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Lightning return stroke current reconstruction or vertical and variable channel shape

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***Abstract*—first a new mathematical approach is presented to evaluate the electric and magnetic field of the lightning, via engineering model with variable shape return stroke channel; next, an inverse procedure is exposed for the reconstruction of both spatial and temporal waveforms of the lightning return stroke current, throughout a numerical field synthesis procedure, based on regularization of ill-posed problems. The approach uses as input data the acquisition of time domain recordings of electric and/or magnetic field generated by the lightning current, at various locations on the ground and transforms these signals into harmonics, by Fourier decomposition. This combination, between the proposed solving procedures and harmonic filtering, yields numerical results that are in good agreement with the testing functions.**

Keywords-lightning, return stroke current, variable channel, ill posed, inverse problem, harmonic reconstruction, field synthesis.

I. INSIGHTS AND CONCEPTS REGARDING LIGHTNING

Lightning return stroke modeling is of interest for a various range of reasons, as part of evaluations into the physics of lightning, as an instrument by which return stroke currents at ground can be identified from nearby or far away measured electromagnetic fields, and hence by which currents of individual lightning strikes or statistical distributions of the stroke currents can be evaluated [1], [2], [3], [4], [5], [6].

In this paper it is presented the mathematical modeling of a direct and then inverse remote sensing procedure in order to identify and reconstruct the spatial and time domain waveform of the lightning return stroke current. It is based on the acquisition of the electric and/or magnetic field generated by the discharge channel, at various locations on the ground and at various frequencies. This may be a mechanism for calculating realistic fields, next to be used in coupling calculations, such as to determine the lightning-induced voltages appearing on power grids or telecommunication lines, when lightning occurs near those lines.

After the identification of a mathematical model of the return stroke current, it becomes possible to evaluate the electric and magnetic field values in any interest area, and subsequently using the transmission line method, to estimate any induced potential in nearby power grids. For this study there must be used Sommerfeld integrals [7], [8]. Some authors consider that a convenient wave solution for both current distribution along the lightning channel and associated electromagnetic fields, can be achieved only when using electromagnetic models. On the other hand, it is well known that engineering models for the return stroke current and fields, consist in both spatial and temporal variation as separated variables.

The theoretical estimation of return stroke currents from remote electromagnetic fields depends on the adopted return stroke model. Expressions relating radiated fields and return stroke channel base currents have been derived for various engineering return stroke models. Engineering return-stroke models have been reviewed in many papers [1], [9]. The problem of determination of the return stroke current from remotely measured electric and/or magnetic fields considerably facilitates the collection of data on the lightning return stroke current without having to instrument towers or trigger the lightning artificially and without the inherent relative in-efficiency of these methods. This is especially true now because of the widespread use of lightning location systems, LLS [4].

II. EXISTING ALTERNATIVE RECONSTRUCTION TECHNIQUES

Several authors have studied the ability of the engineering models to predict the electromagnetic field radiated by return strokes; recently in [9] there are mentioned two primary approaches of evaluation: The first approach involves using a typical channel-base current waveform and a typical return stroke propagation speed as model inputs and then comparing the model-predicted electromagnetic fields with typical observed fields; The second approach involves using the channel-base current waveform and the propagation speed measured for the same individual event and comparing computed fields with measured fields for that same lightning.

The second approach is able to provide a more definitive answer regarding model validity, but it is feasible only in the case of triggered-lightning return strokes or when natural lightning strikes to tall towers where channel-base current can be measured. When trying to reconstruct the return stroke current, if the measured and the calculated field do not agree, the channel current spatial and temporal parameters are changed. The procedure is repeated until the matching becomes satisfactorily good. Some previous researches that were identified [9], [10], [11], [12], propose different versions of trial and error procedures for the reconstruction of the return current, using the measured field values, during lightning occurrence. Another alternative is to directly solve the integral equations with the help of the collocation method, using Cebasev or Geigenbauer base functions [13], [14], [15].

