First lab

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Über dem Umgang mit ausgedruckten Ergebnissen bei einer numerischen Rechner:
Der (naive) Anfänger glaubt an jede einzelne Ziffer.
Der (erfahrene) Programmierer vertraut auf die Hälfte der Stellen.
Der (wissende) Pessimist mißtraut sogar noch dem Vorzeichen.[**?**]

Example

Floating point on 64 bits

$$x = 1./n$$
$$x = (n+1)\,x - 1$$
$$n := 3; x := 1./n$$
$$\text{30 times } x := (n+1)x - 1$$
$$\text{read } x : x = -21.$$
$$4^{30} = 1.1529 \times 10^{18}$$

Example

$$\sin x \approx \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\text{error} < \frac{\left|x^{2n+3}\right|}{(2n+3)!} < 10^{-15}$$

$$x := 46.$$

$$\sin x \approx 605. \qquad \text{with error} < 10^{-15}$$

Example

$$I_n = \int_0^1 \frac{x^n}{x+5} dx$$

$$I_0 = \ln(6/5) \approx 0.18232$$

$$I_n = \frac{1}{n} - 5I_{n-1} = \int_0^1 \frac{\left(x^n + 5x^{n-1}\right) - 5x^{n-1}}{x+5} dx$$

$$I_{30} = -36,668. * **$$

$$5^{30} = 9.3132 \times 10^{20}$$

Example

$$10^8 + 4... + 4 + ... + 4 = 10^8$$

$$10^8 + 5 = 10^8 + 10$$

$\varepsilon - machine$: standard IEEE

Definition

ε-machine is the smallest positive number that added by 1.0 change the value of 1.0.

Example

$$\int_0^1 \frac{dx}{x^4+1} = \sqrt{2}\left(\frac{1}{8}\pi + \frac{1}{8}\ln\left(\sqrt{2}+2\right) - \frac{1}{8}\ln\left(2-\sqrt{2}\right)\right) = 0.86697$$

Example

$$\int_0^1 \exp\left(-t^2/2\right) dt = \tfrac{1}{2}\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\tfrac{1}{2}\sqrt{2}\right) = 0.85562$$

C++

Case Sensitive

operations: +,-,* (or space),/,^

( )

{, } List (array, vector)

[ ] : arg. of function $Sin[x]$

[[ ]]: index $A[[i, j]]$ el. of A from row i and column j

$a = 2$ Head[a]->Integer

$a = 2.$ Head[a]->Real

$a =$. ($Clear[a]$) Var. a does not exist more Head[a]->Symbol

$a := 2. + 3$ (delayed)

Mathematica works symbolic (formal)+rational

$E = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$

$I = Sqrt[-1]$

$Pi \approx \frac{22}{7}$

$e^{j\pi} + 1 = 0$

For[init.,cond.,k++,body]

Login: student

Parola:student

Disasters caused by computer

https://w3.ual.es/~plopez/docencia/itis/parliament.htm

http://www-users.math.umn.edu/~arnold/disasters/patriot.html

http://www-users.math.umn.edu/~arnold/disasters/ariane.html

https://www.joelonsoftware.com/2007/09/26/explaining-the-excel-bug/

https://www.theregister.co.uk/2007/09/26/excel_2007_bug/

http://www-users.math.umn.edu/~arnold/disasters/sleipner.html

$$\binom{49}{6} = 13983816$$

$$2^{16} - 1 = 65535$$

$$20000/180/60 = 1.8519$$