 0 \quad \rightarrow \quad \boxed{\frac{c}{2} = x_R - \frac{l}{2}}$$

Conclusion:
The middle of the beam divides into two equal parts the $c$ distance between the $R$ resultant and $P_j$ point load (the force acting in the dangerous section) of the convoy.
Observation:
Since next to the resultant $R$ will be two neighbouring forces in the mobile convoy, tests will have to be made with both, in order to find out the position of the dangerous section and the value of $M_{max\ max}$. The value of $M_{max\ max}$ will be greater if the resultant $R$ is greater and the distance $c$ (to the neighbouring point load $P_j$) is less.

Example of use:

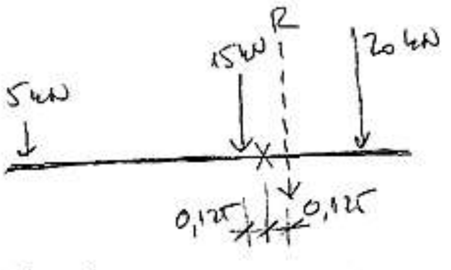



$R = 5 + 15 + 20 = 40$ kN

To find out the position of the $R$ resultant force, a bending moment equilibrium equation may be used (from any position). In this case, the position of the first force (the one with a value of 5 kN) was considered:

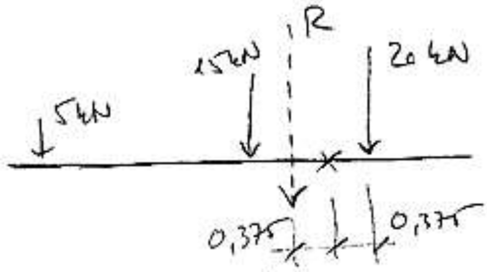
$$5\cdot0 + 15\cdot2 + 20\cdot3 = R\cdot d_R \text{ from where } d_R = 90 / 40 = 2.25 \text{ m}$$

(meaning that $R$ will be at a quarter of the distance between the forces of 15 kN and 20 kN).
After calculating the value and the position of $R$, the two possibilities shall be considered: the middle of the beam at equal distances between the 15 kN point load and $R$, respectively between $R$ and the 20 kN point load (the middle of the beam was marked with X in the two sketches below):




$V_A = (7 - 0{,}125)\cdot40/14 = 19{,}642$ kN

$M = V_A\cdot(7 - 0{,}125) - 5\cdot2 =$
$= 19{,}642\cdot6{,}875 - 5\cdot2 =$
$= 135{,}039 - 10 = 125{,}04$ kNm




$V_A = (7 + 0{,}375)\cdot40/14 = 21{,}071$ kN

$M = V_A(7 + 0{,}375) - 5\cdot3 - 15\cdot1 =$
$= 21{,}071\cdot7{,}375 - 15 - 15 =$
$= 155{,}399 - 30 = 125{,}399$ kNm

$M_{max\ max} = 125{,}40$ kNm

In conclusion, the dangerous cross-section will be under the 20 kN point load (0.375 m to the right of the middle of the beam), the maximum maximorum moment value resulting in 125.4 kNm.