Our proposed procedure implies the following: apply Fourier series to the time domain signal of the electric or magnetic field, and extract some N components from it - amplitudes and phases; if available, apply Fourier series to the time domain signal of the channel base current, and extract some N components from it - amplitudes and phases; at this moment a correlation can be performed between the frequency domain of the electric/magnetic recorded fields and of the return stroke current waveform; to each current harmonic will correspond an electric/magnetic field harmonic, linked by the first kind Fredholm integral equation; this approach has a physical meaning also and it relies on the superposition method; pass then from the analytical integral equation to a linear system of equations, through numerical meshing of the spatial variable - channel height on one side, and range of horizontal sensors on the other side; this numerical system of equations has a severely ill-posed solution, a fact expressed by its condition number.

The modular algorithms on the present approach can be successfully applied for lightning return stroke current reconstruction, next to be used in power engineering electro-magnetic compatibility problems, in the research of the radiated lightning electromagnetic pulse and its coupling with the overhead lines and other metallic structures.

III. Mathematical approach of the return stroke

There is a wide range of electric or magnetic field synthesis applications, that have to be modeled with Fredholm integral equations of the first kind, as ill-posed inverse problems. It is the case of: magnetic resonance imaging coils, for uniform fields; position identification of ships from their gravitational magnetization; underwater determination of corrosion for ocean platforms [16]; lightning return stroke current identification from field measurements [10], etc.

For the engineering models, the cause and effect relations between the spatial-temporal expression of the ascendant leader and the electric and magnetic generated field may be modeled through Fredholm integral equations of the first kind. The modeling may also assume that the return stroke current distribution can be summed up by the individual contributions of the impulse currents which propagate upward with different speeds.

If some adopts as modeling hypothesis the fact that the soil has a flat shape, of homogeneous material and with perfect conductivity, then the kernels of the Fredholm integral equations of the first kind are expressed by rational functions, weighted by decaying exponentials. If it is admitted a close to reality hypothesis, that of nonhomogeneous and finite resistivity soil, the expression of the kernels depends on Bessel functions [17].

Here are some of the hypothesis that are used in the modelization of the lightning: the lightning channel is represented as being 3D variable, along which current propagates as a moving front; the soil is homogeneous, has a flat shape and perfect conductivity. The next paragraphs aim to reveal the mathematical expressions of the field components in cylindrical coordinates, both for electric and magnetic fields, as Fredholm integral equations of the first kind.

Above it can be seen a given approximate geometry model of the lightning reconstruction, which can be generalized to any spatial curve as lightning channel:

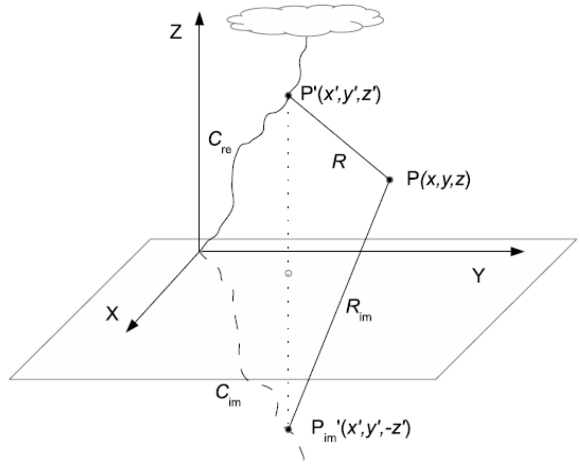

Figure 1. Geometrical parameters used in calculating return stroke fields.

Where: H - channel height; $i(r',t)$ - return stroke current spatial and time dependence; $P(x,y,z)=\mathbf{r}$ - is the coordinate of point in space where the potentials are computed, evaluation location of sensors; $\mathbf{r}'$ - is the coordinate of a point of the source of the current.

Starting from the Maxwell equations in a linear, homogeneous and isotropic medium, we have the solution as magnetic vector potential and the equation to link the field and potential:

$$\mathbf{A}(r,t)=\frac{\mu}{4\cdot\pi}\cdot\int_{V}\frac{\mathbf{A}\left(r',t-\frac{|r-r'|}{c}\right)}{|r-r'|}dv' \qquad (1)$$

$$\mathbf{E}(r,t)=c^{2}\cdot\int_{0}^{t}\nabla(\nabla\cdot\mathbf{A})d\tau-\frac{\partial\mathbf{A}}{\partial t} \qquad (2)$$

These are inhomogeneous solutions expressed by the magnetic vector and electric potentials, dependent one to each other. We also assume that the current distribution and the charge distribution are zero at t = 0.

In the case of the lightning return stroke the source is moving along a curve starting on earth and going up, let C be this general curve with h being the height parameter:

$$\mathbf{r}'(h) = x'(h)\cdot\mathbf{i} + y'(h)\cdot\mathbf{j} + z'(h)\cdot\mathbf{k}, \quad h\in[0;H] \tag{3}$$

And the radius from the variable channel to the observation point to be:

$$R = |\mathbf{R}| = |\mathbf{r}-\mathbf{r}'| = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2} \tag{4}$$

After a series of integrations and substitutions and the use images method to add the contribution of the earth, we obtain the expression of the electric field in respect with this variable spatial curve of the lightning channel, as equation to describe the phenomena:

$$\mathbf{E} = \mathbf{E}_{static} + \mathbf{E}_{induction} + \mathbf{E}_{radiation} \tag{5}$$

Where:

$$\mathbf{E}_{static} = -\frac{1}{4\cdot\pi\cdot\varepsilon_0}\int_{-H}^{H}\frac{R^2\cdot\mathbf{r}'-3\cdot(\mathbf{r}'\cdot\mathbf{R})\cdot\mathbf{R}}{R^5}\left(\int_0^t I(|h|,\tau-R/c)d\tau\right)dh$$

$$\mathbf{E}_{induction} = -\frac{1}{4\cdot\pi\cdot\varepsilon_0}\int_{-H}^{H}\frac{R^2\cdot\mathbf{r}'-3\cdot(\mathbf{r}'\cdot\mathbf{R})\cdot\mathbf{R}}{c\cdot R^4}\cdot I(|h|,\tau-R/c)dh$$

$$\mathbf{E}_{radiation} = -\frac{1}{4\cdot\pi\cdot\varepsilon_0}\int_{-H}^{H}\frac{R^2\cdot\mathbf{r}'-(\mathbf{r}'\cdot\mathbf{R})\cdot\mathbf{R}}{c^2\cdot R^3}\cdot\frac{\partial I(|h|,\tau-R/c)}{\partial t}dh \tag{6}$$

Are the static, the inductive and the radiation components of the electric field. This expression for the electric field was deduced in [24] for the case where the curve C is a segment of a line, using Fourier transform; thus we now have a general equation to use.

Similar considerations and mathematical steps were used to deduce the magnetic field formula:

$$\mathbf{H}(\mathbf{r},t) = \frac{1}{4\cdot\pi}\cdot\int_{-H}^{H}\left[\frac{1}{R^3}\cdot I(|h|,t-R/c) + \frac{1}{c\cdot R^2}\cdot\frac{\partial I}{\partial t}(|h|,t-R/c)\right]\mathbf{r}'\times\mathbf{R}\,dh \tag{7}$$

Up to now, although the relations are valid for any variable spatial channel curve, so as these relations to be numerically computable, let us introduce some issues regarding the return stroke current. General expression of the current:

$$I(h',t') = u(t'-t_d)\cdot i(h',t'-t_d) \tag{8}$$

With the arguments $t' = t - R/c$. The time delay t_d is the duration for the current to arrive from the ground (h=0) to the point of the channel corresponding to h and can be computed with the line integral:

$$t_d = \int_0^h \frac{ds}{v} \tag{9}$$

Where v is the propagation speed of the current, a function of height. Next, if we consider that the lightning channel is composed of n linear segments obtained by joining the points $P_k(x_k, y_k, z_k)$, $k = 0,\ldots,n$, the first point is $P_0(x_0, y_0, z_0)$ the origin and denote by $\mathbf{r}'_k = x_k\cdot\mathbf{i} + y_k\cdot\mathbf{j} + z_k\cdot\mathbf{k}$, $\mathbf{R}_k = \mathbf{r} - \mathbf{r}'_k$ also $\mathbf{l}_k = \mathbf{r}'_k - \mathbf{r}'_{k-1}$ the tangent vector of $\mathbf{r}'$ corresponding to the segment $P_{k-1}P_k$.

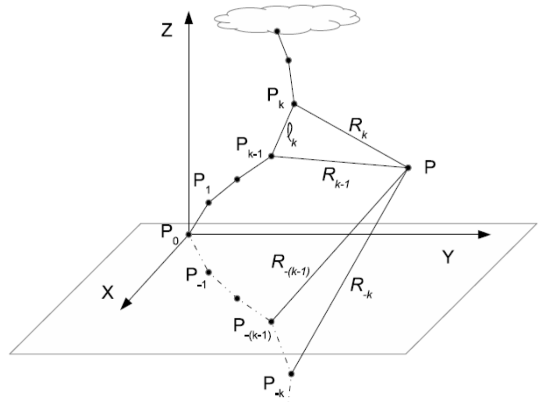

Figure 2. Lightning channel numerical discretization

Denote by v_k the speed of the current corresponding to each segment. Consider $t_0 = R_0/c$ the time for lighting to travel from P_0 to P. Thus, we can compute:

$$t_k = \frac{l_1}{v_1} + \ldots + \frac{l_k}{v_k} + \frac{R_k}{c} \tag{10}$$

Having these considerations, each of the field expressions can now be numerically evaluated very easy as follows:

$$\mathbf{E}_{static} = E_1 + \ldots + E_n \tag{11}$$

Where E_j is the integral corresponding to the segment $P_{j-1}P_j$, $j = 1,\ldots,n.$ Each segment has the equation:

$$\mathbf{r}'_j = \left[x_{j-1} + \frac{x_j - x_{j-1}}{z_j - z_{j-1}}\cdot(h - z_{j-1})\right]\cdot\mathbf{i} + \left[y_{j-1} + \frac{y_j - y_{j-1}}{z_j - z_{j-1}}\cdot(h - z_{j-1})\right]\cdot\mathbf{j} + h\cdot\mathbf{k} \tag{12}$$

We also obtain:

$$R = \sqrt{R_{j-1}{}^2 - 2\cdot\frac{h - z_{j-1}}{z_j - z_{j-1}}\cdot\mathbf{R}_{j-1}\cdot\mathbf{l}_j + l_j{}^2\cdot\left(\frac{h - z_{j-1}}{z_j - z_{j-1}}\right)^2} \tag{13}$$

Using relations (12) and (13), we can use the final computable integral (14):

$$E_j = \frac{-1}{4\cdot\pi\cdot\varepsilon_0}\cdot\int_{z_{j-1}}^{z_j}\frac{R^2\cdot\mathbf{l}_j - 3\cdot\left(\mathbf{R}_{j-1}\cdot\mathbf{l}_j - l_j{}^2\cdot\frac{h - z_{j-1}}{z_j - z_{j-1}}\right)\cdot\mathbf{R}}{(z_j - z_{j-1})\cdot R^5}\cdot\int_0^t I(h,\tau - R/c)d\tau dh$$

These mathematical expressions of the field components in cylindrical coordinates, both for electric and magnetic fields, as Fredholm integral equations of the first kind are used as the engineering models.

We found that a miss interpretation persists regarding the frequency domain of these radial, axial and polar field equations. They do not represent the Fourier transform of the time domain field expressions, but formally written with complex numbers relations. Thus, (1) converts in (2), and the same with the other components:

$$E_z(r,z,\omega)=\frac{1}{4\cdot\pi\cdot\varepsilon_0}\cdot\int_{-H}^{H}\left[\begin{array}{l}G_z(r,z,z',\omega)\cdot\\ \cdot\exp\left(-j\cdot\omega\cdot\frac{\sqrt{r^2+(z'-z)^2}}{c}\right)\cdot I(z',\omega)\end{array}\right]dz' \quad (15)$$

In this case, the kernel function G_z incorporates the static, induction and radiation contributions, as in correspondence with the relations from (14). The return stroke current (RSC) as a function (16) and (17), shows dependence to an initial peak value at the channel base, a spatial attenuation along the channel and to the propagation speed of the current, both for time and frequency dependence:

$$i(h',t)=u(t-|h'|/v)\cdot P(h')\cdot i_0(t-|h'|/v) \quad (16)$$

$$I(h',\omega)=I(0,\omega)\cdot P(h')\cdot\exp(-j\cdot\omega\cdot|h'|/v) \quad (17)$$

Where: $I(0,\omega)$ frequency converted channel base current (CBC); *P(h')* - spatial attenuation of the return stroke current.

IV. Regularization of ill-posed equations

Taking account that these described inverse Fredholm *ill posed* integral equations, consist in computation the cause from the effect, it is expected that small noise in the right-hand side measured field components, are likely to generate numerical RSC's highly contaminated by undesired high-frequency oscillations [13], [18].

Thus, if one by standard numerical procedures evaluates the solution, this has three major inconvenient characteristics: imprecision, instability to small input field modifications and physical inconsistence. The ill-posed electromagnetic inverse problems, ill-posed EIP, are very well detailed in the literature, especially for Fredholm integral equations [13], [19].

For this reason in our study there has been adopted the concept of *Working Regularization Algorithm* (WRA) [13], as a functional mixture of three factors: a regularization algorithm, a parameter choice method and the implementation of these methods. For efficiency we take into account any available mathematical structure in the problem (singularity, symmetry, sparse).

By using the condition number in the initial evaluation, we can show a clear connection between the solution instability and the condition number, as related to any perturbation that may occur in the measured field, or in the problem structure – the kernel matrix. Thus the noise acts on the effect – vector as:

$$\varepsilon_X[\%]=\frac{\|X'-X\|_2}{\|X\|_2}\leq K_A\cdot\frac{\|u'-u\|_2}{\|u\|_2}=K_A\cdot\varepsilon_u[\%] \quad (18)$$

u' - perturbed effect vector; X' - the resulted solution (attenuation function) as related to the perturbed effect; K_A - condition number;

Minimizing a Tikhonov functional [13], [18], expressed with the help of vector norms (18), it is nothing but a constrain method, which limits the uncontrolled growth of the solution:

$$f_{Tikhonov}=\arg\min\left\{\|A\cdot X-u\|_2+\alpha\cdot\|C\cdot X\|_2\right\} \quad (19)$$

Where: A - matrix system; u - field vector; α – regularization parameter; $A\cdot X=u$ the system of equations originated from the integral equation (14). The term $\alpha\cdot\|C\cdot X\|$ consists in a penalty applied to the solution, in order not to allow its instability. Also, the operator C may embed geometrical and physical constrains for the solution. This regularization procedure and its derivations may be regarded as a penalty method [13].

Truncated singular value σ_i decomposition (TSVD), applied as a regularization method, with the limitation of certain terms that enter in the sum, as related to a singular value stated as threshold, it is interpreted as being a projection method; an evaluation of a vector by summing up of other vectors, without undesired components.

$$X^{(k)}=\sum_{i=1}^{k}f_i\cdot U^{<i>T}\cdot u\cdot V^{<i>} \quad (20)$$

Where: f_i - filter factors; U,V - singular matrices; k - truncation coefficient which acts as regularization parameter.

Thus, the regularization effects as a penalty method, or as a projection one. We classified these regularization procedures as following: Tikh - penalty method, based on Tikhnov theory reflected by relation (19); DVST on/off - projection method, truncated singular value decomposition with on/off filter factors $f_i=1/\sigma_i$ *if* $i\leq k$ *or* 0 *if* $i>k$; DVSTA - projection method, damped truncated decomposition of the singular values, based on relation (20) with $f_i=\sigma_i/(\sigma_i^2+\alpha)$ *if* $i\leq k$ *or* $i>k$; other standard methods: GCS - conjugate gradient method; TRA - algebraic reconstruction technique [20]; GCV - generalised crossed validation method expressed as the minimum of $GCV(\alpha)=\alpha\cdot\partial\|C\cdot X\|^2/\partial\alpha^2$ for the choice of the regularization parameter; LC - *L* shape curve function a dependent variation between the error and solution norms as introduced in (19) to which the corner represents the optimum regularization parameter [21].

For each of the above regularization methods, the original contribution of the authors is related to the definition and evaluation of the filtering factors. The threshold from which the filtering starts, it is by itself a regularization parameter, in relation with the decomposition of the time domain signal.

Both WRA penalty and projection methods consist initially in a harmonic analysis for the norms of the singular vectors *V*, from the decomposition, and afterwards in a filtering of those singular vectors that have a lower norm, than an imposed limit, if they may be affected by the amplification due to the singular values σ_i, in the solution reconstruction.

After computing the solution, by any of these methods, an error evaluation is performed, using the relations:

$$\Sigma_{solution}(z')=\frac{|P_{test}(z')-P(z')|}{|P(z')|}\cdot 100\ [\%] \quad (21)$$

$$\Sigma_{effect}(r) = \frac{\left|E_{mas}(r,z,\omega) - E_{calc}(r,z,\omega)\right|}{\left|E_{mas}(r,z,\omega)\right|} \cdot 100\ [\%] \qquad (22)$$

Some error causes that appear for electric or magnetic field measurement: LLS devices; current reflections in instrumented towers [15]. In the evaluations it is also tested the sensitivity of the solution, when noise occurs in the field vector.

V. NUMERICAL RESULTS AND DISCUSSIONS

Not having any available data about the height dependent attenuation function *P(h')*, numerically the vector *X*, this can be evaluated for each frequency from the Fourier spectrum by solving the above integral equation (14) for a vertical channel and the other related field integrals. Then the numerical solutions can be compared with the proposed test functions MTLE and MTLL, as only these have non unitary spatial attenuation dependence.

There were used several input data regarding the location of the field sensors (range of 50 to 5000 m), height of the measurement sensors (0 to 15 m), height of the current channel (1 to 7.5 km) and the sampling frequencies of the measured fields (20 to 500 harmonics, related to a maximum duration of 10^{-3} sec). All of these numerical cases, applied to the Fredholm integral eq. models, vertical electrical field (14) and related horizontal electric and azimuth magnetic field strengths, lead to ill-posed and very severely ill-conditioned initial systems of equations and required regularization.

Let us consider the results for a 7.5 km channel height and an initially imposed CBC, as with indicated expression and parameters given in [4].

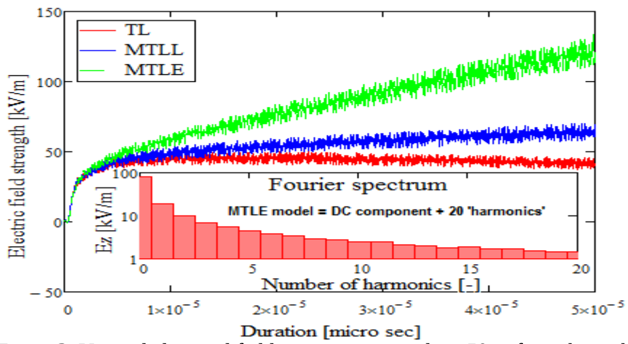

Figure 3. Vertical electrical field component sample at 50 m from the strike

In fig. 3 it can be seen the result of evaluating directly the electric field strength for the TL, MTLL and MTLE models [4], [5], [11], both in time domain and the frequency spectrum for MTLE model, at a distance of 50 m from the strike location, for a duration of 50 µs. This is accounted for the scenario with only one electric field sensor for the remote sensing procedure to identify the spatial distribution of the current along the channel height.

Using these electrical field values, with a 5% added noise as fig. 3 shows, we determined the attenuation function by the WRA. In fig. 4 it is represented a sample result for the identification of the MTLE (Test 1) model:

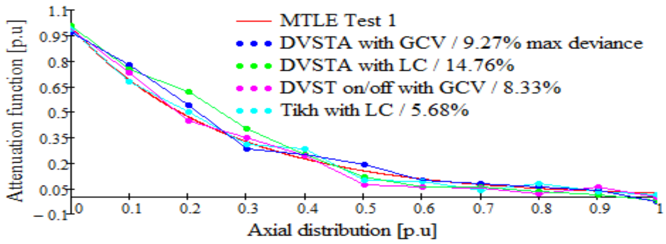

Figure 4. Inverse reconstruction of return current spatial distribution with projection and penalty methods

Having the reconstructed MTLE model spatial attenuation function with the Tikhonov using the *L* curve criterion and 75 \`harmonics\` in not perfectly fitting the CBC (correspondence with the Fourier spectrum of the E_z field), it can be evidenced the return stroke current, at different heights, as in fig. 5. The more \`harmonics\` evaluated, the lower will be the fluctuation in the solution. We assume that the model evidenced discontinuity in the RSC at different heights may be due to dispersion of the current, using the support of [16].

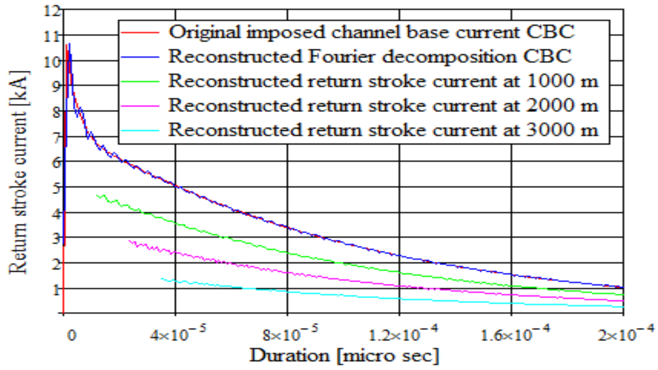

Figure 5. Return stroke current identification for different attenuation functions

Then it was performed the combination of the field equations, for the scenario of both electric and magnetic measured field components, with only one sensor and applied regularization procedures. A sample of the solution errors yields the optimum approach, for the location of the sensor at 500 m, and MTLE:

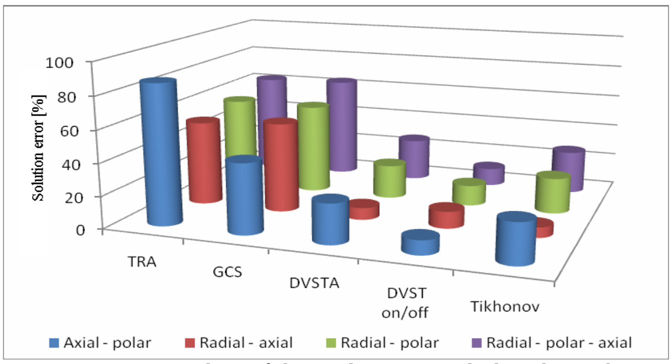

Figure 6. Sensitive analysis of the regularization applied on the combination of the integral models for a single location electric and magnetic field sensors

In the reconstruction of the MTLL (Test 2) model we also achieved reasonable performance as related to other reported results [9] - [12]. Regarding the experimental aspects of the present study, it is worth mentioning that we used simulated values as a testing approach. It is our intention to handle also natural or triggered lightning recordings, provided by LINET Germany. More data have to be evaluated in order to adequately validate the models and to improve them in order to reproduce as closely as possible experimental values.

We find out that without regularization, only for higher frequencies it is expected to have an improvement in the stability of the solutions, if using single frequency recordings but multiple field sensor locations.

Using the presented assumptions we explored the solution behaviour for each of the proposed testing conditions. When taking into account what is the best frequency spectrum, for which to reconstruct the spatial attenuation function, it should be noticed that in the range 1 kHz – 1 MHz (with added DC component) the errors reach minimum values.

VI. CONCLUDING REMARKS

The present work focuses on the synthesis of lightning return stroke currents, from remotely measured generated fields. After the identification of a mathematical model of the return stroke current, it becomes possible to evaluate the electric and magnetic field values in any interest area. As the problem proves to be severely ill-posed, we proposed a WRA as a group of regularization procedures, all based on the harmonic filtering of the singular vectors.

The effectiveness of the algorithms has been proved especially for on/off DVST and DVSTA; also, for Tikhonov regularization in the combination of two type of field measurements, radial – axial. In order to verify the robustness of the inverse procedure, we added noise to the free term of the system, *i.e.* to the field measurements.

The vertical lightning channel is no more acceptable, as one takes into account that real lightning is characterized by tortuosity and branching, in order to be able to justify the fine structure of the fields radiated by lightning discharges whose time-domain behavior exhibits a noisy shape with rich spectral [16], [17]. These features are investigated and exploited to improve the return stroke current reconstructions.

The author's contributions relate to the an original mathematical approach of variable channels, introduction and validation of the Fourier frequency decomposition of the field time domain signals, and numerical regularization in this lightning return stroke current problem reconstruction.

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